Optimal Operation of Ship Electrical Power System with Energy Storage System and Photovoltaics: Analysis and Application

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Abstract: - The extensive electrification of ship power systems has become a very appealing alternative for the development of more efficient and environmentally friendly ships. Renewable energy sources (RES) and energy storage systems (ESS) will have a key role in such systems as they can lead to fuel consumption reduction and increase overall ship efficiency. However, the power production from RES is stochastic and hardly predicted making necessary the combined operation with ESS. In this paper the operation of a ship power system equipped with photovoltaics (PVs) and ESS is analysed from the economic point of view and analytic formulas are obtained for system marginal cost for different case studies. The proposed method leads to simple analytic mathematical relations that could be easily exploited for the technical-economical assessment of ESS and RES integration to ship electric power systems.

Key-Words: - Economic dispatch, ship power system, photovoltaic panels, energy storage system, system marginal cost

1 Introduction

Nowadays, worldwide concern about air quality and greenhouse gas emissions, has led to stricter regulations in ship industry, as Annex VI of the MARPOL Convention [1].

The need for more efficient ships has been the incentive for turning all energy subsystems aboard (including power generation units) into more efficient ones [2] and for the exploitation of green power technology. Moreover optimal power management systems that can be summarized in "take the best out of each unit but having resolved any technical problems emerged" are becoming of great importance in the modern power systems. In this context, the extensive electrification of ship power systems, widely known as All Electric Ship (AES), has become a very appealing alternative. AES introduces novel concepts and combined with optimal power management can lead to more efficient ships [3-8].

AES allows ship power plant configurations with numerous combinations of diesel engines, gas turbines, steam power plants, fuel cells, energy storage systems (ESS) and possibly electric power production from renewable energy sources (RES). The large variety of AES power system components enables conformity with ship energy efficiency directives, not attainable for each component alone.

The proper operation of all ship electric power system components including RES and ESS can

lead to fuel consumption reduction and increase ship overall efficiency. However, the power production from RES is stochastic and hardly predicted. Photovoltaics (PVs) are a promising option for RES onboard applications; however, the above described technical challenges and difficulties in conjunction with their increased cost and space requirements must be overcome for their successful integration. Appropriate ESS is required in order to cope with problems like uncertainty in availability and high variability in their production. Moreover, ESS can greatly contribute to load demand management and generally to the global energy management of the ship with several consequent positive effects e.g. possible reduction of prime movers, further operation cost reduction etc. Several energy storage technologies like high power flywheels, super capacitors, SMES, high energy REDOX, flow batteries etc. are available nowadays. However, batteries seem to be the most suitable for ship power system applications. In any case, an ESS for onboard integration must be assessed taking into account technical and economic requirements during design phase and also its capability to ensure maximum ship power system reliability and redundancy.

In this paper the operation of a ship power system equipped with PVs and ESS is analyzed from the economical point of view. Analytic formulas are obtained for system marginal cost for three case studies. More specifically: a) Ship electric power generation system comprising only thermal units.

b) Ship electric power generation system comprising thermal units and PVs.

c) Ship electric power generation system comprising thermal units, PVs and ESS.

System marginal cost is formulated for different ship power system configurations taking into account the energy and power balances are taken into account for a specific time period of ship power system operation. Finally, the developed full algorithm is presented and is applied to a typical cruiser power system and the obtained results are commented.

2 Basic Information on Ship Electric Power System Operation with Photovoltaic Panels and Energy Storage System

2.1 General

The economically optimum operation of a conventional electric power system for a specific system load is known as economic dispatch and it has been thoroughly analyzed in the literature [9-10]. Next, the optimum operation of ship electric power system supported by PVs and ESS will be analyzed based on the same principles of conventional economic dispatch.

2.2 System load chronological curve

The chronological load curve of the ship power system defines the hourly or 15-minutes average power demand over a specific time horizon, T. Time period T is divided into M intervals, DT_j , with j=1, 2, ..., M. In each time interval DT_j the respective ship's load demand P_{Dj} is considered constant and equal to its average value. Hence, it is calculated by eq. (1):

$$\int_{t_{j-1}}^{t_j} p_{\mathrm{D}}(t) \cdot dt = P_{\mathrm{D}j} \cdot DT_j \quad [\mathrm{kWh}] \tag{1}$$

As the time interval DT_j tends to zero, the P_{Dj} will tend to the instantaneous demand load demand $p_D(t)$, as it becomes apparent from Fig. 1. However, for practical reasons time interval DT_j varies from 15 min to 1 hour.

2.3 Ship thermoelectric power system

It is assumed that the thermal system consists of N thermal generating units connected to the same bus. Let us assume that the i^{th} unit produces output active power $P_{\text{TH}i,j}$ during the j^{th} time interval, lower limited by the technically minimum active power, $P_{\min THi}$, and upper limited by the technically maximum active power $P_{\max THi}$ of the unit. The previous constraints are formulated as:

 $P_{\min \text{TH}i} \le P_{\text{TH}i,j} \le P_{\max \text{TH}i}$ [kW] (2) Where *i*=1, 2,..., *N* and *j*=1, 2,..., *M*.

The respective fuel cost is obtained by the function $F_{THi}(P_{THi,j})$, which is usually a second or third order polynomial of $P_{THi,j}$. The ohmic losses can be safely assumed negligible, as the onboard distribution network between generators and loads is not extended but limited in a few meters of three-phase cables and electric bus-bars.



Fig. 1. Continuous system load chronological curve and its averaged form over specific time intervals.

2.4 The photovoltaic panels system

The capacity of photovoltaic systems integrated in ship electric power systems is limited by the available space onboard while their power production depends on solar irradiation which varies during the day. The photovoltaic system cannot produce power, P_{PVj} , during the j^{th} time interval larger than its technically maximum active power P_{maxPV} while at night is out of operation.

$$0 \le P_{\text{PV}_i} \le P_{\text{maxPV}} \quad [kW] \quad (3)$$

The produced active power follows an approximately sinusoidal time function during the daylight, as it is shown in Fig.2. In each time interval DT_j the active power produced by the PVs, P_{PVj} , is considered constant, and calculated by:

$$\int_{t_{j-1}}^{t_j} p_{\text{PV}}(t) \cdot dt = P_{\text{PV}j} \cdot DT_j \text{ [kWh]} \quad (4)$$

The simplified single-line diagram of the ship electric power system in case of not using ESS is shown in Fig. 3. The dc output voltage of the photovoltaic panels is converted to ac voltage current through a proper dc/ac power converter. The active power P_{CVj} injected to the grid by the power converter can be calculated from the dc power P_{PVj} produced by the PVs and the use of the power converter performance coefficient η_{CV} , as follows:

$$P_{\rm CV_{\it i}} = \eta_{\rm CV} \cdot P_{\rm PV_{\it j}} \qquad [\rm kW] \qquad (4)$$



Fig. 2. Chronological photovoltaic system output active power curve during period T and its averaged form over specific time intervals.



Fig. 3. Ship electric power system with 4 generators connected at the same bus and PVs.

The above calculation stands under the assumption that this power can be absorbed by the ship power system. It is a logical assumption to subtract the active power P_{CVj} produced by the PVs from load demand P_{Dj} if the remaining load demand, $P'_{Dj} = P_{Dj} P_{CVj}$, is bigger than the minimum active power of the thermal units in operation otherwise it should be set equal to the minimum active power of the thermal units.

$$P_{\mathrm{D}j}^{\prime} = \max\left\{ \left(P_{\mathrm{D}j} - P_{\mathrm{CV}j} \right), P_{\mathrm{minTH}} \right\} [\mathrm{kW}] \quad (6)$$

 P'_{Dj} is dispatched to the operating thermal units based on the classical economic dispatch algorithm without considering ship power system ohmic losses [9-10].

2.4 The energy storage system

ESS can greatly contribute to the optimal operation of the ship electric power system. Moreover, it improves ship power system safety and reliability due to its operational flexibility. In the following analysis an appropriate ESS, such as vanadium redox flow battery [11-12], is used to store the energy EPV produced by the PVs or excessive energy from electric generators during the time period T and supply it during low load periods. The energy produced by the PVs over the time interval, T, is:

$$E_{\rm PV} = \sum_{j=1}^{M} P_{\rm PVj} \cdot DT_j \quad [kWh]$$
(7)

When it is required, the ESS can supply the ship electric power system with the stored energy E_{PV} through a proper dc/ac power converter, as it is shown in Fig. 4.



Fig. 4. Ship electric power system with 4 generators connected at the same bus and PVs with ESS.

Thus, the ESS can be considered as an equivalent generator, which can produce active power $P_{\text{ESS}j}$ during the *j*th time interval, under the constraint that the total respective produced energy, E_{ESS} , over the examined time period *T* equals to:

$$E_{\text{ESS}} = \sum_{j=1}^{M} P_{\text{ESS}j} \cdot DT_j = \eta_{\text{ESS-CV}} \cdot E_{\text{PV}} \quad [\text{kWh}] \quad (8)$$

Where, $\eta_{\text{ESS-CV}}$ is the total performance coefficient of the ESS, the PVs and their dc/ac power converter.

ESS output active power, $P_{\text{ESS}j}$, is lower limited by zero and upper limited by the technically maximum active power P_{maxESS} of the dc/ac converter. Thus,

$$0 \le P_{\text{ESS}i} \le P_{\text{max}\,\text{ESS}} \qquad [kW] \qquad (9)$$

Moreover, the ESS energy injected to the electric system until time point t_j should satisfy the following constraints:

$$0 \le \sum_{k=1}^{J} P_{\text{ESS}k} \cdot DT_k \le \eta_{\text{ESS}-PV} \cdot \sum_{k=1}^{J} P_{\text{PV}k} \cdot DT_k \text{ [kWh](10)}$$

Where $i=1, 2, \dots, M$.

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3 Ship Electric Power System Optimal Operation

3.1 Optimal operation of ship electric power system with PVs without energy storage system

The total hourly fuel cost of the power system in the j^{th} time interval DT_j , denoted with $F_{tot,j}$, is obtained as the sum of the fuel costs of the units:

$$F_{tot,j} = \sum_{i=1}^{N} F_{\mathrm{TH}i} \left(P_{\mathrm{TH}i,j} \right) \quad [\pounds/h] \qquad (11)$$

Where *j*=1, 2,..., *M*.

Active power balance constraint in each time interval DT_i is given by:

$$\sum_{i=1}^{N} P_{\text{TH}i,j} = P'_{\text{D}j} \qquad [\text{kW}] \qquad (12)$$

The total hourly fuel cost $F_{tot,j}$ in j^{th} time interval is not related with its values for previous time intervals, as the load demand P'_{Dj} depends exclusively on the produced active power P_{PVj} by the PVs and the load demand P_{Dj} at the j^{th} time interval DT_{j} .

The target is to determine the power production levels of the thermal units subject to the active power balance constraint such that each total hourly fuel cost $F_{tot,j}$ and the total hourly fuel cost are both minimized during the examined time period, T. This means that for each time interval DT_j the total hourly fuel cost $F_{tot,j}$ should be minimized taking into consideration the constraint of eq. (12). This problem can be solved with the well-known Lagrange method. Thus, eq. (11) is modified as:

$$L_{tot,j} = \sum_{i=1}^{N} F_{\text{TH}i} \left(P_{\text{TH}i,j} \right) - \lambda_j \cdot \left(\sum_{i=1}^{N} P_{\text{TH}i,j} - P_{\text{D}j}^{\prime} \right) [\text{€/h}] (13)$$

The optimality conditions are obtained by setting ∂L_{res}

the partial derivatives of
$$\frac{\partial L_{tot,j}}{\partial P_{\text{TH}i,j}}$$
 equal to 0.

$$\frac{\partial L_{tot,j}}{\partial P_{\text{TH}i,j}} = 0 \Rightarrow \frac{dF_{\text{TH}i} \left(P_{\text{TH}i,j} \right)}{dP_{\text{TH}i,j}} - \lambda_j \cdot 1 = 0 \Rightarrow$$

$$\lambda_j = \frac{dF_{\text{TH}i} \left(P_{\text{TH}i,j} \right)}{dP_{\text{TH}i,j}} \quad [\epsilon/\text{kWh}] \text{ for } i = 1, 2, ..., N \quad (14)$$

$$\Rightarrow \lambda_j = \frac{dF_{\text{TH}i} \left(P_{\text{TH}i,j} \right)}{dP_{\text{TH}i,j}} = ... = \frac{dF_{\text{TH}N} \left(P_{\text{TH}N,j} \right)}{dP_{\text{TH}N,j}} \quad (15)$$

According to this result the load should be dispatched to the units so that the respective incremental costs at time interval DT_i are equal to λ_i . The Lagrange coefficient, λ_i , is known as system marginal cost (SMC) and it varies over the period Tas ship electric load also varies. After the determination of λ_i and the power production levels of the units $P_{\text{TH}i,j}$ the technical constraints of the units, as given in ineq. (2), should be checked for each time interval DT_{j} . If an inequality is not valid, i.e. $P_{\text{TH}2,i} < P_{\text{minTH}2}$, the output active power should be set to equal to the violated limit, i.e. $P_{\text{TH}2,j}$ = $P_{\min TH2}$. Next, the load demand P'_{Di} should be reset to $P'_{D_i} = P'_{D_i} - P_{TH2,i}$ and the optimization process repeated for the rest of the units, i.e. without considering unit 2.

3.2 Optimal operation of ship electric power system with PVs and energy storage system

The total fuel cost F_{tot} of the power system for the examined study period *T*, is calculated by:

$$F_{tot} = \frac{1}{T} \cdot \sum_{j=1}^{M} \sum_{i=1}^{N} F_{\text{TH}i} \left(P_{\text{TH}i,j} \right) \cdot DT_j \quad \text{[\pounds/h]} \quad (16)$$

In this case, it is not possible to minimize separately the hourly fuel cost $F_{tot,j}$ for the j^{th} time interval DT_j , as done in paragraph 3.1. This happens, because the energy balance involves the energy from ESS that can be calculated according to eq. (8) only over the time period T. The active power balance constraint for each time interval DT_j can be written as:

$$\sum_{i=1}^{N} P_{\text{TH}i,j} + P_{\text{ESS}j} = P_{\text{D}j} \quad [kW] \quad (17)$$

The power generation levels of the thermal units should be determined for each time interval DT_j such as the total equivalent hourly fuel cost, F_{tot} , is minimized, the respective active power balances (eq. (17)) and the total energy balance of ESS (eg. (8)) are fulfilled. This minimization problem can be solved by integrating the eq. (17) and eq. (8) into eq. (16) with the unknown Lagrange multipliers, i.e. λ_j with j=1, 2, ..., M for eq. (17) and w for eq. (8). Thus, eq. (16) is modified as:

$$L_{tot} = \frac{1}{T} \cdot \sum_{j=1}^{M} \sum_{i=1}^{N} F_{\text{TH}i} \left(P_{\text{TH}i,j} \right) \cdot DT_{j}$$
$$- \sum_{j=1}^{M} \lambda_{j} \cdot \left(\sum_{i=1}^{N} P_{\text{TH}i,j} + P_{\text{ESS}j} - P_{\text{D}j} \right) \cdot \frac{DT_{j}}{T}$$
$$- w \cdot \left(E_{\text{ESS}} - \sum_{j=1}^{M} P_{\text{ESS}j} \cdot DT_{j} \right) \qquad [\pounds/h] \quad (18)$$

Each constraint of eq. (17) has been multiplied with the factor DT_j/T , so as the first two terms of eq. (18) be similar to the respective terms of eq. (13). The use of this multiplication factor maintains the constraint of eq. (17) active and the property of Lagrange multiplier λ_j to reflect system marginal cost during time interval DT_j .

The optimality conditions are obtained by setting the partial derivatives $\frac{\partial L_{tot}}{\partial P_{\text{TH}i,j}} = 0$ equal to 0. More

specifically, the optimality conditions are:

$$\frac{\partial L_{tot}}{\partial P_{\text{TH}i,j}} = 0 \Rightarrow \frac{dF_{\text{TH}i} \left(P_{\text{TH}i,j} \right)}{dP_{\text{TH}i,j}} \cdot \frac{DT_j}{T} - \lambda_j \cdot \frac{DT_j}{T} = 0 \Rightarrow$$
$$\lambda_j = \frac{dF_{\text{TH}i} \left(P_{\text{TH}i,j} \right)}{dP_{\text{TH}i,j}} \quad [\pounds/\text{kWh}] \text{ for } i=1, 2, \dots, N \quad (19)$$
$$\frac{\partial L_{tot}}{\partial P_{\text{ESSj}}} = 0 \Rightarrow -\lambda_j \cdot \frac{DT_j}{T} + w \cdot DT_j = 0 \Rightarrow$$

$$w = \frac{\lambda_j}{T} [\epsilon/kWh/h]$$
 for $j=1, 2, ..., M$ (20)

Finally it stands that,

$$\Rightarrow \lambda_1 = \lambda_2 = \dots = \lambda_M \triangleq \lambda \tag{21}$$

According to eq. (19) the load should be dispatched to the units so as the respective incremental costs in time interval DT_j are equal to λ_j .

Also, according to eq. (21) the system marginal $\cot \lambda_j$ at the different time intervals should be equal to λ so as F_{tot} is minimized. Theoretically, each thermal unit should operate at the same operating point during the under study time period T and the load variations be served by the ESS. It is noted that the Lagrange coefficient w is the equivalent marginal cost of the stored energy per hour, which should be also constant for all time intervals.

After the determination of λ_j and w, the power generation levels of the units $P_{\text{TH}i,j}$ (*M*·*N* unknown variables) and the power levels of the energy storage system $P_{\text{ESS}j}$ (*M* unknown variables), the *M*·*N* inequalities from the thermal units technical constraints and *M* inequalities related with the ESS technical constraints should be checked over the time period *T*. As it has been already mentioned in §3.1, if a violation occurs for a unit then the output active power will be set equal to the respective violated limit. Next, the optimization process will be executed without the respective unit taken into consideration and its output power subtracted from the load demand.

4 System Marginal Cost Comparison for Different Ship Power System Configurations

Next, it is assumed that the fuel cost of the i^{th} thermal unit, $F_{THi}(P_{THi,j})$, is a second order polynomial.

$$F_{\text{TH}i}\left(P_{\text{TH}i,j}\right) = a_i + b_i \cdot P_{\text{TH}i,j} + c_i \cdot P_{\text{TH}i,j}^2 \quad [\pounds/h] \quad (22)$$

The derivative of $F_{THi}(P_{THi,j})$ with respect to $P_{THi,j}$ is calculated as:

$$\lambda_{j} = \frac{dF_{\text{TH}i}(P_{\text{TH}i,j})}{dP_{\text{TH}i,j}} = b_{i} + 2 \cdot c_{i} \cdot P_{\text{TH}i,j} \quad [\pounds/\text{kWh}] (23)$$

According to eq. (23) the generating level of the i^{th} unit in the j^{th} time interval can be calculated as a function of the respective system marginal cost λ_j :

$$P_{\text{TH}i,j} = \frac{\lambda_j - b_i}{2 \cdot c_i} \qquad [\text{kW}] \qquad (24)$$

In the following analysis it is assumed that the technical constraints the thermal units will not be activated.

In case of a ship power system comprising thermal units and PVs the active power balance (eq. (12)) should be valid for each time interval DT_i :

$$\sum_{i=1}^{N} \frac{\lambda_j - b_i}{2 \cdot c_i} = P_{\mathrm{D}_j}^{\prime}$$

Consequently, the respective system marginal cost λ_{PVi} equals to:

$$\lambda_{PVj} \triangleq \lambda_j = \frac{P'_{Dj} + \sum_{i=1}^{N} \frac{b_i}{2 \cdot c_i}}{\sum_{i=1}^{N} \frac{1}{2 \cdot c_i}} \quad [\pounds/kWh] \quad (25)$$

Where *j*=1, 2, ..., *M*.

If no PVs are installed then the thermal units will exclusively supply the load and eq. (25) becomes:

$$\lambda_{\text{TH}j} \triangleq \lambda_j = \frac{P_{\text{D}j} + \sum_{i=1}^{N} \frac{b_i}{2 \cdot c_i}}{\sum_{i=1}^{N} \frac{1}{2 \cdot c_i}} [\text{€/kWh}] \qquad (26)$$

It is inferred from eq. (6) that the load demand P'_{Dj} (load demand P_D minus PV production), is always smaller or equal to the original load demand P_D . Consequently, the respective system marginal cost λ_{PVj} is always smaller or equal to the system marginal cost λ_{THj} that corresponds to the case of no PVs installed onboard.

(6)
$$\Rightarrow P'_{\mathrm{D}j} \leq P_{\mathrm{D}j} \Rightarrow \lambda_{\mathrm{PV}j} \leq \lambda_{\mathrm{TH}j}$$
 (27)

In case that PVs and ESS are installed onboard the active power balance should be valid for each time interval DT_j . The output active power of the ESS is calculated as:

$$\sum_{i=1}^{N} \frac{\lambda_j - b_i}{2 \cdot c_i} + P_{\text{ESS}j} = P_{\text{D}j} \Longrightarrow$$
$$P_{\text{ESS}j} = P_{\text{D}j} - \lambda_j \cdot \sum_{i=1}^{N} \frac{1}{2 \cdot c_i} + \sum_{i=1}^{N} \frac{b_i}{2 \cdot c_i} \text{ [kW] (28)}$$

The equivalent system marginal cost, λ_{ESS} , for the examined study period *T* can be calculated with the exploitation of the ESS energy balance. More specifically, if eq. (8), (21) & (28) are combined, then:

$$\left\{ \lambda_{\text{ESS}} \triangleq \lambda_{1} = \lambda_{2} = \dots = \lambda_{M} \right\} \Longrightarrow$$

$$E_{\text{ESS}} = \sum_{j=1}^{M} \left\{ P_{\text{D}j} - \lambda_{\text{ESS}} \cdot \sum_{i=1}^{N} \frac{1}{2 \cdot c_{i}} + \sum_{i=1}^{N} \frac{b_{i}}{2 \cdot c_{i}} \right\} \cdot DT_{j} \Longrightarrow$$

$$\lambda_{\text{ESS}} = \frac{\sum_{j=1}^{M} P_{\text{D}j} \cdot DT_{j} + \sum_{i=1}^{N} \frac{b_{i}}{2 \cdot c_{i}} \cdot T - E_{\text{ESS}}}{\sum_{i=1}^{N} \frac{1}{2 \cdot c_{i}} \cdot T} \quad [\pounds/\text{kWh}] (29)$$

If the time intervals DT_j are considered equal to T/M, then eq. (29) is modified as:

$$\lambda_{\text{ESS}} = \frac{\frac{1}{M} \cdot \sum_{j=1}^{M} P_{\text{D}j} + \sum_{i=1}^{N} \frac{b_i}{2 \cdot c_i} - \frac{E_{\text{ESS}}}{T}}{\sum_{i=1}^{N} \frac{1}{2 \cdot c_i}} \quad [\pounds/\text{kWh}] (30)$$

It is noted that the marginal costs $\lambda_{\text{TH}j}$ and $\lambda_{\text{PV}j}$ refer to the time interval DT_j , while the equivalent system marginal cost λ_{ESS} to the examined period T as a whole. For comparison reasons the respective equivalent system marginal costs λ'_{TH} and λ'_{PV} , which refer to the time period T, are obtained as follows:

$$\lambda'_{\mathrm{TH}} \stackrel{\wedge}{=} \frac{1}{T} \cdot \sum_{j=1}^{M} \lambda_{\mathrm{TH}j} \cdot DT_{j}$$

$$\stackrel{DT_{j}=T/M}{\Longrightarrow} \lambda'_{\mathrm{TH}} = \frac{1}{M} \cdot \sum_{j=1}^{M} \lambda_{\mathrm{TH}j} \quad [\mathbb{C}/\mathrm{kWh}] \quad (31)$$

$$\lambda'_{\mathrm{PV}} \stackrel{\wedge}{=} \frac{1}{T} \cdot \sum_{j=1}^{M} \lambda_{\mathrm{PV}j} \cdot DT_{j}$$

$$\stackrel{DT_{j}=T/M}{\Longrightarrow} \lambda'_{\mathrm{PV}} = \frac{1}{M} \cdot \sum_{j=1}^{M} \lambda_{\mathrm{PV}j} \quad [\mathbb{C}/\mathrm{kWh}] \quad (32)$$

If no PVs are installed onboard then the respective equivalent system marginal cost λ'_{TH} becomes:

$$\lambda'_{\text{TH}} = \frac{\frac{1}{M} \cdot \sum_{j=1}^{M} P_{\text{D}j} + \sum_{i=1}^{N} \frac{b_i}{2 \cdot c_i}}{\sum_{i=1}^{N} \frac{1}{2 \cdot c_i}} \ [\pounds/\text{kWh}] \ (33)$$

It is safely concluded from eq. (30) and (33) that the respective equivalent system marginal cost λ_{ESS} corresponding to the use of PVs and ESS is always smaller than the equivalent system marginal cost λ'_{TH} .

In case of PVs installed onboard the respective equivalent system marginal cost λ'_{PV} [\notin /kWh] is obtained after series of calculations as follows:

$$\lambda'_{PV} = \frac{\frac{1}{M} \cdot \sum_{j=1}^{M} P_{Dj} + \sum_{i=1}^{N} \frac{b_i}{2 \cdot c_i} - \frac{\eta_{CV}}{\eta_{ESS-CV}} \cdot \frac{E_{ESS}}{T}}{\sum_{i=1}^{N} \frac{1}{2 \cdot c_i}}$$
(34)

It is concluded from eq. (30) and (34) that the equivalent system marginal cost λ_{ESS} is larger than the equivalent system marginal cost λ'_{PV} , as the performance coefficient, η_{CV} , of the PVs' dc/ac converter is larger than the performance coefficient η_{ESS-CV} of the PVs, the ESS and their dc/ac power converter. However, this would happen if the active power produced by the PVs was available at the proper time interval DT_j and it was enough to counterbalance ship electric load variance.

In fact, ship power system operation technical constraints are expected to render the equivalent system marginal cost λ_{ESS} smaller as the use of the ESS practically can reduce the peak load demand.

Without using ESS peak load demand smoothing cannot be ensured by only the PVs themselves. In the above analysis all thermal units onboard have been maintained in operation (operating with the minimum technical active power if necessary), while in reality some generating units could be disconnected when total load demand is below certain levels according to spinning reserve constraint.

5 Computational Algorithms for System Marginal Cost 5.1 General

Next, it is assumed that the fuel cost of the i^{th} generating unit, $F_{\text{THi}}(P_{\text{THi},j})$, is well fitted by a polynomial of third order of P_{THi} :

$$F_{\text{TH}i}\left(P_{\text{TH}i,j}\right) = a_i + b_i \cdot P_{\text{TH}i,j} + c_i \cdot P_{\text{TH}i,j}^2 + d_i \cdot P_{\text{TH}i,j}^3 \quad (35)$$

Where, a_i , b_i , c_i , d_i are the proper economic coefficients.

In this case system marginal cost, λ_{j} , is calculated as:

$$\lambda_j = b_i + 2 \cdot c_i \cdot P_{\text{TH}i,j} + 3 \cdot d_i \cdot P_{THi,j}^2$$
 for $i = 1, ..., N$ (36)

The generating level of the i^{th} unit can be calculated from eq. (36) as a function of the respective system marginal cost λ_i :

$$P_{\text{TH}_{i,j}} = \begin{cases} \frac{-c_i + \sqrt{c_i^2 + 3 \cdot d_i \cdot (\lambda_j - b_i)}}{3 \cdot d_i}, & d_i \neq 0\\ \frac{\lambda_j - b_i}{2 \cdot c_i}, & d_i = 0 \end{cases}$$
(37)

Classic Gauss-Seidel technique is used to obtain the arithmetic solution of the problem.

5.2 Optimal operation of ship electric power system equipped with PVs

More specifically, the determination of $\lambda_{PV,j}$ is achieved as following:

1) The produced active power by PVs P_{PVj} is determined by suitable forecasting tools or alternatively by the next simplified approach:

$$P_{\rm PV}(t) = P_{\rm maxPV} \cdot \sin\left(\frac{2 \cdot \pi \cdot (t - t_{sr})}{24}\right) \quad (38)$$

Where $t_{sr} \le t \le 24 - t_{sr}$, *t* is the sun time, t_{sr} is the sunrise sun time.

- 2) The active power P_{CVj} injected to the grid by the dc/ac converter is calculated by eq. (4).
- Suitable set of generators is chosen for each time interval so as to satisfy spinning reserve requirement. Hence, ship electric power generation system can cover demand load even

in case the largest generator/power supplier is suddenly shut down.

- 4) The remaining load, P'_{Dj} , is calculated taking into account the constraint of eq. (6).
- 5) $\lambda_{PVj}(0)$ and $\lambda_{PVj}(1)$ are initialized for the j^{th} time period.
- 6) Power produced by the thermal units $P_{\text{TH}i,j}$ is determined based on eq. (37).
- 7) Imbalance of the active power is calculated as:

$$\varepsilon^{(k)} = \sum_{j=1}^{N} P_{\text{TH}i,j}^{(k)} - P_{\text{D}j}^{\prime}$$
(39)

8) Proceed with the next iteration:

(40)

9) If *k*=1, step 6 is executed else process continues with step 10.

k = k + 1

- 10) If $|\varepsilon^{(k)}| \leq convergence$ limit then step 12 is executed else process continues with step 11.
- 11) Based on Newton-Raphson method the $\lambda_{PVj}^{(k)}$ is determined as:

$$\lambda_{\mathrm{PV}j}^{(k)} = \lambda_{\mathrm{PV}j}^{(k-1)} - \frac{\lambda_{\mathrm{PV}j}^{(k-1)} - \lambda_{\mathrm{PV}j}^{(k-2)}}{\varepsilon^{(k-1)} - \varepsilon^{(k-2)}} \cdot \varepsilon^{(k-1)}$$
(41)

Afterwards, step 6 is executed.

12) After the determination of λ_{PVj} and the generating levels of the units, $P_{THi,j}$, the *N* inequalities deriving from the technical constraints are checked.

The calculation of system marginal cost in case of using only the thermal units, λ_{TH_j} , is realized by ignoring steps (1) to (4) and using demand load P_{D_j} .

5.3 Optimal operation of ship electric power system with PVs and ESS

The determination of w and λ_j can be done in a similar way with that of paragraph 5.2 with some necessary modifications due to the presence of ESS. It should be noted that the theoretical analysis of paragraph 4 is valid, if the same power plants are operated during time period *T*. Practically, it is easily concluded that "*peak shaving*" should be applied at the time intervals DT_j with the highest load demand. Peak shaving constitutes the generators' loading transfer between time intervals so as more economic operation is achieved. It requires precise load forecasting for the time period *T*, so as the energy E_{ESS} provided by the ESS can cover the highest load.

Consequently, the respective algorithm becomes: 1) The produced active power by PVs, P_{PVj} , is determined by the user or calculated typically by eq. (38). The energy produced by the PVs E_{PV} is calculated by eq. (7), while the energy produced by the ESS, E_{ESS} , is calculated by the eq. (8). The constraints in inequality (10) are satisfied practically in case of the complete charge of ESS for the first time and afterwards the cycle of recharge and charge is performed for each typical period T. This leads to a larger and more expensive ESS, but it can be used as an emergency back-up supply for vital loads increasing system reliability.



Fig. 5. Chronological load curve and load demand duration curve before and after the "peak shaving".

2) The load demand duration curve is constructed by using the chronological load demand curve for

the typical period *T* by ranking load in decreasing order. Then peak shaving is applied by the use of the energy produced by the ESS $P_{\text{ESS}j}$, as it is presented in Fig. 5. Then the remaining load demand covered by the thermal power plants $P_{Dj}^{\prime\prime}$ is given by:

$$P_{\rm Dj}^{\prime\prime} = \sum_{i=1}^{N} P_{\rm TH_{i,j}}^{\prime\prime} = P_{\rm Dj} - P_{\rm ESSj}$$
(42)

Eq. (42) is subject to the inequality (9).

- 3) The new chronological load curve is reconstructed.
- 4) The number of the operating generators for each time interval is chosen as to satisfy spinning reserve requirement.
- 5) $\lambda_{\text{ESS}j}^{(0)}$ and $\lambda_{\text{ESS}j}^{(1)}$ are initialized for the *j*th time period.
- 6) The generating levels of the units $P_{\text{TH}i,j}$ are determined based on eq. (37).
- 7) The active power imbalance $\varepsilon^{(k)}$ is calculated:

$$\varepsilon^{(k)} = \sum_{j=1}^{N} P_{\text{TH}_{i,j}}^{\prime\prime}{}^{(k)} - P_{\text{D}_{j}}^{\prime\prime}$$
(43)

- 8) Proceed with the next iteration (eq. (40)).
- If k=1, step 6 is executed else continue with step 10.
- 10) Convergence check: if $|\varepsilon^{(k)}| \le$ convergence limit then step 12 is executed else we continue with step 11.
- 11) $\lambda_{\text{ESS}j}^{(k)}$ is determined based on Newton-Raphson method (similarly to eq. (41)). Afterwards, step 6 is executed.
- 12) After the determination of $\lambda_{\text{ESS}j}$ and the generating levels of the units, $P_{\text{TH}i,j}$, the *N* inequalities from the technical constraints are checked.

6 Application of the Proposed Methods

6.1 Case study

The developed methodology is applied to the typical daily load demand of a cruiser, which is presented in Fig. 6 with a 15-minutes time step, maximum load demand of 11900 kW, average load of 7770 kW and minimum load of 3000 kW, approximately.

The cruiser is equipped with 8 generators of nominal active power of 2 MW with 0.8 inductive power factor. A simple thermal-electric power system without transmission losses is assumed with the following fuel cost functions and technical minimum/maximum power production for the eight power plants:

$$F_{\text{TH},1}(P_{\text{TH},1}) = 18.1 + 0.058 \cdot P_{\text{TH},1} + 4 \cdot 10^{-5} \cdot P_{\text{TH},1}^2$$

$$\begin{split} F_{\text{TH},2}\left(P_{\text{TH},2}\right) &= 18.0 + 0.0585 \cdot P_{\text{TH},2} + 4 \cdot 10^{-5} \cdot P_{\text{TH},2}^2 \\ F_{\text{TH},3}\left(P_{\text{TH},3}\right) &= 17.9 + 0.059 \cdot P_{\text{TH},3} + 4 \cdot 10^{-5} \cdot P_{\text{TH},3}^2 \\ F_{\text{TH},4}\left(P_{\text{TH},4}\right) &= 18.1 + 0.058 \cdot P_{\text{TH},4} + 4 \cdot 10^{-5} \cdot P_{\text{TH},4}^2 \\ F_{\text{TH},5}\left(P_{\text{TH},5}\right) &= 18.15 + 0.0582 \cdot P_{\text{TH},5} + 3.98 \cdot 10^{-5} \cdot P_{\text{TH},5}^2 \\ F_{\text{TH},6}\left(P_{\text{TH},6}\right) &= 18.0 + 0.0584 \cdot P_{\text{TH},6} + 4.02 \cdot 10^{-5} \cdot P_{\text{TH},6}^2 \\ F_{\text{TH},7}\left(P_{\text{TH},7}\right) &= 17.95 + 0.0592 \cdot P_{\text{TH},7} + 3.99 \cdot 10^{-5} \cdot P_{\text{TH},7}^2 \\ F_{\text{TH},8}\left(P_{\text{TH},8}\right) &= 18.1 + 0.0581 \cdot P_{\text{TH},8} + 4.01 \cdot 10^{-5} \cdot P_{\text{TH},8}^2 \\ &\qquad 200 \leq P_{\text{TH},1}, \dots, P_{\text{TH},8} \leq 2000 \text{ [kW]} \end{split}$$

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Where, the measurement unit of the fuel cost functions is the monetary unit (m.u.) per hour and P_{THi} is measured in kW.

The cruiser is assumed to comprise photovoltaic panels of 600kW, which can produced for a typical day of 12-h sunshine 4584kWh. This number is obtained by integrating the respective curve of Fig. 2 and based on eq.(38). The performance coefficient $\eta_{\rm CV}$ of the dc/ac converter is considered 0.9, while the total coefficient of ESS, PVs and their dc/ac converter, $\eta_{\rm ESS-CV}$, is assumed 0.83.



Fig. 6 Typical daily load chronological demand curve.

6.2 Thermal power system

The execution of the classical economic dispatch algorithm (section 5.1 with the $P_{PV}=0$) converged after two iterations. The results for the number of thermal units on operation, the thermal units' power dispatch and the system marginal cost are presented in Figs 6 to 8, respectively. The total operation cost is 24025 m.u. per day while the total electric energy produced by the thermal units is 186475 kWh.

6.3 Thermal power system with PVs

The initial load demand $P_{\rm D}$, the power produced by the PVs $P_{\rm PV}$ and the load demand $P'_{\rm D}$, which is covered by the thermal plants, are presented in Fig. 9. The execution of the classical economic dispatch algorithm converged after two iterations. The results for the number of thermal units in operation, the thermal units' power dispatch and the system marginal cost are presented in Figs 10 up to 12, respectively. The total operation cost is 23414 m.u. per day while the total electric energy produced by the thermal power plants and by PVs is 182351 kWh and 4124 kWh, respectively.

If the photovoltaic panels are used then the number of power plants in operation is decreased by one for 5.21% of the time period (1.25 h per day). Also, the mean system marginal cost decreases by 0.73% and the total cost by 2.54%.



Fig. 6. Number of thermal units in operation (ship electric system comprising only thermal units).



Fig. 7. Thermal units power production (ship electric systems comprising only thermal units).



Fig. 8. System marginal cost (ship electric system comprising only thermal units).



Fig. 9. Typical daily total chronological load demand curve for a cruiser with and without PVs and power produced by PVs.



Fig. 10. Number of thermal units in operation (ship electric system comprising thermal units and PVs).



Fig. 11. Thermal units power production (ship electric systems comprising thermal units and PVs).

6.4 Thermal power system with PVs and ESS The total load demand duration curve is constructed from the chronological load curve of Fig. 6 and it is presented in Fig. 13. Next, the peak shaving is applied by ESS exploitation. In this study, ESS rated power is 600 kW (the same with the PVs) while its energy storage capacity is 4125 kWh (it corresponds to the maximum energy production by PVs).



Fig. 12. System marginal cost (ship electric system comprising thermal units and PVs).

The initial load demand $P_{\rm D}$, the power production by ESSs $P_{\rm ESS}$ and the load demand $P''_{\rm D}$, which is covered only by the thermal plants, are presented in Fig. 14.

The execution of the economic dispatch algorithm converged after two iterations. The results for the number of thermal units in operation, the thermal units' power dispatch and the system marginal cost are presented in Fig. 15 up to 17, respectively. The total operation cost is 23413 m.u. per day, while the total electric energy produced by the thermal units and by PVs and ESS is 182674 kWh and 3801 kWh, respectively.

If the PVs and ESS are used, the number of thermal units in operation is decreased by one for 3.21% of the time period (0.75 h per day), the mean system marginal cost is decreased by 0.82% and the total cost by 2.55%.

In case of $\eta_{\text{ESS-CV}}=0.8$ the total operation cost is 23437 m.u. per day, while the total electric energy produced by the thermal power plants is 182810 kWh and by PVs and ESS is 3665 kWh. The number of power plants in operation is decreased by one for the 3.21% of the time period (0.75 h per day), the mean system marginal cost is decreased by 0.78% and the total cost by 2.45%.



Fig. 13. Typical daily total load demand duration curve of a cruiser (with and without PVs-ESS).



Fig. 14. Typical total chronological load demand curve of a cruiser (with and without PVs-ESS) and power produced by PVs-ESS.



Fig. 15. Number of thermal units in operation (ship electric system comprising thermal units, PVs and ESS).



Fig. 16. Power produced by the thermal units (ship electric system comprising thermal units, PVs and ESS).

The last results indicate that the performance coefficients of PVs and of PVs-ESS as well as the time synchronization of PVs power production with the peak of the chronological load demand curve play a key role. In both cases that either PVs or PVs and ESS are used the average system marginal cost and the total daily cost have been decreased compared with the basic scenario of using only thermal units.



Fig. 17. System marginal cost (ship electric system comprising thermal units, PVs and ESS).

7 Conclusions

In this paper the operation of a ship electric power system equipped with PVs and ESS is analyzed from the economical point of view. Analytic formulas of system marginal cost of the produced energy are obtained for three case studies:

- a) Ship electric power generation system comprising only thermal units.
- b) Ship electric power generation system comprising thermal units and PVs.
- c) Ship electric power generation system comprising thermal units, PVs and ESS.

The energy and power balances are taken into account for a specific time period of ship electric power system operation.

From the comparison of the obtained results it is concluded that system marginal cost is smaller in case of PVs installed onboard without using ESS. However, this would be valid only if the active power produced by the PVs was available at the time interval DT_i and it was enough to counterbalance load demand variance. However, ship power system operation technical constraints are expected to render the equivalent system marginal cost λ_{ESS} , practically smaller as the use of the ESS can safely smooth the load demand variation. Consequently, peak shaving is exploited in the application of the proposed methods for the case study of a typical cruiser, where the performance coefficients of PVs system and of PVs-ESS system have a key role

The proposed method is a convenient tool, as it leads to simple analytic mathematical relations, which could be exploited in different ways for the technical-economical assessment of ship electric systems employing energy storage and green technology. References:

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