Addressing the Sustainable Energy Conversion in Electric Railway Vehicles with Traction Synchronous Motors

CORNELIA A. BULUCEA¹, DORU A. NICOLA¹, DANIEL C. CISMARU¹, NIKOS E. MASTORAKIS², CARMEN A. BULUCEA³, CONSTANTIN BRINDUSA⁴.

¹University of Craiova, Faculty of Electrical Engineering, Craiova 200440, Romania; ²Technical University of Sofia, Industrial Engineering Department, Sofia, Bulgaria & Military Institutions of University Education (ASEI), Hellenic Naval Academy, Piraeus 18539, Greece; ³University of Medicine and Pharmacy of Craiova, Craiova 200349, Romania; ⁴University Constantin Brancusi of Targu Jiu, Targu Jiu, Romania.

E-Mails: abulucea@em.ucv.ro; dnicola@em.ucv.ro; dcismaru@em.ucv.ro; mastor@tu-sofia.bg; carmen.bulucea@umfcv.ro; rinstalctin@yahoo.com

Abstract: This paper addresses some aspects illustrating sustainable energy conversion processes during the operation of electric railway vehicles with traction synchronous motors. Increasing efforts are being expended to enhance the sustainability of transportation technologies and systems, and supporting transport systems’ sustainability is a key way of thinking towards cleaner production. In line with this idea, this paper addresses the operation sustainability of electric railway vehicles with traction synchronous motors, highlighting the chain of interactions among the main electric equipment on an electrically driven railway system. The paper supports the findings that electric traction drive systems using traction synchronous motors powered by network-side converters and machine-side converters enhance the sustainable operation of railway trains.

Key-Words: electromagnetic torque, high speed electric train, railway vehicle, sustainability dynamics, synchronous motor

1 Introduction
In line with the framework of cleaner production, to address meaningfully many of the problems facing railway vehicles, conditions for the performance of sustainable transportation systems must be formulated [1-5]. Correspondingly, sustainability concepts can help understand the efficiencies of electrically driven systems and guide improvement efforts [1-5].

According to the term defined by United Nation Environmental Programme in 1990, cleaner production requires a new way of thinking about processes and products, and about how they can be made less harmful to humans and the environment [6-7]. Within the conceptual framework of cleaner production improvements of technology and changes in equipment design should suggest better choices in use of materials and energy in manufacturing and operation of industrial systems [5-7]. Improved control and automatisation represent key concepts of cleaner production strategies [6-8]. Green products and systems are generally produced in a manner that consumes fewer natural resources or uses them in a more sustainable way, and they may involve high exergy efficiency in their manufacture and operation [4-5].

The sustainability of an electric transportation system is based on technical performance, safety, energy and exergy efficiency, mitigated environmental impact, economics and societal acceptance [1-3]. Costs should reflect value, which is doubtless associated with sustainability aspects [1-5]. Addressing the sustainability of traction and braking operation of the electric vehicles constitutes a challenge in electric transportation research.

2 Structure of Main Electric Circuits of Railway Vehicles with Traction Synchronous Motors
In order to be used as a traction motor on high speed trains (namely, the French TGV), the synchronous motor (being an electric motor with rotating field) is needing a variable frequency three-phase supply [9-15]. It results implicitly the necessity of the machine converter (MC) in the power circuit. The MC is a current inverter (CI) with a three-phase bridge (with
6 thyristor branches linked as a three-phase bridge) which operates being commutated by the counter-electromotive forces (CEMF) \((-e_0)\) induced in the three-phase stator winding of the supplied synchronous motor [9-11].

One could notice that problems related to the natural commutation appear solely at very low speed, when the CEMF amplitude is small (and consequently, insufficient for ensuring the commutation). In such situation (from the starting regime up to 5-10% of the maximum speed \(v_{\text{max}}\)) the commutation adjustment is imposed. The devices for the assisting commutation are quite simple and don’t have a hard weight (for instance, a weight of 70 kg for a 6000 kW locomotive). Moreover, by using the signals regarding the rotor position and speed at the inverter control, the synchronous motor will drive (or pilot) the current inverter which is supplying it, this way being eliminated the risks of synchronism loss, whatever is the rotor speed. Hence, the traction synchronous motor is a self-piloted motor [9-15].

As a pattern, the operation of a self-piloted synchronous motor supplied by a current inverter source is similar to the DC motor with fixed armature and rotating inductor. According to this scenario, the speed regulation of the self-piloted synchronous motor will be ensured similarly to the DC motor with separate excitation [17-18]. Concretely, it is performed the change of voltage of the DC intermediary circuit with single phase bridges with phase regulation (mixed bridges or fully controlled bridges) in the case of the supply from a AC contact line, or there are utilized electronic DC drives (DC/DC converters) in the case of the supply from a DC contact line. In order to obtain high speeds there is performed the field weakening [9-15, 18-20].

Unlike the electric railway vehicle with induction motors, during the electric power transmission on the vehicle with synchronous motors the amplitude regulation functions are completely separated from those of frequency regulation. Actually, the voltage variation is enabled as in classic traction with DC motors by the electronic DC drives (DC/DC converters) or by mixed single-phase half-controlled bridge rectifier by asymmetric type. Conversely, the frequency variation is provided by the current inverters (CI) through which the traction synchronous motors are powered.

Consequently, the electric trains with traction synchronous motors entail compulsory two conversion stages of the electric energy. Specifically, it is imposed the presence of the network converter (NC) both when the railway electric vehicle is powered from an AC contact line (see Figure 1) and when the vehicle is supplied from a DC contact line (see Figure 2).

As a traction motor, the synchronous motor is operating in a self-piloted regime, the control pulses of the current inverter thyristors (synchronized with the induced voltages) being correlated with the rotor position due to the position transducer PT. In the operation at low speed, when the counter-electromotive forces can not anymore ensure the commutation of the current inverter thyristors it is carried out the “assisted commutation” performed by the auxiliary device AC.

In the case of electric locomotives and urban trains with traction synchronous motors supplied from an AC contact line, the network converter NC is encompassing a system of single-phase rectifiers (full or part controlled) connected in series (in order to improve the power factor). In contrast, when the electric railway vehicle is powered by a DC line, the network converter is a classic chopper (with operation in 1 or 2 quadrants).

Consequently, in both situations the structure of the network converters is similar to classic solutions with rectifiers and choppers. Conversely it is imposed to enhance the analysis of the system inverter –
synchronous motor, underlining that its structure is identical for both supply types from the contact line.

In contrast with the case of electric locomotives and urban trains with traction induction motors, the utilization of synchronous motors as electric traction motor does not raise problems related to the electric braking safety. The electric braking might be easily accomplished, since when the contact line power is cut the traction synchronous motors, being excited now from the locomotives batteries will operate as synchronous generators, delivering the electric energy (converted to DC power by the converter bridge) on the electrical braking resistance conveniently sized.

For economic reasons, so far the synchronous motor has been used as traction motor only at high power (800 kW and 1.1 MW) on locomotives and electric trains TFV-A type.

3 Electromagnetic Torque Developed by Synchronous Motor

Within the framework of cleaner production, the exergy concept, which is a measure of energy quality, can be used to enhances understanding and help improve the efficiencies of technical systems which convert energy and matter [1-5]. According to the laws of thermodynamics, energy is never destroyed during a process, although it can change from one form to another, but exergy can be destroyed. While energy is a measure of quantity only, exergy is a measure of quantity and quality or usefulness. Energy is often thought of as motion and exergy as work [1-5]. Thus, exergy has an important role to play in increasing efficiencies of energy systems and technologies, within energy optimization and engineering modeling studies.

Since exergy is a measure of the potential of a system to do work, the electromagnetic torque \( M \) developed by an electrical motor can be interpreted as the driving force of useful work, i.e. the electric motor output exergy [1-5].

The electromagnetic torque \( M \) (respectively, the electromagnetic power \( P_M = M \cdot \Omega \)) developed by any synchronous motor is determined by calculation on basis of the active powers’ balance, as follows:

\[
P_M = P_1 - P_M = 3 \cdot U \cdot I \cdot \cos \varphi - 3 \cdot R_l \cdot I^2 \tag{1}
\]

\[
M = \frac{P_M}{\Omega} = \frac{3}{\omega} 
( U \cdot I \cdot \cos \varphi - R_l \cdot I^2 ) \tag{2}
\]

For generalization, we consider the case of salient pole synchronous motor, referring to the classic phasor diagram depicted in Figure 3, where the components of the current \( I \) and voltage \( U \) on the axes \( d \) and \( q \) are:

\[
I_d = I \cdot \sin \Psi; \quad I_q = I \cdot \cos \Psi \tag{3}
\]

\[
U_d = U \cdot \sin \theta; \quad U_q = U \cdot \cos \theta \tag{4}
\]

If there is taken into consideration that \( \varphi = \psi + \theta \), and the function \( \cos (\psi + \theta) \) is trigonometrically developed as:

\[
I \cdot \cos \varphi = I \cdot \cos \Psi \cdot \cos \theta - I \cdot \sin \Psi \cdot \sin \theta = I_d \cdot \cos \theta - I_q \cdot \sin \theta \tag{5}
\]

and in the expression of active power \((U \cdot I \cdot \cos \varphi)\) the components \( U_d \) and \( U_q \) are highlighted:

\[
U_d \cdot U_q - I_d \cdot U_d - I_d \cdot U_q \tag{6}
\]

then the formula (2) for the calculation of electromagnetic torque \( M \) becomes:

\[
M = \frac{3}{\omega} \cdot (I_q \cdot U_q - I_d \cdot U_d - R_l \cdot I^2) \tag{7}
\]

Using again the phasor diagram (Fig.3.a), by projection of the phasors’ contour on the axes \( d \) and \( q \), the voltage equations in scalar form of components are obtained as:

\[
U_d = -R_l \cdot I_d + X_q \cdot I_q \tag{8}
\]

\[
U_q = E_{op} - R_l \cdot I_q + X_d \cdot I_d \tag{8}
\]

If further the term \( I_q \cdot U_q - I_d \cdot U_d \) is evaluated as below:

\[
I_q \cdot U_q - I_d \cdot U_d = E_{op} \cdot I_q + (X_d - X_q) \cdot I_d \cdot I_q + R_l \cdot I^2 \tag{9}
\]

then the expression of the electromagnetic torque \( M \) given by (7) will take the compact form:

\[
M = \frac{3}{\omega} \cdot [E_{op} \cdot I_q + (X_d - X_q) \cdot I_d \cdot I_q] \tag{10}
\]

At the non-salient synchronous motor, when \( X_q = X_d = X \), the component caused by the magnetic anisotropy in the formula (10) will not appear anymore. Actually, the final expression of the electromagnetic torque will depend on the type of the power supply of the synchronous motor (voltage source or current source).
3.1 Electromagnetic Torque Developed by the Synchronous Motor Supplied from a Three-Phase Sinusoidal Currents’ Source

In the case of powering from a three source of sinusoidal currents the current phasor $I$ (through the reference stator winding) is specified by its magnitude $|I|=I$ and initial phase (through the position angle) $\psi$ to the axis $q$ of the rotor. It means that implicitly there are specified the components $I_d$ and $I_q$ (3) of the stator current $I$.

In these supplying conditions the electromagnetic torque $M_I=M$ will be determined with relation (10) in which there are performed the substitutions specified by (3), obtaining the expression as follows:

$$M_I = \frac{3P}{2\omega} \left[ E_{op} \cdot I \cdot \cos \Psi + (X_d - X_q) I^2 \cdot \sin \Psi \cdot \cos \Psi \right] \cos \Psi$$

And the final form:

$$M_I = \frac{3P}{2\omega} \left[ E_{op} \cdot I \cdot \cos \Psi + \frac{1}{2} I^2 (X_d - X_q) \cdot \sin 2\Psi \right] \cos \Psi$$

On the whole, one could notice the presence of two components $M'_I$ and $M''_I$, so that based on relation (12) it can be written $M_I = M'_I + M''_I$. The main component $M'_I = \frac{3P}{2\omega} E_{op} \cdot I \cdot \cos \Psi$ depends on the excitation degree (namely, the magnitude of current $I_e$), while the secondary component $M''_I = \frac{3P}{2\omega} \left[ \frac{1}{2} I^2 (X_d - X_q) \cdot \sin 2\Psi \right]$ is determined by the magnetic asymmetry of the rotor, being present (and non-zero) even in the absence of the synchronous motor excitation. At $I=ct.$, $f=ct.$ and excitation current $I_e=ct.$ (when $E_{op}=ct.$), the electromagnetic torque $M_I$ depends solely on the load angle $\psi$.

For the salient pole synchronous motor, the characteristic $M_I = f(\psi)$ is represented in Figure 4 by a thick line.

![Fig.4 Characteristic representation $M_I = f(\psi)$ of synchronous motor with salient poles at powering from a three-phase source of sinusoidal currents](image)

In the case of non-salient pole motors, in which $X_d=X_q=X_s$ the secondary component will not appear ($M''_I=0$) and the load characteristic $M_I = f(\psi)$ is represented just by the main component with a cosine variation.

One could note that if in formula (12) of the torque $M_I$ there are performed the substitutions:

$$E_{op} = \omega \cdot (w_1 \cdot k_{s1} \cdot \Phi_{om}) \cdot \cos \Psi; \quad X_d = \omega \cdot L_d \quad \text{and} \quad X_q = \omega \cdot L_q$$

then the electromagnetic torque expression $M_I$ (at the supply in current) becomes:

$$M_I = 3P \cdot (w_1 \cdot k_{s1} \cdot \Phi_{om}) \cdot I \cdot \cos \Psi + \frac{3P}{2} I^2 (L_d - L_q) \cdot \sin 2\Psi$$

In relations (13) and (14) the physical significance of the new quantities are:

- $w_1$ = the number of turns, in series, on a stator phase of synchronous motor;
- $k_{s1}$ = the winding factor of a stator phase;
- $\Phi_{om}$ = the maximum value of the main magnetic flux, depending on the magnitude of excitation current $I_e$, namely $\Phi_{om}=f(I_e)$;
- $L_d$ = the longitudinal inductance of stator winding
- $L_q$ = the transverse inductance of stator winding of synchronous motor.

Particularly, for $\psi=0$ (when $I_d=0$ and $I_q=I$), the expression (14) of the electromagnetic torque $M_I$ becomes:

$$M_I = 3P \cdot w_1 \cdot k_{s1} \cdot \Phi_{om} \cdot I = K \cdot \Phi_{om} \cdot I$$

videlicet $M_I$ will get a form that is mathematically identical to the electromagnetic torque $M=\Phi \cdot I$ developed by a DC motor (with collector) with separated excitation.

In relation (15), with $K$ has been denoted the constructive constant $K = \frac{3P \cdot w_1 \cdot k_{s1}}{\sqrt{2}}$ of synchronous motor.

Expression (14) highlights that at $I_e=ct.$, $I=ct.$ and $\psi=ct.$ the electromagnetic torque $M_I$ is constant (and nonzero) and has the same magnitude regardless of the numerical value of the frequency $f$ of the stator currents.

Precisely the existence of nonzero electromagnetic torque at the powering in current (by magnitude $I$ phase shift $\psi=ct.$ related to the counter-electromotive forces $E_{op}$) led to the development of synchronous motor applications in electric traction.

On the electric locomotives and urban trains the synchronous motor is powered from a current inverter (thyristorised), with natural commutation, determined exactly by the counter-electromotive forces $E_{op}$ (induced in the stator windings of the synchronous motor).

In order to ensure the phase shift $\psi$ (of the current fundamental $I$ leading the counter-electromotive...
forces $E_{eq}$), the control of thyristors’ conduction is based on the signals from a position transducer solidary with the rotor. Consequently the synchronous motor is piloting.

4 Sustainable Operation of Self-Piloted Synchronous Motor

On the electric railway vehicles the most convenient source of three-phase currents (to powering the synchronous motor) is the current inverter with natural commutation. The reason of this choice is related to low price, high reliability and smallsize (due mainly to the absence of the auxiliary extinction circuits).

In this context one could remind that the synchronous motor operation is characterized by a three-phase system of induced counter-electromotive forces ($-e_{OA}$, $-e_{OB}$, $-e_{OC}$) which might ensure the commutation of the three-phase bridge thyristors, just as the three-phase alternating voltage network ensured the commutation of rectifier bridge thyristors.

However, to ensure the natural commutation of the thyristorised bridge, the induced counter-electromotive forces must be sufficiently large values, even at low speed, which requires synchronous motor to run over-excited (from start up to rated speed). The over-excitation of synchronous motor will lead to winding currents $I$ that are leading the voltage $U$ (the phase shift $\phi$ being by capacitive type, like in Figure 3b), in this case the motor providing also through the terminals the reactive power necessary to the inverter bridge thyristors.

In order to ensure the capacitive phase shift of the current, the control of bridge thyristor ignition will be based on signals from a rotor position transducer. This way is correlated the position of current phasor $I$ to the axis $d$ of the rotor.

On the opposite side (the DC side) the autonomous current inverter (CI) must be connected to a DC variable source. Usually, a variable DC source is made up of adjustable DC voltage source (of chopper type or single-phase rectifier bridge with phase adjustment) connected in series with a smoothing coil with iron core, with the inductance $L_0$ of large size.

Accordingly, the configuration of self-piloted synchronous motor powered from an autonomous current inverter is depicted in Figure 5. The representation entails the over-excited synchronous motor (SM), a three-phase thyristorised bridge (CI), a position transducer (PT) and a smoothing coil (with the parameters $R_0$ and $L_0$).

Since the supply is of current powered type, it results that the reversibility of assembly SM+CI can only be achieved by changing the sign (by reversing the polarity) of the average voltage $V_{ds}$ (on the DC side), therefore by controlling the passing of three-phase bridge (CI) in rectifier regime.

4.1 Detection of Rotor Position

In contradistinction to the thyristorised bridge connected to three-phase network (thereupon the firing pulses of thyristors were synchronized with the alternating voltages of the network), for the current case of thyristorised bridge (through that the self-piloted synchronous motor is powered) the generation of firing pulses of thyristors must be correlated with the rotor position. Specifically, in the case of a synchronous motor with $2p$ magnetic poles, the rotor position controller must generate $6\cdot p$ „control orders” (at each complete rotation of the rotor) for the ignition of thyristors of bridge CI. It should be noted that „the control orders” of the thyristor ignition are given evolutionary (with time) as the rotor rotates.

As discussion at this point, one could highlight that, in such circumstances, it is necessary to provide a device that will help operation of the thyristorised bridge, since that needs to piloting of knowing angular position of the rotor. To this purpose, the traction synchronous motor shall be provided with:

- a toothed steel disc, solidary with the rotor, precisely positioned relative to inductor poles, and
- a set of sensors, set on the stator and precisely positioned relative to the stator core slots.

The successive passing of teeth and slots of the toothed disc under the position sensors shall allow finding at any time the exact rotor position. Depending on the rotor position as detected, further is established the control of the thyristors supplying phase stator windings so that the electromagnetic torque developed by the motor to be the maximum possible.

In principle, the stator sensors might be delayed in order to change the phase of control orders (of the ignition of thyristors) relative to the position of pole wheel. This way it is possible to control the phase angle $\psi$ between the current and the induced counter-electromotive force on each phase. Nevertheless, on the modern traction synchronous motor the angle $\psi$ is...
electronically controlled, by an electronic phase shift of signals generated by the position sensors. Usually, the device to detecting the rotor position is located inside the traction synchronous motor, being situated on the opposite side to the rotor shaft end.

4.2 Current Diagrams

Study of joint operation of current inverter and synchronous motor in self-piloted regime can be performed solely in the context of supply scheme from Figure 5. This is the easiest powering at variable frequency of synchronous motor [9-11].

The synchronous motor will be over-excited and the three-phase induced counter-electromotive forces (-e0A, -e0B, -e0C) of frequency f=ω/2π will be able to drive the commutation of the current inverter thyristors at any speed Ω>$Ω_{\min}$ without the risk of losing synchronism between rotor and rotating magnetic field.

Specifically, at any speed “n” of the rotor of self-piloted synchronous motor (corresponding to the frequency f=p/n/60 of stator currents) the induced counter-electromotive forces (e0A, e0B, e0C) of frequency f=ω/2π will be able to drive the commutation of the current inverter thyristors at any speed Ω>$Ω_{\min}$ without the risk of losing synchronism between rotor and rotating magnetic field.

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Express the fundamental currents of stator phasors (iA, iB, iC) as space phasors (Ia, Ib, Ic) given by (18). From this relationship, the space phasor iA, will take the form:

\[
I_a = \frac{\sqrt{2}}{\pi} I_0 \cdot e^{j(\frac{\pi}{2} + \psi)}
\]

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\[
I_a = \frac{\sqrt{2}}{\pi} I_0 \cdot e^{j(\frac{\pi}{2} + \psi)}
\]
there are obtained the effective values $I_{1d}$ and $I_{1q}$ of the orthogonal components:

$$I_{1d} = -I_1 \cdot \sin \psi = -\left( \frac{\sqrt{6}}{\pi} I_0 \right) \sin \psi$$

$$I_{1q} = I_1 \cdot \cos \psi = \left( \frac{\sqrt{6}}{\pi} I_0 \right) \cos \psi$$

If further in expression (10) of the electromagnetic torque $M$ there are performed the substitutions $I_d=I_{1d}$ and $I_q=I_{1q}$, and, in addition there are used the substitutions (13), finally will result for $M=M_I$:

$$M_I = 3p \left[ \frac{\sqrt{6}}{\pi} I_0 \cdot (X_0 \omega) \cdot \cos \psi - \frac{3}{\pi} I_0 \cdot (L_d - L_q) \cdot \sin 2 \psi \right]$$

where the average value of current $I_0$ (which is established in the DC intermediary circuit, see Figure 5) is calculated from the relationship:

$$I_0 = \frac{U_{ld} - V_{da}}{R_0}$$

The sign „-“ between the components of the torque $M_I$ (13) is caused by the over-excited operation (with the current fundamentals leading the counter-electromotive forces), that means with a capacitive shift (see Figure 3) of the self-piloted synchronous motor.

The sole problem of self-piloted synchronous motor is the starting regime. At low rotational speeds, under 5% of $\Omega_{\text{max}}$, the counter-electromotive force (-e_{0p}) becomes unable to ensure the commutation of the thyristors of the current inverter (CI). In this speed range it is applied the assisted commutation, realized by the device AC (see Figure 1 and Figure 2) through ingenious company solutions.

4.5 Considering the Non-Sinusoidal Currents

In very deed, the phase currents $i_a$, $i_b$, $i_c$ are strongly distorted as against the ideal sinusoidal shape. Actually, under the simplifying assumptions adopted, each phase current ($i_a$, $i_b$ or $i_c$ from Figure 6) consists of „rectangular blocks“ of amplitude $\pm I_0$ (and duration equal to $2\pi/3$) separated by intervals (by duration $\pi/3$) in which the currents are null.

Moreover, the fundamentals of phase currents (denoted by $i_{a1}$, $i_{b1}$ and $i_{c1}$ in Figure 6) have equal amplitudes (of magnitude $\frac{2\sqrt{3}}{3} I_0$) and they are shifted, one lagging another, with the same angle of $2\pi/3$, forming a symmetrical three-phase system of sinusoidal currents, by direct succession. One could notice that by $I_0$ is denoted the DC intensity from the DC intermediary circuit.

In the fixed stator referential $a\beta$, with the real axis Ox overlapped to reference stator winding axis “A” and with the temporal origin (ot=0) chosen at the moment when its current is passing through a maximum (when $i_a = -\frac{2\sqrt{3}}{3} I_0$), the space phasor $i_{\text{a}}$ of stator current fundamentals, calculated with the formula:

$$i_{\text{a}}(\omega t) = \frac{2}{3} \left( i_{a1} + a \cdot i_{b1} + a^2 \cdot i_{c1} \right) = \frac{\sqrt{6}}{\sqrt{3}} I_0 \cdot e^{\omega t} \quad (25)$$

is a rotating phasor (with the angular speed $\omega$=c$\omega$) with the top-point describing the circle of radius equal to $\frac{2\sqrt{3}}{3} I_0$ (represented by dashed line in Figure 7).

In the same time interval, still in the fixed stator referential $a\beta$, the space phasor $i_{\text{s}}$ (of non-sinusoidal currents) is completely different. For this reason, analytically it will proceed to an evaluation of this space phasor on each angular interval (by width $\pi/3$), in which the stator currents $i_a$, $i_b$ and $i_c$ remain constant. Thereby:

1. In the first interval $0 \leq \omega t \leq \pi/3$ when $i_a = +I_0$, $i_b = 0$ and $i_c = -I_0$ the space phasor $i_s$ is fixed (position „0“ in Figure 7) and has the expression:

$$i_s(\omega t) = \frac{\sqrt{6}}{\sqrt{3}} I_0 \cdot e^{\omega t} \quad (26)$$

2. In the interval $(k+1)$ by width $\pi/3$, when $\omega t$ might be written as:

$$\omega t = k \cdot \frac{\pi}{3} + \omega t' \quad 0 \leq \omega t' < \frac{\pi}{3}; \quad k = 1, 2, 3... \quad (27)$$

the space phasor $i_s$ remains fixed (in position „k”, k=1,2,3… in Figure 7) on whole duration $0 \leq \omega t' < \pi/3$.

Broadly speaking, for any $\omega t$ by type (27), the expression of space phasor $i_s$ can be written as:

$$i_s(\omega t) = \frac{2}{\sqrt{3}} I_0 \cdot e^{k \frac{\pi}{3} + \omega t'} \quad \text{with} \quad k = 0, 1, 2, 3... \quad (28)$$
During each period $\omega T=2\pi$, the space phasor $i_s(\omega t)$ of non-sinusoidal currents will successively scan all 6 fixed positions (denoted by ‘0’, ‘1’, ‘2’, ‘3’, ‘4’, ‘5’ in Figure 7). At each position it will remain motionless for $t' = \frac{\pi}{3} \leq t' \leq \frac{\pi}{3} + \frac{\pi}{6}$, that time being numerically equal to the duration between two successive commutations of the bridge, then instantly “jumps” to the next position.

Geometrically, over the duration of each period $\omega T=2\pi$, the “jumping” space phasor $i_s(\omega t)$ performs a complete rotation, like the rotating phasor $I_s(\omega t)$ of current fundamentals. Accordingly, the virtual angular speed $\omega_{vir}$ (fictitious average) of “jumping” phasor of non-sinusoidal currents

$$\omega_{vir} = \frac{2\pi}{T} = 2\pi f = \omega$$

is numerically equal to the rotation angular speed $\omega$ of the rotating phasor of stator current fundamentals. Consequently, this common value $\omega$ of angular speeds will be used at changing the referential of space phasors.

In the coordinates (d,q) rotating with angular speed $\omega$, the space phasor $i_{ds}$ will become $i_{dq}$, while the space phasor $i_{qs}$ will become $i_{qs}$. With expression (25) for $i_s$, the relation of phasor $i_{ds}$ given by (29) gets the form:

$$i_{ds} = \sqrt{2} \left( -\frac{\sqrt{6}}{\pi} \right) I_0 \cdot e^{j\omega t} \cdot e^{-j\omega t} = \frac{2\sqrt{3}}{\pi} I_0 \cdot e^{-j\omega t}$$

Absolutely similar, for any $\omega t$ as the form:

$$i_{ds} = \sqrt{2} \left( \frac{\sqrt{6}}{\pi} \right) I_0 \cdot e^{j\omega t} \cdot e^{-j\omega t} + \frac{2\sqrt{3}}{\pi} I_0 \cdot e^{-j\omega t}$$

when, in fixed coordinates, the space phasor $i_q$ has the expression (28), it will result that, in synchronous rotating coordinates, the space phasor $i_{ds}$ (30) will be established by:

$$i_{ds} = \frac{2\sqrt{3}}{\sqrt{3}} I_0 \cdot e^{j\omega t} \cdot e^{-j\omega t} = \frac{2\sqrt{3}}{\pi} I_0 \cdot e^{-j\omega t}$$

If the initial position $\omega_0$ is chosen accordingly to relation (21), meaning $\omega_0 = -\left( \frac{\pi}{2} + \psi \right)$, then in rotating coordinates (d,q), the space phasors $i_{ds}$ and $i_{ds}$ become:

$$i_{ds} = \frac{2\sqrt{3}}{\pi} I_0 \cdot e^{j\omega t} \cdot e^{j\psi} = \frac{2\sqrt{3}}{\pi} I_0 \cdot e^{j\psi}$$

respectively:

$$i_{ds} = \frac{2\sqrt{3}}{\sqrt{3}} I_0 \cdot e^{j\omega t} \cdot e^{j\psi} = \frac{2\sqrt{3}}{\sqrt{3}} I_0 \cdot e^{j\psi}$$

Space phasors $i_{ds}$ and $i_{ds}$ are geometrically represented in Figure 8, in the interval $0 \leq \omega t < \frac{\pi}{3}$ (on the time duration $0 \leq t < \frac{\pi}{3} \cdot \frac{T}{6}$), meaning that exactly the duration $T/6$ between two successive commutations of the bridge.

Figure 8 emphasizes that during the time between two successive commutations $T/6$ the space phasor $i_{ds}$ rotates in opposite sense, with the angular speed $\omega = \omega_t$, its extremity describing the arc between the points 1 and 2 corresponding to an angle at the centre of $\pi/3$ electrical radians.

![Figure 8. Oscillations of phasor $i_{ds}$ around $i_{ds}$ in case of rectangular currents](image-url)

Actually, at the moment $t'=0$, the phasor $i_{ds}$ starts from initial position 1 (corresponding to angle $\psi + 30^\circ$) towards the final position 2 (corresponding to angle $\psi - 30^\circ$), reached after $t'=T/6$. At each commutation of the bridge, the space phasor $i_{ds}$ instantly returns from position 2 to position 1, then the oscillation around the fixed phasor $i_{ds}$ is repeated in an identical manner.

Accordingly, the electromagnetic torque $M_t$ developed by the self-piloted synchronous motor will oscillate.

In order to evaluate the electromagnetic torque $M_t$, will first be established the effective values $I_d(t')$ and $I_q(t')$ of the orthogonal components of phasor $I_{ds}$:

$$I_d = \sqrt{2} \left( I_d(t') + j \cdot I_q(t') \right) = j \cdot \frac{2}{\sqrt{3}} I_0 \cdot e^{j(\psi - \frac{\pi}{6} - \alpha^t)}$$

then, by identifying follows immediately:

$$I_d(t') = -\frac{\sqrt{3}}{\sqrt{3}} I_0 \cdot \sin(\psi + \frac{\pi}{6} - \alpha^t)$$

$$I_q(t') = \frac{\sqrt{3}}{\sqrt{3}} I_0 \cdot \cos(\psi + \frac{\pi}{6} - \alpha^t)$$
If in expression (10) of the electromagnetic torque \( M \) there are performed the substitutions \( I_d = I_d(t') \) and \( I_q = I_q(t') \) specified by (36), and additionally there are used the substitutions (13), then the torque \( M_t \) will get the form:

\[
M_t(t') = 3p \left[ A K_0 \omega \left( \frac{1}{L_0} \int_0^1 \left( L_0 - L_0 \sin 2y + \frac{\pi}{6} - \alpha t \right) \right) \right] \quad (37)
\]

This expression of the torque is valid on each interval \( 0 < \omega t' < \pi/3 \) between two successive commutations of the bridge. Overall, the torque \( M_t \) results as irregular.

The non-uniformity of electromagnetic torque \( M_t \) is caused by the variation of the orthogonal components \( I_d(t') \) and \( I_q(t') \) accordingly to (36), this variation being determined by the oscillation of space phasor \( i_{dq} \) around the fixed phasor \( i_{dq} \). On each period \( T \) (of non-sinusoidal variation of currents) the electromagnetic torque \( M_t \) performs 6 pulsations of the type described by relationship (37).

In order to attenuate pulsations of electromagnetic torque one is searching for ways to limit the oscillation amplitude of phasor \( i_{dq} \) (for instance between \( \psi+15^\circ \) and \( \psi-15^\circ \)). For this goal, the stator winding of synchronous motor is divided in two distinct three-phase windings (spatially shifted with \( \pi/6 \) electrical radians), each being separately supplied from a current inverter. If the two inverters provide current pulses shifted from each other by \( \pi/6 \) electric radians, then the torque \( M_t \) will be positive; hence the pulsation period of the electromagnetic torque equal to \( T/12 \). This results implicitly that the amplitude of torque pulsation will be less.

4.6 Intermediate Circuit Approach

Further we will focus again on the configuration in Fig.5 that is depicting the self-piloted synchronous motor powered by a current inverter with natural commutation

One could note that the assembly current inverter and synchronous motor (CI+SM) can be seen as a three-phase thyristorized bridge connected to the terminals of a three-phase synchronous machine (with \( \sqrt{3} U \) as the effective value of the voltage between phases).

In line with this idea, on the d.c. side of the three-phase bridge, between the terminals \( K \) and \( A \) (reverse arrow in Fig.5) will appear the unilateral voltage \( u_{dK}(\omega t) \) with the ideal average value \( V_{KA} = V_{dK0} \) of magnitude:

\[
V_{dK0} = \frac{3\sqrt{2}}{\pi} \cdot (\sqrt{3} \cdot U) \cdot \cos \alpha = 2.34 \cdot U \cdot \cos \alpha \quad (38)
\]

The ideal average value \( V_{dK0} \) of the rectified three-phase voltage corresponds to the ideal non-load operation, and does not take into consideration the voltage drops \( \Delta V_d \) that include the direct voltage drops on the bridge branches in conduction, the inductive voltage drops caused by the commutation phenomenon, and the ohmic voltage drops on the resistances of different parts of the circuit.

At the load operation the voltage drops \( \Delta V_d \) can be considered proportional to the direct current \( I_d \) so that the average voltage \( V_{KA} = V_d' \) (according to the sign rule of the generators) will be as the form:

\[
V_{KA} = V_{dK0} - \Delta V_d = (1 - \Delta V_d) \cdot 2.34 \cdot U \cdot \cos \alpha = (1 - \Delta V_d) \cdot V_{dK0} \quad (39)
\]

One could highlight that in previous relation by \( \Delta V_d \) have been denoted the total voltage drops, in which the most important is the inductive voltage drop \( \Delta V_{d0} \) given by the expression:

\[
\Delta V_{d0} = \frac{3}{\pi} (\omega L_0) \cdot I_{dK} \quad (40)
\]

We must note that in Fig.5 between the terminals \( K \) and \( A \) is indicated the voltage \( V_{KA} = V_{dK} \) (according to the sign rule of the generators), with the magnitude:

\[
V_{KA} = V_{dK} = (1 - \Delta V_d) \cdot 2.34 \cdot U \cdot \cos \alpha \quad (41)
\]

The results described above impose highlighting that:

- at operation in traction regime (when the three-phase bridge operates as an inverter) the control angle \( \alpha \) has values in quadrant II, \( 90^\circ < \alpha < \alpha_{lim} \) (with \( \alpha_{lim} < 180^\circ \)), and the voltage \( V_{KA} = V_{dK} \) given by the relation (41) will be positive;
- under such circumstances, the balance equation of the voltages in the d.c. intermediate circuit (written with average quantities) will be:

\[
U_{ld} = V_{dK0} + R_0 \cdot I_{dK} \quad (42)
\]

Mathematically, equation (42) is of the form \( U = E + R \cdot I \). By identification of terms it results that \( V_{dK} \equiv E \), meaning the voltage \( V_{dK} \) (at the terminals \( A-K \) of assembly \( CI+SM \)) is acting like a counter electromotive force \( E \) in the circuit \( M-A-K-N \) of Fig.5. As a consequence, relative to the average components, the physical system in Fig.5 (composed by the intermediate circuit + the three-phase thyristorised bridge + synchronous motor) behaves just like a d.c. motor with separate excitation. This electric motor has the counter-electromotive force \( E \equiv V_{dK} \) and the internal resistance \( R = R_0 \). Accordingly, in the system depicted in Fig.5 the current inverter (CI) with the position transducer (PT) perform the functions of the mechanical collector of the d.c. machine.

In the same context, the smoothing coil of the intermediate circuit (with the inductance \( L_0 \) of large size) acts to withhold the variable components of the
supply voltage of the system in Fig.5, so that to the terminals M-N to remain solely the direct voltage U_{id}.

The purpose of the intermediate circuit powering consists in maintaining the constant magnitude of the direct current (I_0=ct.) at any value of the running speed. To this effect, the supply voltage U_{id} of the intermediate circuit must be adjusted depending on the speed, according to the law U_{id}=f(v), which is graphically represented in Fig.9 by the diagram a-b-c-d. This diagram is built on the basis of relations (42), (41) and the variation law of the synchronous motor phase voltage U=U(Ω).

![Diagram](image)

Fig.9 Diagrams U_{id}=f(v) and I_0=f(v) of intermediate circuit

Actually, at v=0 the voltage U_{id} has the minimum value R_0 I_0, then it must be linearly increased with the speed according to the straight line a-b-c. At the rated speed v_N the intermediate circuit supply voltage will reach the maximum U_{id}=U_C=U_{idmax}. The subsequent increase of speed (at I_0=ct.) might be obtained solely by the weakening of main flux of synchronous motor Φ_{0(IE)}. One could note that this weakening is obtained along with the decrease of excitation current I_E, while the intermediate circuit remains powered at maximum voltage U_{id}=U_{idmax}=ct.

One could highlight some explanatory notes.

The first ascertainment emphasizes that, in case of electric railway vehicles powered from a single-phase industrial frequency a.c. contact line, the network converter is composed of two single-phase rectifier bridges (a fully controlled bridge and a bridge half-controlled) connected in series and sequentially controlled in terms of speed, as below:
- at very low speeds 0≤v≤v_m (see Fig.9) the intermediate circuit is powered by the fully controlled rectifier bridge, and the current inverter (CI) will have assisted commutation;
- at low speeds (over v_m) when v_m<v≤v_b (see Fig.9) the current inverter is passing in operation with natural commutation. The thyristorised rectifier bridge begins to function as a mix bridge (in order to improve the power factor) and will be controlled to increase the voltage U_{id} up to b_1b=U_b=0.5 U_{idmax} at the speed v_b. At the speed v_b (equal to 0.5 v_N) the thyristorised rectifier bridge will be fully open;
- at speeds in the range v_b<v≤v_N occurs mixed rectifier bridge which (connected in series with the fully controlled bridge) will increase the voltage U_{id} from the value U_b to the maximum U_{idmax} (at the speed v_N);
- at higher speeds v_N<v≤v_M, the two rectifier bridges connected in series, fully opened, are powering the intermediate circuit with the voltage U_{id}=U_{idmax}=ct.

The second statement focuses on the electric braking regime. The overexcited synchronous machine will operate as autonomous synchronous generator, and the three phase bridge CI (at the same current direction through the thyristors) will now operate as controlled three phase rectifier with α≈0 (meaning, nearly to a three-phase diode bridge).

The voltage V_{AK}=V_{da}≈0 (results negative for α≈0) so that the electric power P_{AK} (changed through the terminals A and K) and calculated with:

\[ P_{AK} = V_{da} \cdot I_0 < 0 \]  

will be negative. That power will be provided to the intermediate circuit in which it should be consumed by dissipation on the braking resistances R_F.

Moreover, one could note that operation in electric braking regime of the self-piloted synchronous machine is completely independent of the presence of contact line voltage, which ensures an efficiency of 100%.

5 Conclusion

The sustainability dynamics in operation of electric railway vehicles has been addressed, highlighting the chain of interactions within the main electric equipment on an electrically driven railway system supplied from a d.c. or an a.c. contact line: the contact line–side converter, the machine–side converter and the traction synchronous motor.

The paper supports the findings that electric traction drive systems using synchronous motors fed by current inverters enhance the sustainable operation of railway trains. Electric traction drive systems using synchronous motors fed by current inverters provide high performance for high speed electric trains, in terms of both train dynamics and environmental issues, while advanced power current inverters ensure optimum traction and minimum energy use. Hence, the paper highlights that in order to synchronize the machine rotating field with the rotor (and avoiding the risk of losing the synchronism at any frequency value of stator currents) the synchronous motor will control the
current inverter which is powering it, becoming this way a self-piloted synchronous motor. Actually, the position of direct axis of synchronous motor rotor will be evolutionary checked in time (through the position transducers) by 6·p times during each complete rotation (at any value of rotor speed), by the ignition control of the thyristor of the current inverter bridge that is supplying the motor.

The physical phenomena within the electric system composed by the contact line-side converter, the machine-side converter and the traction synchronous motor are complex and its overall behaviour suggests that cleaner production offers an appropriate framework for assessing the sustainability of an electric railway system. Hence, the paper emphasizes that at electrically driven railway systems with traction synchronous motors the functions of amplitude adjustment are completely separate from those of the frequency control. As sequel, the result is a simpler construction for both the power part and the control electronics of static converters.

References:


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