Hybrid Distributed State Estimation Algorithm with Synchronized Phasor Measurements

CAMILO A. VILLARREAL, JORGE W. GONZÁLEZ, GABRIEL J. LOPEZ, IDI A. ISAAC, HUGO A. CARDONA
School of Electrical Engineering
Universidad Pontificia Bolivariana
Circular 1 No. 70-01
COLOMBIA
camilo.villareal@upb.edu.co

Abstract: - This paper proposes a hybrid distributed state estimation algorithm with synchronized phasor measurements. The implementation is easy since it initially uses a conventional or traditional state estimation and then adds a non-iterative second linear step calculation, which can be implemented by software without hardware investments.

Distributed algorithm accuracy was verified in a well-known power system. Even though it was a little less than the state estimation made to the complete system, it offers a faster computation. Hence, for applications that require real-time awareness of the power system, the distributed scheme results are a great alternative that can significantly reduce the bandwidth requirements, like time delays in the processes involved in the supervision and control of the power system. The proposed algorithm makes use of a central coordinator and assumes that each area of the system has at least one synchronized phasor measurement that allows finding the synchronization angles with respect to the angular reference of the system.

Key-Words: - State Estimation, Distributed State Estimation, Hybrid State Estimation, PMU, SCADA, Synchronized Phasor Measurements.

1 Introduction

The power system operators have always had the responsibility to execute multiple actions in real-time, including dispatch scheduling, generation exchange, service continuity, quick and safe restoration of the system after emergency conditions, among others [1]. Many of these actions are a result of the planning departments after performing multiple load flow studies. However, events leading to collapses are frequently unexpected and cannot be planned. Therefore, it is very important that the operators know the system status at all times accurately, to take the most appropriate preventive or even corrective actions. For this, the states estimation (SE) becomes the basic source of network information data.

In [2], [3] and [4] the basics of conventional methodology of traditional SE are presented. However, it is well known that traditional methodologies produce error characterization of the system, since the states are inferred from unsynchronized measurements, creating uncertainty about the true state of the system at a given time.

Synchronized phasor measurement units (PMUs) are being installed in many power systems worldwide in several cases, with the intention of using the system data to increase the features of the estimators of states [5]. Thus, many algorithms and methodologies for SE with PMUs have emerged nowadays.

In [6], SE algorithm using only synchronized phasor measurements is presented. Combination of traditional measurements with synchronized phasor data for a completely new SE is given in different ways in [7], [8], [9], [10], among others. Similarly, in [11] and [12] addition of PMUs measurements to traditional estimators, as a final step of linear calculation to increase accuracy of the estimation, is proposed. Also, in [13] it was implemented and tested an industrial SE for the New York Power Authority (NYPA).

Moreover, it is conventional that the states of a system can be calculated by a central estimator which collects all available measurements. However, due to the large size of the electrical system, extensive computation time is required.

For the above situation, a possible solution is to distribute the SE between areas of the power system using synchronized phasor measurements to improve the efficiency of the calculation. Distributed SE methods for individual areas performing an estimation with local measures and communicating with a central coordinator that organizes the states of the complete system is presented in [14] and [15]. In contrast, a decentralized algorithm is proposed in [16], in which each area performs its estimation in parallel. In [17] a method of distributed SE with PMUs using grid computing to calculate the reference angles of each area is shown. In [18], it is proposed a distributed SE with PMUs and a central coordinator, without using power injection measurements.

This paper presents a hybrid distributed SE algorithm using synchronized phasor measurements, which is easy to apply. The reason is that it is not required any hardware investment since traditional states estimator is used, but adding a second linear calculation step using PMU measurements. The latter can be simply implemented by software. Along with this solution, coordination of the estimated states is centrally performed according to the angular reference for each area.

The algorithm is tested on the IEEE 14 bus system with a software developed for this purpose.

In Section 2, basic concepts are briefly introduced. Section 3 formulates the hybrid algorithm of distributed state estimation with synchronized phasor measurements. Finally, Section 4 presents the results obtained with this algorithm.

2 Background

2.1 Phasor Measurement Units (PMU)

The phasor measurement units were developed in the mid-80's and are devices that measure, in real time, phasor representations of sinusoidal signal voltages, currents and frequency in the electrical power system. These representations are synchronized with the same time stamp and are called Synchrophasors.

A synchrophasor is a phasor representing sinusoidal signals referenced to the nominal system frequency (50/60Hz) and the specific time reference given. The PMU uses, as the time reference, the UTC (Universal Time Coordinated) signal obtained from a GPS transmitter.

2.2 State Estimation

The states of a power system are given by all voltage magnitudes and its phase angles in the network busbars. Conventional estimators, e.g., SCADA, use asynchronous measured power flows, injections of active and reactive power and also voltage magnitudes to determine the phase angles of the system. These estimators also monitor the status of network elements to determine its actual topology. The algorithms used are based on iterative solutions that use the method of weighted least squares (that consist of minimizing the error of estimated data with respect to experimental) to obtain a reliable function of estimated states.

Even though at present, state estimators are capable of solving large systems, they are vulnerable to problems of convergence, affecting the reliability and accuracy, especially when the power system is stressed.

2.3 State Estimation with PMU

In recent years, the power system operators have recognized the benefits of using synchronized phasor measurements in state estimation. The increased accuracy and increased sampling frequency (10-60 phasors per second), along with the fact that all measurements are taken with the same time stamp, are key improvements. The benefits would be reflected in better and more accurate estimates of the states, which would have better and more reliable safety margins in real time. This can drive to a more economical operation and safety.

PMUs can measure phasors of positive sequence, and considering that system state vector consists of the magnitudes of positive sequence voltages and respective phase angles according to a reference (Slack Bus), it follows that the state vector of the system could directly be measured with the use of PMUs, rather than estimating this from SCADA measurements.

2.4 Distributed State Estimation with PMUs

The growth of power systems and bandwidth limitations that may arise when seeking to obtain all measurements at a central point, have created the need to consider the management of the form of distributed systems.

There are two ways to perform distributed state estimation. One is using a central coordinator that is responsible for consolidating the results of local estimates of each area of the system. The other way is without a central coordinator, so that each area performs a local estimate and, among them, communicate and coordinate the system states [17] [19].

Similarly, in [20] distributed SE concept is conducted further into the power system, proposing a three-phase SE called SuperCalibrator operating at substation level and requiring at least one PMU for each substation.

3 Hybrid Distributed State Estimation Algorithm

The power system is divided into areas in which a local state estimation is performed. This is done by a hybrid state estimation method with PMUs that is a combination of the traditional measurements (SCADA) with PMU phasor measurements. Subsequently the results of every area are consolidated in a central coordinator.

In each area, hybrid state estimation composed by three steps are performed:

1. Conventional states are estimated by the method of weighted least squares (WLS).
2. States estimated in step 1 are transformed from polar to rectangular form, and are used as inputs to the state estimator with phasor measurements.
3. The states are estimated with a linear and non-iterative model using phasor measurements, and those estimated by the traditional systems are inputs

The first step is to estimate the system states as commonly done now in the traditional estimators with the method of weighted least squares (WLS). Those methods consider a set of measurements $[\boldsymbol{z_t}]$, having the values of unsynchronized active and reactive power in the network. The power injections and voltage magnitudes in buses, which are commonly taken by the SCADA system network operator are also extracted. It is assumed that the measurement errors are eliminated and do not take part of $[\boldsymbol{z_t}]$. The measurements are nonlinear functions of the state vector $[\boldsymbol{E}]$, which consists of all positive sequence voltages of the buses of the power system. Therefore,

$$[z_t] = [h_t(E)] + [\varepsilon_t] \tag{1}$$

Where $[\boldsymbol{h_t}]$ represents nonlinear functions of the state vector $[\boldsymbol{E}]$ expressed in polar coordinates, and $[\boldsymbol{\varepsilon_t}]$ is the vector of measurement errors.

Using the technique of weighted least squares WLS, an iterative solution for the state vector is given by:

$$[E_{k+1}] = [E_k] + [G_t(E_k)][H_t^T R_t^{-1}][z_t - h_t(E_k)] \tag{2}$$

Where $[\boldsymbol{G_t(E_k)}]$ is the gain matrix, which is obtained by:

$$[G_t(E_k)] = [H_t^T(E_k) R_t^{-1} H_t(E_k)]^{-1} \tag{3}$$

The Jacobian matrix $[\boldsymbol{H_t}]$ is found by calculating the partial derivatives of $[\boldsymbol{h_t}]$ with respect to$[\boldsymbol{E}]$

$$[H_t(E)] = \left[\frac{\partial h_t(E)}{\partial (E)}\right] \tag{4}$$

Variance matrix of the measurements$[\boldsymbol{R_t}]$ is a diagonal matrix:

$$[R_t] = diag\{[\sigma_1^2\ \sigma_2^2\ \dots\ \sigma_m^2]\} \tag{5}$$

It is considered that a correct estimate is obtained when $[\boldsymbol{E_{k+1}}] - [\boldsymbol{E_k}]$ is less than the convergence criterion.

At this point, it should be made a synchronization of calculated states according to the system reference. For this purpose, the calculated angles are outdated according to the difference that exists between the slack bus area and slack bus power system. Thus:

$$\boldsymbol{\theta_{sinc} = \theta_{slack_{area}} - \theta_{slack_{system}}} \tag{6}$$

$$\therefore [\boldsymbol{E}] = \begin{bmatrix} \boldsymbol{V_1} \\ \vdots \\ \boldsymbol{V_n} \\ \boldsymbol{\theta_2 - \theta_{sinc}} \\ \vdots \\ \boldsymbol{\theta_n - \theta_{sinc}} \end{bmatrix}$$

The second step of the method is to integrate PMU measurements to generate an improvement in the accuracy of the system. This is based on the model proposed in [12] and [11], which consists of using a linear model composed by a traditional state estimation enhanced by phasor measurements:

$$[M] = [H][V] + [\varepsilon] \tag{7}$$

Where $[\boldsymbol{H}]$ is the coefficient matrix of the Jacobian matrix, $[\boldsymbol{V}] = [\boldsymbol{V_R}\ \boldsymbol{V_I}]^{\boldsymbol{T}}$ is the state vector of the real and imaginary parts of the voltages in buses and $[\boldsymbol{\varepsilon}]$ is the vector of measurement errors.

The measurement vector $[\boldsymbol{M}]$ consists of the estimated states traditionally $[\boldsymbol{V_R V_I}]_{\boldsymbol{ES}}^{\boldsymbol{T}}$ and measurements of voltages and currents of the PMUs $[\boldsymbol{V_R}\ \boldsymbol{V_I}]_{\boldsymbol{PMU}}^{\boldsymbol{T}}$, $[\boldsymbol{I_R}\ \boldsymbol{I_I}]_{\boldsymbol{PMU}}^{\boldsymbol{T}}$. For the implementation of the model it is required that all measurements are in rectangular form, opposite to polar form using the traditional method. This is why the suffixes R and I are used, which denote the real and imaginary parts of the voltage and current measurements.

$$\begin{bmatrix} \begin{bmatrix} V_R \\ V_I \end{bmatrix}_{ES} \\ \begin{bmatrix} V_R \\ V_I \end{bmatrix}_{PMU} \\ \begin{bmatrix} I_R \\ I_I \end{bmatrix}_{PMU} \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \\ H_{31} & H_{32} \\ H_{41} & H_{42} \\ H_{51} & H_{52} \\ H_{61} & H_{62} \end{bmatrix} \begin{bmatrix} V_R \\ V_I \end{bmatrix} + \begin{bmatrix} \varepsilon_{VR}^{ES} \\ \varepsilon_{VI}^{ES} \\ \varepsilon_{VR}^{PMU} \\ \varepsilon_{VI}^{PMU} \\ \varepsilon_{IR}^{PMU} \\ \varepsilon_{II}^{PMU} \end{bmatrix} \tag{8}$$

Equation 8 shows the model to be used. It should be noted that all measurements are decomposed into real and imaginary parts. Regarding the error ε, it is assumed to have a zero mean value and correlated to a diagonal covariance matrix ***R***.

$$R = diag\{[\sigma^2_{|VR\_ES|}\ \sigma^2_{|VI\_ES|}\ \sigma^2_{|VR_{PMU}|}\ \sigma^2_{|VI_{PMU}|}\ \sigma^2_{|IR_{PMU}|}\ \sigma^2_{|II_{PMU}|}] \tag{9}$$

Where each element of ***R*** is a diagonal submatrix. For example:

$$\sigma^2_{|VR\_ES|} = diag\{[\sigma^2_{|VR|1}\ \sigma^2_{|VR|2}\ \sigma^2_{|VR|3} \dots \sigma^2_{|VR|N}]\} \tag{10}$$

The variance **σ** must correspond to rectangular components of phasors. Then, it is necessary to transform the variances used in the traditional estimator.

In matrix ***H*** each element corresponding to the voltage phasor measurements $[\boldsymbol{V_R}\ \boldsymbol{V_I}]^{\boldsymbol{T}}_{\boldsymbol{PMU}}$ is a vector of zeros with a one in the column associated with the state variable.

$H_{11} = I, H_{12} = 0, H_{21} = 0, H_{22} = I,$ are submatrices NxN, with N being the number of buses of the system. I is an identity matrix and 0 is a null matrix.

The elements in ***H*** corresponding to the phasor current measurements $[I_R\ I_I]^T_{PMU}$ depend on the type of transmission lines, and are the partial derivatives of currents with respect to system voltages.

$$H_{51} = \frac{\partial I_R^{PMU}}{\partial V_R}; \quad H_{52} = \frac{\partial I_R^{PMU}}{\partial V_I} \tag{11}$$

$$H_{61} = \frac{\partial I_I^{PMU}}{\partial V_R}; \quad H_{62} = \frac{\partial I_I^{PMU}}{\partial V_I} \tag{12}$$

For instance, if the transmission line can be described by a π model, elements of ***H*** would be:

$$\frac{\partial I_{ijR}}{\partial V_{iR}} = \frac{\partial I_{ijI}}{\partial V_{iI}} = G + G_{i0} \tag{13}$$

$$\frac{\partial I_{ijR}}{\partial V_{jR}} = \frac{\partial I_{ijI}}{\partial V_{jI}} = -G \tag{14}$$

$$\frac{\partial I_{ijR}}{\partial V_{iI}} = -\frac{\partial I_{ijI}}{\partial V_{iR}} = -(B + B_{i0}) \tag{15}$$

$$\frac{\partial I_{ijR}}{\partial V_{jI}} = -\frac{\partial I_{ijI}}{\partial V_{jR}} = B \tag{16}$$

According to the latter, states can be estimated directly and not iteratively using the following weighted least squares solution:

$$[V] = (H^T R^{-1} H)^{-1} H^T R^{-1} M \tag{17}$$

Finally, the results are consolidated at the central coordinator. This is also responsible for updating the angles of synchronization for each area.

4 Simulation Results

The algorithm was tested and verified in the ASPUPB2 software developed in the research group of Transmission and Distribution of Electric Power at Universidad Pontificia Bolivariana [21] [22]. IEEE testing system 14 buses was used and divided into three areas as shown in Fig. 1.

Additionally, the software runs a load flow of the IEEE system. Then produces the results of voltages and power flows, adding them with a random error to simulate measurements. These quantities are used for the estimation of states in a way of convenience, with or without PMUs and distributed or not.

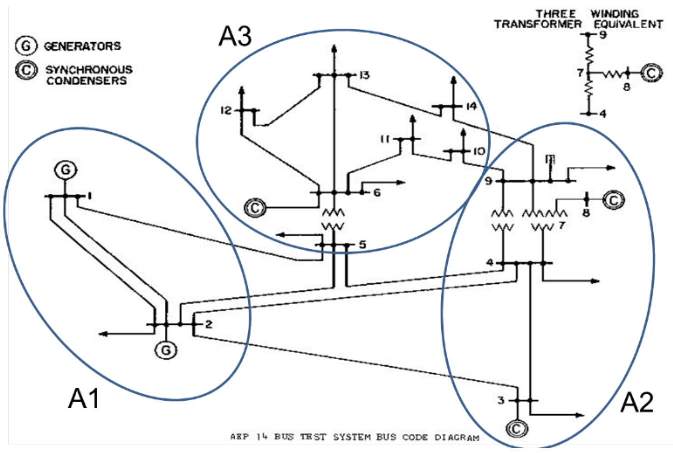

Fig. 1 IEEE 14 Bus Test Case. [23]

The simulation was performed with PMUs installed in "Slack buses" of each area (buses 1, 3 and 6). θ_{sinc}, are available for each area in Table 1.

Table 1. Synchronization Angles for Areas.

	A1	A2	A3
θ_{sinc}	0°	-12.72°	-14.22°

In Table 2, the results of the distributed state estimation and of the entire system with the theoretical values of IEEE 14 Bus Test Case are compared. Furthermore, in Table 3 percentage errors of each of the states for the two types of estimates are showed.

In terms of the obtained results, it can be said that both cases are accurate, being the minor one the distributed case because it is less redundant.

Errors in voltages are less than 1%, and for angles do not exceed 5%. This can be translated into a maximum difference of 0.8° (degrees) with respect to the theoretical value. Threshold values of 1% for voltage error and 5% for angle error are common [24].

Table 2. Estimation Results Comparison between Complete and Distributed Estimation.

Bus	Area	Exact		Complete Estimation		Distributed Estimation	
		Voltage	*Angle*	*Voltage*	*Angle*	*Voltage*	*Angle*
1	A1	1.06	0	1.0602	0	1.0607	0
2	A1	1.045	-4.98	1.0452	-4.98	1.0457	-4.99
3	A2	1.01	-12.72	1.0102	-12.73	1.0065	-12.72
4	A2	1.019	-10.33	1.018	-10.33	1.0167	-10.38
5	A3	1.02	-8.78	1.0202	-8.76	1.0202	-8.52
6	A3	1.07	-14.22	1.0702	-14.82	1.0699	-14.22
7	A2	1.062	-13.37	1.062	-13.63	1.0588	-13.88
8	A2	1.09	-13.36	1.0899	-13.63	1.0859	-13.88
9	A2	1.056	-14.94	1.0568	-15.29	1.054	-15.64
10	A3	1.051	-15.1	1.0519	-15.49	1.0549	-14.58
11	A3	1.057	-14.79	1.0575	-15.28	1.0584	-14.53
12	A3	1.055	-15.07	1.0554	-15.65	1.0551	-15.03
13	A3	1.05	-15.16	1.0507	-15.72	1.0508	-15.07
14	A3	1.036	-16.04	1.0358	-16.45	1.0387	-15.64

Table 3. Percentage Error Comparison between Complete and Distributed Estimation.

Bus	Area	Complete Estimation		Distributed Estimation	
		% Error Voltage	*% Error Angle*	*% Error Voltage*	*% Error Angle*
1	A1	0.0189	0.0000	0.0660	0.0000
2	A1	0.0191	0.0000	0.0670	0.2008
3	A2	0.0198	0.0786	0.3465	0.0000
4	A2	0.0981	0.0000	0.2257	0.4840
5	A3	0.0196	0.2278	0.0196	2.9613
6	A3	0.0187	4.2194	0.0093	0.0000
7	A2	0.0000	1.9447	0.3013	3.8145
8	A2	0.0092	2.0210	0.3761	3.8922
9	A2	0.0758	2.3427	0.1894	4.6854
10	A3	0.0856	2.5828	0.3711	3.4437
11	A3	0.0473	3.3130	0.1325	1.7579
12	A3	0.0379	3.8487	0.0095	0.2654
13	A3	0.0667	3.6939	0.0762	0.5937
14	A3	0.0193	2.5561	0.2606	2.4938

Additionally, execution time for the algorithm is also examined. For area 1 it took 9 ms; for area 2, 115 ms was the computing time. For area 3, 35 ms was the execution time. This means that a total distributed state estimation takes about 115 ms since computation takes place in the areas in parallel. Similarly, for the entire system it took a total of 278 ms, more than double of the time for distributed state estimation. It is important to mention that execution time may vary according to computers and systems used. Therefore, the previous values are indicative, and are used to show the improvement given by the proposed method.

5 Conclusions

This paper proposed a hybrid distributed state estimation algorithm with synchronized phasor measurements, easy to implement since it initially uses the conventional or traditional state estimation. Then, the method adds a second linear step calculation, which is non-iterative, and can be applied by software without hardware investments.

Distributed algorithm accuracy was verified through simulations in a power system. Even though it was a little less than the state estimation made to the complete system, it offers a faster computation. Hence, for applications that require real-time awareness of the power system, the distributed scheme results a great alternative that can significantly reduce the bandwidth requirements, like time delays in the processes involved in the supervision and control of the power system.

The proposed algorithm makes use of a central coordinator and assumes that each area of the system has at least one synchronized phasor measurement that allows finding the synchronization angles with respect to the angular reference of the system.

Tests on more extensive and real systems will be performed in the future, also with real world data.

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