Load Flow Solution for Unbalanced Radial Power Distribution using Monte-Carlo Simulation

MINI THOMAS
Prof. Electrical Department
JamiaMiliaIslamia University
New Delhi
INDIA

RAKESHRANJAN
Director,
IITB
Sonipat, Haryana
INDIA

ROMA RAINA
PhdScholor
JamiaMiliaIslamia University
New Delhi
INDIA
romaraina@yahoo.com

Abstract: - In modern power system, voltage stability is a major concern. To maintain voltage stability, it is desirable to assess effect of any unforeseen events and identify the nodes which are more sensitive. The most important task for distribution engineer is to efficiently simulate the system so that effective corrective actions can be taken. Load flow analysis is one of the techniques to simulate the system. This paper presents a probabilistic modeled load flow solution for three phase Radial Power Distribution System (RDS). Probabilistic model uses Monte Carlo Simulation (MCS) and considers input parameters as random variables. Input uncertainties are addressed using a specific shape of probability distribution function and simulation is run for 500 trails. Simulation calculates system losses, nodal voltages and voltage stability index (SI) for all the nodes. The simulation results obtained incorporates the uncertainty and provides distribution of output which can be used as input for corrective action and planning purposes

Key-Words: -Branch Voltage, Three phase load flow, Voltage Stability Index, Radial power distribution, Monte Carlo, Probability distributions.

1 Introduction

In a three phase AC power distribution system power flows from the substation to the load points through different networks, buses and branches. The flow of active and reactive power is called power flow or load flow. A systematic mathematical approach for determination of bus node voltages, currents, branch losses and active & reactive power flows through different branches is known as load flow studies. Load flow study is widely used by power distribution engineer for planning and operation of three phase distribution system.

Distribution system deliver power to a variety of loads, i.e. residential, industrial, and commercial, etc., which are typically subjected to daily load variations over a wide range. The loading patterns of these loads peak at different hours of the day and connected feeders/lines, as well as the substation of the RDS, become heavily loaded. Since the voltage stability of the system is largely influenced by the loading patterns, it is adversely affected during these peak load conditions.

To maintain voltage stability, it is important to predict any unforeseen events and identify sensitive distribution system nodes.

The voltage instability can be addressed by techniques like reconfiguration, addition of capacitor banks etc., however success depends on efficient load flow simulation and output which can address uncertainty.

In past, many solution methods have been developed on load flow distribution networks. In early research work, direct solution methodologies using the impedance matrix of the unbalanced networks was proposed by S. K. Goswami& S. K. Basu [1]. Similarly Zbus Gauss approach was proposed by T. H. Chen [2], Current injection method was proposed by P.A.N, Garcia, [3] However, above methods considers constant input parameters (e.g. load & line data). In real power distribution system scenario, input parameters will have significant uncertainties. Carpinelli proposed probabilistic three-phase load flow using a multi-linear simulation algorithm [4]. Caramia proposed a probabilistic solution method based on Monte-Carlo simulation applied to the nonlinear three-phase load flow equations including wind farms, thereby taking into account all load and line unbalances [5]. M. A. Golkar presented a new probabilistic linearization method for the load flow study of radial distribution system [6]. Peñuela presented an approach for probabilistic analysis of unbalanced three-phase weakly meshed distribution systems considering uncertainty in load demand [7].

This paper presents probabilistic modeled load flow solution, which addresses input uncertainties as a specific shaped probability distribution function and load data is modeled as random variables. The proposed load flow algorithm is a simple and computationally fast mathematical model.Simulation result provides probable nodal voltage distribution including real & reactive power loses. The results help in assessing the system performance and the impact due to uncertainty. The simulation also calculates voltage stability index for all the nodes and provides the distribution of the same.

2 Methodology

Load flow solution using monte-carlo simulation is done using following steps:

1. Define the load flow & stability index formulas & calculation algorithm.
2. Defines the monte-carlo simulation principle.
3. Provides simulation inputs by modeling of input data as random variables.
4. Run simulation
5. Evaluate the outputs

2.1 Calculation algorithm for load flow & stability index

2.1.1 Load Flow calculation algorithm

Load flow calculation algorithm used in the paper is based on concept described by R. Raina, M Thomas, R. Ranjan [8] and is further modified to suite the probabilistic model (for Monte Carlo simulation). The algorithm calculates the total real and reactive system power loss, nodal voltages and stability index.

The proposed load flow calculation algorithm uses the basic systems analysis method and circuit theory and requires only the recursive algebraic equations to get the voltage magnitudes, currents & power losses values at all the nodes. This load flow methodology also evaluates the total real and reactive power fed through any node.

The calculation uses Carson & Lewis matrix method, which takes into account the self and mutual coupling effects of the unbalanced three phase line section. Using concept of simple circuit theory, the relation between the bus voltages and the branch currents in Fig.1 can be expressed as:

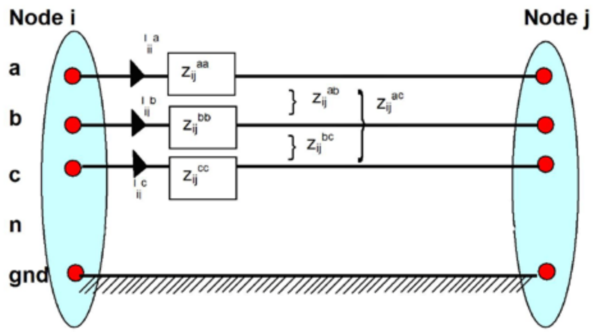


Fig. 1 – Three phase four wire line model

$$\begin{bmatrix} V_i^{ag} - V_j^{ag} \\ V_i^{bg} - V_j^{bg} \\ V_i^{cg} - V_j^{cg} \end{bmatrix} = \begin{bmatrix} V_{ij}^{a} \\ V_{ij}^{b} \\ V_{ij}^{c} \end{bmatrix} = \begin{bmatrix} Z_{ij}^{aa} & Z_{ij}^{ab} & Z_{ij}^{ac} \\ Z_{ij}^{ba} & Z_{ij}^{bb} & Z_{ij}^{bc} \\ Z_{ij}^{ca} & Z_{ij}^{cb} & Z_{ij}^{cc} \end{bmatrix} \begin{bmatrix} I_{ij}^{a} \\ I_{ij}^{b} \\ I_{ij}^{c} \end{bmatrix} \quad (1)$$

Where:

V_i^{ag}	=	Voltage of phase a at node i with respect to ground
V_i^{ab}	=	Voltage drop between two phases a and b at node i.
V_{ij}^{a}	=	Voltage Drop between nodes i and j in phase a.
I_{ij}^{a}	=	Current through phase a between nodes i and j.
z_{ij}^{aa}	=	Selfimpedance between nodes i and j in phase a.
z_{ij}^{ab}	=	Mutual impedance between phase a and b between nodes i and j.
Pi^a, Qi^a, Si^a	=	Real, reactive and complex power loads at phase a at ith bus.
Sij^{phase}	=	Complex power at phase (a, b and c) between nodes i and j.
$PLij^{phase}$	=	Real power loss in the line between node i and j.
$QLij^{phase}$	=	Reactive power loss in the line between node i and j.
$SLij^{phase}$	=	$PLij^{phase} + jQLij^{phase}$

Rewriting (1)

$$\begin{bmatrix} V_j^{a} \\ V_j^{b} \\ V_j^{c} \end{bmatrix} = \begin{bmatrix} V_i^{a} \\ V_i^{b} \\ V_i^{c} \end{bmatrix} - \begin{bmatrix} Z_{ij}^{aa} & Z_{ij}^{ab} & Z_{ij}^{ac} \\ Z_{ij}^{ba} & Z_{ij}^{bb} & Z_{ij}^{bc} \\ Z_{ij}^{ca} & Z_{ij}^{cb} & Z_{ij}^{cc} \end{bmatrix} \begin{bmatrix} I_{ij}^{a} \\ I_{ij}^{b} \\ I_{ij}^{c} \end{bmatrix} \quad (2)$$

Following equations (3), (4) & (5)gives the branch currents between the nodes i and j:

$$I_{ij}^{a} = \frac{\left(P_{ij}^{a}+jQ_{ij}^{a}\right)}{V_j^{a}} \quad (3)$$

$$I_{ij}^{b} = \frac{\left(P_{ij}^{b}+jQ_{ij}^{b}\right)}{V_j^{b}} \quad (4)$$

$$I_{ij}^{c} = \frac{\left(P_{ij}^{c}+jQ_{ij}^{c}\right)}{V_j^{c}} \quad (5)$$

The real and reactive power losses in the line between buses i and j are written as;

$$SL_{ij}^{a} = PL_{ij}^{a} + jQL_{ij}^{a} = \left(V_i^{a} * I_{ij}^{a}\right) - \left(V_j^{a} * I_{ji}^{a}\right)$$

$$SL_{ij}^{b} = PL_{ij}^{b} + jQL_{ij}^{b} = \left(V_i^{b} * I_{ij}^{b}\right) - \left(V_j^{b} * I_{ji}^{b}\right)$$

$$SL_{ij}^{c} = PL_{ij}^{c} + jQL_{ij}^{c} = \left(V_i^{c} * I_{ij}^{c}\right) - \left(V_j^{c} * I_{ji}^{c}\right)$$

The algorithm computes the real & reactive power and uses the formula given in equation no. (6). Receiving end power at any phase, say phase A, of line between the nodes i and j is expressed as:

$$P_{ij}^{a} + jQ_{ij}^{a} = \left[\sum P_{kj}^{a} + jQ_k^{a}\right] + \left[\sum PL_{mnj}^{a} + jQL_{kmn}^{a}\right] \quad (6)$$

k = index of all nodes fed through the line between nodes i& j.
m,n = Nodes fed through the line between i, j

Fig. 2.shows the typical load flow calculation algorithm used with monte-carlo simulation. The details of formulas and computing method are in [8].

2.1.1 Stability Index calculation algorithm

The proposed simulation also calculates the voltage stability index (SI) for all the nodes using load flow results. There are several methods to estimate or predict the voltage stability condition of a power system. The paper utilizes the voltage stability index defined by N.C.Sahoo, K.Prasad [9] to indicate the voltage stability condition at each bus of the system. Stability index for the bus j (SI) is defined as:

$$SI = V_i^2 - 4\left(P_{ij}R_{ij} + Q_{ij}X_{ij}\right) \quad (7)$$

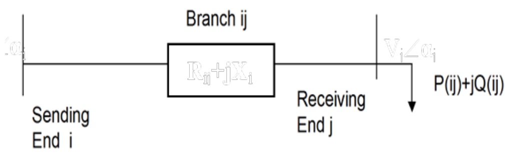


Fig.3 Electrical equivalent of one branch

The proposed load flow calculation algorithm uses the basic systems analysis method and circuit theory and requires only the recursive algebraic equations to get the voltage magnitudes, currents & power losses values at all the nodes. This load flow methodology also evaluates the total real and reactive power fed through any node.The value of SI varies from 0 to 1. For stable operation of the RDS, stability Index (SI) should be nearing one. If the SI is nearing 0, this reflects unstable bus.

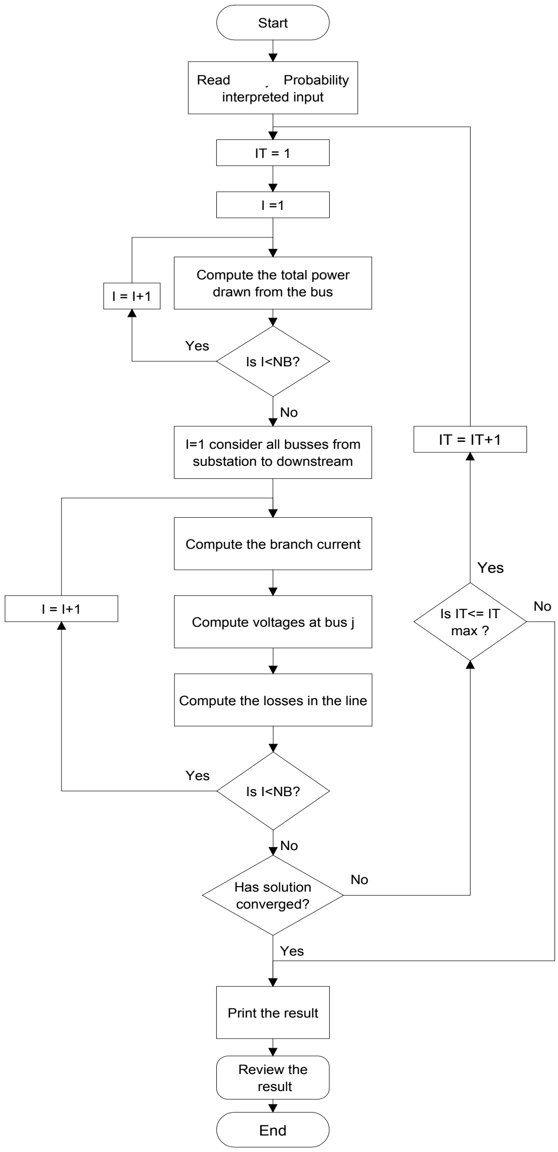


Fig. 2. Typical monte-carlo load flow calculation chart used with probability interpreted inputs

2.2 Monte-carlo simulation method (MCS)

Monte Carlo simulation is a computational algorithm that relies on repeated random sampling to compute the results and is used for simulating systems with many degrees of freedom or with significant uncertainty in inputs.

The MCS principle is described in Fig. 4. Uncertain input parameter is considered as a random variables P and numbers of realizations Pi of P are generated and load flow algorithm is run for each of them producing an output Ri. The simulation is run for 500 trails and set of outputs Ri represents the set of realizations of the random variable R. The statistical properties of R are therefore computed from the realizations Ri.

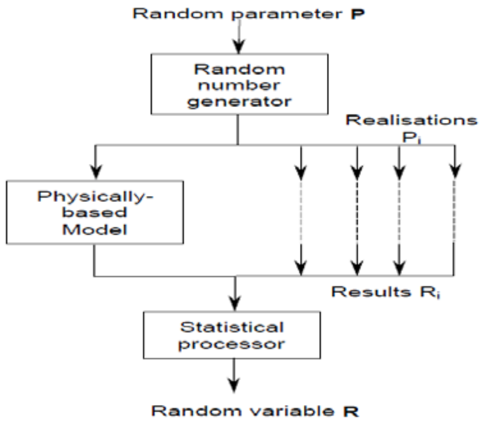


Fig. 4. Sketch for Monte-carlo Simulation method

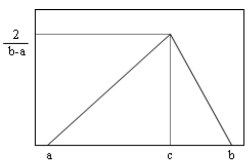
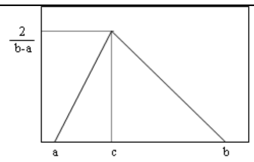
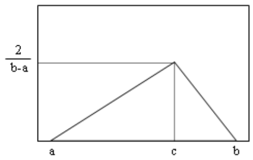
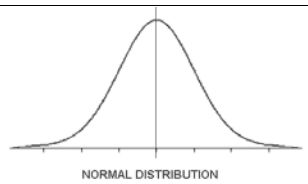
2.3 Modeling of Input data as variables

This simulation is run on a typical 19 bus distribution system from the D. Thukram, H.MW.Banda, and J. Jerome [10] shown in Fig. 5.

The Monte-carlo principle is based on considering input parameters as random variables and with defined distribution shape. Probability density function describes the likelihood of same future events. For simulation purpose connected load is assumed to be varying based on Table 1 probability distribution shape.

Input connected load data for the feeder are given in Table 2, Conductor data for the feeders are given in Table 3 & Table 4.

Table – 1- Probability distribution for connected load

Nodes	Probability Distribution shape: Shape	Probability Distribution shape: Data
2, 7, 13, 18,19		a = 20% b = 100% c = 130%
4, 10, 16		a = 90% b = 100% c = 140%
5, 12, 15		a = 15% b = 100% c = 105%
3, 6, 8, 9, 11, 14, 17		Mean = 1.0 SD = 0.1

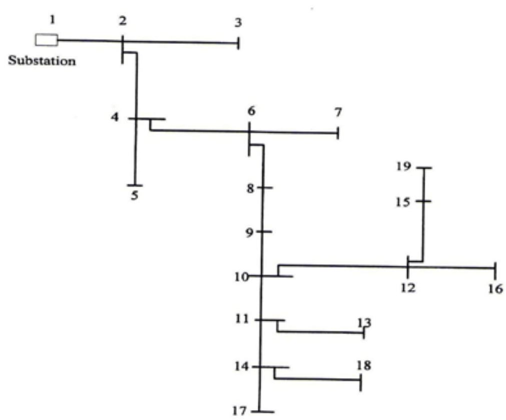


Fig. 5. Shows a practical 19 bus distribution feeder used for the modelling and simulation purpose.

Table – 2. Load data

Node	Phase Load in kVA: A	B	C
2	64	32	64
3	68	32	60
4	25	35	40
5	40	32	28
6	26	19	18
7	60	50	50
8	46	33	21
9	76	92	82
10	21	26	16
11	46	46	68
12	60	50	50
13	27	33	40
14	19	19	25
15	27	30	43

Node	Phase Load in kVA		
	A	B	C
16	48	64	48
17	40	30	30
18	33	33	34
19	54	62	44

Table – 3- Conductor Data

Conductor type	Resistance PU/Km	Reactance PU/Km
1	0.008600	0.003700
2	0.012950	0.003680

Table – 4 -Conductor Code & Distances

Sending End Node(IR)	Receiving End Node(IR)	Conductor Code	Distance in Km
1	2	1	3
2	3	2	5
2	4	1	1.5
4	5	2	1.5
4	6	1	1
6	7	2	2
6	8	1	2.5
8	9	1	3
9	10	1	5
10	11	1	1.5
10	12	1	1
11	13	2	5
11	14	1	3.5
12	15	1	4
12	16	2	1.5
14	17	1	6
14	18	2	5
15	19	1	4

3 Simulation Results

The simulation is run for 500 trails and distribution of results are plotted as frequency distribution plot & cumulative distribution function (CDF) plot.

Frequency distribution plot shows the frequency or count of the output within a particular group or data interval. Cumulative distribution function (CDF) describes the probability of an output with-in the frequency distribution and can be found as value less than or equal to output.Figure 6 & 7 shows the frequency distribution and cumulative density function plot for Real and Reactive power. Power values corresponding to 90% (0.9) cumulative probability are 2130 kW/1030 KVAr. The value of 90% (0.9) cumulative probability signifies that for a simulation run of 500 trails, 90% time values were less than 2130 kW/1030 KVAr. This value can be used as an input for further planning and corrective action purpose.

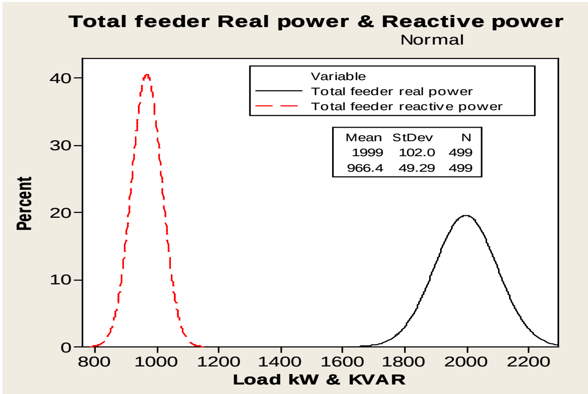


Fig. 6. Distribution of Feeder Real and Reactive power

Figure 8 & 9 shows the frequency distribution and cumulative density function diagram of Real and Reactive power loss and also shows value corresponding to 90% cumulative probability.

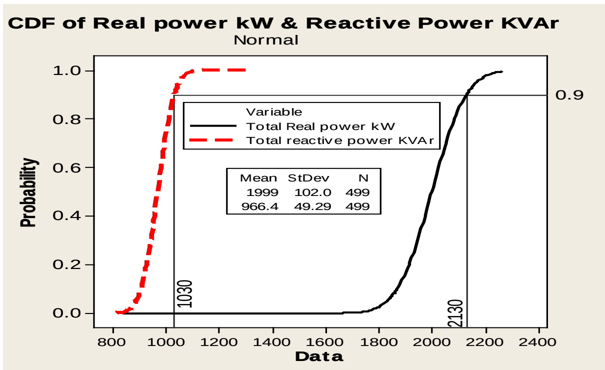


Fig. 7.CDF of Real & Reactive Power

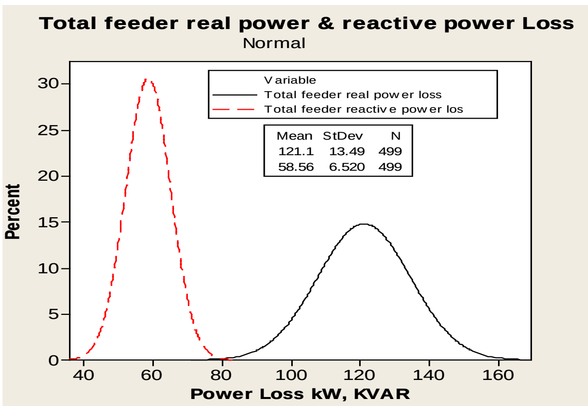


Fig. 8. Distribution of Real and Reactive power

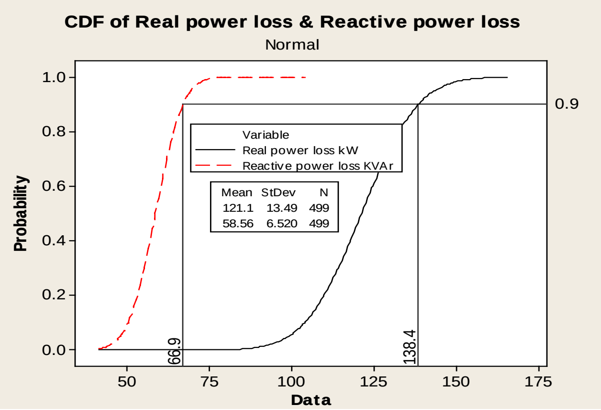


Fig. 9.CDF of Real & Reactive Power loss

Table -5 provides the above data in a tabular format with additional data related to minimum and maximum values of total system power.

Table – 5- Distribution Data for Real & Reactive power

Data	Distribution Mean	Std. Div.	Min.	Max.	90% Cum prob. value
Real Power kW	1999.5	102.0	1683.2	2264.8	2130
Reactive Power KVAr	966.42	49.29	813.56	1094.6	1030

Table –6 Distribution Data for Voltage Magnitudes

Node	Phase A			Phase B			Phase C		
	Mean	StDev	90% prob	Mean	StDev	90% prob	Mean	StDev	90% prob
2	0.979	0.0011	0.977	0.980	0.001	0.979	0.985	0.0015	0.983
3	0.974	0.0013	0.972	0.978	0.0011	0.976	0.980	0.001	0.979
4	0.970	0.0016	0.968	0.971	0.0015	0.969	0.974	0.0016	0.972
5	0.969	0.0017	0.967	0.970	0.0016	0.968	0.971	0.0015	0.969
6	0.964	0.0019	0.962	0.965	0.0018	0.963	0.968	0.0019	0.965
7	0.963	0.0021	0.960	0.964	0.002	0.962	0.965	0.0018	0.963
8	0.953	0.0025	0.949	0.953	0.0025	0.950	0.959	0.0024	0.956
9	0.940	0.0032	0.936	0.939	0.0033	0.935	0.947	0.0032	0.943
10	0.922	0.0044	0.917	0.921	0.0046	0.915	0.933	0.0041	0.928
11	0.920	0.0045	0.914	0.919	0.0047	0.913	0.920	0.0046	0.915
12	0.921	0.0046	0.915	0.919	0.0047	0.913	0.921	0.0046	0.915
13	0.918	0.0047	0.912	0.916	0.0049	0.910	0.918	0.0047	0.913
14	0.917	0.0047	0.911	0.916	0.0048	0.910	0.918	0.0047	0.912
15	0.918	0.0049	0.912	0.916	0.0052	0.909	0.919	0.0048	0.913
16	0.920	0.0046	0.914	0.917	0.0048	0.911	0.919	0.0047	0.913
17	0.915	0.0048	0.908	0.914	0.0048	0.908	0.916	0.0048	0.910
18	0.915	0.005	0.909	0.914	0.0051	0.907	0.916	0.0048	0.910
19	0.916	0.0052	0.910	0.914	0.0056	0.907	0.916	0.0052	0.909

Table 6 shows the simulation distribution results for all nodal voltages based on 500 trails. The minimum nodal voltage corresponding to 90% cumulative probability is also calculated. Minimum nodal voltage value corresponding to 90% cumulative probability means that for 500 trains, 90% time nodal voltage value will be more than calculated value. This can be used for future planning / corrective action purposes.

Figure 10 is the scatter plot diagram for the mean voltage and nodes. The plot provides information about relative nodal voltages for nodes. For distribution data used, we can note that mean voltage drops for downstream nodes. The mean voltage value can be used for future planning / corrective action purposes.

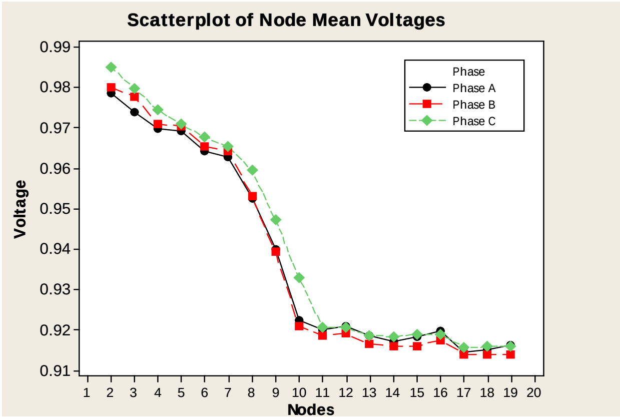


Fig. 10. Scatter plot of Node Mean Voltages

Table –7 Distribution Data for Stabilty Index

Node	Phase A			Phase B			Phase C		
	Mean	StDev	90% prob	Mean	StDev	90% prob	Mean	StDev	90% prob
2	0.799	0.0104	0.786	0.812	0.0094	0.800	0.802	0.0098	0.790
3	0.954	0.0023	0.951	0.959	0.002	0.956	0.967	0.0031	0.963
4	0.908	0.0048	0.902	0.909	0.0047	0.903	0.921	0.0054	0.914
5	0.939	0.0032	0.935	0.942	0.003	0.938	0.949	0.0032	0.944
6	0.880	0.0064	0.872	0.881	0.0063	0.873	0.891	0.0063	0.882
7	0.925	0.0045	0.919	0.928	0.0041	0.923	0.933	0.0042	0.927
8	0.876	0.0064	0.868	0.876	0.0065	0.867	0.883	0.0064	0.874
9	0.828	0.0093	0.816	0.822	0.0097	0.809	0.836	0.0094	0.824
10	0.864	0.0073	0.855	0.862	0.0075	0.852	0.877	0.0072	0.867
11	0.845	0.0084	0.834	0.842	0.0086	0.831	0.863	0.008	0.853
12	0.807	0.0127	0.791	0.799	0.0132	0.782	0.828	0.0118	0.813
13	0.844	0.0085	0.833	0.840	0.0089	0.829	0.843	0.0089	0.832
14	0.834	0.0091	0.822	0.832	0.0092	0.820	0.835	0.0091	0.823
15	0.843	0.0091	0.831	0.839	0.0095	0.826	0.842	0.0091	0.830
16	0.836	0.0087	0.825	0.829	0.0092	0.817	0.836	0.0088	0.824
17	0.830	0.009	0.819	0.831	0.009	0.819	0.835	0.0087	0.824
18	0.837	0.0091	0.825	0.835	0.0093	0.823	0.839	0.0091	0.827
19	0.843	0.009	0.832	0.839	0.0094	0.827	0.844	0.0087	0.833

Table 7 shows the simulation distribution results for Stability Index for all nodes based on 500 trails. Minimum Stability Index corresponding to 90% cumulative probability is also calculated which can be used for future planning / corrective action purposes.

Figure 11 is the scatter plot diagram for the mean Stability Index and nodes. The result shows that node 2, 9 and 12 are more sensitive nodes.

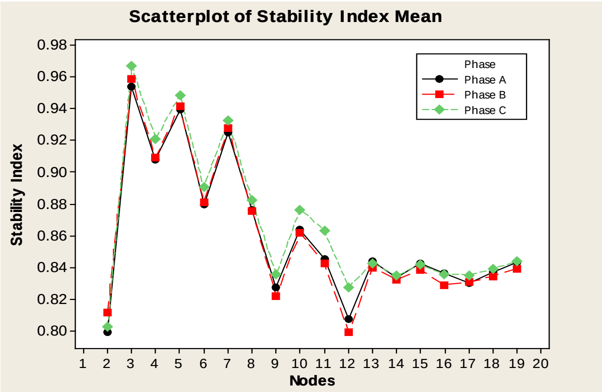


Fig.11. Scatter plot of Node Mean Stability index

4 Conclusion

This paper presents probabilistic model for solving three phase unbalanced distribution system. The simulation is based on Monte Carlo method which relies on repeated random sampling to compute the results and address the uncertainty and degrees of freedom associated with input data. Further connected load is assumed to be varying based on pre-defined probability distribution shape, provide more reliable distribution of output data.

The proposed technique addresses all possible input combinations and provides output snapshot. It also calculates values corresponding to 90% (0.9) cumulative probability which ccan be used for future planning and corrective action purposes including reconfiguration and capacitor sizing and placing.

References:

[1] S.K.Goswami and S.K.Basu, “Direct solution of distribution system,” *Proc. Inst.Elect.Eng.,Gen.,Trnsm.,Distr.*, vol.138,no.1.pp.78-88,Jan 1991.

[2] T.H. Chen, “Distribution system power flow analysis-arigid approach,” *IEEE Trans.Power Del.*, vol.6, no.3,pp.1146-1152, Jul1991.

[3] P.A.N, Garcia, “Three phase power flow calculationsusing the current injection method,” *IEEE Trans. Power Sys.*,vol.15, no.2, pp.508-514, May 2000

[4] Carpinelli, G. Di Vito, V. and Varilone, “Multi-linear Monte Carlo simulation for probabilistic three-phase load flow”, *European Transactions on Electrical Power*, volume 17, pages 1–19, Jan/Feb2007. doi: 10.1002/etep.102

[5] Caramia, P. Carpinelli, G. Pagano, M. Varilone, P, “Probabilistic three-phase load flow for unbalanced electrical distribution systems with wind farms”, *Renewable Power Generation, IET*, Volume: 1 Issue: 2, On page(s): 115 – 122, June 2007. Digital Object Identifier:10.1049/iet-rpg:20060013

[6] M. A. Golkar, “A new probabilistic load-flow method for radial distribution networks", *European Transactions on Electrical Power*,Volume 13, Issue 3, pages 167–172, May/June 2003, doi: 10.1002/etep.4450130305.

[7] Peñuela, C.A.Mauricio, G.E.Mantovani, J.R.S, “Probabilistic analysis of the distributed power generation in weakly meshed distribution systems “,*Transmission and Distribution Conference and Exposition: Latin America (T&D-LA), 2010 IEEE/PES*, page(s): 171 - 177 , 8-10 Nov. 2010 , Digital Object Identifier: 10.1109/TDC-LA.2010.5762878

[8] Mini S Thomas, RakeshRanjan, Roma Raina, “fuzzy modeled load flow solution for unbalanced radialPower distribution system”, *Proceedings of the IASTED International Conference, Power and Energy Systems (EuroPES 2011)*, June 22 - 24, 2011 Crete, Greece. doi:0.2316/P.2011.714-003

[9] N.C.Sahoo, K.Prasad, “A fuzzy genetic approach for network reconfiguration to enhance voltage stability in radial distribution systems”, *Energy conversion and Management* 47(3288-3306),2006

[10] D. Thukram, H.MW.Banda, and J. Jerome, A robust three phase power flow algorithm for radial distribution systems,*”JournalofElectricalPowerSystems Research”*,vol.50,no. 3,pp.227-236,June 1999