An Accurate 3D Stress State for a Correct Evaluation of Failure Indexes in Aerospace Structures

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Abstract: This paper proposes a three-dimensional exact shell model for the correct evaluation of the 3D stress state of multilayered composite and sandwich structures for aerospace applications. The model is based on the equilibrium equations written in general orthogonal curvilinear coordinates which are valid for spherical shells. Such equations automatically degenerate in those for cylindrical shells and plates allowing a general and unified formulation. The equations are solved in closed form supposing simply supported structures and applied harmonic loads. The partial differential equations are solved by means of the exponential matrix method and a layer-wise approach is considered for the multilayered plates and shells. Layer-wise approaches allow the zigzag form of the displacement field in the thickness direction. In-plane stresses can be discontinuous at each layer interface. On the contrary, transverse stresses must be continuous at each layer interface for equilibrium reasons. Displacements must be continuous in the thickness direction for compatibility reasons. Therefore, displacements and transverse shear/normal stresses are continuous functions in the thickness direction. Moreover, displacements and transverse stresses have discontinuous first derivatives in the thickness direction with correspondence to each interface because the mechanical properties change in each layer (zigzag effect). The fulfillment of all these requirements is a crucial point in the development of the present 3D shell model. The obtained 3D stress state allows the determination of failure parameters by means of several mathematical models such as the Von Mises's Criterion, the Maximum Stress Criterion, the Tsai-Wu's Criterion, the Tsai-Hill's Criterion, the Hoffman's Criterion, the Hashin's Criterion, the Puck's criterion and the LaRC04 criterion.

Key–Words: 3D exact shell model, failure indexes, composite and multilayered structures, 3D stress state, layer-wise approach, zigzag effects, interlaminar continuity.

1 Introduction

Multilayered composite and sandwich structures have an extensive application range in aerospace, marine and automotive fields due to their high values of the stiffness/weight ratio and the strength/weight ratio. For this reason, the accurate prediction of failure parameters for composite and sandwich structures plays a fundamental role. Research activity has focused its attention on the determination of failure parameters by means of several mathematical models. Both glass and carbon fiber reinforced materials in cross-ply and angle-ply configurations can be also investigated. The present work proposes a three-dimensional exact shell model [1]-[8] which is able to define a complete and exhaustive 3D stress state useful to calculate correct failure parameters for multilayered composite and sandwich plates and shells.

The Tresca's criterion and the Von Mises' criterion are the most known ones for the failure investigation of isotropic structures [9]. The Tresca's criterion is the most conservative one, but it requests the introduction of stresses in the principal axes. The Von Mises' criterion is invariant with respect to the reference system. The most important failure criteria for composite structures can be found in [10]-[12]. The fundamental features, the advantages and disadvantages have been analyzed at the World Wide Failure Exercise organised by Hinton and Soden [13]. An exhaustive comparison between the most important failure criteria for composite structures has been proposed in [14] and [15]. The criteria used in these two interesting works were the max stress, Tsai-Wu's, Tsai-Hill's, Hoffman's and Hashin's criteria. Further interesting criteria which are worthy to be mentioned are the LaRC02 criterion developed by NASA [16] and the Puck's criterion widely used by the European Space Agency [17]-[19].

The three-dimensional shell model here proposed is based on the three-dimensional equilibrium equations written in general orthogonal curvilinear coordinates valid for spherical shells. Such equations automatically degenerate in those for cylinders, cylindrical shell panels and plates by means of opportune considerations made on the radii of curvature. The equations are exactly solved considering simply supported structures and harmonic forms for applied loads. The second order differential equations are reduced to first order differential equations, and then they are solved by means of the exponential matrix method. A layer-wise approach is applied for the considered multilayered plates and shells. The proposed 3D shell model is a generalization of the models already proposed by Messina [20] for plates in orthogonal rectilinear coordinates, by Soldatos and Ye [21] for cylinders in cylindrical coordinates and by Fan and Zhang [22] for doubly-curved shells. The use of a layer-wise approach and the 3D point of view allow the zigzag form of the displacement field in the thickness direction. The equilibrium conditions can be directly imposed in the model in order to obtain transverse stresses which are continuous at each layer interface. The compatibility conditions are directly introduced in the model in order to have displacements which are continuous in the thickness direction. Therefore, displacements and transverse shear/normal stresses are continuous functions in the thickness direction. Moreover, displacements and transverse stresses have discontinuous first derivatives in the thickness direction with correspondence to each interface because the mechanical properties change in each layer of the multilayered structure (zigzag effect). The fulfillment of all these requirements is a crucial point in the development of the present 3D shell model in order to obtain a correct 3D stress state which will be a fundamental input in the correct evaluation of the most important failure parameters used in the aerospace field.

The present paper is divided in two main parts, the first part presents the most important failure parameters in order to understand the main inputs to give for their correct evaluations. The second part shows the main details of the proposed 3D shell model and some results about the static analysis of multilayered structures in order to propose the 3D stress state given by the developed model. Future developments will be the introduction of the obtained 3D stress state in the main failure parameters proposed in the literature in order to have an exhaustive comparison between the most important failure criteria.

2 Failure parameters

A possible classification of failure parameters can be made between classical criteria for homogeneous materials and advanced criteria for composite materials. In classical criteria we can include those criteria valid for continuous, homogeneous and isotropic materials such as the Rankine-Navier's criterion, the Tresca's criterion, the Coulomb-Mohr's criterion and the Von Mises' criterion. In advanced criteria we include those criteria valid for advanced composite and sandwich structures such as the Tsai-Hill's criterion, the Hoffman's criterion, the Tsai-Wu's criterion, the Hashin's criterion, the Puck's criterion and the LaRC02 criterion.

Rankine-Navier's criterion

It is the maximum stress criterion and it is usually used for fragile materials. The failure occurs when the maximum or minimum principal stress reaches the limit [9]. The equivalent stress is given by:

$$\sigma_e = max\{\sigma_1, r\sigma_2\}\,, \tag{1}$$

where σ_1 is the maximum principal stress ($\sigma_1 > 0$) and σ_2 is the minimum principal stress ($\sigma_2 < 0$). The parameter r is the ratio between the ultimate tensile stress σ_t and the ultimate compressive stress σ_c:

$$r = \frac{\sigma_t}{\sigma_c}\,. \tag{2}$$

Tresca's criterion

It is also known as the maximum shear stress criterion and it was originally proposed by Coulomb in 1773 and then it was developed by Tresca in 1868 during his studies about the plastic deformations [9]. This criterion can be applied to ductile materials and the failure occurs when the maximum shear stress reaches the yield value for a specimen subjected to a tensile state. The maximum yield shear stress is:

$$\tau_s = \frac{1}{2}\sigma_s\,. \tag{3}$$

The maximum shear stresses in the principal planes are:

$$\tau_{ij} = \frac{1}{2}(\sigma_i - \sigma_j)\,, \quad i,j = 1,2,3\,, \quad i \neq j\,. \tag{4}$$

The equivalent stress is given by:

$$\sigma_e = max\left\{|\sigma_i - \sigma_j|\right\}\,. \tag{5}$$

Coulomb-Mohr's criterion

Some ductile materials have a different behavior in tensile and in compression state. This criterion is able to consider this no-symmetric behavior [9]. For this reason, the equivalent stress is:

$$\sigma_e = max\{\sigma_1, r\sigma_2, \sigma_1 + r\sigma_2\}\,, \tag{6}$$

the Eq.(6) is modified for fragile materials as:

$$\sigma_e = \{\sigma_1, r\sigma_2, (1+r)\sigma_1 + r\sigma_2\}\,. \tag{7}$$

The meaning of r, σ_1 and σ_2 is the same already seen for the Rankine-Navier's criterion.

Von Mises' criterion

The Von Mises' criterion is also known as the criterion of maximum distortion energy. It was developed by Huber in 1904, Von Mises in 1913 and Hencky in 1925. This criterion is valid for ductile materials and it is based on the feature that a ductile material has a greater strength when subject to a hydrostatic state in place of a tensile state [9]. The energy for unite of volume given by the principal stresses σ_1, σ_2 and σ_3 is:

$$U_{tot} = \frac{1}{2E}[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\nu(\sigma_1\sigma_2+\sigma_1\sigma_3+\sigma_2\sigma_3)]\,, \tag{8}$$

the equivalent stress can be obtained from the energy given in Eq.(8):

$$\sigma_e = \sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2-(\sigma_1\sigma_2+\sigma_1\sigma_3+\sigma_2\sigma_3)}\,. \tag{9}$$

Tsai-Hill's criterion

The Tsai-Hill's criterion was proposed by Tsai in 1966 starting from that by Hill in 1950. It was developed for general orthotropic materials and it can be considered as a generalization of the Von Mises' theory [10]-[12]. However, in the present case the total deformation energy is considered in place of the distortion energy. This feature is due to the fact that in an anisotropic material, it is not possible to separate the distortion energy from the energy connected with the volume variation. For the failure index (FI), Hill proposed:

$$\begin{aligned} FI &= A(\sigma_1-\sigma_2)^2 + B(\sigma_2-\sigma_3)^2 \\ &+C(\sigma_3-\sigma_1)^2 + D\sigma_4^2 + E\sigma_5^2 + F\sigma_6^2\,, \end{aligned} \tag{10}$$

where parameters *A, B, C, D, E, F* can be experimentally calibrated. These parameters have been defined by Tsai. The weak point of the method is that it has been developed starting from a theory valid for symmetric ductile materials.

Hoffman's criterion

The limitations of the Tsai-Hill's criterion have been overcome by Hoffman in 1967 which added the linear terms in Eq.(10):

$$\begin{aligned} FI &= H_1(\sigma_1-\sigma_2)^2 + H_2(\sigma_2-\sigma_3)^2 \\ &+H_3(\sigma_3-\sigma_1)^2 + H_4\sigma_1 + H_5\sigma_2 + H_6\sigma_3 + \\ &+H_7\sigma_4^2 + H_8\sigma_5^2 + H_9\sigma_6^2\,. \end{aligned} \tag{11}$$

In the Tsai-Hill's criterion the parameters were 6, in the Hoffman's criterion they are 9. In general, the Hoffman's criterion is more accurate than the Tsai-Hill's criterion. The two criteria are coincident in the case of tensile strength which is equal to compressive strength [10]-[12].

Tsai-Wu's criterion

Tsai and Wu developed their criterion in 1971 using a quadratic relation between the stresses in order to be more general as possible [10]-[12]. In this case, 12 parameters must be defined for the evaluation of the following failure index (FI):

$$FI = \sum_{i=1}^{3} F_i\sigma_i + \sum_{i=1}^{6} F_{ii}\sigma_i^2 + \sum_{i=1}^{2}\sum_{j=i+1}^{3} F_{ij}\sigma_i\sigma_j\,. \tag{12}$$

Hashin's criterion

In 1980, Hashin developed a criterion valid for composite materials because it is able to differentiate between fiber failure and matrix failure, and also between tensile and compressive stress status. The formulation is based on quadratic relations which are a good compromise between the simplicity of the solution and its accuracy [14], [15].

- Tensile fiber (f) failure mode, ($\sigma_1 \geq 0$)

$$FI_f = \left(\frac{\sigma_1}{X_T}\right)^2 + \left(\frac{\sigma_4}{R}\right)^2 + \left(\frac{\sigma_6}{T}\right)^2\,. \tag{13}$$

- Compressive fiber (f) failure mode, ($\sigma_1 < 0$)

$$FI_f = -\frac{\sigma_1}{X_C}\,. \tag{14}$$

- Tensile matrix (m) failure mode, ($\sigma_2 + \sigma_3 \geq 0$)

$$\begin{aligned} FI_m &= \left(\frac{\sigma_2}{Y_T} + \frac{\sigma_3}{Z_T}\right)^2 \\ &+\frac{1}{S^2}(\sigma_5^2 - \sigma_2\sigma_3) + \left(\frac{\sigma_4}{R}\right)^2 + \left(\frac{\sigma_6}{T}\right)^2\,. \end{aligned} \tag{15}$$

- Compressive matrix (m) failure mode, ($\sigma_2 + \sigma_3 < 0$)

$$FI_m = \frac{1}{Y_C}\left[\left(\frac{Y_C}{2S}\right)^2 - 1\right]\sigma_2 \qquad (16)$$
$$+\frac{1}{Z_C}\left[\left(\frac{Z_C}{2S}\right)^2 - 1\right]\sigma_3 +$$
$$\frac{1}{4S^2}(\sigma_2+\sigma_3)^2 + \frac{1}{S^2}(\sigma_5^2 - \sigma_2\sigma_3)$$
$$+\left(\frac{\sigma_4}{R}\right)^2 + \left(\frac{\sigma_6}{T}\right)^2 .$$

The meaning of coefficients in Eqs.(13)-(16) is proposed in [14] and [15]. This criterion is an important step if compared with the other presented criteria because it is based on a new approach of the phenomenon. It shows some difficulties in the case of predominant matrix behavior.

Puck's criterion

The Puck's criterion was formulated by Puck and Schürmann in 1998 and it can be considered as a generalization of the Coulomb-Mohr's criterion. In particular, it is based on the hypothesis that the failure is associated only to stresses acting in the plane where failure happens. As already seen in the Hashin's criterion, there is a distinction between the fibre failure and the matrix failure. The first one is here indicated as FF (Fiber Failure), the second one is here indicated as IFF (Inter-Fiber Failure). There is a good correlation with the experimental tests but a large number of parameters is requested, and these parameters are often not easy to be identified. More details about the Puck's criterion can be found in [17]-[19].

LaRC02 criterion

The LaRC02 criterion has been proposed in 2002 by the NASA research centre of Langley after the World Wide Failure Exercise (WWFE). The WWFE was useful to analyse the state of the art about the failure parameters for composite materials. A deep study was conducted about the advantages and disadvantages of failure parameters developed for composite materials. For this reason, NASA decided to develop a new failure parameter with a maximum efficiency for composite materials. The LaRC02 criterion is a further development and refinement of Hashin's and Puck's criteria. Further details about the theory and the formulation can be found in [16]. The great capability of this model is strictly connected with the physical bases used for its development.

2.1 Principal stresses

In order to use some of the failure indexes proposed in the previous parts, it is sometimes fundamental to find the principal stresses to introduce in the above equations. The stress state will be defined by means of the 3D shell model presented in the next section. The six stress components $\sigma_{\alpha\alpha}$, $\sigma_{\beta\beta}$, $\sigma_{\alpha\beta}$, $\sigma_{\alpha z}$, $\sigma_{\beta z}$ and σ_{zz} in orthogonal curvilinear coordinates will be used to define the principal stresses in accordance with the following procedure.

In order to calculate the principal stresses it is necessary to solve the eigenvalue problem for the stress tensor. In particular, it is fundamental the solution of the following equation:

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0\,, \qquad (17)$$

where the invariants are calculated as

$$I_1 = \sigma_{\alpha\alpha} + \sigma_{\beta\beta} + \sigma_{zz}\,, \qquad (18)$$
$$I_2 = \sigma_{\alpha\alpha}\sigma_{\beta\beta} + \sigma_{\alpha\alpha}\sigma_{zz} + \sigma_{\beta\beta}\sigma_{zz} \qquad (19)$$
$$-\sigma_{\alpha\beta}^2 - \sigma_{\alpha z}^2 - \sigma_{\beta z}^2\,,$$
$$I_3 = \sigma_{\alpha\alpha}\sigma_{\beta\beta}\sigma_{zz} + 2\sigma_{\alpha\beta}\sigma_{\alpha z}\sigma_{\beta z} \qquad (20)$$
$$-\sigma_{\alpha\alpha}\sigma_{\beta z}^2 - \sigma_{\beta\beta}\sigma_{\alpha z}^2 - \sigma_{zz}\sigma_{\alpha\beta}^2\,.$$

I_1, I_2 e I_3 are invariants because they do not depend on the used reference system adopted in the point.

3 3D shell model

The equilibrium equations are written using a general orthogonal curvilinear coordinate system (α, β, z) valid for plates and shells with constant radii of curvature. These equations are solved in exact form by means of simply supported boundary conditions, harmonic form for applied loads and the use of the exponential matrix method for the solution of the differential equations in z. The three differential equations written for the static analysis of multilayered spherical shells (embedding N_L physical layers) with constant radii of curvature R_α and R_β are:

$$H_\beta\frac{\partial\sigma_{\alpha\alpha}^k}{\partial\alpha} + H_\alpha\frac{\partial\sigma_{\alpha\beta}^k}{\partial\beta} + H_\alpha H_\beta\frac{\partial\sigma_{\alpha z}^k}{\partial z} + \qquad (21)$$
$$\left(\frac{2H_\beta}{R_\alpha} + \frac{H_\alpha}{R_\beta}\right)\sigma_{\alpha z}^k = 0,$$
$$H_\beta\frac{\partial\sigma_{\alpha\beta}^k}{\partial\alpha} + H_\alpha\frac{\partial\sigma_{\beta\beta}^k}{\partial\beta} + H_\alpha H_\beta\frac{\partial\sigma_{\beta z}^k}{\partial z} + \qquad (22)$$
$$\left(\frac{2H_\alpha}{R_\beta} + \frac{H_\beta}{R_\alpha}\right)\sigma_{\beta z}^k = 0,$$
$$H_\beta\frac{\partial\sigma_{\alpha z}^k}{\partial\alpha} + H_\alpha\frac{\partial\sigma_{\beta z}^k}{\partial\beta} + H_\alpha H_\beta\frac{\partial\sigma_{zz}^k}{\partial z} - \qquad (23)$$

$$\frac{H_\beta}{R_\alpha}\sigma^k_{\alpha\alpha} - \frac{H_\alpha}{R_\beta}\sigma^k_{\beta\beta} + \left(\frac{H_\beta}{R_\alpha} + \frac{H_\alpha}{R_\beta}\right)\sigma^k_{zz} = 0,$$

in above equations, $(\sigma^k_{\alpha\alpha}, \sigma^k_{\beta\beta}, \sigma^k_{zz}, \sigma^k_{\beta z}, \sigma^k_{\alpha z}, \sigma^k_{\alpha\beta})$ are the six stress components. Displacements u^k, v^k and w^k are considered through α, β and z directions, respectively. Each quantity has a dependence from the k physical layer. R_α and R_β are the radii of curvature evaluated in the mid-surface Ω_0 of the whole multilayered structure. Parametric coefficients H_α and H_β for shells with constant radii of curvature continuously vary through the thickness direction z of the multilayered structure:

$$H_\alpha = (1 + \frac{z}{R_\alpha}), H_\beta = (1 + \frac{z}{R_\beta}), H_z = 1. \quad (24)$$

Shells and plates are considered as simply supported. Therefore, the three displacement components can be written in harmonic form:

$$u^j(\alpha, \beta, z, t) = U^j(z)cos(\bar{\alpha}\alpha)sin(\bar{\beta}\beta), \quad (25)$$
$$v^j(\alpha, \beta, z, t) = V^j(z)sin(\bar{\alpha}\alpha)cos(\bar{\beta}\beta), \quad (26)$$
$$w^j(\alpha, \beta, z, t) = W^j(z)sin(\bar{\alpha}\alpha)sin(\bar{\beta}\beta). \quad (27)$$

$U^j(z)$, $V^j(z)$ and $W^j(z)$ are the displacement amplitudes in α, β and z directions, respectively. $\bar{\alpha} = \frac{m\pi}{a}$ and $\bar{\beta} = \frac{n\pi}{b}$, where m and n are the half-wave numbers and a and b are the shell dimensions in α and β directions, respectively (evaluated at the mid-surface Ω_0). j indicates the mathematical layers used to approximate the curvatures and/or the functionally graded laws in each k physical layer.

The analyzed plates and shells have simply supported edges and they can be loaded at the top and/or at the bottom of the whole laminated structure using the following conditions:

$$\sigma_{zz} = p_z\,, \quad \sigma_{\alpha z} = p_\alpha\,, \quad \sigma_{\beta z} = p_\beta \quad (28)$$
$$\text{for} \quad z = -h/2, +h/2\,,$$
$$w = v = 0,\ \sigma_{\alpha\alpha} = 0 \quad (29)$$
$$\text{for} \quad \alpha = 0, a\,,$$
$$w = u = 0,\ \sigma_{\beta\beta} = 0 \quad (30)$$
$$\text{for} \quad \beta = 0, b\,,$$

p_z, p_α and p_β are harmonic mechanical loads that can be applied at the top or at the bottom of the structure in z, α and β direction, respectively:

$$p^j_\alpha(\alpha, \beta, z) = P^j_\alpha(z)cos(\bar{\alpha}\alpha)sin(\bar{\beta}\beta)\ , \quad (31)$$
$$p^j_\beta(\alpha, \beta, z) = P^j_\beta(z)sin(\bar{\alpha}\alpha)cos(\bar{\beta}\beta)\ , \quad (32)$$
$$p^j_z(\alpha, \beta, z) = P^j_z(z)sin(\bar{\alpha}\alpha)sin(\bar{\beta}\beta)\ , \quad (33)$$

P^j_α, P^j_β and P^j_z indicate the load amplitudes.

Via the substitution of harmonic forms of displacements and loads and the use of the exponential matrix method [1]-[8], it is possible to obtain a final linear algebraic system. From the solution of this system, the three displacement components and their first derivatives in z can be calculated. These variables allow to obtain the strain components by means of the geometrical relations and the stress components by means of the constitutive equations. In order to approximate the curvature terms, the structures have been divided in M=100 mathematical layers and an order of expansion N=3 has been set for the exponential matrix calculation.

An example is here given to demonstrate the great capability of such a method. A sandwich cylindrical shell panel with two external isotropic skins and an internal soft core is considered. The radii of curvature are $R_\alpha = 10m$ and $R_\beta = \infty$, the dimensions are $a = \frac{\pi}{3}R_\alpha$ and $b = 20m$. The two external skins have thickness $h_1 = h_3 = 0.1h$ where h is the total thickness. The internal soft core has thickness $h_2 = 0.8h$. The load is applied in z direction at the top with amplitude $P_z = 1Pa$ and half-wave numbers $m = n = 1$. The external skins have Young modulus $E = 73GPa$ and Poisson ratio $\nu = 0.3$. The internal core has Young modulus $E = 180MPa$ and Poisson ratio $\nu = 0.37$. The structure is moderately thick with a thickness ratio $R_\alpha/h = 10$. The results are given in Figures 1-6 in terms of the following no-dimensional stress amplitudes:

$$(\bar{\sigma}_{\alpha\alpha}, \bar{\sigma}_{\beta\beta}, \bar{\sigma}_{\alpha\beta}) = \frac{10^3(\sigma_{\alpha\alpha}, \sigma_{\beta\beta}, \sigma_{\alpha\beta})}{P_z(R_\alpha/h)^2}\,, \quad (34)$$

$$(\bar{\sigma}_{\alpha z}, \bar{\sigma}_{\beta z}) = \frac{10^3(\sigma_{\alpha z}, \sigma_{\beta z})}{P_z(R_\alpha/h)}\,, \quad \bar{\sigma}_{zz} = \sigma_{zz}\,. \quad (35)$$

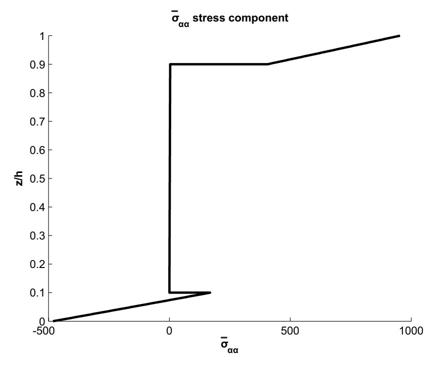

Figure 1: No-dimensional stress component $\bar{\sigma}_{\alpha\alpha}$ through the thickness of a sandwich cylindrical shell.

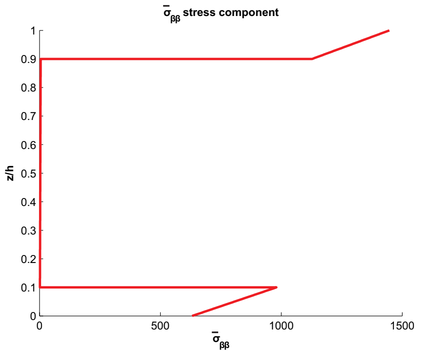

Figure 2: No-dimensional stress component $\bar{\sigma}_{\beta\beta}$ through the thickness of a sandwich cylindrical shell.

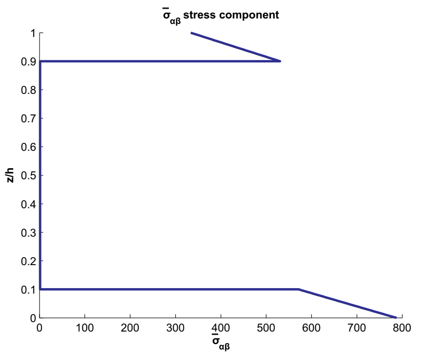

Figure 3: No-dimensional stress component $\bar{\sigma}_{\alpha\beta}$ through the thickness of a sandwich cylindrical shell.

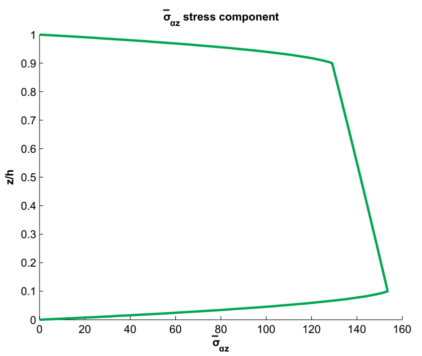

Figure 4: No-dimensional stress component $\bar{\sigma}_{\alpha z}$ through the thickness of a sandwich cylindrical shell.

Figures 1-6 clearly demonstrate the 3D capability of the proposed shell model to capture the stress state of a multilayered structure. Figures 1 and 2 give the in-plane normal stresses, the typical zigzag form is shown due to the presence of the external skins and the internal soft core. These stresses can be discontinuous at the interfaces. Similar considerations can be

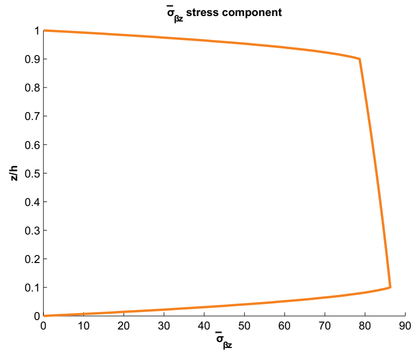

Figure 5: No-dimensional stress component $\bar{\sigma}_{\beta z}$ through the thickness of a sandwich cylindrical shell.

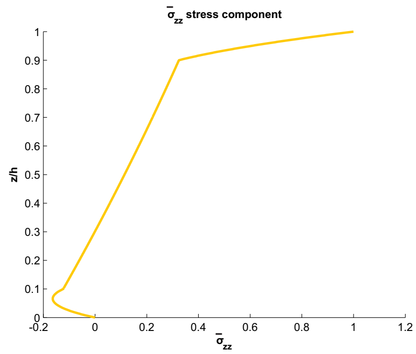

Figure 6: No-dimensional stress component $\bar{\sigma}_{zz}$ through the thickness of a sandwich cylindrical shell.

made for the in-plane shear stress in Figure 3. Figures 4 and 5 propose the transverse shear stresses, these quantities are continuous at each interface because the equilibrium conditions have been successfully imposed in the 3D shell model. The loading conditions at the external surfaces are correctly determined ($P_\alpha = P_\beta = 0$ at the top and bottom surfaces). Figure 6 shows the transverse normal stress through the thickness. It is continuous at each interface and it follows the boundary loading conditions ($P_z = 1Pa$ at the top and $P_z = 0$ at the bottom).

For the sake of brevity, further results will be proposed at the conference. First of all, an exhaustive validation of the proposed 3D shell model will be given. The correct values for the number M of mathematical layers and the order of expansion N for the exponential matrix will be determined by means of comparisons with further results in the literature. New benchmarks will be considered including plate, cylinder, cylindrical shell and spherical shell geometries. Each geometry can be single- or multi-layered including composite materials and sandwich configurations. For each proposed case, the 3D shell model will give the opportune stress state in order to use the appropriate failure parameter for the correct failure analysis of the investigated structure.

4 Conclusions and future developments

A general 3D shell model has been proposed in this work. The model is able to analyze different geometries embedding several isotropic, orthotropic and composite layers. For each simply supported structure, a correct and accurate 3D stress state can be determined in terms of the six stress components in the structural reference system. Such components can be used to calculate the principle stresses to determine the most appropriate failure parameters for a refined and accurate failure analysis of advanced structures. Plates, cylinders, cylindrical and spherical shell panels will be analyzed in single- or multi-layered configuration determining an accurate failure analysis in the cases of isotropic, orthotropic and composite layers.

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