Maintenance Scheduling of Thermal Power Units in Electricity Market

ADNAN MUHAREMOVIC
Independent System Operator in Bosnia and Herzegovina
Hamdije Cemerlica 2, Sarajevo
BOSNIA AND HERZEGOVINA
a.muharemovic@nosbih.ba

SMAJO BISANOVIC
Public Enterprise Elektroprivreda of Bosnia and Herzegovina
Vilsonovo setaliste 15, Sarajevo
BOSNIA AND HERZEGOVINA
s.bisanovic@elektroprivreda.ba

MENSUR HAJRO
Faculty of Electrical Engineering
University of Sarajevo
Zmaja od Bosne bb, Sarajevo
BOSNIA AND HERZEGOVINA
mensur.hajro@etf.unsa.ba

Abstract: – The objective of this paper is to present a decision-based function to determine the optimal maintenance scheduling strategy of thermal power units taking into account the particular obligations of Generation Company, such as bilateral contracts. Opportunities for energy selling at the electricity market as well as a detailed modeling of the power plants are considered in the optimization problem. The deterministic constrained combinatorial optimization problem that considers maximization of profit is solved using the interior point cutting plane method. This method possesses the advantages of both, the interior point method and the cutting plane method, and becomes very promising approach for the large-scale discrete optimization problem. Furthermore, in order to improve the robustness and efficiency of this method, a new general optimal base identification method is developed and introduced to deal with various types of optimal solutions. The implementation and performances of proposed solution technique are presented. The effectiveness of the approach is tested on the realistic size case study, and numerical results are demonstrated and discussed.

Key-Words: – Maintenance scheduling; electricity market; bilateral contracts; market prices, interior point cutting plane method, optimal base identification method.

1 Introduction

The maintenance activities for Generation Company (GenCo) present one of important tasks that have significant reflections on its profit and efficiency. It is particularly emphasized in liberalized ambient where GenCos are faced with numerous challenges with respect to ensuring reliable electricity supply at profit-efective rate. One of these challenges concerns the planned preventative maintenance of company's power generating units. Maintenance scheduling as critical technical task requires carefully planning and analysis to guarantee system reliability and economic benefits for the GenCo. Because all power units must be maintained and inspected, the planners in GenCo must schedule planned outages during the year. Several factors entering into this scheduling analysis includes: weekly (or daily) power profile (bilateral contracts), market prices, amount of maintenance to be done on all power units, capacity of units, availability of maintenance crews, elapsed time from the last maintenance activities, technological restrictions and season limits, obligations toward System Operator (SO) regarding to ancillary services. All these factors must be included into GenCo’s objective for profit maximization.

The maintenance scheduling is hard, complex combinatorial optimization problem that has been studied widely in past. Traditional optimization techniques such as integer programming [1,2], decomposition methods [2,3,4], goal programming [5] have been used to solve this problem. Modern evolutionary techniques, as genetic algorithm [6,7], simulated annealing [7,8], memetic algorithm [9], tabu search [7,10,11] and fuzzy sets theory [12,13] have been applied to the problem. The maintenance scheduling of thermal power units should be optimized in terms of the objective function under series of constraints. The selection of objectives and constraints depends on the particular needs of maintenance scheduling problem, the data available, an accuracy to be sought and chosen methodology for solving this problem. There are generally two categories of objectives in maintenance scheduling problem: based on costs [1,3,4,7,14] or on profit [11,15,16,17] and based on reliability [8,10,12]. The most common objective based on costs is to minimize the total operating costs over the planning period (horizon). This minimization often requires many approximations or computationally intensive simulation to yield a solution. It was reported in literature that minimization of the total operating costs (or production costs that is the main part of the operating costs for thermal units) is an insensitive the objective for maintenance scheduling problem [6,8,14]. A number of reliability indices, such as expected lack (shortage) of reserve, expected energy not supplied and loss of load probability, which are based on power system measures has been used as reliability criteria for the formulation of objective function [8,10,14]. The maintenance timetable should satisfy set of constraints related to power units (maintenance window constraints), prevent the simultaneous maintenance of set of units (exclusion constraints), restrictions the start of maintenance on some units after period of maintenance of other units (sequence constraints), system constraints (balance constraints, transmission constraints), crew constraints, etc.

In recent literature the maintenance scheduling problem has been oriented toward new relations in electric power sector. In a number of electricity markets, deregulation of the power industry has given GenCos the independence to maintain power units in decentralized manner with a minimum regulatory intervention for system security purposes only. The maintenance periods of time for power units are scheduled either by profit-seeking GenCos only, or by coordination between profit-seeking GenCos and reliability-concerned SO, and the extent of coordination depends on the market environment and actual legislative. Although the coordination procedure how SO adjust individual GenCos' maintenance schedules and how each GenCo responds to adjusted schedule is important, it is not a main concern of this paper and one can investigate more about this subject. An applicable procedure that conciliates objective for GenCos, to schedule their units for maintenance in order to maximize their profit, and SO requirement that ensures adequate security throughout the weeks of year, is determined through multiple interaction between GenCos and SO and given in [15,16]. In this paper the maintenance scheduling problem is analyzed from the GenCo point of view. In order to ensure adequate level of security, in this paper we assume simple interaction of the SO toward the GenCo taking into account minimal level of reserve requirement. This requirement can be a part of SO's total policy, contained in its plan of ancillary services. For minimal level of reserve GenCo will have benefits through price of capacity in reserve (this revenue is not analyzed in this paper).

The maintenance scheduling is an active research area in power system optimization. The complexity introduced by planning concepts such as multiple and contradictory objectives, associated with the combinatorial nature of the problem, lead to the perception of limitation of traditional methods. Rounding off based method has quick computation speed, but it significantly degrades optimality and may be impossible to obtain a feasible solution. Standard integer (binary) programming methods, as branch and bound algorithm, are non-polynomial. Consequently, it is slow and intractable for large-scale problems. Heuristic algorithms, as genetic algorithms, simulated annealing, tabu search and fuzzy sets theory, are very computationally time-consuming provided that the size of the search space is huge.

In recent years, interior point method (IPM) has been widely used for solving optimization problems in electric power system, for its fast convergence characteristics and dealing with inequalities conveniently. Generally, IPM deals with problems in which all variables are continuous. Interior point cutting plane method (IPCPM) appeared as a powerful tool to deal with mixed integer programming that enlarges the application area of IPM. In 1992, IPCPM was proposed by Mitchell and Todd to solve the perfect match problem [18]. Mitchell and Borchers solve linear ordering problems by IPCPM [19]. The comparison with simplex cutting plane method (SCPM) [20,21,22] shows that it has remarkable advantages as problem size increases. Ding, et al. use IPCPM to solve large-scale, discrete and nonlinear mixed integer optimal power flow problems [23,24].

If the optimal solution of the relaxation problem solved by IPM is a degenerate solution or convex combination solution, the cutting planes will fail to be generated, and the IPCPM will fail. A new base identification method is presented. The improved algorithm can find optimal base for various types of optimal solutions. Efficiency of base identification procedure is improved. Large computation time may be consumed in matrix rank calculation and row-column transformation. The perturbation method and some linear algebra techniques are introduced to IPCPM that can significantly improve computation efficiency.

The focus of this paper is development of comprehensive model for maintenance scheduling strategy of thermal power units taking into account the particular obligations of the GenCo from long-term bilateral contracts, as well as determination of power profile for selling on electricity market based on forecasted prices. The main contributions of this paper are as follows:

1. presentation of the approach that is flexible and robust to be used in the maintenance scheduling of thermal power units;
2. development of a hybrid model that combines energy sales through bilateral contracts and energy sales on the market for the maximization of GenCo's profit with smaller uncertainty from market price volatility;
3. application of the interior point cutting plane method for mixed-integer linear programming approach that guarantees convergence to the optimal solution and computational efficiency in large-scale case studies.

This paper is organized as follows. Section 2 provides the notation used throughout the paper. In Section 3 optimal maintenance scheduling problem is modeled as deterministic programming problem. Section 4 details the principle of IPCPM and its application for the problem. In Section 5 results from a realistic size case study are presented and discussed. Section 6 states all of the conclusions of this paper.

2 Notation

The notation used throughout the paper is stated below:

Indexes:

- i thermal unit index
- k thermal power plant index
- m bilateral contract index
- t time period (week) index

Constants:

- θ number of hours in week ($\theta = 168$)
- $\pi_m^c(t)$ price of bilateral contract *m* in period t [$/MWh]
- $\pi^s(t)$ market price of energy in period t [$/MWh]
- $a_{i,k}$ fixed operating cost of unit i in plant k [$/h]
- $b_{i,k}$ linear cost term in cost characteristic of unit i in plant k [$/MWh]
- $c_{i,k}$ quadratic cost term in cost characteristic of unit i in plant k [$/MW^{2}h]
- $d_{i,k}$ variable O&M cost of unit i in plant k [$/MWh]
- $C_{i,k}^M$ maintenance cost of unit i in plant k [$/MW]
- $ET_{i,k}$ earliest maintenance start of unit i in plant k
- $LT_{i,k}$ latest maintenance start of unit i in plant k
- $M_{i,k}$ duration of maintenance for unit i in plant k
- N_k number of units in plant k that can be maintained simultaneously
- $O_{a,b}$ number of periods during that maintenance of units a and b should overlap
- $p_m^c(t)$ power from bilateral contract *m* in period t [MW]
- $\overline{P}_{i,k}$ capacity of unit i in plant k [MW]
- $\underline{P}_{i,k}$ minimum output of unit i in plant k [MW]
- $R_0(t)$ minimum reserve level assigned to GenCo from the SO in period t [MW]
- $S_{a,b}$ number of periods required between the end of maintenance of unit a and the beginning of maintenance of unit b

Variables:

- $P_{i,k}(t)$ power generated by unit i in plant k in period t [MW]
- $p^s(t)$ power for bid on market in period t [MW]
- $v_{i,k}(t)$ 0/1 variable, equal to one if unit i in plant k is online in period t, otherwise zero
- $y_{i,k}(t)$ 0/1 variable, equal to one if unit i in plant k is being maintained in period t, otherwise zero

Numbers:

- I_k number of thermal units in plant k
- K number of thermal power plants
- M number of bilateral contracts
- T number of periods of the planning horizon.

3 Problem Formulation

3.1 Objective function

Important point in maintenance scheduling of thermal power units presents selection of objective function. It depends of long-term GenCo's strategic parameters, its obligations toward SO, regulatory agreements and etc. Because of that, maintenance scheduling is essentially multi-objective task with conflicting objectives. In this paper the objective is to maximize profit for the GenCo. The expected profit for the GenCo is calculated as a difference between expected revenues and operating costs. Operating costs include costs of energy production and maintenance costs. The bilateral sales contracts with particular energy patterns and price profiles are included in this objective. Also, in objective (1) the market clearing prices for each period are known.

The objective function for GenCo is expressed as profit maximization and formulated as follows:

$$\max \sum_{t=1}^{T} \left\{ \theta \sum_{m=1}^{M} \pi_m^c(t) p_m^c(t) + \theta \pi^s(t) p^s(t) - C_o(t) \right\} \tag{1}$$

$$\begin{aligned} C_o(t) = \theta \sum_{k=1}^{K} \sum_{i=1}^{Ik} a_{i,k} v_{i,k}(t) + b_{i,k} P_{i,k}(t) + c_{i,k} P_{i,k}^2(t) + \\ + \theta \sum_{k=1}^{K} \sum_{i=1}^{Ik} d_{i,k} P_{i,k}(t) + \sum_{k=1}^{K} \sum_{i=1}^{Ik} C_{i,k}^M \overline{P}_{i,k} y_{i,k}(t) \end{aligned} \tag{2}$$

In equation (1) the first term is related to revenue from bilateral contracts between the GenCo and other market players (load serving entities, traders, distribution companies). The amount of power that the GenCo has agreed to serve in period t as result of bilateral contract *m* is $p_m^c(t)$ and the price that the GenCo will be paid is $\pi_m^c(t)$. With this contract, the GenCo's revenue increases for $\pi_m^c(t) p_m^c(t)$. The second term represents expected revenue from selling power $p^s(t)$ on market with forecasted price $\pi^s(t)$ in period t. The third term represents the total costs $C_o(t)$ consist of production costs (fuel costs) $FC_{i,k}(t)$, variable O&M costs $d_{i,k}$ and maintenance costs $C_{i,k}^M$, as stated in (2). The fuel costs are represented by quadratic function:

$$FC_{i,k}(t) = a_{i,k} + b_{i,k} P_{i,k}(t) + c_{i,k} P_{i,k}^2(t) \tag{3}$$

In order to use *state-of-the-art* techniques for linear programming, quadratic fuel cost functions are modelled by by piecewise linear approximation [25]. This ensures formulation of problem as mixed-integer linear programming model that can be solved faster than original mixed-integer nonlinear programming model. The representation of this approximation is stated in Section 5.

3.2 Bilateral contracts and energy for market

In the newly restructured electricity market, the GenCo and other market players (load serving entities, distribution companies) can sign long-term bilateral contracts to cover players needs, which are derived from the demand of their customers. These bilateral contracts cover the real physical delivery of electrical energy. The actors agree on different prices, quantities, or different qualities of electrical energy. Also, duration of the contracts may differ, from medium-term (weekly, monthly) to long-term (yearly, few years). How much of their capacity and demand GenCo and players will contract through bilateral contracts, and how much they will leave open for market transactions, is their strategic and fundamental question. Basically, their reasons for contracting bilateral contracts are follows. Because of price volatility, market power risk and possible constraints in transmission network, the GenCo will estimate how much of its capacity will be contracted through bilateral contracts, and how much of capacity will be offered on the market. Bilateral contracts reduce risk for the GenCo because its capacities may go unused as a result of not finding buyers or transportation capacity on the market. Also, load-serving entities, distribution companies, as other party in bilateral contracts with the GenCo, face with risk on the market because of price volatility. Additionally, for large consumers whose load needs high reliable electric energy, the bilateral contracts give guarantee that their load will be always supplied. The bilateral contracts define that certain amount of energy during number of hours will be delivered at given time in the future, at agreed prices and at defined locations. The GenCo must take these bilateral contracts into consideration when scheduling its units [25].

Usually, bilateral contracts have a discrete power pattern during certain number of periods as well as corresponding price pattern. Power $p_m^c(t)$ and price $\pi_m^c(t)$ in period *t* are constant. This implies that revenue from all bilateral contracts is constant. According to forecasted weekly prices on market, GenCo has possibility to sell a part of its remaining production on the market. Level of power for bid on the market in period t, $p^s(t)$, depends of market price in period t, $\pi^s(t)$. The revenue from selling power on the market is $\pi^s(t) p^s(t)$. The variables $p^s(t)$ are optimization variables.

Prices and power quantities relevant for the bilateral contract can be obtained by systematic negotiation scheme [26] throughout the GenCo and its contract partners can reach a mutually benefit and tolerable risk. Negotiation for prices and power quantities will converge only if both sides can find price mix that provides an acceptable compromise between the risks and benefit (usually, part of the portfolio management).

3.3 Maintenance constraints

The following relations represent set of constraints that must be satisfied in maintenance scheduling problem. Also, minimal request on reserve level determined by the SO is here taken in consideration as obligation for the GenCo.

a) Minimum and maximum power output: The power output for each online unit must be within declared range represented by its minimum and maximum power output:

$$\underline{P}_{i,k} v_{i,k}(t) \le P_{i,k}(t) \le \overline{P}_{i,k} v_{i,k}(t) \quad \forall i,\ \forall k,\ \forall t \quad (4)$$

The unit cannot be online if it is in maintenance that ensured by constraint:

$$v_{i,k}(t) + y_{i,k}(t) \le 1 \quad \forall i,\ \forall k,\ \forall t \quad (5)$$

If the unit undergo maintenance in period t, $y_{i,k}(t) = 1$, constraint (5) ensures that $v_{i,k}(t) = 0$, because of that constraint (4) ensures the output of the unit is set to zero during maintenance. The power output of the unit can be equal to zero if the unit is not online and is not undergo maintenance.

b) Contracted arrangements and power for market: The total power generated in thermal units must be enough to covers the contracted load patterns and power determined for the market for each period:

$$\sum_{k=1}^{K} \sum_{i=1}^{I_k} P_{i,k}(t) = \sum_{m=1}^{M} p_m^c(t) + p^s(t) \quad \forall t \quad (6)$$

c) Requirement on minimum of reserve: Available capacity of units must satisfied requirement on minimal level of reserve imposed by the SO for each period:

$$\sum_{k=1}^{K} \sum_{i=1}^{I_k} \overline{P}_{i,k}(1 - y_{i,k}(t)) - \sum_{m=1}^{M} p_m^c(t) - p^s(t) \ge R_0(t) \quad (7)$$

d) Maintenance duration: For each unit must be ensured the necessary number of time periods for its maintenance during the horizon. The constraint (8) ensures this request:

$$\sum_{t=1}^{T} y_{i,k}(t) = M_{i,k} \quad \forall i,\ \forall k \quad (8)$$

e) Continuous maintenance period: This constraint ensures that the maintenance for each unit must be finished once when begins:

$$y_{i,k}(t) - y_{i,k}(t-1) \le y_{i,k}(t + M_{i,k} - 1) \quad \forall i,\ \forall k,\ \forall t \quad (9)$$

f) Earliest and latest maintenance start time: Planner in the GenCo determines earliest and latest maintenance start time for each thermal power unit taking into consideration specific unit maintenance requirements, appropriate season limits (heating, working feasibility, crew availability). Suppose $T_{i,k} \subset T$ is the set of periods when maintenance unit i in plant k may start, so:

$$T_{i,k} = \left\{ t \in T :\ ET_{i,k} \le t \le LT_{i,k} \right\} \quad \forall i,\ \forall k \quad (10)$$

g) Number of units in the plant that can be maintain simultaneously: The next constraint limits the number of units in one plant that can be maintained at the same time:

$$\sum_{i=1}^{I_k} y_{i,k}(t) \le N_k(t) \quad \forall k,\ \forall t \quad (11)$$

h) Incompatible pairs of units: The requirement that some units cannot be maintained at the same time is easily stated by binary constraints (12). If units *a* and *b* (in the same plant or in other plants) cannot undergo maintenance during the same period, this is stated as follows:

$$y_{a,k}(t) + y_{b,k}(t) \le 1 \quad \forall t \quad (12)$$

i) Maintenance priority: If power unit *a* must be maintained before unit *b*, that following constraint must be satisfied:

$$\sum_{\tau=1}^{t} y_{a,k}(\tau - 1) \ge y_{b,k}(t) \qquad \forall t,\ \left\{ y_{a,k}(t) = 0, \text{for } (\tau - 1) \le 0 \right\} \quad (13)$$

j) Separation among consecutive maintenance outages: If between finish of maintenance of unit *a* and begin of maintenance of unit *b* (in the same plant or in different plants) is needed separation of $S_{a,b}$ periods, than following constraints must be satisfied [15]:

$$\sum_{\tau=1}^{t} y_{a,k}(\tau - M_{a,k} - S_{a,b}) \ge y_{b,k}(t) \quad \forall t \quad (14)$$

$$\sum_{\tau=1}^{t} M_{a,b,k}^{\min} y_{a,k}(\tau - M_{a,k} - S_{a,b}) \le \sum_{\tau=1}^{t} M_{a,b,k}^{\max} y_{b,k}(\tau)$$
$$\forall t,\ \left\{y_{a,k}(t) = 0, \text{for } (\tau - M_{a,k} - S_{a,b}) \le 0\right\} \quad (15)$$

$$M_{a,b,k}^{\min} = \min\left\{M_{a,k}, M_{b,k}\right\},\ M_{a,b,k}^{\max} = \max\left\{M_{a,k}, M_{b,k}\right\}.$$

k) Overlap in maintenance outages: If during period in that unit *a* finishes the maintenance before unit *b* and if duration the maintenance of unit *b* must overlap specified number of periods $O_{a,b}$, than following constraints must be satisfied [15]:

$$\sum_{\tau=1}^{t} y_{a,k}(\tau - M_{a,k} + O_{a,b}) \ge y_{b,k}(t) \quad \forall t \quad (16)$$

$$\sum_{\tau=1}^{t} M_{a,b,k}^{\min} y_{a,k}(\tau - M_{a,k} + O_{a,b}) \le \sum_{\tau=1}^{t} M_{a,b,k}^{\max} y_{b,k}(\tau)$$
$$\forall t,\ \left\{y_{a,k}(t) = 0, \text{for } (\tau - M_{a,k} + O_{a,b}) \le 0\right\} \quad (17)$$

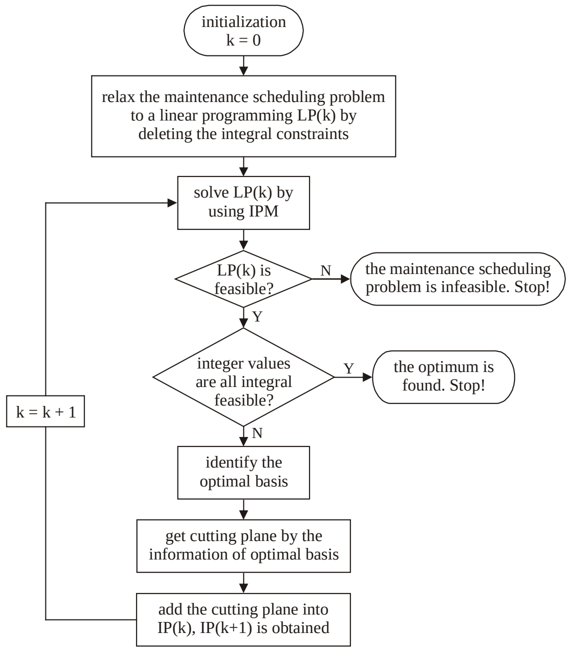
Unit *a* and unit *b* can be in the same power plant, or in different plants. If is $O_{a,b} = M_{b,k}$, that unit *a* and unit *b* finish maintenance simultaneously.

4 Methodology and Algorithm

4.1 IPCPM principle and its implementation

In traditional cutting plane method (CPM), the linear programming relaxations have been solved using the simplex method. Simplex method has an exponential-time characteristic, which restricts its real-world application. Contrarily, IPM searches optimum inside the feasible region, its iteration numbers do not obviously change as the scale of system increases, so it is superior to the simplex in convergence and calculation speed [27]. Generally, cutting plane method for (0/1) mixed-integer linear programming requires solving a large number of linear programming relaxations, so it is obvious that replacing simplex algorithm with IPM will improve calculation efficiency. Based on this idea, IPCPM in many theoretical studies and practical applications were shown as very promising tool for solving large-scale discrete optimal problems.

The main computational principle of IPCPM and its implementation for the maintenance scheduling problem is given in Fig. 1. Obviously, two points are very important for IPCPM success. Firstly, how to generate cutting plane without simplex tableau? It is necessary to modify the classical techniques for generating cutting planes from the optimal tableau [23]. Secondly, how to identify the base variables in IPCPM? The base identification in IPCPM is as important as simplex tableau generation in SCPM. In [23] shown how is obtained base information from the matrix $\mathbf{DA^T}(\mathbf{AD}^2\mathbf{A^T})^{-1}\mathbf{AD}$ under non-degenerate hypothesizes. Unfortunately, most of linear programming problems are degenerate and many problems have multiple optimums, which limit the IPCPM applications.



ig. 1: The calculation flowchart of IPCPM for the maintenance scheduling problem.

4.2 Failure reason analysis

Cutting plane can be generated once the optimal base is obtained. For traditional SCPM, the optimal solution converges to the vertex point of convex hull. Optimal base can be obtained based on the simplex tableau and then the cutting planes are generated. However, for IPCPM, if relaxed linear program is degenerate or has multiple solutions, the base cannot be correctly identified. There are two cases in that the optimal base cannot be identified: (i) the optimal solution is degenerate and (ii) the linear relaxation programming has multiple optimal solutions.

Assume that the linear programming problem has the standard form: $\min\{\mathbf{c^T x} : \mathbf{Ax} = \mathbf{b}, \mathbf{x} \ge 0\}$, and its dual problem form: $\max\{\mathbf{b^T y} : \mathbf{A^T y} + \mathbf{s} = \mathbf{c}, \mathbf{s} \ge 0\}$, where is $(\mathbf{c}, \mathbf{x}, \mathbf{s}) \in R^n$, $(\mathbf{b}, \mathbf{y}) \in R^m$, $\mathbf{A} \in R^{m \times n}$, and $\mathbf{A}$ is assumed to have full row rank.

A simple problem is used to shown the failure reason of IPCPM caused by the above two cases:

$$\max\ 2x_1 + 4x_2 \quad \text{(a)}$$
$$\text{s.t. } x_1 + 2x_2 + x_3 = 8 \quad \text{(b)}$$
$$x_1 + x_4 = 8 \quad \text{(c)}$$
$$x_2 + x_5 = 3 \quad \text{(d)}$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

The problem (a) – (d) is equivalent to:

$$\max\ 2x_1 + 4x_2 \quad \text{(e)}$$
$$\text{s.t. } x_1 + 2x_2 \leq 8 \quad \text{(f)}$$
$$x_1 \leq 8 \quad \text{(g)}$$
$$x_2 \leq 3 \quad \text{(h)}$$
$$x_1, x_2 \geq 0$$

It is a multiple solution for linear programming problem. There are three types of solutions:

1) non-degenerate solution x'=(2,3,0,6,0), number of nonzero elements is equal to 3 (it equals to number of equality constraints);
2) degenerate solution x''=(8,0,0,0,3), number of nonzero elements is less than 3;
3) convex combination solution $x''' = \alpha x' + (1-\alpha)x''$ and for $\alpha \in (0,1)$, e.g. x'''=(4.5,1.75,0,3.5,1.25) when α=7/12, the amount of nonzero elements is greater than 3.

The geometric meanings of the above optimal solution types are shown in Fig. 2. Line AC represents (h); line CD represents (g); line BD represents (f); the region area (convex polytope) enclosed by ABD0 is the feasible region constructed by constraints (f), (g) and (h). Because the line BD parallels the line represented by objective function, any point on line BD is the optimal solution.

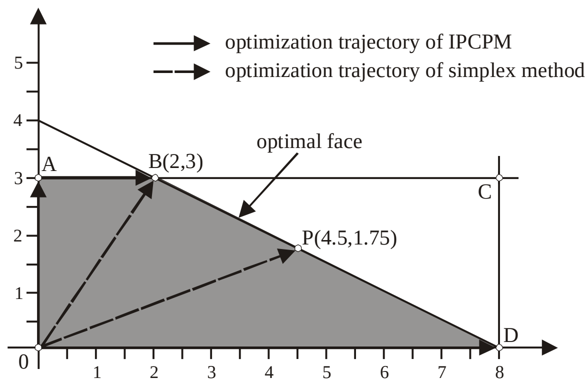


Fig. 2: Schematic diagram of optimal solutions.

As the simplex method searches for optimal solutions through vertices, the solution is definitely vertex point of the optimal face. The vertex solution is corresponding to the non-degenerate solution (point B) or degenerate solution (point D) shown in Fig. 2. Gomory cutting planes can be generated from the final simplex tableau. Obviously, a simplex tableau is not available when the interior point method is used. Interior point method searches for optimal solution through the interior of feasible region, and all three types of solutions may be obtained. As shown in Fig. 2, the optimal solution is more likely to converge to any point pertaining to line BD. In other words, the optimum is convex combination solution. In this situation, IPCPM would fail as the optimal base cannot be identified.

4.3 The improvement of IPCPM

An optimal base identification method is developed [28] and its procedure can be described as in Fig. 3.

The index set $\{1,\ldots,n\}$ is partitioned into subsets $\Im_1$, $\Im_2$ and $\Im_3$, and defined as follows:

$\Im_1 = \{j \mid x_j > 0, s_j = 0\}$, $\Im_2 = \{j \mid x_j = 0, s_j = 0\}$,
$\Im_3 = \{j \mid x_j = 0, s_j > 0\}$, (s_j – dual slack variables).

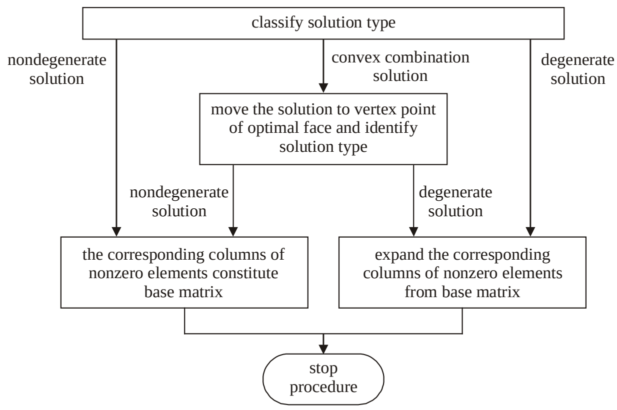


Fig. 3: Diagram of optimal base identification.

The columns of matrix **A** and elements of vector **c** are classified as three parts:

$$\mathbf{A}_1 = \{A_{*j} \mid j \in \Im_1\},\ \mathbf{c}_1 = \{c_j \mid j \in \Im_1\}$$
$$\mathbf{A}_2 = \{A_{*j} \mid j \in \Im_2\},\ \mathbf{c}_2 = \{c_j \mid j \in \Im_2\}$$
$$\mathbf{A}_3 = \{A_{*j} \mid j \in \Im_3\},\ \mathbf{c}_3 = \{c_j \mid j \in \Im_3\}$$

where $\mathbf{A_1}$, $\mathbf{A_2}$ and $\mathbf{A_3}$ are composed of the corresponding columns of index sets $\Im_1$, $\Im_2$ and $\Im_3$

of matrix **A** separately; $\mathbf{c}_1$, $\mathbf{c}_2$ and $\mathbf{c}_3$ are composed of the corresponding elements of index sets $\Im_1$, $\Im_2$ and $\Im_3$ of vector **c** separately.

The procedure is as follows:

① Classify the type of solution: non-degenerate solution, degenerate solution or convex combination solution. The index sets $\Im_1$, $\Im_2$ and $\Im_3$ are obtained:

i) if it is a non-degenerate solution, the index set of nonzero elements is the index set of optimal base, let $\mathbf{B} = \mathbf{A}_1$, go to ⑩

ii) if it is a degenerate solution, let $\mathbf{B} = \mathbf{A}_1$, go to ⑥

iii) if it is convex combination solution, go to ②

②The movement of primal optimal solution:

i) solve equation $\mathbf{A}_1\mathbf{z} = 0$. As $\mathbf{A}_1$ is column linearly dependent, **z** may have multiple solutions. Choose one of nonzero vector **z**;

ii) solve $\mathbf{x}_1' = \mathbf{x}_1 + t\mathbf{z} \ge 0$, calculate the range of scalar t: $t_{\min} \le t \le t_{\max}$.

③Choose anyone of $t_{\min}$ and $t_{\max}$, let $\mathbf{x}_1' = \mathbf{x}_1 + t\mathbf{z}$. For certain $x_{1j}' = 0$ (x_{1j}' represents the j[th] element of $\mathbf{x}_1'$), x_{1j}' is removed from $\mathbf{x}_1'$ and added into $\mathbf{x}_2$, the column corresponding to x_{1j}' is removed from $\mathbf{A}_1$ and added into $\mathbf{A}_2$, let $\mathbf{x}_1 = \mathbf{x}_1'$, $\mathbf{x} = (\mathbf{x}_1', \mathbf{x}_2, \mathbf{x}_3)$.

④ If $\mathbf{A}_1$ is column dependent, go to ② else let $\mathbf{B} = \mathbf{A}_1$, go to next step.

⑤If $rank(\mathbf{B}) < rank([\mathbf{A}_1, \mathbf{A}_2])$ go to ⑧ else go to next step.

⑥ Add the columns of $\mathbf{A}_2$ which are linearly independent of $\mathbf{B}$ into $\mathbf{B}$.

⑦If $rank(\mathbf{B}) = m$, go to ⑩ else go to next step (*m* is number of linear independent constraints).

⑧The movement of dual optimal solution:

i) solve equation $\mathbf{B}^{\mathbf{T}}\mathbf{z} = 0$ and choose anyone of nonzero vector $\mathbf{z}$;

ii) according to $\mathbf{A}_3^{\mathbf{T}}\mathbf{y}' \le \mathbf{c}_3$, calculating the range of t: $t_{\min} \le t \le t_{\max}$.

⑨Choose anyone of $t_{\min}$ and $t_{\max}$, let $\mathbf{y}' = \mathbf{y} - t\mathbf{z}$ certainly *j* satisfies $a_{3j}^{\mathbf{T}}\mathbf{y}' = \mathbf{c}_{3j}$ ($\mathbf{a}_{3j}$ is j[th] column of $\mathbf{A}_3$, c_{3j} is j[th] element of $\mathbf{c}_3$), $\mathbf{a}_{3j}$ is removed from $\mathbf{A}_3$ and added into $\mathbf{A}_2$ and **B**, meanwhile **B** should be kept column linearly independent, go to ⑦

⑩Stop procedure, **B** is optimal base matrix.

5 Case Study

To illustrate the effectiveness of the proposed model we have presented an illustrative case study. The model has been implemented and solved with C++ language on PC based platform with GenuineIntel processor clocking at 3.20 GHz with 3 GB of RAM.

5.1 Input data

Input data from realistic case study are presented in this section. The GenCo generation system consists of five thermal power plants with total 20 power units. Table 1 and Table 2 show list of thermal units with its capacities, maintenance parameters, fuel cost coefficients, O&M and maintenance costs. The length of the planning horizon is 52 weeks, and maintenance schedule for each unit will occur just once during the planning horizon.

Table 1: The parameters of thermal power units

plant *k*	unit *i*	unit *no.*	Pmin	Pmax	*M*	*ET*	*LT*
TPP #1	1	1	265	310	5	23	32
	2	2	265	310	6	18	26
	3	3	120	220	6	1	20
	4	4	115	180	4	1	44
	5	5	65	90	4	1	44
TPP #2	1	6	100	155	3	1	50
	2	7	120	180	5	1	36
	3	8	120	180	5	1	36
TPP #3	1	9	360	450	7	24	38
	2	10	255	330	4	22	29
	3	11	215	270	4	18	42
	4	12	160	240	6	16	29
TPP #4	1	13	420	450	3	34	40
	2	14	265	320	5	20	28
	3	15	220	340	6	20	35
	4	16	210	255	5	1	45
TPP #5	1	17	155	230	5	1	45
	2	18	130	180	4	18	29
	3	19	120	160	5	12	40
	4	20	120	160	5	18	36

Table 2: Fuel cost coefficients, O&M costs and maintenance costs of thermal power units

plant	unit	*a*	*b*	*c*	*d*	C^M
TPP #1	1	90	9.64	0.0395	0.76	126
	2	90	9.64	0.0395	0.76	126
	3	122	11.04	0.0673	0.44	117
	4	104	12.60	0.0883	0.43	104
	5	130	16.02	0.0831	0.36	93
TPP #2	6	210	10.44	0.0639	0.30	104
	7	156	13.09	0.0761	0.34	104
	8	156	13.09	0.0761	0.34	104
TPP #3	9	555	4.77	0.0234	0.82	137
	10	287	7.34	0.0493	0.77	126
	11	135	9.93	0.0534	0.53	117
	12	255	11.04	0.0678	0.48	117
TPP #4	13	540	5.91	0.0263	0.79	137
	14	297	10.05	0.0471	0.72	126
	15	303	11.13	0.0398	0.81	117
	16	167	16.04	0.0477	0.69	117
TPP #5	17	136	12.18	0.0701	0.51	117
	18	144	14.01	0.0826	0.49	104
	19	202	13.89	0.0931	0.52	104
	20	202	13.89	0.0931	0.52	104

Using of piecewise linear approximation for quadratic fuel cost characteristics given in Eq. (3), complete model is then presented as mixed-integer linear programming (MILP) formulation of the maintenance scheduling problem that ensures an efficient solution using IPCPM. The piecewise linear approximation of cost characteristics (variable costs) is formulated as follows [25,29]:

$$VC_{i,k}(t) = \sum_{n=1}^{N} F_n(i,k) d_n(i,k,t), \quad \forall i,\ \forall k,\ \forall t$$

$$P_{i,k}(t) = \underline{P}_{i,k} v_{i,k}(t) + \sum_{n=1}^{N} d_n(i,k,t), \quad \forall i,\ \forall k,\ \forall t$$

$$0 \le d_n(i,k,t) \le d_n^s(i,k), \quad \forall i,\ \forall k,\ \forall t,\ n = 1,2,\ldots,N$$

where N is the number of blocks of the piecewise linear variable cost function, $F_n(i,k)$ represents the slope of block n of the variable cost of thermal unit i, $d_n(i,k,t)$ represents the power produced by unit i in period t using n^{th} power block, $d_n^s(i,k)$ is size of the n^{th} power block for unit i.

Variable costs have been modelled using the piecewise linear approximation with three blocks as shown in Table 3.

Table 3: Piecewise linear approximation of fuel cost characteristic of thermal power units

plant	unit	T_1 (MW)	T_2 (MW)	F_1 (\$/MWh)	F_2 (\$/MWh)	F_3 (\$/MWh)
TPP #1	1	280	295	31.168	32.353	33.538
	2	280	295	31.168	32.353	33.538
	3	153.3	186.7	29.435	33.922	38.409
	4	136.7	158.3	34.822	38.649	42.475
	5	73.3	81.7	27.516	28.901	30.286
TPP #2	6	118.3	136.7	24.392	26.735	29.078
	7	140	160	32.876	35.920	38.964
	8	140	160	32.876	35.920	38.964
TPP #3	9	390	420	22.320	23.724	25.128
	10	280	305	33.716	36.181	38.646
	11	233.3	251.7	33.871	35.829	37.787
	12	186.7	213.3	34.544	38.160	41.776
TPP #4	13	430	440	28.265	28.791	29.317
	14	283.3	301.7	35.877	37.603	39.330
	15	260	300	30.234	33.418	36.602
	16	225	240	36.790	38.221	39.652
TPP #5	17	180	205	35.664	39.169	42.674
	18	146.7	163.3	36.863	39.616	42.369
	19	133.3	146.7	37.475	39.958	42.441
	20	133.3	146.7	37.475	39.958	42.441

In Table 3 constants T_1 and T_2 mean upper limit of blocks 1 and 2 of thermal unit variable cost.

Table 4 shows forecasted weekly prices on the market. It should be noted that price profile should be obtained by appropriate forecasting procedures.

Table 4: Weekly forecasted market prices

t	$\pi^s(t)$	t	$\pi^s(t)$	t	$\pi^s(t)$	t	$\pi^s(t)$
1	44.44	14	37.84	27	40.59	40	36.19
2	45.87	15	37.62	28	40.26	41	34.87
3	44.99	16	41.58	29	42.68	42	36.63
4	44.44	17	36.74	30	41.47	43	41.58
5	47.08	18	41.47	31	34.76	44	47.96
6	44.11	19	46.09	32	36.08	45	47.85
7	46.09	20	45.65	33	37.84	46	52.36
8	42.9	21	42.46	34	35.97	47	53.13
9	41.14	22	46.86	35	34.65	48	51.26
10	39.93	23	46.75	36	35.64	49	54.89
11	40.15	24	49.17	37	38.83	50	58.63
12	39.16	25	47.08	38	34.87	51	66.22
13	40.37	26	43.01	39	33.66	52	56.32

The request on minimum reserve determined by the SO, that GenCo must satisfy as an obligation, assumed to be value of 250 (MW) for each week.

The GenCo has two yearly bilateral contracts, for example, with large consumers. First contract has power pattern that is constant during certain number of weeks with corresponding price pattern and second contract with constant power during year with fixed price. The contracted power profiles and prices are presented in Table 5.

Table 5: Bilateral contracts with power profile (MW) and prices profile (\$/MWh)

CONTRACT #1							
t	1-8	9-24	25-28	29-32	33-40	41-49	50-52
$p_1^c(t)$	2300	2150	2000	1700	1750	2200	2300
$\pi_1^c(t)$	45.5	38.8	38.0	38.4	36.9	42.5	52.9
CONTRACT #2: $p_2^c(t) = 1250$ (BASE LOAD); $\pi_2^c(t) = 45.8$							

The results of following test cases are analysed:

- **case #1**: only constraints (1) – (11);
- **case #2**: case #1 plus incompatible pairs of units (units 4 and 5 in TPP#1 and units 7 and 8 in TPP#2 cannot be maintained at the same time);
- **case #3**: case #2 plus maintenance priority (in TPP#3, unit 9 must be maintained before unit 13 in TPP#4);
- **case #4**: case #3 plus separation among consecutive maintenance outages (after finishing maintenance of unit 16 in TPP#4 and beginning maintenance of unit 20 in TPP#5, separation of 5 weeks is needed);

- **case #5:** case #4 plus overlap in maintenance outages (maintenance of unit 14 in TPP#4 must begin 3 weeks before unit 9 in TPP#3 finish its maintenance).

5.2 Test results and analysis

For specified test cases, Table 6 shows total costs, profit and total energy for market. Maximum profit and the biggest energy amount for the market are obtained in test case #1 that considers only basic constraints (4) – (11). The lowest costs and the smallest energy amount for the market are obtained in test case #5 characterized with the smallest value of profit compared with other cases. It can be seen from Table 6 how different set of constraints assigned to maintenance scheduling problem affects total costs of GenCo's generation system, and how affects its profit. In all cases, total energy from both bilateral contracts is equal 29,114,400 (MWh).

Table 6: The global results for different test cases – with bilateral contracts

test case	total costs ($)	profit ($)	total energy for market (MWh)
#1	872,372,944.5	677,634,841.3	7,110,600
#2	875,677,007.3	676,948,698.9	7,173,533
#3	873,235,053.5	676,893,648.6	7,109,390
#4	875,219,021.6	676,636,072.0	7,167,367
#5	866,444,939.5	673,087,691.8	6,916,560

It is illustrative to consider the influence of bilateral contracts on GenCo's profit for the above test cases. Table 7 shows total costs, profit, and the total generated energy for market without bilateral contracts. Introduction of bilateral contracts further complicates finding optimal maintenance schedule. However, always it ensures an additional profit for the GenCo. Furthermore, plant generation becomes far less sensitive to fluctuations of market prices, simply because bilateral contracts stipulate energy production and placement.

Table 7: The global results for different test cases – without bilateral contracts

test case	total costs ($)	profit ($)	total energy for market (MWh)
#1	865,154,473.0	581,259,694.5	35,987,717
#2	864,898,156.6	580,971,102.9	35,980,997
#3	865,154,473.0	581,259,694.5	35,987,717
#4	864,155,260.8	580,158,658.1	35,956,082
#5	864,580,583.7	576,363,503.8	35,972,597

Let's assume that market prices have deviations in the range of ±5%, as reason, for example, errors in forecast prices. The increment of this deviations is –1% (decrease prices), i.e. +1% (increase prices). The elements from two bilateral contracts remain the same and they are given in Table 5.

Next, we focus on test case #5. Table 8 shows profit for every incremental price change on the market, as well as profit deviation for a given price change. Results in Table 8 lead to the conclusion that existence of bilateral contracts play the role of shock absorber in the profit function, meaning that change of profit happens slowly than change of market prices. Essentially, profit deviations are attenuated when compared to the market prices changes. For instance, 5% price decrease causes only 2.04% decrease in the profit. The sensitivity is somewhat larger in the case of price increase, where 5% market price increase causes 2.25% increase in the profit.

Table 8: The profit deviations with changes in the market prices

market prices deviation	**–5%**	**–4%**	**–3%**	**–2%**	**–1%**	**0%**
profit (mil. $)	659.3	662.1	664.8	667.6	670.9	**673.1**
profit deviation (%)	–2.04	–1.64	–1.24	–0.82	–0.32	**0**

market prices deviation	**1%**	**2%**	**3%**	**4%**	**5%**
profit (mil. $)	676.2	678.9	682.0	684.8	688.2
profit deviation (%)	0.51	0.86	1.33	1.74	2.25

Obtained maintenance schedule of the thermal power units for test cases #1 and #5 can be seen in Tables 9 and 10.

Tables 9 and 10 show total production of thermal units $P^T(t)$, power for market $p^S(t)$, total reserve $R(t)$ and power in maintenance $P^M(t)$ for each week t. Shown in Tables 9 and 10, resulting schedule during the horizon satisfied all specified constraints and ensured profit maximization obtained by IPCPM described in Section 4. Schedules for test cases #2, #3 and #4 are obtained in similar manner, where given constraints are satisfied for each test case. In analyzed test cases maintenance constraints, weekly power profile from bilateral contracts and weekly forecasted market prices are dominant factors for high number of contemporaneous power plants in maintenance condition. The effect of constraints in the maintenance scheduling problem significantly impacts both total production, as well as the power offered to the market.

Table 9: The maintenance schedule for case #1

week	units in maintenance	$P^T(t)$ (MW)	$p^S(t)$ (MW)	R(t) (MW)	$P^M(t)$ (MW)
1	–	4760	1210	250	0
2	–	4760	1210	250	0
3	–	4760	1210	250	0
4	–	4760	1210	250	0
5	–	4760	1210	250	0
6	–	4760	1210	250	0
7	–	4760	1210	250	0
8	–	4705	1155	305	0
9	–	4512	1112	498	0
10	–	4433	1033	577	0
11	–	4452	1052	558	0
12	3	4182	782	608	220
13	3	4283	883	507	220
14	3,4,5	3885	485	635	490
15	3,4,5,6	3660	260	705	645
16	3,4,5,6,12	3812	412	313	885
17	3,4,5,6,12	3500	100	625	885
18	12	4327	927	443	240
19	12	4520	1120	250	240
20	12	4520	1120	250	240
21	12	4520	1120	250	240
22	–	4760	1360	250	0
23	–	4760	1360	250	0
24	–	4760	1360	250	0
25	–	4760	1510	250	0
26	2	4417	1167	283	310
27	2	4160	910	540	310
28	2,10,14	3597	347	453	960
29	2,10,14,18	3593	643	277	1140
30	2,10,14,18	3522	572	348	1140
31	1,2,10,14,18	2950	0	610	1450
32	1,8,14,15,18,20	2950	0	570	1490
33	1,8,15,20	3407	407	613	990
34	1,8,9,15,20	3000	0	570	1440
35	1,8,9,15,20	3000	0	570	1440
36	7,8,9,13,15,20	3000	0	250	1760
37	7,9,13,15	3028	28	562	1420
38	7,9,13,16,17,19	3000	0	285	1725
39	7,9,11,16,17,19	3000	0	465	1545
40	7,9,11,16,17,19	3000	0	465	1545
41	11,16,17,19	3450	0	645	915
42	11,16,17,19	3473	23	622	915
43	–	4567	1117	443	0
44	–	4760	1310	250	0
45	–	4760	1310	250	0
46	–	4760	1310	250	0
47	–	4760	1310	250	0
48	–	4760	1310	250	0
49	–	4760	1310	250	0
50	–	4760	1210	250	0
51	–	4760	1210	250	0
52	–	4760	1210	250	0

Table 10: The maintenance schedule for case #5

week	units in maintenance	$P^T(t)$ (MW)	$p^S(t)$ (MW)	R(t) (MW)	$P^M(t)$ (MW)
1	–	4760	1210	250	0
2	–	4760	1210	250	0
3	–	4760	1210	250	0
4	–	4760	1210	250	0
5	–	4760	1210	250	0
6	–	4760	1210	250	0
7	–	4760	1210	250	0
8	–	4705	1155	305	0
9	–	4512	1112	498	0
10	–	4433	1033	577	0
11	–	4452	1052	558	0
12	3	4182	782	608	220
13	3	4283	883	507	220
14	3	4090	690	700	220
15	3	4020	620	770	220
16	3	4380	980	410	220
17	3	4020	620	770	220
18	–	4567	1167	443	0
19	–	4760	1360	250	0
20	–	4760	1360	250	0
21	–	4630	1230	380	0
22	–	4760	1360	250	0
23	16	4505	1105	250	255
24	9,16	4055	655	250	705
25	9,16,18	3875	625	250	885
26	2,9,16,18	3565	315	250	1195
27	2,9,16,18	3370	120	445	1195
28	2,9,14,18	3297	47	453	1260
29	2,9,10,12,14	3077	127	283	1650
30	2,9,10,12,14	3032	82	328	1650
31	2,7,10,12,14	3008	58	622	1380
32	1,7,10,12,14,19	2950	0	520	1540
33	1,5,7,12,19,20	3337	337	533	1140
34	1,5,7,12,15,19,20	3000	0	530	1480
35	1,5,7,15,19,20	3098	98	672	1240
36	1,5,8,15,19,20	3113	113	657	1240
37	8,15,20	3808	808	522	680
38	4,8,13,15,17	3000	0	630	1380
39	4,8,11,13,15,17	3000	0	360	1650
40	4,6,8,11,13,17	3000	0	545	1465
41	4,6,11,17	3450	0	725	835
42	6,11,17	3648	198	707	655
43	–	4567	1117	443	0
44	–	4760	1310	250	0
45	–	4760	1310	250	0
46	–	4760	1310	250	0
47	–	4760	1310	250	0
48	–	4760	1310	250	0
49	–	4760	1310	250	0
50	–	4760	1210	250	0
51	–	4760	1210	250	0
52	–	4760	1210	250	0

Table 11: Model dimensions for case #5

# of binary variables	# of real variables	# of constraints
2288	3484	7208

Table 12: Evolution cutting planes through iteration progress for case #5

iteration ID	cutting plane	iteration ID	cutting plane	iteration ID	cutting plane
1	1145	6	148	11	25
2	986	7	97	12	14
3	759	8	66	13	6
4	431	9	43	14	3
5	257	10	38	15	0

Table 11 summarizes calculation dimensions for case #5. The presented maintenance scheduling problem has many binary and continuous variables that are typical for MILP. Because the complexity of the presented formulation, it cannot be solved by original IPCPM. In fact, the main reason is failing to find the cutting plane, as it is analyzed in section 4.2. The optimal solution is obtained by using improved IPCPM and number of cutting planes during 15 iterations is listed in Table 12.

6 Conclusion

In restructured power systems and liberalized market, maintenance scheduling problem has new characteristics different from those in traditional environment. In presented maintenance scheduling model, GenCo's interest is to maximize profit, combining energy sales through bilateral contracts and energy sales on market. GenCo is responsible for performing necessary maintenance of its power units in order to sustain its position on the market. The maintenance periods of time for power units are scheduled either by profit-seeking GenCos only, or by coordination between profit-seeking GenCos and reliability-concerned System Operator and extent of coordination depends on the market characteristics. Although the coordination procedure how System Operator adjust the individual GenCos' maintenance schedules and how each GenCo responds to the adjusted schedule is important, it is not a main concern of this paper and one can investigate more about this subject. In this paper the maintenance scheduling problem is analyzed from the GenCo point of view.

To solve this maintenance scheduling problem, the interior point cutting plane method is used. This method possesses advantages of both, interior point method and cutting plane method, and becomes very promising approach for large-scale and discrete optimization problems. The new base identification method is presented to solve problem of degenerate solutions and convex combination solutions. The improved algorithm can solve difficulties brought by multiple solutions.

The presented model has been successfully tested on the realistic size case study. Numerical results have revealed the accuracy and computationally efficient performance of the presented formulation.

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