A New Theoretical Approach on Teenage Cyclist-Vehicle Crash

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Abstract: Data obtained in teenage cyclist impact against three vehicles (sedan, SUV and Pick Up), tested in previous works, are analyzed to verify the theoretical results by the coupling of momentum conservation principle and energy conservation principle. The speed of vehicle and thorax are compared with theoretical ones, obtaining an excellent agreement. Following the results found in the literature, particular emphasis is done on teenager cyclist thorax and head speeds, indicating that the cyclist speed can be theoretically until 1,41 times the impact speed, while greater values, until 2, are obtained in the simulations. A small slowing of the vehicle is found theoretically and in the simulations, due to kinetic energy transfer. A small component of the vehicle speed exists in the orthogonal plane to the motion, due, for example, to the compression of the rider body on the bonnet. A parameter is determined using all the data in term of thorax speed, in all the impact speeds and in all the relative positions; its value does not appear depending on the vehicle mass. It allows the subsequent determination of the best values of three geometric actual parameters identified in front of the vehicle: bumper height, bonnet height and bonnet inclination angle, by interpolation with a second order curve, by making the conclusion that the frontal part of the vehicle may be designed in order to reduce the injury.

Key-Words: teenage bicyclist, vehicle impact, front vehicle shape, mass influence, theory of impact

1 Introduction

Many works are found in literature on the impact between vehicle and teenager [1] [4] or adult pedestrian [2] [3] [5] [10] [18] [23] [28] [29] [30] [31] [32], also numerous works study the impact between the vehicle and the adult cyclist [9] [11] [13] [16] [17] [20] [21] or both cyclist and pedestrian [6] [7] [8] [12] [22] [39], more recently also the papers on the accident vehicle - teenage cyclist are found in literature [14] [15] [19] [35] [36] [37]. In [21] Authors indicate that car-mounted countermeasures designed to mitigate pedestrian injury have the potential to be effective even for bicyclists. In general multibody technique is the applied method for numerical simulation; the most widely used programs are MADYMO, Aprosys, PC Crash, while Simpack is used in [30] and Sim Wise

is effectively used in this paper. Studies give an idea of the vehicle front shape in order to reduce injuries, that may arise due to the impact [18] [29], but neither these works are frequent in literature. In particular the works [21] [22] [23] also address the crash between SUV vehicle against cyclist or pedestrian, but other papers on Pick up - cyclist impact are not found, over the paper [42]. This extends the results already achieved in the papers [14] [15] [37] where the injuries caused by the energy impact of a teenage cyclist with a sedan car are taken into account and analyzed. Analogous crash is studied in case of SUV [35] [36], instead than a normal sedan, in order to fill the gap in literature: references are found only to an adult or to a child, in many cases without taking into account the type of vehicle.

The influence of the front of the vehicle on the injury of the cyclist or pedestrian is a topic that is not frequently found in the literature. In [33] [34], Authors investigate the deploying time (or response time) of an active hood lift system (AHLS) of a passenger vehicle activated by gunpowder actuator, while in [38] four vehicle types including large and compact passenger cars, minivans and light trucks are simulated according to their frequency of involvement in real world accidents. The influences of various vehicle front shape and compliance parameters are analysed. Moreover, the possible countermeasures on basis of vehicle front design, to mitigate the injury severity of the pedestrians, are discussed.

This research group carried out a campaign of virtual simulations with the availability of the virtual models: sedan [14], SUV [35] and Pick Up [42] against teenage cyclist, in order to quantify the injury caused to the head and chest on the basis of criteria such as HIC and 3 ms [24] [25]. An attempt to obtain the best values of the front vehicle parameters is reported in [43] using the speed data of side crash examined previously and the results of the head speed.

In this paper the chest speed data of the previous simulations are analysed in order to quantify the influence of the front part of the vehicle on the injury of the cyclist. The result is obtained using all the crash data and a theoretical approach is given for the study of vehicle crash, with very good results. Theoretical approach allows understanding the influence of the vehicle mass; a criterion is given to determine of the best value of some parameters on the vehicle front part. Other papers, except [28] [41], are not found in literature on these scopes.

2 Implementation of Virtual Models

The paper [1] is very useful for the study of the anthropomorphic model of the human figure of a teenager, while the paper [3] has analogous value for the adult; the book [26] and the paper [27] are very useful for the chassis design and the geometry of the bike (fig. 1). The implementation of the virtual model of the bike is the same for SUV, Pick Up and sedan simulations [14] [35] [36].

The information on pitch, height, length was provided by the manufacturers. Autodesk 3D Studio Max software is used to get the STL model, which was subsequently imported into Sim Wise (fig. 2, 3 and 4), attributing the masses, the centres of gravity and inertia moments of the individual components, the elastic constant of the suspension springs; these parameters are essential for the proper conduct of the tests and the acquisition of results.

Virtual simulations, performed with Sim Wise, allowed quantifying the injuries in head and chest of the teenager cyclist during the impact.



Figure1: cyclist model in Sim Wise



Fig.2: Pick-up in Sim Wise



Fig. 3: SUV in SIM Wise



Fig. 4 – sedan in Sim Wise

3 Vehicle- cyclist crash simulations

This paragraph illustrates the simulations to assess the damage produced on teenage cyclist varying the impact condition, in order to calculate the speed of the head and of the chest for the evaluation of injuries. Dynamic of impact of teenage cyclist – vehicle is reconstructed by Sim Wise. The relative positions between the vehicle and the cyclist are the same for all the vehicles [14] [15] [35] [36] [37]. They are three: in the first the teenage cyclist is positioned on the roadway with the side facing the vehicle about to occur (side impact); in the second case the cyclist is located opposite the vehicle about to occur (frontal impact), while in the third and last case the cyclist is placed behind the vehicle (telescoping or rear impact). Crash tests are executed at four different speeds: 20 km/h, 30 km/h, 40 km/h and 50 km/h.

- The parameters measured during the tests are:
- Acceleration in the head gravity centre;

- Acceleration in the chest gravity centre.



Figure 3: Head and chest acceleration in the side impact with Pick Up at 40km/h



Fig. 4 – Chest and head speed in the frontal impact with sedan at 20 km/h

Fig. 3 shows an example of acceleration performances of the head and thorax. In previous papers this data are analysed and elaborated in order to quantify the injury of the head and the thorax.

3.1 Head impact speed

Also these parameters are measured in the simulations:

- resulting speed of the head gravity center;

- resulting speed in the thorax gravity center.

Fig. 4 shows an example of the trend of the head and thorax speed versus the time. A greater draft number is reported in paper [43]. Table 1 shows the obtained results with the values of the maximum impact speed of the thorax and of the head, with the contact time.

Table 1: max impact speed of thorax and of head and the contact time (F=front, S= side, R= rear).

		Impact	V _{max}	V _{max}	т
Vehicle	Pos.	speed	head	thorax	I contact
		[km/h]	[m/s]	[m/s]	[IIIS]
Sedan		20	10,10	11,44	1408
Sedan		30	12,35	12,20	544
Sedan		40	16,18	16,05	360
sedan		50	19,13	19,25	240
SUV		20	7,67	10,20	352
SUV	Б	30	10,04	9,96	168
SUV	Г	40	16,93	16,32	352
SUV		50	21,73	20,40	264
PickUP		20	12,10	12,30	528
Pick UP		30	16,88	16,80	200
Pick UP		40	18,12	17,92	208
Pick UP		50	27,74	25,90	64
Sedan		20	8,29	8,11	272
sedan		30	11,13	11,96	208
sedan		40	16,03	18,49	176
sedan		50	16,98	17,26	176
SUV		20	9,18	8,43	272
SUV		30	10,29	10,76	272
SUV	S	40	17,77	18,28	200
SUV		50	20,11	19,50	192
Pick UP		20	8,51	10,10	96
Pick UP		30	12,88	13,06	64
Pick UP		40	21,58	20,78	208
Pick UP		50	26,13	26,20	32
sedan		20	9,42	9,11	984
sedan		30	10,11	10,98	1348
sedan		40	10,17	9,82	600
sedan		50	15,87	14,45	176
SUV		20	7,19	7,25	640
SUV		30	9,81	9,33	544
SUV	R	40	9,57	10,13	472
SUV		50	13,39	13,96	552
Pick UP		20	4,69	4,57	160
Pick UP		30	10,67	10,69	248
Pick UP		40	14,99	15,01	232
Pick UP		50	20,61	19,41	128

Figures 5, 6 and 7 show the trend and the visual comparison for the chest, while the analogous results for the head are reported in [43].

The figures show that the sedan gives the best performance under all conditions of impact compared to other vehicles. This depends on the gentler shaping of the front and on the low mass; at low speed collision the speed of impact of the chest takes on greater values than other vehicles. In general the resulting speed of the cyclist chest is greater if the vehicle has high front. The following considerations can be done:



Fig. 5: maximum chest speed in the frontal impact.



Fig. 6: maximum chest speed in the side impact.



Fig. 7: maximum chest speed in the telescoping.

- The impact speed of the chest of the teenage cyclist increases varying the considered vehicle.

- The mass of the coming up vehicle is important for the injury gravity only at greater or equal speed than 40 km/h [41].

Table 2 – Vehicle speed after the impact a	nd
relative slowing	

VehiclePosImpact speed $[km/h]$ Postimpact speed $[km/h]$ Slowing $[km/h]$ sedan2018.41,60sedan3028.431,57sedan4037.982,02sedan5047.742,26SUV2019.70,30SUV3029.660,34SUV5049.040,96Pick UP2019.30,70Pick UP4039.220,78Pick UP5048.521,48sedan3028.731,27sedan3028.731,27sedan3028.731,27sedan3028.731,27SUV5048.231,77SUV2019.570,43SUVS4039.20,80SUVS5049.180,82Pick UP5048.691,31sedan2018.621,38sedan3029.470,53Pick UP5048.691,31sedan3028.961,04sedan3028.961,04sedan3029.680,32SUVS4039.210,79Sedan3029.680,32SUVS4039.770,23SUVS4039.770,23SUVSUVS4039.470,53 <th></th> <th></th> <th></th> <th></th> <th></th>					
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ikm/h] ikm/h] ikm/h] ikm/h] ikm/h] sedan 20 18.4 1,60 sedan 30 28.43 1,57 sedan 40 37.98 2,02 sedan 50 47.74 2,26 SUV 50 47.74 2,26 SUV 50 49.04 0,96 Pick UP 40 39.27 0,73 SUV F 40 39.27 0,73 SUV S0 29.09 0,91 Pick UP 40 39.22 0,78 Pick UP 50 48.52 1,48 sedan 20 18.94 1,06 sedan 20 19.57 0,43 SUV S 40 38.02 1,98 sedan 50 48.23 1,77 SUV S 40 39.2 0,80 SUV S 40 39.2 0,80	Vehicle	Pos	speed	speed	[km/h]
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$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	sedan		40	38.02	1,98
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sedan 50 48.46 1,54 SUV 20 19.74 0,26 SUV 30 29.68 0,32 SUV 8 40 39.77 0,23 SUV 50 49.64 0,36 Pick UP 20 19.9 0,10 Pick UP 30 29.64 0,36 Pick UP 40 39.47 0,53	sedan		40	39.21	0,79
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	Pick UP		40	39.47	0.53
Pick UP 50 49.19 0.81	Pick UP		50	49.19	0,81

Comparison of the results of the paper [1] shows that the teenage pedestrian head speed is greater than the teenage cyclist, since a part of the impact energy is dissipated by the bike chassis.

- The obtained result for the chest bear out the head results reported in [43].

Table 2 shows the values of the vehicle speed after the impact; a small slowing is present since a small part of the vehicle kinetic energy is used to increase the cyclist speed.

4. Theoretical validation of the speed results

Several simulations of vehicle – pedestrian crash are executed in [38] by the multibody program MADYMO: a simulation technique is applied giving the relative speed of the head respect to vehicle, while the technique adopted in this work gives the absolute values, so that no numerical or qualitative comparison is possible.

Indicating with m_v the mass of the vehicle and m_c the mass of the rider, an instant before the impact, if the speed of the cyclist is neglected, the momentum of the system is given by m_vV . The two bodies begin to move with speed V_v due to the initially inelastic impact, so that the momentum conservation states:

$$m_v V = (m_c + m_v) V_v.$$
 (1)

Since m_v is very greater than m_c , the solution $V_v \approx V$ is fairly reliable, so that after the impact instant the rider has a relative speed very close to the absolute speed of the vehicle before the impact.

In reality the speed V_v is actually a few less than the speed V, because during the impact phase, which endures small milliseconds, strong impulsive forces are awakened, then strong accelerations vary the kinematic behaviour of the vehicle-cyclist system. These accelerations are strongly dependent on the vehicle mass and are in part neglected by examining the relative motion. The dynamic behaviour of the system is modified looking directly the relative speeds.

In effect, in the case of elastic collision, the literature considers suitable both the momentum conservation principle and kinetic energy conservation principle, but the validity of both the principles can be questioned in the case of quasi elastic or inelastic collision. Given that the motion is supposed in the x direction, V is the vehicle initial impact speed that has a known value. In general the six components of the speed are different by zero in the final instant. Momentum conservation is:

$$m_{c}V_{xc} = -m_{v}(V_{xv} - V)$$

$$m_{c}V_{yc} = -m_{v}V_{yv}$$

$$m_{c}V_{zc} = -m_{v}V_{zv}$$
(2)

The minus sign can be changed in relationship (2), for the effective sense of the speed, but this does not compromise the final result. Squaring the relationships (2) and summing, remembering that is:

$$V_{c} = \sqrt{V_{xc}^{2} + V_{yc}^{2} + V_{zc}^{2}}$$

$$V_{v} = \sqrt{V_{xv}^{2} + V_{yv}^{2} + V_{zv}^{2}}$$
(3)

the following relationship is obtained:

$$m_c^2 V_c^2 = m_v^2 \left(V_v^2 + V^2 - 2V_{xv} V \right)$$
(4)

Given that both V_c and V_v are unknown, another equation can be obtained applying the energy conservation principle. One has:

$$\frac{1}{2}m_c V_c^2 = -\frac{1}{2}m_v \left(V_v^2 - V^2\right)$$
(5)

The energy conservation principle is written neglecting other forms of energy (potential gravitational, rotational kinetic of the rider, potential elastic of the suspension springs, etc.).

The definition of the following dimensionless variables is now convenient:

$$x = \frac{V_v}{V} \qquad y = \frac{V_c}{V} \tag{6}$$

that are named normalized speeds. Putting:

$$V_{xv} = \beta V_v \tag{7}$$

where β is a direction cosine of the vector V_v; dividing the relationship (4) by V², one obtains:

$$m_c^2 y^2 = m_v^2 \left(x^2 + 1 - 2\beta x \right)$$
(8)

And, in analogous way, relationship (5) becomes:

$$m_{c}y^{2} = -m_{v}(x^{2} - 1)$$
(9)

Relationships (8) and (9) form a system of two equations in three unknowns x, y and β . In general the textbooks of physics indicate that a third condition is necessary to resolve the system in the case of bi-dimensional or three-dimensional collision, but a more effective solution is not given. In effects the solution of the system, for example versus β , shows that the solution is real if:

$$1 \ge \left|\beta\right| = \pm \sqrt{1 - \frac{m_c^2}{m_\nu^2}} \tag{10}$$

In the examined case the system can be resolved in a conditioned way, optimizing the result in order to obtain an only solution. The technique of Lagrange multipliers [42] is applied, assuming the variable β obtained by (8) as function:

$$\beta = \frac{x^2 + 1 - \frac{m_c^2}{m_v^2} y^2}{2x}$$
(11)

under the condition (9). Lagrangian is then written:

$$L(x, y, \lambda) = \frac{x^2 + 1 - \frac{m_c}{m_v^2} y^2}{2x} - \lambda (m_c y^2 + m_v (x^2 - 1))$$
(12)

where λ is Lagrange multiplier. The optimum condition is obtained equalling to zero both the first derivative respect to x and y.

$$\frac{\partial \hat{L}(x, y, \lambda)}{\partial x} = \frac{\partial \hat{L}(x, y, \lambda)}{\partial y} = 0$$
(13)

that represents a condition assumed naturally by the system. The relationships (13) and (9) form a system of three equations with three unknowns x, y and λ which solution is:

$$\lambda = -\frac{m_c}{2xm_v^2}$$

$$x = \pm \sqrt{\frac{m_v - m_c}{m_v + m_c}}$$

$$y = \pm \sqrt{\frac{2m_v}{m_v + m_c}}$$
(14)

 λ value is not useful for the final solution. At last relationship (11) allows β calculation; one obtains:

$$\beta = \pm \sqrt{1 - \frac{m_c^2}{m_v^2}} \tag{15}$$

where β and x have the same sign, and the case y<0 has not interesting in the examined case. Particular cases: the cases $m_c = 0$ or $m_v = 0$ are not particular, since the original equations should be different. The case:

$$m_c = m_v \quad \beta = 0 \quad x = 0 \quad y = 1$$
 (16)

indicates that all the energy is transferred to the second mass and the first mass is stopped, if the two masses are equal.

Instead the equations have to be reconsidered in the case $m_c > m_v$, but this case in not interesting for this work purpose. However the mathematical development is studied and verified for the case of vehicle – cyclist crash, but its application can be larger.

Substantially the procedure allows calculating the results so that the solutions of the system constituted by (8) and (9) are real and agreeing. By the geometrical view point the condition is obtained that the curves represented by the above relationships are tangent in the point having the coordinates (14).

The third relationship (14) shows that the speed of the cyclist (or pedestrian) can be greater than the vehicle impact speed, since the normalized speed tends to 1.41 if $m_c \ll m_v$. However the vehicle mass effect is greater as soon as the impact speed V increases.

Tables 3, 4 and 5 show the normalized impact speed for sedan, SUV and Pick Up respectively. One can note that the normalized speed x assumes values a few lower than 1, so that the small reduction of speed may not be neglected, and that the cyclist speed after the impact is greater than the vehicle impact speed. The normalized speed y assumes values between 1,2 and 2 about, against the theoretical value 1.41 above indicated. This is due to non-fully elastic collision, and to the fact that the energy conservation (5) does not take in account other energy forms.

Pos.	Impact speed [km/h]	V _{max head} normal.	V _{max chest} normal.	Post impact speed normal.
	20	1.818	2.0592	0.92
Б	30	1.482	1.464	0.947667
I.	40	1.4562	1.4445	0.9495
	50	1.37736	1.386	0.9548
	20	1.4922	1.4598	0.947
c	30	1.3356	1.4352	0.957667
3	40	1.4427	1.6641	0.9505
	50	1.22256	1.24272	0.9646
R	20	1.6956	1.6398	0.931
	30	1.2132	1.3176	0.965333
	40	0.9153	0.8838	0.98025
	50	1.14264	1.0404	0.9692

Table 3: normalized speeds for the sedan

Table 4: normalized speeds	for the	SUV
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	Impact			Post
Dog	angad	$V_{max head}$	V _{max chest}	impact
1 05.	[km/h]	normal.	normal.	speed
	[גווו/וו]			normal.
	20	1.3806	1.836	0.985
Б	30	1.2048	1.1952	0.988667
Г	40	1.5237	1.4688	0.98175
	50	1.56456	1.4688	0.9808
	20	1.6524	1.5174	0.9785
c	30	1.2348	1.2912	0.988
3	40	1.5993	1.6452	0.98
	50	1.44792	1.404	0.9836
R	20	1.2942	1.305	0.987
	30	1.1772	1.1196	0.989333
	40	0.8613	0.9117	0.99425
	50	0.96408	1.00512	0.9928

The solution (14) and (15) allows studying the crash by the theoretical view point. In particular allows better understanding the influence of the vehicle mass that was previously studied using HIC parameter [41]. In that case the influence of the mass was individuated qualitatively only for the greater speeds.

Table 6 shows the cyclist and vehicles masses and the directional cosine β calculated by (15); the

positive value is chosen. Its value is very close to one, but the rounding up jeopardizes the result.

	Impact			Post
Doc	speed	V _{max head}	V _{max chest}	impact
1 05.	[km/h]	normal.	normal.	speed
				normal.
	20	2.178	2.214	0.965
Б	30	2.0256	2.016	0.969667
Г	40	1.6308	1.6128	0.9805
	50	1.99728	1.8648	0.9704
	20	1.5318	1.818	0.9825
c	30	1.5456	1.5672	0.982333
3	40	1.9422	1.8702	0.972
	50	1.88136	1.8864	0.9738
R	20	0.8442	0.8226	0.995
	30	1.2804	1.2828	0.988
	40	1.3491	1.3509	0.98675
	50	1.48392	1.39752	0.9838

Table 5: normalized speeds for the Pick Up

Table 6: masses and directional cosine

	mass [kg]	β
cyclist	45	
sedan	968	0.998919
SUV	2900	0.99988
Pick Up	3085	0.999894

Fig. 8 shows the comparison of the sedan results with the theoretical ones. Relationships (8) (momentum) and (9) (kinetic) are shown highlighting the tangency condition.

In superposition the figure shows the values obtained by the numerical simulation: the values of the post impact normalized speed of the vehicle are reported in abscissa, while the values of the chest normalized speed are reported in ordinate. In a more correct way, the values of the impact speed of the cyclist gravitational centre should be used, but the error is very small. A very good agreement of the numerical results with the theoretical one can be noted, indicating the goodness of the executed procedure.

Fig. 9 and 10 show the analogous comparison of SUV and Pick Up results respectively. Also in both these cases the numerical results is very closed to theoretical one.

Finally the theoretical validation shows:

- All the data of the three vehicles, in the three condition of impact analyzed, are very close to the optimal condition determined in the previous.
- The speed of the cyclist (or pedestrian) at the end of the contact is very greater than the speed of the coming up vehicle (until two times).



Fig. 8 – Theoretical comparison of sedan results.



Fig. 9 - Theoretical comparison of SUV results.



Fig. 10 - Theoretical comparison of Pick Up results.

- SUV vehicle has the best behavior, among the three vehicles studied, since the speed values are all more closed to the optimal condition (fig. 9).

The study of acceleration from a theoretical point of view deserves further investigation. One can observe that the mean vehicle acceleration can be calculated dividing the slowing in table 2 by the time of contact in table 1; the mean cyclist acceleration can be calculated dividing the cyclist speed by the time contact.

5. Best values evaluation and discussion

Table 7 shows the geometric characteristic of the examined vehicles; fig. 11 shows the geometrical position. In the paper [43] a procedure is shown to calculate the best values of the parameters indicated in figure. The case of lateral impact was used, following the ideas order in [38]. Only the results for V = 30 km/h were used, because the results at the other speed (40 and 50 km/h) were discarded due to the influence of the mass or to the too low speed (20 km/h) to arouse high injuries in the cyclist. The results at speed 30 km/h were manipulated to obtain the best values of the examined parameters.

In this paper the chest speed values are used, instead than the head speed. The following procedure is useful to eliminate the mass influence as far as possible: relationships (14) allow the following coordinate transform:

$$y = y' \sqrt{\frac{2m_{\nu}}{m_{\nu} + m_c}}$$

$$x = x' \sqrt{\frac{m_{\nu} - m_c}{m_{\nu} + m_c}}$$
(17)

In this way the optimal point of the previous has coordinate (x'=1, y'=1) for whatever vehicle. Substituting (17) in relationship (9), the derivative in the point (1, 1) is:

$$\frac{dy'}{dx'}(1,1) = a = -\frac{m_v - m_c}{2m_c}$$
(18)

that is indicated by a in the following. The tangent equation at both the kinetic and momentum curve in the optimal point (1,1) can be constructed; the following relationship is obtained:

$$y'=1+a(x'-1)$$
 (19)

The tangents are drawn in fig. 12; their slope a is strongly dependent on the vehicle mass.

All the coordinates of the three vehicles and for the three different positions (front, side and rear) are converted by using the relationships (17). The corresponding points are shown in the same fig. 12. The figure shows that the simulation results are thickened around the point (1,1) and are positioned along the tangent line of the previous. The figure confirms the excellent concordance of the theoretical result with the numerical simulation.



Figure 11 – evaluation parameters

Table 7- geometric characteristics of the vehicles

	Bonnet	Bumper	Bonnet	
vehicle	height	height	angle	γ
	[mm]	[mm]	[degrees]	
sedan	847	390	20	0.934
SUV	999	556	14	0.798
Pick Up	1087	659	11	0.976
Best				
values	960	516	16	≈0.78



Fig. 12 – global representation of the numerical result

A criterion is carried out to determine the best values of the vehicle front shape. The following quantity is calculated for every point of the simulation results:

$$\frac{\Delta y'}{\Delta x'} = \frac{y'-1}{x'-1} \tag{20}$$

The values are used to calculate to derivative ratio γ :

$$\gamma = \frac{1}{na} \sum \frac{y'-1}{x'-1} \tag{21}$$

Where n (=12) is the number of simulations for every vehicle and a is the derivative in (18). The quantity is calculated for every vehicle, without distinction on the relative position of impact; the SUV result at 20 km/h in the front position is discarded for a too high quantity (20); so that n=11. Respect to the method applied in [43], the use of γ parameter avoids the discarding of the results that are influenced by the mass; for the rest the ideas order is substantially the same: the values γ are interpolated by a parabola, and the minimum condition is considered like optimum value.

 γ values are reported in table 7, with the best values of bonnet height, bumper height and bonnet angle, obtained with this criterion. Optimal values are a few greater than the values calculated in [43]. The interpolations for the bonnet and bumper height are shown in fig. 13; the interpolation for bonnet angle is shown in fig. 14.



Figure 13 – derivative ratio versus the heights of bonnet and bumper



Figure 14 – derivative ratio versus the bonnet angle

Results obtained in this work cannot be considered definitive; best results can be obtained by increasing the number of examined vehicles, and also considering the offset of impact in the lateral crash. They may be used as the starting data for the mathematical procedures that can lead to improve the result.

Theory set in this paper shows an excellent validity in the case of teenage cyclist crash, and can found other application in other impact condition.

6 Conclusions

Impact performances with a teenager cyclist of three vehicles tested in previous works are analysed in this work. The values of the simulations are compared with an original theoretical procedure based on the application of momentum and energy conservation principle. The results show that the system vehicle - cyclist assumes naturally a condition of optimum that can be calculated with simplicity. The analysis of chest and head speeds indicates that the cyclist (or pedestrian) assumes a speed that can be very greater than that of the vehicle coming up; this endures a vehicle slowing that cannot be neglected for the study of the crash phenomenon. Moreover the theoretical study shows that the vehicle speed has a small component along the orthogonal plane to the motion, due to a value of the directional cosine a few lower than 1, that cannot be neglected for analogous reason.

A literature research allows the identification of the following determinants geometric parameters of the frontal part of the vehicle: bumper height, bonnet height, bonnet angle; the criterion used for the optimal values determination is based on the derivative ratio γ , and is applied on the absolute speed of the thorax in the impact instant. The adopted procedure ensures a neglecting influence of the vehicle mass and is derived by the theoretical formulation of the previous. This procedure cannot be defined as shape optimization, because the classical procedures cannot be applied for the skimpiness of available values. A similar procedure, not shown in the work, can be applied to the HIC values, obtaining small variations of the best values. The vehicles offering the best shape performance, among those examined, is the SUV: the speed of impact of the head and thorax is close enough to the minimum; the representative point of this vehicle occupies a very close position to the minimum in the determination with HIC.

Next development of this work is the application of the theory to the pedestrian, in order to verify the impact condition in a similar way. Moreover the result can be improved inserting the results of other vehicles types.

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