Generalization of PER Model for the Whole Liner Height Analysis of Asymmetric Shaped Charges

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Abstract: - This paper presents a model that explains the liner collapse phenomenon for the whole liner height analysis of conical shaped charges in presence of geometrical asymmetries. In presence of geometrical asymmetries, the velocities of the elements lying on the two arms of the liner may not be symmetric. Therefore, the elements colliding at the core will have different distances from the nominal axis of symmetry, and of course will not meet on the axis. Some solution methods which are applicable to asymmetric models, converge for conditions such as incomplete traversing of the wave front over the liner, and therefore are not applicable for general description of jet and slug formation. With those methods the detonation wave can traverse 72% to 90% of the liner height for a typical shaped charge. In fact, because of ill-conditioning and/or numerical errors, convergence is not guaranteed with other solution methods near the end positions of the liner (near the base). In the proposed model, a generalized form of classical jet formation theory and its proper solution method is developed for analysis of the whole liner in asymmetric conditions. The present method can solve the generalized form of the PER model to 100% of the liner height. Thus the determination of the asymmetry effects of the whole liner height is easily possible. In other words, in the present paper a different method has been proposed that determines the core coordinate and its motion for the whole liner height. In the present model, the specifications of the jet and slug, such as off-axis jet velocity is validated with previous works.

Key-Words: - Asymmetric shaped charge, Jet, Slug, Liner, Detonation, Collapse phenomenon.

1 Introduction

A planar shaped charge liner is made up of two thin plates joined at an angle of 2α . The outer part of the liner is enclosed by explosive material. When the explosive is detonated, the detonation wave front impacts both plates, and thus the liner elements accelerate. The elements from two plates collide at one individual moving point (core) that may be on the nominal axis of symmetry of the liner. At this point, a plane fast jet and a low velocity slug are produced.

Birkhoff [1] and Pugh et al [2], developed some models for the description of symmetric liners. Because of liner or explosive asymmetries or other imperfections, the core may not be on the axis and has an off-axis velocity that reduces the penetrability of the jet. First, in 1979, Aseltine [3] proposed a model for the description of asymmetries in shaped charges, but due to simplifications, it was not applicable for later investigations.

Since then, a good deal of attention has been paid to developing predictions of the asymmetric jet and slug formation. However, experimentation and testing are good methods for tolerance design in shaped charges. But this expensive method must be accompanied by an appropriate theory [4]. In 1984, asymmetry parameters and their effects on penetration were investigated by Hirsch [5]. In his model, core motion and its off-axis velocity and outgoing flows from the core are important parameters affecting the phenomenon. In 1987, Mayseless et al [6] continued earlier works and compared the effects of asymmetries on peripherally and point initiated shaped charges.

For the first time, Pack et al [7], proposed a closed form solution and simple model that in fact was a generalization of the Birkhoff model. For instance, in this model the velocities of the elements in each side of the liner are constant and do not vary along it, but there is a very small difference between these two velocities. In 1992 the earlier model was completed by the modification of some simplifications. For example, some formulas were proposed for the description of the phenomenon in three dimensions [8]. The model was completed with consideration of the acceleration of element motion [9]. Heider [10] also suggested another model and used the streamline concept.

During this time, some investigations were carried out regarding the collision of two asymmetric flows [11-15]. Contemporarily, other investigators worked on the essential tolerances for the manufacturing of shaped charges [16-19]. In 2010, Ayisit [20] evaluated jet and slug behavior in various asymmetric shaped charges by means of Autodyn software. Some studies on simulation of collapse phenomenon and design parameters of shaped charges and jet penetrability were done [21-23].

The present work is a generalization of classic jet formation theory for planar shaped charges which explains the collapse phenomenon in the presence of asymmetries. The developed relations are used for conical shaped charges by the use of some simple approximate formulas.

2 Theory

This model is a generalization of the PER theory, thus, by means of its basic equations and definition of asymmetry parameters; analysis of the core, slug and jet behavior is done.

The *X* axis is taken along the nominal axis of symmetry (Fig.1) and *Y* axis perpendicular to it at the apex. A detonation wave front is modeled as a plane wave, moving with a constant speed *U*.

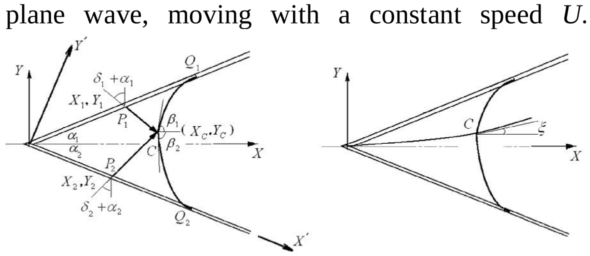

Fig.1 The colliding elements of the two arms at point C

It is assumed that the velocity of the elements at P_1 and P_2 change instantaneously from zero to a constant velocity of V_{01} and V_{02} in the directions of P_1C and P_2C the elements meet together at C. On the other hand, it is assumed that the collapse of each element on each arm is not affected by neighboring elements. In the time interval during which P_1 (or P_2) reaches the position C, elements on the P_1Q_1 (or P_2Q_2) line, will reach the CQ_1 (or CQ_2) curve, respectively.

Also during this time interval, C moves with V_C velocity and the detonation wave sweeps arm 1 (or 2) and moves from P_1 (or P_2) to Q_1 (or Q_2).

The element lying on arm 1 collides with the other element lying on arm 2, at core C (See Fig.1). Thus, the coordinates of moving point C are :

$$X_C = X_1 + V_{01}(t - T_1)Sin(\delta_1 + \alpha_1)$$

$$= X_2 + V_{02}(t - T_2)Sin(\delta_1 + \alpha_1)$$

$$Y_C = X_1 tan\alpha_1 - V_{01}(t - T_1)Cos(\delta_1 + \alpha_1)$$

$$= -X_2 tan\alpha_2 + V_{02}(t - T_2)Cos(\delta_2 + \alpha_2) \qquad (1)$$

Where V_{01} and V_{02} are projection velocities of elements and δ_1 and δ_2 are Taylor angles.

Of course, the following parameters are well defined:

$$\begin{cases} V_{01} = V_{01}(X_1), T_1 = \frac{X_1}{U}, \delta_1 = \delta_1(V_{01}) = \delta_1(X_1) \\ V_{02} = V_{02}(X_2), T_2 = \frac{X_2}{U}, \delta_2 = \delta_2(V_{02}) = \delta_2(X_2) \end{cases} \qquad (2)$$

For the solution of equations (1), some methods can be proposed.

As the first method, reforming equations (1) by differentiating them with respect to time and integrating them again can be rearranged in the form of [8]:

$$[X_1, X_2, X_C, Y_C]^T = E_{4\times4}[\dot{X}_1, \dot{X}_2, \dot{X}_C, \dot{Y}_C]^T \qquad (3)$$

The above equations can be solved by the fourth or sixth order Runge-Kutta method and will result in vector $[X_1, X_2, X_C, Y_C]$ as a function of t.

For the introduced shaped charge in [8], this method gives the solution up to 90% of liner height. Because of numerical problems, solution by this method stops at this height.

As the second method for a desired X_2, four other unknown variables t, X_1, X_C, Y_C can be determined. On the other hand, t can be obtained as a function of X_1 and X_2 :

$$t = \frac{X_2 - V_{02}T_2 sin(\delta_2 + \alpha_2) - X_1 + V_{01}T_1 sin(\delta_1 + \alpha_1)}{V_{01}sin(\delta_1 + \alpha_1) - V_{02}sin(\delta_2 + \alpha_1)} \qquad (4)$$

Using equation (4), t can be eliminated from (1) and X_1, X_C and Y_C in (1) can be determined for the specified parameter X_2.

This method for introduced shaped charge in [8], will converge up to a maximum of 72% of the liner height. It seems that this inefficiency is caused by the existence of similar sinus and cosine terms on both sides of equations 1 and the fact that V_{01} and V_{02} are regressive functions of X_1 and X_2 in conventional shaped charges.

With a decrease in V_{01} and V_{02} near the liner base

and thus, a decrease in δ_1 and δ_2, the weight of sinus and cosine terms will reduce and therefore the convergence algorithm diverge.
If the similarity of the terms can be eliminated, convergence will be successful.
Accordingly, the equations (1) will be rearranged in a new coordinate system, such as (X',Y'), which in fact, is caused from the clockwise rotation of the (X,Y) coordinate system.

$$X'_C = \frac{X_1}{cos\alpha_1}cos(\alpha_1+\alpha_2) + V_{01}(t-T_1)sin(\delta_1+\alpha_1+\alpha_2)$$

$$= \frac{X_2}{cos\alpha_2} + V_{02}(t-T_2)sin\delta_2$$

$$Y'_C = \frac{X_1}{Cos\alpha_1}sin(\alpha_1+\alpha_2) + V_{01}(t-T_1)cos(\delta_1+\alpha_1+\alpha_2)$$

$$= V_{02}(t-T_2)cos\delta_2 \qquad (5)$$

With this rearrangement, convergence will be easily attainable.
So, $[X_1, X_2, X_C, Y_C]^T$ as a function of time will be determined and the velocity of the core and its direction of motion (ξ) is completely defined.

$$V_{CX} = \frac{dX_C}{dt}, \quad V_{CY} = \frac{dY_C}{dt}, \quad \xi = tan^{-1}\frac{V_{CY}}{V_{CX}} \qquad (6)$$

By using determined variables (X_1, X_2, X_C, Y_C, t), angles β_1, β_2 can be calculated.

$$\left(V'_{01} = \frac{dV_{01}}{dX_1}, \quad \delta'_1 = \frac{d\delta_1}{dX_1}, \quad V'_{02} = \frac{dV_{02}}{dX_2}, \quad \delta'_1 = \frac{d\delta_2}{dX_2}\right)$$

$$tan\beta_1 = tan\alpha_1 - V'_{01}(t-T_1)cos(\delta_1+\alpha_1) + \frac{V_{01}}{U}cos(\delta_1+\alpha_1) + V_{01}\delta'_1(t-T_1)sin(\delta_1+\alpha_1)/\left[1+V'_{01}(t-T_1)sin(\delta_1+\alpha_1) - \frac{V_{01}}{U}sin(\delta_1+\alpha_1) + V_{01}\delta'_1(t-T_1)cos(\delta_1+\alpha_1)\right]$$

$$tan\beta_2 = tan\alpha_2 + V'_{02}(t-T_2)cos(\delta_2+\alpha_2) - \frac{V_{02}}{U}cos(\delta_2+\alpha_2) + V_{02}\delta'_2(t-T_2)sin(\delta_2+\alpha_2)/\left[1+V'_{02}(t-T_2)sin(\delta_2+\alpha_2) - \frac{V_{02}}{U}sin(\delta_2+\alpha_2) + V_{02}\delta'_2(t-T_2)cos(\delta_2+\alpha_2)\right] \qquad (7)$$

Two triangles shown in Fig.2 illustrate the relative values and directions of motion of the elements and the core.

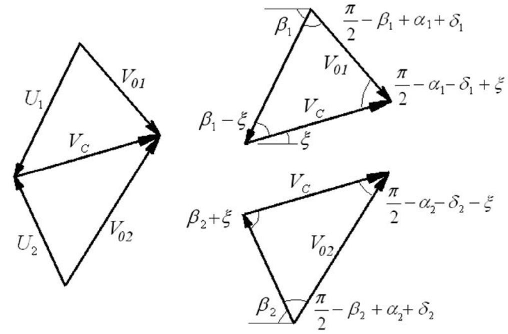

Fig.2 Relative values of velocities and directions of the elements and core motion

According to Fig.2, two quantities ξ, V_c which are common in the two triangles. These two quantities are useful for the determination of the elements velocity relative to the core (U_1, U_2):

$$\begin{cases} \dfrac{V_{01}}{sin(\beta_1-\xi)} = \dfrac{V_C}{sin(\frac{\pi}{2}-\beta_1+\delta_1+\alpha)} \\ \dfrac{V_{02}}{sin(\beta_2+\xi)} = \dfrac{V_C}{sin(\frac{\pi}{2}-\beta_2+\delta_2+\alpha)} \end{cases} \qquad (8)$$

$$\frac{sin(\beta_2+\xi)}{sin(\beta_1-\xi)} = \frac{V_{02}cos(\beta_2-\delta_2-\alpha)}{V_{01}cos(\beta_1-\delta_1-\alpha)} = B$$

$$\xi = tan^{-1}\left(\frac{Bsin\beta_1 - sin\beta_2}{Bcos\beta_1 + cos\beta_2}\right) \qquad (9)$$

$$V_C = V_{01}\frac{cos(\beta_1-\delta_1-\alpha)}{sin(\beta_1-\xi)} = V_{02}\frac{cos(\beta_2-\delta_2-\alpha)}{sin(\beta_2+\xi)} \qquad (10)$$

$$\begin{cases} U_1 = \dfrac{V_{01}}{Sin(\beta_1-\xi)}Sin(\frac{\pi}{2}-\alpha_1-\delta_1-\xi) \\ U_2 = \dfrac{V_{02}}{Sin(\beta_2+\xi)}Sin(\frac{\pi}{2}-\alpha_2-\delta_2-\xi) \end{cases} \qquad (11)$$

Areas of cross sections for two flows entering the core for a planar shaped charge are:

$$\begin{cases} A_1 = \dfrac{wh_1\dot{X}_1}{U_1} \\ A_2 = \dfrac{wh_2\dot{X}_2}{U_2} \end{cases} \qquad (12)$$

In equations (12) $\dot{X}_1$ and $\dot{X}_2$ are values of the traversing velocity of the detonation wave on arms 1 and 2, while h_1, h_2 are the thicknesses of arms 1 and 2, respectively and w is the out of plane dimension for each arm. Also φ_1 and φ_2 for the two flows are:

$$\begin{cases} \varphi_1 = \beta_1 - \xi \\ \varphi_2 = -\beta_2 - \xi \end{cases} \qquad (13)$$

Finally by the use of determined variables $(U_1, U_2, \varphi_1, \varphi_2, A_1, A_2)$, the specifications of the flows 3 and 4 can be concluded (Fig.3).

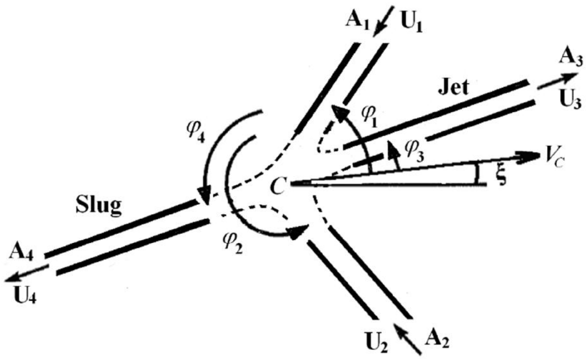

Fig.3 The asymmetric collision of the two flows and their quantities

$$U_3 = U_4 = \sqrt{\frac{A_1 U_1^3 + A_2 U_2^3}{A_1 U_1 + A_2 U_2}} \quad (14)$$

$$\varphi_3 = tan^{-1}\left(\frac{A_1 U_1^2 \sin\varphi_1 + A_2 U_2^2 \sin\varphi_2}{A_1 U_1^2 \cos\varphi_1 + A_2 U_2^2 \cos\varphi_2}\right) \quad (15)$$

$$\varphi_4 = \varphi_3 + \pi \quad (16)$$

$$A_3 = \frac{1}{2}\left(\frac{A_1 U_1 + A_2 U_2}{U_3} - \frac{\sqrt{A_1^2 U_1^4 + 2A_1 A_2 U_1^2 U_2^2 \cos(\varphi_1 - \varphi_2) + A_2^2 U_2^4}}{U_3^2}\right) \quad (17)$$

$$A_4 = \frac{1}{U_3}(A_1 U_1 + A_2 U_2) - A_3 \quad (18)$$

These results are used for the determination of jet and slug quantities:

$$\begin{cases} V_{jX} = U_3 \cos(\xi + \varphi_3) + V_C \cos\xi \\ V_{jY} = U_3 \sin(\xi + \varphi_3) + V_C \sin\xi \end{cases}, \varphi_j = tan^{-1}\left(\frac{V_{jY}}{V_{jX}}\right) \quad (19)$$

$$\begin{cases} V_{sX} = V_C \cos\xi - U_3 \cos(\xi + \varphi_3) \\ V_{sY} = V_C \sin\xi - U_3 \sin(\xi + \varphi_3) \end{cases}, \varphi_s = tan^{-1}\left(\frac{V_{sY}}{V_{sX}}\right) \quad (20)$$

Also the planar jet and slug thicknesses are determined from $h_j = A_3 / w$, $h_s = A_4 / w$,where w is the out of plane thickness of these flows.

If a conical liner is divided into 2N parts in circumferential direction (Fig.4), the specifications of flows 1 and 2 $(U_1, U_2, \varphi_1, \varphi_2, A_1, A_2)$ for each reciprocal couple part can be determined by the preceding formulas (8,9,11) except for A_1 and A_2 [8] :

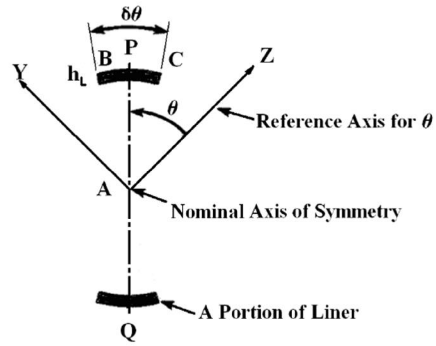

Fig.4 Division of a conical liner in θ direction

$$\begin{cases} \dot{M}_{1\iota} = \rho A_{1\iota} U_{1\iota} = \frac{\rho\pi}{2N}(2X_1 h_{1\iota} \tan\alpha \sec\alpha + h_{1\iota}^2 \sec^2\alpha)\dot{X}_1 \\ \dot{M}_{2i} = \rho A_{2i} U_{2i} = \frac{\rho\pi}{2N}(2X_2 h_{2i} \tan\alpha \sec\alpha + h_{2i}^2 \sec^2\alpha)\dot{X}_2 \end{cases} \quad (21)$$

Where $\dot{M}_{1i}$ and $\dot{M}_{2i}$ are mass rates of a reciprocal couple part of a conical liner in circumferential direction.

Of course V_{01} and V_{02} must be computed from proper Gurney formulas and appropriate liner, explosive and confinement elements masses [9]. For each circumferential section, $U_1, U_2, \varphi_1, \varphi_2$ can also be calculated similar to equations (8), (9), (11) for planar shaped charge. After this step and the determination of the parameters of flows 1 and 2 as functions of X_1 and X_2, specifications of the core motion and flows 3 and 4 can be calculated. This is done by means of a weighted averaging method [8]. This method is applicable for the computation of jet and slug motion. Due to the fact that the core position for each couple element is not necessarily the same, this procedure is not exact.

3 Result and discussion

As the basic step for the introduced shaped charge [8], presented in table.1, for specific conditions, such as a completely symmetric manner, concluded results from the present method were compared with the Birkhoff and PER models. In these comparisons all coincidences of the jet and slug quantities with the two basic models are very good.

For example, considering the parameters of table.1 with $V_{01} = V_{02} = 0.41\ cm/\mu\text{sec}$, the following results are concluded which are coincident to the results of Birkhoff model.

$$M_{jet} = 83.94\ gr, M_{Slug} = 505.72\ gr, M_{total} = 589.66\ gr$$

For symmetric conical shaped charge with the following projection velocities $(V_{01} = V_{02} = 0.41 - 0.0131X)$, the jet and slug masses will be 127.43 and 462.57gr which are equal to the results of the PER model.

Table.1 Nominal parameters of sample shaped charge

Detonation velocity	0.8 cm/μsec
Cone height	20 cm
Cone half angle	22.5 deg
Density of liner material (cooper)	8.9 gr/cm3
Liner thickness	0.114 cm

Also for asymmetric wedge shaped charge with small asymmetry, coincidence of results in the present model and P.C. model [7] is very good. In specific conditions such as constant V_{01} and V_{02}, and small asymmetries, off axis velocity and core motion which result from the present model and the P.C. model are completely identical .

Now, the present methods are compared with the Brown-Curtis-Cook (BCC) method [8] for a typical shaped charge geometry.

As stated in [8] , $V_1(X_1)$ and $V_2(X_2)$ are:

$$\begin{cases} V_{01}(X_1) = U(A+BX_1)(1+\frac{P_C\cos\theta}{100}) \\ V_{02}(X_1) = U(A+BX_2)(1-\frac{P_C\cos\theta}{100}) \end{cases} \quad (22)$$

Since A and B are not specified in [8], they are obtained by trial and error procedure: $A=0.41$ and $B=-0.0131/Cm$.

Using these values for A and B (and $P_C=1$), velocities V_j , V_s and V_c will have the same forms as [8] until $t=80\ \mu s$ (90% of liner height).

But, the proposed method can show these quantities for more extended time and for the whole liner height.

Upon determining A and B, one can evaluate and compare the behavior of the present and BCC methods. According to the whole liner height analysis, the last element on arm 1 ($X_1=20cm$) will collide with the element at $X_2=19.15cm$ from arm 2 at about $t=100\ \mu\text{sec}$.

Other elements of arm 2 whose $X_2>19.15cm$ will not collide with reciprocal elements in arm 1 because of no element, in arm 1 remains. Jet and slug masses till $t=80\ \mu\text{sec}$ are $M_j=90.6\,\text{gr}$ and $M_s=373.4\,\text{gr}$. But these quantities have the values of $M_j=122.4\,\text{gr}$ and $M_s=454.6\,\text{gr}$ in $t=100\ \mu\text{sec}$. This comparison shows that a noticeable mass of jet and slug is added when the detonation wave reaches the liner base.

Before $t=80\ \mu\text{sec}$, 18 cm of arm 1 and 17.17 cm of arm 2 have been traversed by detonation wave. In other words, until $t=80\ \mu\text{sec}$, 90% of liner height from arm 1 and 86% of liner height in arm 2 will affect the jet and slug formation.

The evaluation of other quantities such as the jet and slug radius, off axis velocity and core motion for any desired P_C are investigated below.

The jet and slug radiuses which result from the present method are shown in Fig.5. In this figure the resulting radiuses from the present model and the B.C.C model are shown. Till $t=80\ \mu s$ these values have very good coincidence to BCC values ($P_C=1$) [8] and for $80<t<100\ \mu s$ the results are a completion of the BCC method.

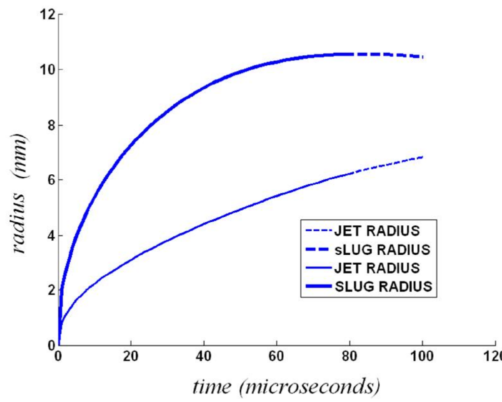

Fig.5 Jet and slug radius results from the present model and the BCC model.

For $P_C=5$ and concluded A and B, off axis velocity and core displacement of the jet are shown in Figures 6 and 7, respectively. In these figures, for $80<t<100\mu s$ the results of the present model and the BCC model generally have the same values. The off axis velocity and core displacement in the two methods also have the same behaviors.

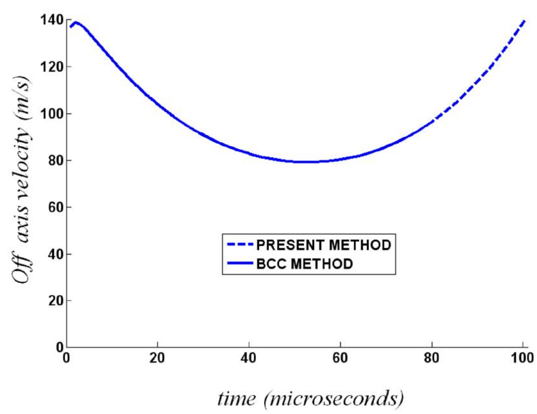

Fig.6 Off axis jet velocity in the present and the BCC model

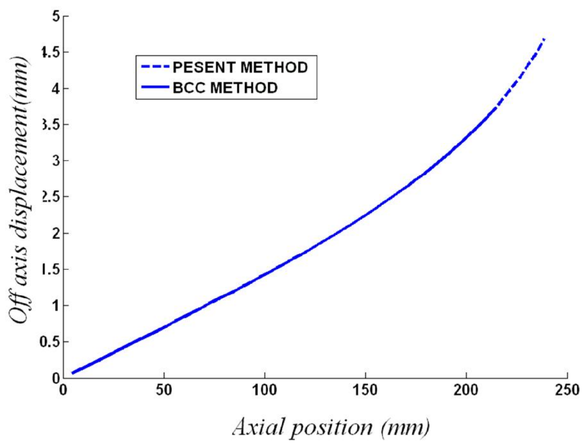

Fig.7 Core motion in the present and the BCC model

4 Conclusion

For analysis of asymmetric shaped charges, several models have been developed. It is logical that by means of kinematic relations of the liner elements motion, one can analyse core, jet and slug motion. However, due to some numerical considerations, the solution of kinematic relations is somewhat complicated. This is because of the nature of the projection velocity distribution which for conventional conical or planar shaped charges is often a regressive function of the axial coordinate. In earlier works on the analysis of asymmetric shaped charges, due to numerical errors, complete traversing of the liner height will not happen. For example, in a typical shaped charge, behavior of the core, jet and slug motion for the elements which have $X \geq 0.9H$ (H is liner height) cannot be determined.

But by means of a change in the coordinate system, traversing of the whole liner height is possible. The method proposed in this paper can solve the kinematic relations much faster than conventional method(s), since the numerical errors are eliminated.

Also, a mathematical model for the analysis of planar and conical asymmetric shaped charges is developed. This model is a generalization of the PER theory in the presence of asymmetry which leads to results of the PER model in case of symmetry. On the other hand, the behavior of a typical asymmetric shaped charge is similar to previous works up to 90% liner height and of course is a completion of them after this height.

Due to the behavior of the off-axis jet velocity near the liner base which is a progressive function, analysis after this point is very important.

On the other hand, it should be noted that about 25% of the jet mass is added near the liner base after 90% of the liner height is traversed.

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