# An Analytical Approach for Optimal Placement and Sizing of Energy Storage 

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#### Abstract

Finding of optimal location and capacity for storage units have been highly noticed in modern power systems. In this paper, the optimal location and capacity of a storage unit is determined by an analytical technique in order to acquire the maximum profit. Target function is a single- variable function in which by its derivation the conditions required for maximization of profit are identified. In target function, all costs and earnings have been introduced as functions of nominal power of storage unit and thereby the capacity of storage unit and location for its installation is derived. This technique has been simulated on a 12-bus distribution network by means of MATLAB software. The authenticity of the derived results from the purposed analytical methods has been verified by review all possible modes for installation and determination of capacity for storage unit in this grid.


Key-words:- energy storage, load peak, optimal allocation and capacity, analytical approach

## 1 Introduction

Although load peak hour includes only a short time interval during a year, Electrical Generation Company shall increase its generation capacity to supply load peak. On the other hand, operator has to enter its high cost generation units into the network so this will raise price of energy consumption for customers. Likewise, all network constraint are placed close to their allowed limits during load peak; therefore, in the case of encountering any defect in grid elements and peak load in another part, operator might have to delete some of this load from the power grid. Under such conditions, the possibility of shutdown of system throughout the power grid will inflict hug loss to economy of a country. Application of energy storage is one of the solutions for this problem.
Energy storage systems, at large scale, cover a wide group of applications in power system. At the one end of spectrum of such applications, there are applications need to energy discharge with high power within a short period of time even up to a fraction of a second so that one of these applications is the improvement of power quality. At the other end of this spectrum, energy
management applications are placed that require energy discharge for long time up to several hours. Load management on power grids is one the foremost type of these applications. Utilization of energy storage system has several advantages, including [1]: Possibility for abundantly use of renewable energies, correction of power factor and improvement of power quality, control of reactive power and voltage, ability to follow up variances of load curve, using as spinning reserve, delay of capital in distribution and transfer systems and environmental advantages.
Electricity cost varies in different hours during day and night. As a result, with respect to electricity variable prices, we can use energy storage units in such a way that to charge it during low load hours when electricity price is low and to inject the saved energy into the grid at load peak hours when this cost is more expensive. Given the type of storage, using these types of systems may control and meet demand at times of load peak situations. Energy storage makes it possible for electricity power to be generated and distributed with its highest capacity thereby causes reduced demand for new lines of generation and distribution. Storing electrical energy for long time may be adapted for creation of balance in voltage under load peak.
Several studies have been carried out about topology of energy storage systems. Reference [2] determines the
appropriate storage unit with respect to wind power plant. An economic load distribution has been used for this purpose in this paper. In reference [3], an economic analytical method has been investigated to find appropriate storage unit. This approach is based on profit of net face value. Genetic algorithm has been adapted to find the maximum profit from the net face value. Reference [4] has identified optimal location and capacity of capacitor and energy storage by means of genetic algorithm. In reference [5], topology has been implemented by using genetic algorithm, and then an appropriate controlling strategy has been determined by dynamic planning. Reference [6] suggests an economic approach in order to determine optimal capacity of energy storage in microgrid. These studies have been conducted in two forms of islanding and connected states of grid. Reference [7] identifies location and optimal capacity of storage that was intended to improve voltage profile. Sensitivity analysis is the presented approach in this paper. Reference [8] reviews the application of storage by aiming at reduced load peak and leveling load curve.
In most of the conducted studies in the field of energy storage supply, this technology has been considered along with Distributed Generation (DG) supplies and they were aimed at improving system performance with these existing supplies. While, during recent years this technology has been highly advanced and they were manufactured in several and appropriate model types with various efficiency rates. Alternately, application of variable prices for electricity consumption during various hours of round-the-clock, may serve as a motive for employing this technology. In this article, this technology has been taken into consideration alone and its cost-effectiveness will be explored. In the present article, location and optimal capacity of storage unit are determined by two analytical methods and reviewing all possible states. This paper is structured in such a way that the presented approach is expressed at part 2 ; the method of revision of all possible states is interpreted in part 3. In part 4, results and simulations are given and part 5 comprises of conclusion.

## 2 Formulation of Problem

As it mentioned above, the consumed load varied during several hours of day and night. With respect to different price of electricity power during various hours of day and night, the energy storage receives energy from the grid when electricity price is low during low load hours and then it delivers the stored energy during peak load hours when electricity price is higher. With
respect to different prices within low load and peak load hours, this may cause rising profit of the grid. Since this unit receives energy during low load hours and injects energy into the grid at peak load hours, the percent of reduced losses is greater during peak load hours and at the same time losses are increased at low load hours. Moreover, installation of storage unit is followed by incurrence some costs. This essay is intended to maximize the profit caused by installation of storage unit. Cost and profit functions are given in the following.

### 2.1 Capital Cost

Installation of storage unit consists of two components i.e. the cost relating to electronic power equipments and cost of storage unit. The cost relating to electronic power equipments is proportional to power of storage unit while the cost regarding storage unit is relevant to energy. The method of calculation of storage system cost is as follows [9]:

Annual cost includes capital annual cost, replacements annual cost, and repair and maintenance annual cost. Annual cost for repair and maintenance is as follows:
$O M C=O M f \times P_{M}$
Where OMF denotes repair and maintenance cost per kilowatt (\$/KW).
Capital cost comprises of three parts including cost of power electronic instruments, storage unit cost, and power plant balance cost.
Cost of electronic power equipments is as follows:
$P C S=P C S U \times P_{M}$
Where, PCSU stands for cost of electronic power equipments per kilowatt ( $\$ / \mathrm{KW}$ ).

Storage unit cost may be acquired by the following expression:
$S U C=\frac{S U C U \times P_{N} \times H}{e f f}$

Where, SUCU is storage unit cost per kilowatt/hour ( $\$ / \mathrm{KWh}$ ); $H$ denotes charge and discharge, and eff is system efficiency:
eff $=\frac{\text { energy }(k w h)_{\text {out during_disc harge }}}{\text { energy_( } k w h)_{-} \text {in_during_charge }^{\text {and }}}$
Balance cost is:
$B O P=B O P U \times H \times P_{M}$
While, BOPU is balance cost per kilowatt (\$/KWh).
Total capital cost is obtained by sum of costs of electronic power equipments, storage unit, and system balance:
$T C C=P C S+S U C+B O P$
Annual capital cost may be written as follows:
$A C=T C C \times C R F$
CRF is capital return coefficient:
$\mathrm{CRF}=\frac{\left(\left(\mathrm{i}_{r}\left(1+i_{r}\right)^{y}\right)\right.}{\left(\left(1+i_{r}\right)^{y}-1\right)}$
$\mathrm{i}_{\mathrm{r}}$ is annual interest rate and $y$ denotes system's use life.
When we use battery in storage unit, it is possible for us to have to replace it within use life of the system. Annual replacement cost per kilowatt/ hour is as follows:
$A=F \times\left[(1+i r)^{-r}+*\left[(1+i r)^{-2 r}+\cdots\right] \times C R F\right.$ (9)

Number of terms in the above expression is the same as number of battery replacement during system's use life. $R$ is substitution (replacement) period:
$\mathrm{r}=\frac{C}{n \times D}$
Where, C denotes number charge/ discharge cycles within time interval of battery's use life.

Annual replacement cost is:
$\mathrm{ARC}=\frac{A \times P_{M} \times H}{e f f}$
As a result, capital cost per day is a follows:
$\mathrm{COD}=\frac{(A C+O M C+A R C)}{D}$

### 2.2 Energy Storage Charge Cost

Storage unit receives energy from the grid at low load hours. This cost is as follows:
$C_{\text {charge }}=P_{M} \times H \times \lambda_{1}$
Where, $\lambda_{1}$ is electricity price during low load hours (\$/KWh).

### 2.3 Cost Incurred Due to Rising Losses

Since storage unit receives energy from the grid at low load hours so it increases losses during these hours in the system. The resultant cost is:
$C_{L_{\text {Charge }}}=\left(P_{L_{\text {Charge }}}-A_{1}\right) . H . \lambda_{1}$
$\mathrm{P}_{\text {Lcharge }}$ is losses during storage charge state and $A_{1}$ denotes losses during low load state before installation of storage unit.

### 2.4 Profit Caused by Sale of Energy

Storage unit delivers energy to the grid in peak load state; therefore, the resultant profit (gain) from sale of this energy is as follows:
$C_{\text {decharge }}=P_{M} . H . \lambda_{2}$
$\lambda_{2}$ is electricity price at peak load hours.

### 2.5 Profit Caused by Reduced Losses

Storage unit injects energy to the grid at peak load hours and this leads to reduction of losses. The profit that is caused by reduction of losses is as follows:
$C_{L_{\text {dec harge }}}=\left(A_{2}-P_{L_{\text {dec harge }}}\right) \cdot H \cdot \lambda_{2}$
$A_{2}$ denotes losses during grid peak load state before installation of storage unit and $\mathrm{P}_{\text {Ldecharge }}$ is losses at peak load state and after installation of storage unit.

This paper is intended to maximize profit (gain) so the target function is:

Benefti $=C_{L_{\text {dec harge }}}+C_{\text {decharge }}-C_{\text {charge }}-$
$C_{L_{\text {charge }}}-C O D$
$C$ is number of charge or discharge in storage unit during its use life.

Losses of the system are as follows [10]:
$P_{L}=\sum_{i=1}^{N} \sum_{j=1}^{N}\left[\alpha_{i j}\left(P_{i} P_{j}+Q_{i} Q_{j}\right)+\beta_{i j}\left(Q_{i} P_{j}-\right.\right.$
PiQj
PiQj
$\alpha_{i j}=\frac{r_{i j}}{V_{i} V_{j}} \cos \left(\delta_{i}-\delta_{j}\right)$
$\beta_{i j}=\frac{r_{i j}}{V_{i} V_{j}} \sin \left(\delta_{i}-\delta_{j}\right)$
Element rij (i.j) belongs to the virtual part of impedance matrix. Vi and Vj denotes the voltages in bus i and bus j ; and $\delta_{i}$ and $\delta_{j}$ denote the voltage angle in buses i and j .

Whereas calculation have been done for both low load and peak Load states in this article, at any rate buses voltage and their equal angles are different. Thus, expressions (19) and (20) are computed separately for any state so that $\alpha_{\mathrm{ij}}$ and $\beta_{\mathrm{ij}}$ are related to peak Load state while $\alpha_{\mathrm{ij}}^{\prime}$ and $\beta_{\mathrm{ij}}^{\prime}$ are ascribed to low load state.
$P_{\mathrm{i}}=\mathrm{P}_{\mathrm{G}_{\mathrm{i}}}-\mathrm{P}_{\mathrm{D}_{\mathrm{i}}}$
$Q_{\mathrm{i}}=\mathrm{Q}_{\mathrm{G}_{\mathrm{i}}}-\mathrm{Q}_{\mathrm{D}_{\mathrm{i}}}$
$\mathrm{P}_{\mathrm{Gi}}$ is reactive power generated in bus $i$ and $\mathrm{P}_{\mathrm{Di}}$ denotes consumed power in bus $i$. as a result, $\mathrm{Q}_{\mathrm{Gi}}$ is the generated reactive power and $\mathrm{Q}_{\mathrm{Di}}$ expresses the consumed reactive power in bus $i$.

In the event that storage unit receives electrical energy, it is considered as a load for the grid. Therefore, in expression of losses, power of bus $i$ in which storage unit has been installed is:
$P_{\mathrm{i}}=-\mathrm{P}_{\mathrm{M}_{\mathrm{i}}}-\mathrm{P}_{\mathrm{D}_{\mathrm{i}}}$
$Q_{i}=-\alpha \mathrm{P}_{\mathrm{M}_{\mathrm{i}}}-\mathrm{Q}_{\mathrm{D}_{\mathrm{i}}}$
Where, $\alpha$ is:
$\alpha=\tan \left(\cos ^{-1}(\varnothing)\right)$
Ø denotes storage power factor.
Storage unit injects energy into the grid at peak load state and it acts like a generator. Therefore, in expression of losses, power of bus $i$ in which storage unit has been installed is:
$P_{\mathrm{i}}=\mathrm{P}_{\mathrm{M}_{\mathrm{i}}}-\mathrm{P}_{\mathrm{D}_{\mathrm{i}}}$
$Q_{\mathrm{i}}=\alpha \mathrm{P}_{\mathrm{M}_{\mathrm{i}}}-\mathrm{Q}_{\mathrm{D}_{\mathrm{i}}}$

### 2.6 The Presented Approach

This problem is intended to maximized profit. If storage unit is installed on bus $i$, target function is changed as follows:
Benefit $=\left(A_{2}-P_{L_{\text {dec harge }}}\right) \cdot H \cdot \lambda_{2}+P_{M_{i}} \cdot H \cdot \lambda_{2}-$
$P_{M_{i}} \cdot H \cdot \lambda_{1}-\left(P_{L_{C h a r g e}}-A_{1}\right) \cdot H \cdot \lambda_{1}-C O D$
Differentiation is one of the methods to find extremum point of a function. If the target function is differentiated relative to $f$, and set it to zero, optimal power of storage unit that has been installed in bus $i$,
will be obtained and this is maximum against power of this profit function.
$C D=\frac{\delta C O D}{\delta P_{M_{i}}}=\frac{\left.\left[(B O P U \times H)+\frac{S U C U \times H}{e f f}+P C S U\right)\right] \times C R F+O M F+\frac{A \times H}{e f f}}{D}$
$-\frac{\delta P_{L_{\text {dec }} \text { arge }}}{\delta P_{M_{i}}}+H \cdot \lambda_{2} \cdot e_{f f}-H \cdot \lambda_{1}-\frac{\delta P_{L_{\text {charge }}}}{\delta P_{M_{i}}}-C D=0$
Derivatives of losses for storage charge and discharge modes are as follows:
$\frac{\delta P_{L_{\text {dec harge }}}}{\delta P_{M_{i}}}=-2 \cdot H \cdot \lambda_{2}\left\{\sum_{j=1}^{N}\left(\alpha_{j i}\left(P_{j}+\alpha Q_{j}\right)+\right.\right.$
$\beta j i(Q j-\alpha P j))-\alpha i i \quad P M i(1+\alpha 2)\}$
$\frac{\delta P_{L_{\text {charge }}}}{\delta P_{M_{i}}}=2 . H \cdot \lambda_{1}\left\{\sum_{j=1}^{N}\left(\alpha_{j i}^{\prime}\left(P_{j}^{\prime}+\alpha Q^{\prime}{ }_{j}\right)+\right.\right.$ $\left.\left.\beta^{\prime} j i\left(Q^{\prime} j-\alpha P^{\prime} j\right)\right)+\alpha^{\prime} i i P M i(1+\alpha 2)\right\}$
$P_{j}$ and $Q_{j}$ are active and reactive powers of $i^{\text {th }}$ bus $d$ at peak load state and $P_{j}^{\prime}$ and $Q_{j}^{\prime}$ active and reactive powers of $i^{\text {th }}$ bus in low load situation.

As a result, $P_{\text {Mi }}$ is:
$P_{M_{i}}=\frac{\left\{2 . H \cdot \lambda_{2} \sum_{j=1}^{N}\left(\alpha_{j i}\left(P_{j}+\alpha Q_{j}\right)+\beta_{j i}\left(Q_{j}-\alpha P_{j}\right)\right)\right\}+H \cdot \lambda_{2} \cdot e_{f f}}{2 \cdot H \cdot \lambda_{2} \cdot \alpha_{i i} \cdot P_{M_{i}}\left(1+\alpha^{2}\right)+2 \cdot H \cdot \lambda_{1} \cdot \alpha_{i i}^{\prime} \cdot P_{M_{i}}\left(1+\alpha^{2}\right)}$
$-\frac{\left\{2 . H \cdot \lambda_{1} \sum_{j=1}^{N}\left(\alpha^{\prime}{ }_{j i}\left(P^{\prime}{ }_{j}+\alpha Q^{\prime}{ }_{j}\right)+\beta^{\prime}{ }_{j i}\left(Q^{\prime}{ }_{j}-\alpha P^{\prime}{ }_{j}\right)\right\}+H \cdot \lambda_{1}+C D\right.}{2 \cdot H \cdot \lambda_{2} \cdot \alpha_{i i} \cdot P_{M_{i}}\left(1+\alpha^{2}\right)+2 \cdot H \cdot \lambda_{1} \cdot \alpha^{\prime}{ }_{i i} \cdot P_{M_{i}}\left(1+\alpha^{2}\right)}$
By using the above expression, the optimal capacity of installed storage unit in bus $i$, will be obtained. Accordingly, optimal capacity is determined for all buses. In any situation, profit value is computed as equal to optimal capacity. The maximum gained profit is equal to the best location and capacity for installation of storage unit.

## 3 Method of Review All Possible States

In this approach, all the possible states are considered for installation of storage unit in such a way that we put storage unit on each of all buses for its installation in this grid and change its capacity from zero to maximum allowed value (total load of grid) at fixed time pace. In any situation after computation, load distribution of target function, which is the same as profit will be calculated and stored in a matrix that its rows are number of buses and its columns are equal to the power injected by storage unit. In this matrix, minimum array
is the best location and capacity for installation of storage unit.

## 4 Results and Simulation

In this section, simulations have been conducted based the given approaches and all possible states were also examined to review the validity of the results derived from them. Simulations have been done on a 12 - bus radial system. Similarly, load curve for different hours
of a day and night were drawn in Fig. 1[11]. According to price of time of use (TOU), simulation has been carried out. The related information for this time of pricing is given in Table I. Lead-acid battery was used for energy storage in this essay.Table II includes information regarding storage unit [12, 13].


Fig. 1. Load curve for a 12-bus radial distribution grid

## Table 1. TOU PRICES (http://www.pge.com/nots/rates/tariffs/ResTOUCurrent)

| period | price(\$/KWh) |
| :---: | :---: |
| off-peak | 0.10074 |
| mid-peak | 0.184 |
| on-peak | 0.28719 |

Table 2. SPECIFICATIONS OF LEAD-ACID BATTERY

| Parameter | Lead- acid |
| :---: | :---: |
| Efficiency | 0.75 |
| Cost of electronic power equipments (\$/KWh) | 175 |
| Storage unit cost (\$/KWh) | 150 |
| Grid connection (balance) cost (\$/KWh) | 50 |
| Repair \& maintenance cost $(\$ / \mathrm{KW} / \mathrm{y})$ | 15 |
| Replacement cost $(\$ / \mathrm{KWh})$ | 150 |
| Number of charges and discharges | 1500 |

The results derived from two analytical methods and review all possible states are given in Table III. Where there is dark line in this Table, storage unit does not become optimal in these buses against any value. The optimal rate of storage unit is characterized for any bus and the given profit is also derived. A bus with maximum profitability may be determined as the
optimal location for installation of storage unit. As it seen here, the maximum profit refers to the mode in which storage unit is installed in bus no 11 and it is 3.458\$. In this mode, rates of power and energy are 0.72 and 0.36 in storage unit, respectively. With respect to Table III, the surveying method may also determine the values of 0.072 and 0.36 as optimal rates and bus
no 11 as the optimal location for installation of storage unit at all possible modes. The given results are similar in both methods. As a result, it may be implied that the purposed analytical method is favorably accurate.
On the other hand, energy storage system may level and adjust load curve. Load curve is shown in Fig. 2 in the case of installing optimal storage unit and as it observed there, network peak load has been reduced and installation of storage unit has caused leveling of load curve.
Using storage unit causes variation in voltage profile. Fig. 3 and 4 show voltage profile at low load and peak load states respectively. In any situation, voltage profile has been drawn before and after installation of storage unit. As it observed in low load state and before
charge state, it is considered as load for the grid and it causes reduction of voltage in buses; however, also in this state voltage in all buses is under its allowed limit. For instance, voltage in bus no 12 is 0.974 in the case of low demand and before installing storage unit and it is 0.957 with the presence of storage unit. As it seen in Fig. 4, in peak load state and with absence of storage unit, voltage exceeds from its allowed limit in some buses but installation of storage unit may improve voltage profile. Since storage unit feeds the network with power during peak demand hours thus it causes improving voltage profile. With respect to Fig. 4, voltage of bus no 12 is 0.93 before installation of storage unit at higher demand, but voltage increases and reaches to 0.954 with the presence of storage unit
installation of storage unit, voltage in all buses is placed within its allowed limit. When storage unit is at

Table 3. SIMULATION RESULTS

| Analytical Method |  |  |  | Reload Flow |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bus. no | PM(MW) | WM(MWh) | Benefit | PM(MW) | WM(MWh) | Benefit |
| 2 | - | - | - | - | - | - |
| 3 | - | - | - | - | - | - |
| 4 | - | - | - | - | - | - |
| 5 | - | - | - | - | - | - |
| 6 | - | - | - | - | - | - |
| 7 | - | - | - | - | - | - |
| 8 | 0.051 | 0.255 | 1.2741 | 0.058 | 0.29 | 1.297 |
| 9 | 0.071 | 0.355 | 2.904 | 0.073 | 0.365 | 2.905 |
| 10 | 0.073 | 0.365 | 3.385 | 0.073 | 0.365 | 3.385 |
| 11 | 0.072 | 0.36 | 3.458 | 0.072 | 0.36 | 3.458 |
| 12 | 0.069 | 0.345 | 3.366 | 0.069 | 0.345 | 3.366 |



Fig. 2. Load curve


Fig. 3. Voltage profile at low load state


Fig. 4. Voltage profile at peak load state

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