Wave Propagation in an Irregular Fluid Saturated Porous Anisotropic Layer Sandwiched between a Homogeneous Layer and Half Space

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Abstract: - The propagation of shear waves in an anisotropic fluid saturated porous layer sandwiched between homogeneous isotropic layer and isotropic half-space with irregularity present at the interface, has been examined. The dispersion equation for shear waves is derived by using the perturbation technique. The effect of wave number and irregularity are studied numerically. Also the dispersion curves for different size of irregularity are compared graphically with the help of MATLAB. This study shows that the phase velocity is significantly influenced by the wave number and the size of irregularity.

Keywords: - Shear waves, Anisotropic layer, Dispersion Equation, Perturbation Technique, MATLAB.

1 Introduction

The earth has a layered structure, and this exerts a significant influence on the propagation of elastic waves. The propagation of elastic waves in homogeneous layer is of considerable importance in earthquake engineering and seismology. The study of wave propagation in elastic medium with different boundaries is of great importance to seismologists as well as to geophysicists to understand and predict the seismic behavior at different margins of earth. The propagation of shear waves has been studied by many authors with assuming different forms of irregularities at the interface. Bhattacharya [2] discussed the dispersion curves for Love wave propagation in a transversely isotropic crustal layer with an irregularity in the interface. Jones [3] discussed wave propagation in a two layered medium. Chattopadhyay et al. [4] studied the propagation of SH guided wave in an internal stratum with parabolic irregularity in the lower interface. Konczak [5] derived dispersion equation for shear waves in a multilayered medium including a fluid saturated porous stratum. The influence of irregularity and rigidity on the propagation of torsional waves has been discussed by Gupta et al. [6]. Love wave propagation in a porous rigid layer lying over an initially stressed half space is discussed by Kundu et al. [7]. For the elastic and viscoelastic waves, a long list of references is available in the monographs of Lamb [8], Victorov [9], Miklowitz [10] and Kolsky [11].

In this paper we have discussed the propagation of shear waves in a transversely isotropic fluid saturated porous layer resting on a homogeneous elastic half space, lying under an elastic isotropic and homogeneous layer with irregularity at the interface. The irregularity is in the form of rectangle. The perturbation technique indicated by Erigen and Samuels [1] has been used. The dispersion curves are depicted by means of graphs for different size of irregularity and different values of common wave velocity. The influence of depth of irregularity on phase velocity and some special cases have been studied.

2 Formulation of the Problem

A transversely isotropic fluid saturated porous layer of thickness H_2 resting on a homogeneous elastic half space, lying under an elastic isotropic and homogeneous layer of thickness H_1 has been considered. The Cartesian coordinate system (x, y, z) is chosen with z-axes taken vertically downward in the half space and x-axes is chosen parallel to the layer in the direction of propagation of the disturbance. We assume the irregularity in the form of a rectangle with length s and depth H'. The origin is placed at the middle point of the interface irregularity. The source of the disturbance is placed on positive z axes at a distance d (d> H') from the origin. Therefore, the upper layer describes the medium M$_1$: $-(H_1+H_2)\le z\le H_2$, the intermediate layer describes the medium M$_2$: $-H_2\le z\le 0$ and the homogeneous elastic half space describes the medium M$_3$: $0\le z<\infty$.The geometry of the problem is shown in figure: 1.

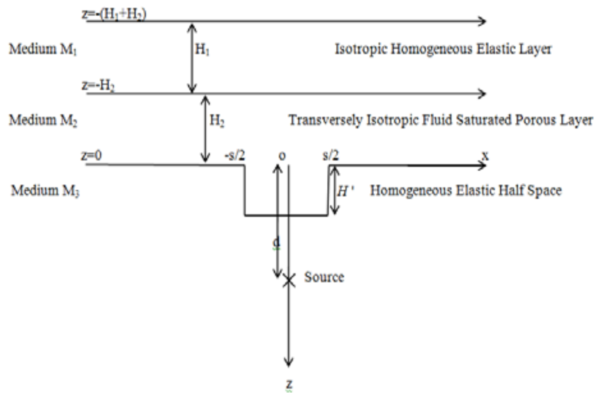

Figure 1: Geometry of the problem

The interface between the layer and half space is defined as

$$z=\varepsilon\, h(x) \tag{1}$$

where
$$h(x)=\begin{cases}0; x\le -\dfrac{s}{2}, x\ge \dfrac{s}{2}\\ f(x); -\dfrac{s}{2}\le x\le \dfrac{s}{2}\end{cases} \tag{2}$$

and $\varepsilon=\dfrac{H'}{s}$ and $\varepsilon\ll 1$.

3 Basic Equations

The basic equations for the medium considered are as follows:

3.1 For Medium M$_1$

The equations of motion, without body force [12] are given by:

$$\sigma^{(1)}_{ij.j}=\rho^{(1)}\ddot{u}^{(1)}_i, \tag{3}$$

where $\sigma^{(1)}{}_{ij}$ are the components of stress tensor, $u^{(1)}_i$ are the components of displacement vector, and $\rho^{(1)}$ is the density. The comma denotes differentiation with respect to position and dot denotes differentiation with respect to time.

The constitutive relations are given by

$$\sigma^{(1)}_{ij}=\lambda^{(1)}e^{(1)}_{kk}\delta_{ij}+2\mu^{(1)}e^{(1)}_{ij}, \tag{4}$$

where $\lambda^{(1)}$ and $\mu^{(1)}$ are Lame's elastic coefficients and δ_{ij} is the Kronecker delta and

$$2e^{(1)}_{ij}=(u^{(1)}_{i,j}+u^{(1)}_{j,i}),\ e^{(1)}_{kk}=u^{(1)}_{k,k}=e^{(1)}. \tag{5}$$

3.2 For Medium M$_2$

The equation of motion for the intermediate fluid saturated porous layer in the absence of body forces are [13]:

$$\sigma^{(2)}_{ij,j}=\rho_{11}\ddot{u}^{(2)}_i+\rho_{12}\ddot{U}^{(2)}_i-b_{ij}(\dot{U}^{(2)}_j-\dot{u}^{(2)}_j) \tag{6}$$

$$\sigma^{(2)}_{,i}=\rho_{12}\ddot{u}^{(2)}_i+\rho_{22}\ddot{U}^{(2)}_i+b_{ij}(\dot{U}^{(2)}_j-\dot{u}^{(2)}_j) \tag{7}$$

where $\sigma^{(2)}_{ij}$ are the components of stress tensor in the solid skeleton, $\sigma^{(2)}=-fp$ is the reduced pressure of the fluid (p is the pressure in the fluid, and f is the porosity of the medium), $u^{(2)}_i$ are the components of the displacement vector of the solid skeleton and $U^{(2)}_i$ are these of fluid.

The stress-strain relations for the transverse-isotropic fluid saturated porous layer are [13]:

$$\sigma^{(2)}{}_{11}=(2C_1+C_2)e^{(2)}_{11}+C_2e^{(2)}_{22}+C_3e^{(2)}_{33}+C_6e^{(2)}$$
$$\sigma^{(2)}{}_{22}=C_2e^{(2)}_{11}+(2C_1+C_2)e^{(2)}_{22}+C_3e^{(2)}_{33}+C_6e^{(2)}$$
$$\sigma^{(2)}{}_{33}=C_3e^{(2)}_{11}+C_3e^{(2)}_{22}+2C_4e^{(2)}_{33}+C_7e^{(2)}$$
$$\sigma^{(2)}{}_{23}=2C_5e^{(2)}_{23}$$
$$\sigma^{(2)}{}_{31}=2C_5e^{(2)}_{31}$$
$$\sigma^{(2)}{}_{12}=2C_1e^{(2)}_{12}$$
$$\sigma^{(2)}=C_6e^{(2)}_{11}+C_6e^{(2)}_{22}+C_7e^{(2)}_{33}+C_8e^{(2)}$$
(8)

$$2e^{(2)}_{ij}=(u^{(2)}_{i,j}+u^{(2)}_{j,i}),$$
$$\text{where } e^{(2)}=divU^{(2)}\equiv U^{(2)}_{j,j}, \tag{9}$$
$$e_{kk}=divu^{(2)}\equiv u^{(2)}_{k,k}$$

and $C_1,C_2,C_3,C_4,C_5,C_6,C_7,C_8$ are the material constants.

3.3 For Medium M$_3$

For the lower non-homogeneous half-space the basic equations of motion, without body force are [12]:

$$\sigma_{ij,j}^{(3)} = \rho^{(3)} \ddot{u}_i^{(3)}, \tag{10}$$

where $\sigma_{ij,j}^{(3)}$ are the components of stress tensor, $u_i^{(3)}$ are the components of displacement vector, and $\rho^{(3)}$ is the density.

The constitutive relations are given by

$$\sigma_{ij}^{(3)} = \lambda^{(3)} e_{kk}^{(3)} \delta_{ij} + 2\mu^{(3)} e_{ij}^{(3)}, \tag{11}$$

where $\lambda^{(3)}$ and $\mu^{(3)}$ are Lame's elastic coefficients and are functions of x, y, z and

$$2e_{ij}^{(3)} = (u_{i,j}^{(3)} + u_{j,i}^{(3)}), \quad e_{kk}^{(3)} = u_{k,k}^{(3)} = e^{(3)}. \tag{12}$$

In this paper, attention is confined to shear waves propagating in the xy-plane. The displacements are parallel to y direction and are independent of the y coordinate. Thus

$$\begin{aligned} u^{(1)} &\equiv w^{(1)} \equiv 0, & v^{(1)} &\equiv v^{(1)}(x,z,t), \\ u^{(2)} &\equiv w^{(2)} \equiv 0, & v^{(2)} &\equiv v^{(2)}(x,z,t), \\ U^{(2)} &\equiv W^{(2)} \equiv 0, & V^{(2)} &\equiv V^{(2)}(x,z,t), \\ u^{(3)} &\equiv w^{(3)} \equiv 0, & v^{(3)} &\equiv v^{(3)}(x,z,t), \end{aligned} \tag{13}$$

and the equations of motion (3), (6), (7) and (10) with the help of (4), (5) and (8), (9) and (11), (12) respectively reduce to the form

$$\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right\} v^{(1)} = \frac{1}{\beta_1^2} \frac{\partial^2 v^{(1)}}{\partial t^2}, \tag{14}$$

$$\left\{\begin{gathered} C_1 \frac{\partial^2}{\partial x^2} + C_5 \frac{\partial^2}{\partial z^2} - \\ \left[\rho_{11}\partial_t^2 + b_{11}\partial_t - \frac{(\rho_{12}\partial_t^2 - b_{11}\partial_t)^2}{\rho_{22}\partial_t^2 + b_{11}\partial_t}\right] \end{gathered}\right\} (v^{(2)}, V^{(2)}) = 0 \tag{15}$$

$$\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right\} v^{(3)} = \frac{1}{\beta_3^2} \frac{\partial^2 v^{(3)}}{\partial t^2}. \tag{16}$$

The appropriate boundary conditions for the considered problem are as:

(i) At the free surface $z = -(H_1 + H_2)$, the shear stress component vanishes, i.e.,

$$\sigma_{32}^{(1)}(x, z = -(H_1 + H_2), t) = 0. \tag{17}$$

(ii) At the interface $z = -H_2$, the displacements are continuous, i.e.,

$$v^{(1)}(x, z = -H_2, t) = v^{(2)}(x, z = -H_2, t). \tag{18}$$

(iii) At the interface $z = -H_2$, the shear stress components are continuous, i.e.,

$$\sigma_{32}^{(1)}(x, z = -H_2, t) = \sigma_{32}^{(2)}(x, z = -H_2, t). \tag{19}$$

(iv) At the interface $z = \varepsilon h(x)$, the displacements are continuous, i. e.,

$$v^{(2)}(x, z = \varepsilon h(x), t) = v^{(3)}(x, z = \varepsilon h(x), t). \tag{20}$$

(v) The stresses are continuous at the interface $z = \varepsilon h(x)$, i.e.,

$$\begin{aligned} &C_5 \frac{\partial v^{(2)}}{\partial z} - C_1 \varepsilon h'(x) \frac{\partial v^{(2)}}{\partial x} \\ &= \mu\left(\frac{\partial v^{(3)}}{\partial z} - \varepsilon h'(x) \frac{\partial v^{(3)}}{\partial x}\right), \end{aligned} \tag{21}$$

where $h'(x) = \dfrac{dh(x)}{dx}$.

Thus equations (14)-(16) with above boundary conditions are the governing equations of the problem considered.

4 Solution of the Problem:

For waves changing harmonically with time t and propagating in x-direction, we obtain

$$v^{(1)}(z,x,t) = v_0^{(1)}(z,x)\exp(i\omega t), \tag{22}$$

$$v^{(2)}(z,x,t) = v_0^{(2)}(z,x)\exp(i\omega t), \tag{23}$$

$$V^{(2)}(z,x,t) = V_0^{(2)}(z,x)\exp(i\omega t), \tag{24}$$

$$v^{(3)}(z,x,t) = v_0^{(3)}(z,x)\exp(i\omega t), \tag{25}$$

where ω is the angular frequency.

Thus equations of motion (14)-(16) take the form of

$$\frac{\partial^2 v_0^{(1)}}{\partial x^2} + \frac{\partial^2 v_0^{(1)}}{\partial z^2} + \frac{\omega^2}{\beta_1^2} v_0^{(1)} = 0, \tag{26}$$

$$\left(C_1 \frac{\partial^2}{\partial x^2} + C_5 \frac{\partial^2}{\partial z^2} + \xi_1^2\right)(v_0^{(2)}, V_0^{(2)}) = 0, \tag{27}$$

$$\frac{\partial^2 v_0^{(3)}}{\partial x^2} + \frac{\partial^2 v_0^{(3)}}{\partial z^2} + \frac{\omega^2}{\beta_3^2} v_0^{(3)} = 0. \tag{28}$$

where

$$\xi_1^2 = \alpha_1 + i\alpha_2,$$
$$\alpha_1 = F\omega^2 / c_G^2, \alpha_2 = R\omega^2 / c_G^2,$$
$$F = F(\omega) = \frac{1+\Omega^2\gamma_{22}C'}{1+(\Omega\gamma_{22})^2}.\frac{\gamma_{22}}{C'},$$
$$R = R(\omega) = \frac{(C'-\gamma_{22})\Omega}{1+(\Omega\gamma_{22})^2}.\frac{\gamma_{22}}{C'}, \tag{29}$$
$$C' = \gamma_{11}\gamma_{22} - \gamma_{12}^2, \gamma_{kl} = \frac{\rho_{kl}}{\rho}(k,l=1,2),$$
$$c_G^2 = (\rho_{11} - \rho_{12}^2/\rho_{22})^{-1}, \Omega = \frac{\rho\omega}{b_{11}}.$$

Ω is the dimensionless frequency and c_G is the velocity of shear wave in the porous layer.

Define the Fourier Transform $\bar{v}_0^{(1)}(z,\eta)$ of $v_0^{(1)}(z,\eta)$

as $$\bar{v}_0^{(1)}(z,\eta) = \int_{-\infty}^{\infty} v_0^{(1)}(z,x)e^{i\eta x}dx \tag{30}$$

And inverse Fourier Transform is given by

$$v_0^{(1)}(z,x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{v}_0^{(1)}(z,\eta)e^{-i\eta x}d\eta, \text{etc.} \tag{31}$$

The Fourier Transform of equations (26)-(28) then are

$$\frac{\partial^2\bar{v}_0^{(1)}}{\partial z^2} + \chi_1^2\bar{v}_0^{(1)} = 0, \tag{32}$$
$$\frac{\partial^2\bar{v}_0^{(2)}}{\partial z^2} + \chi_2^2\bar{v}_0^{(2)} = 0, \tag{33}$$
$$\frac{\partial^2\bar{V}_0^{(2)}}{\partial z^2} + \chi_2^2\bar{V}_0^{(2)} = 0, \tag{34}$$
$$\frac{\partial^2\bar{v}_0^{(3)}}{\partial z^2} - \chi_3^2\bar{v}_0^{(3)} = 0. \tag{35}$$

where

$$\chi_1^2 = \left(\frac{\omega^2}{\beta_1^2} - \eta^2\right), \chi_2^2 = \frac{C_1}{C_5}\left(\frac{\xi_1^2}{C_1} - \eta^2\right), \chi_3^2 = \left(\eta^2 - \frac{\omega^2}{\beta_3^2}\right)$$

The solution of equations (32)-(35) is

$$\bar{v}_0^{(1)} = A\cos\chi_1 z + B\sin\chi_1 z, \tag{36}$$
$$\bar{v}_0^{(2)} = C\cos\chi_2 z + D\sin\chi_2 z, \tag{37}$$
$$\bar{V}_0^{(2)} = \bar{C}\cos\chi_2 z + \bar{D}\sin\chi_2 z, \tag{38}$$
$$\bar{v}_0^{(3)} = E\exp(-\chi_3 z), \tag{39}$$

where $A, B, \bar{A}, \bar{B}, D$ are functions of η.

Thus, by inverse Fourier Transform, we obtain

$$v_0^{(1)}(z,x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}(A\cos\chi_1 z + B\sin\chi_1 z)e^{-i\eta x}d\eta, \tag{40}$$
$$v_0^{(2)}(z,x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}(C\cos\chi_2 z + D\sin\chi_2 z)e^{-i\eta x}d\eta, \tag{41}$$
$$V_0^{(2)}(z,x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}(\bar{C}\cos\chi_2 z + \bar{D}\sin\chi_2 z)e^{-i\eta x}d\eta, \tag{42}$$
$$v_0^{(3)}(z,x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}(Ee^{-\chi_3 z} + \frac{2}{\chi_3}e^{\chi_3 z}e^{-\chi_3 d})e^{-i\eta x}d\eta, \tag{43}$$

where the second term in the integrand of $v_0^{(3)}(z,x)$ is introduced due to the source in the lower half space.

The relations between the constants $\bar{C}, \bar{D}$ and C, D are provided by eq. (15).

We set the following approximations due to small value of ε

$$A \cong A_0 + A_1\varepsilon, B \cong B_0 + B_1\varepsilon, C \cong C_0 + C_1\varepsilon,$$
$$D \cong D_0 + D_1\varepsilon, E \cong E_0 + E_1\varepsilon. \tag{44}$$

Since the boundary is not uniform, the terms A, B, C, D, E in equation (44) are also functions of ε. Expanding these terms in ascending powers of ε and keeping in view that ε is small and so retaining the terms up to the first order of ε, A, B, C, D, E can be approximated as in equation (44). In physical situations, when the depth H' of the irregular boundary is too small with respect to the length of the boundary s, the above assumptions are justified. Further for small ε

$$e^{\pm\alpha\varepsilon h} \cong 1 \pm \alpha\varepsilon h, \cos\chi_1\varepsilon h \cong 1, \sin\chi_1\varepsilon h \cong \chi_1\varepsilon h$$

where α is any quantity.

Defining Fourier Transform of h(x) as

$$\bar{h}(\lambda) = \int_{-\infty}^{\infty} h(x)e^{i\lambda x}dx, \tag{45}$$

And the inverse Fourier Transform is

$$h(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{h}(\lambda)e^{-i\lambda x}d\lambda, \tag{46}$$

Therefore, $$h'(x) = \frac{-i}{2\pi}\int_{-\infty}^{\infty}\lambda\bar{h}(\lambda)e^{-i\lambda x}d\lambda \tag{47}$$

Now, by using boundary conditions (17)-(21) along with equations (40)-(41) and (43)-(44) we obtain a system of ten equations (after equating the absolute term (terms not containing ε) and the coefficients of ε):

$$A_0 \sin(H_1+H_2)\chi_1 + B_0 \cos(H_1+H_2)\chi_1 = 0 \quad (48)$$

$$\begin{aligned} &A_0 \cos H_2\chi_1 - B_0 \sin H_2\chi_1 \\ &- C_0 \cos H_2\chi_2 + D_0 \sin H_2\chi_2 = 0 \end{aligned} \quad (49)$$

$$\begin{aligned} &\mu\chi_1\left[A_0 \sin H_2\chi_1 + B_0 \cos H_2\chi_1\right] \\ &- C_5\chi_2\left[C_0 \sin H_2\chi_2 + D_0 \cos H_2\chi_2\right] = 0 \end{aligned} \quad (50)$$

$$C_0 - E_0 = \frac{2}{\chi_3} e^{-\chi_3 d} \quad (51)$$

$$\mu\chi_3 E_0 + C_5\chi_2 D_0 = 2\mu e^{-\chi_3 d} \quad (52)$$

$$A_1 \sin(H_1+H_2)\chi_1 + B_1 \cos(H_1+H_2)\chi_1 = 0 \quad (53)$$

$$\begin{aligned} &A_1 \cos H_2\chi_1 - B_1 \sin H_2\chi_1 \\ &- C_1 \cos H_2\chi_2 + D_1 \sin H_2\chi_2 = 0. \end{aligned} \quad (54)$$

$$\begin{aligned} &\mu\chi_1\left[A_1 \sin H_2\chi_1 + B_1 \cos H_2\chi_1\right] \\ &- C_5\chi_2\left[C_1 \sin H_2\chi_2 + D_1 \cos H_2\chi_2\right] = 0. \end{aligned} \quad (55)$$

$$E_1 - C_1 = R_1(k) \quad (56)$$

$$\mu\chi_3 E_1 + C_5\chi_2 D_1 = R_2(k) \quad (57)$$

where $R_1(k)$ and $R_2(k)$ are given by appendix-A.

Solving the above system of equations for $A_0, B_0, C_0, D_0, E_0, A_1, B_1, C_1, D_1, E_1$ and the corresponding values are given in Appendix-A.

The displacement in the anisotropic layer is

$$v_0^{(2)} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{4\mu e^{-\chi_3 d}\left\{1+\varepsilon(R_2-\mu\chi_3 R_1)e^{\chi_3 d}/4\mu\right\}\left[B_2+B_3\right]}{E(k)}e^{-ikx}\right)dk, \quad (58)$$

where B_2 and B_3 are given by appendix-A.

Now from equations (1), (2) and (45), we have

$$\bar{h}(\lambda) = \frac{2s}{\lambda}\sin\frac{\lambda s}{2}. \quad (59)$$

Using values of $R_1(k)$ and $R_2(k)$ as given in appendix-A, we obtain

$$(R_2 - \mu\chi_3 R_1) = \frac{s}{\pi}\int_{-\infty}^{\infty}\begin{bmatrix}\phi(k-\lambda)\\ +\phi(k+\lambda)\end{bmatrix}\frac{1}{\lambda}\sin\frac{\lambda s}{2}d\lambda \quad (60)$$

where $\phi(k-\lambda)$ is given in Appendix-A.

Using asymptotic formula of Willis [14] and Tranter [15] and neglecting the terms containing 2/s and highest powers of 2/s for large s, we obtain

$$\int_{-\infty}^{\infty}\begin{bmatrix}\phi(k-\lambda)\\ +\phi(k+\lambda)\end{bmatrix}\frac{1}{\lambda}\sin\frac{\lambda s}{2}d\lambda \cong \frac{\pi}{2}.2\phi(k) = \pi\phi(k). \quad (61)$$

Now using equations (60) and (61), we obtain

$$R_2 - \mu\chi_3 R_1 = s\phi(k) = \frac{H'}{\varepsilon}\phi(k) \quad (62)$$

Therefore the displacement in the anisotropic layer is

$$v_0^{(2)} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4\mu e^{-\chi_3 d}}{E(k)\left[1-H'\psi(k)e^{\chi_3 d}\right]}\left[B_2+B_3\right]e^{-ikx}dk, \quad (63)$$

where $\psi(k) = \dfrac{\phi(k)}{4\mu}$.

The value of this integral depends entirely on the contribution of the poles of the integrand. The poles are located at the roots of the equation

$$E(k)\left\{1-H'\psi(k)e^{\chi_3 d}\right\} = 0 \quad (64)$$

This equation is the dispersion equation for SH waves.

If c is the common wave velocity of wave propagating along the surface, then we can set in equation (64) $\omega = ck$ (ω is the circular frequency and k is the wave number), $\chi_1 = P_1 k, \chi_2 = P_2 k, \chi_3 = P_3 k$ where

$$P_1 = \sqrt{\frac{c^2}{\beta_1^2}-1}, P_3 = \sqrt{1-\frac{c^2}{\beta_3^2}}$$

$$P_2 = \sqrt{\left(\frac{1}{C_5}\left(\frac{c^2}{c_G^2}.F(\omega)-C_1\right)+i\frac{1}{C_5}.\frac{c^2}{c_G^2}.R(\omega)\right)}$$

Solving equation (64), we obtain

$$\frac{\mu P_1}{C_5 P_2}\left(\begin{matrix}\left[\tan P_1 kH_2 - \tan P_1 k(H_1+H_2)\right]\\ \left\{\begin{matrix}(1+P_3 kH')\left[C_5 P_2 + \mu P_3 \tan P_2 kH_2\right]\\ -H'P_2 k(C_5 P_2 \tan P_2 kH_2 - \mu P_3)\end{matrix}\right\}\end{matrix}\right) = \left(\begin{matrix}\left[1+\tan P_1 kH_2 \tan P_1 k(H_1+H_2)\right]\\ \left\{\begin{matrix}H'P_2 k(C_5 P_2 + \mu P_3 \tan P_2 kH_2)\\ +(1+P_3 kH')\left[C_5 P_2 \tan P_2 kH_2 - \mu P_3\right]\end{matrix}\right\}\end{matrix}\right) \quad (65)$$

Since the quantity P_2^2 is complex, so we have

$$P_2 = k_1 + ik_2, \quad (66)$$

where

$$k_{1,2} = \left\{ \frac{1}{2} \left(\sqrt{ \left\{ \frac{1}{C_5}\left(\frac{c^2}{c_G^2}.F(\omega) - C_1 \right) \right\}^2 + \left(\frac{1}{C_5}.\frac{c^2}{c_G^2}.R(\omega) \right)^2 } \pm \left\{ \frac{1}{C_5}\left(\frac{c^2}{c_G^2}.F(\omega) - C_1 \right) \right\} \right) \right\}^{\frac{1}{2}} \tag{67}$$

Thus, the equation (65) is complex and its real part gives the dispersion equation for shear waves.

5 Numerical Results

In order to investigate the effect of irregularity present in the transversely isotropic fluid saturated porous layer and to compare the results numerically between the phase velocity and the wave number, we will use the values of elastic constants given by Haojiang Ding et al. [16] for medium M_2 and Konczak [7] for medium M_1 and M_3. And by using MATLAB, we obtain the following graph for different values of common wave velocity c for two special cases as:

Case I: - When H_2=0, H_1=H, that is the wave propagation in elastic homogeneous layer lying over a homogeneous half space:

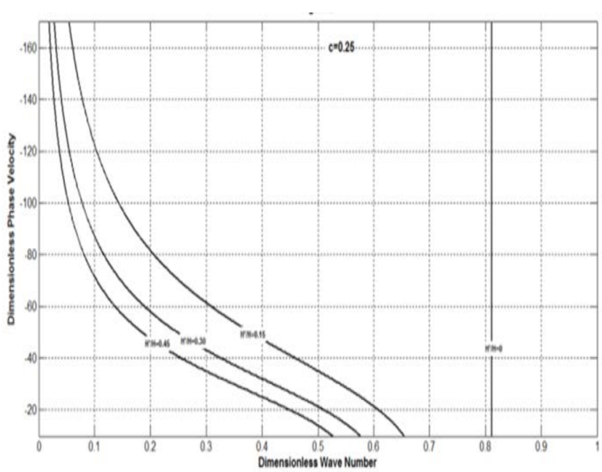

Figure 2: Variation of the dimensionless phase velocity (c/c_G) against the dimensionless wave number (kH) for different values of H'/H (0, 0.15, 0.30, 0.45) and common wave velocity **c=0.25**.

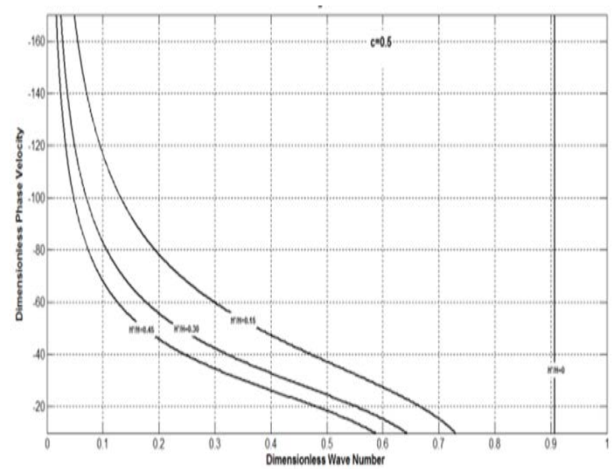

Figure 3: Variation of the dimensionless phase velocity (c/c_G) against the dimensionless wave number (kH) for different values of H'/H (0, 0.15, 0.30, 0.45) and common wave velocity **c=0.5**.

Figure 4

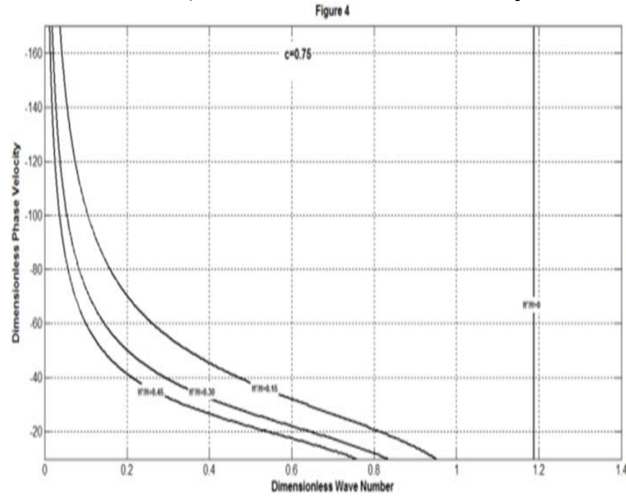

Figure 4: Variation of the dimensionless phase velocity (c/c_G) against the dimensionless wave number (kH) for different values of H'/H (0, 0.15, 0.30, 0.45) and common wave velocity **c=0.75**.

Case II: - When H_1=0, H_2=H, that is the wave propagation in a transversely isotropic fluid saturated porous layer lying over a homogeneous half space:

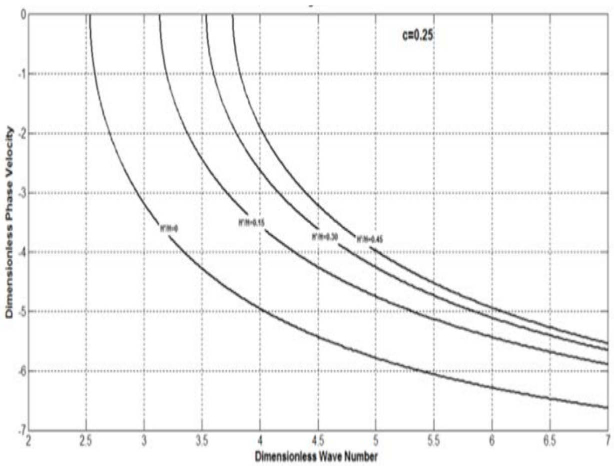

Figure 5: Variation of the dimensionless phase velocity (c/c_G) against the dimensionless wave number (kH) for different values of H'/H (0, 0.15, 0.30, 0.45) and common wave velocity **c=0.25**.

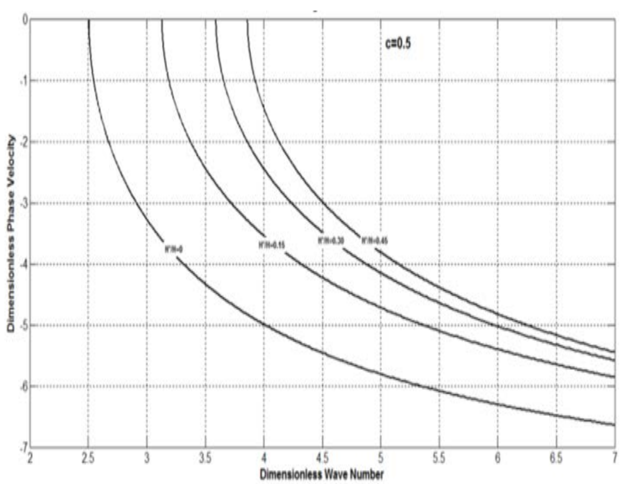

Figure 6: Variation of the dimensionless phase velocity (c/c_G) against the dimensionless wave number (kH) for different values of H'/H (0, 0.15, 0.30, 0.45) and common wave velocity **c=0.5**.

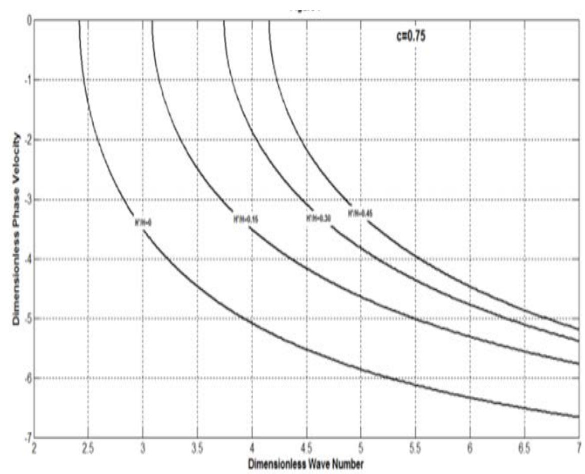

Figure 7: Variation of the dimensionless phase velocity (c/c_G) against the dimensionless wave number (kH) for different values of H'/H (0, 0.15, 0.30, 0.45) and common wave velocity **c=0.75**.

The dimensionless phase velocity (c/c_G) is plotted against the dimensionless wave number (kH) in Figures 2-7. It is clear from above figures that the phase velocity decreases with increase in wave number and also increase in the value of H'/H.

6. Conclusions

Propagation of shear waves in a transversely isotropic fluid saturated porous layer with irregularity over a homogeneous isotropic half space and lying under an elastic isotropic and homogeneous layer has been studied. The Eringen's perturbation method is applied to find the dispersion equation and displacement field in the layer. The effect of dimensionless wave number on dispersion curve is shown graphically for different cases. Variation of phase velocity for different ratio of irregularity depth to the layer width is studied and shown graphically. It has been observed that:

- In general the phase velocity of shear waves in transversely isotropic fluid saturated porous layer over a homogeneous half space with irregularity decreases with the increase in wave number.
- Phase velocity is a function of wave number as well as layer width and depth of irregularity.

Thus, it is concluded that the phase velocity in transversely isotropic fluid saturated porous layer sandwiched between isotropic layer and half space with irregularity at the interface is significantly affected by not only the depth of irregularity , but also by wave number and ratios of the depth of the irregularity to layer width and layer structure.

7. Acknowledgments

The authors are grateful to the reviewers and Editor for constructive suggestions for the improvement of this paper.

Appendix-A

$$R_1(k)=\frac{1}{2\pi}\int_{-\infty}^{\infty}[(D_0\chi_2+E_0\chi_3-2e^{-\chi_3 d})]^{\eta=k-\lambda}\,\bar{h}(\lambda)d\lambda$$

$$R_2(k)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\begin{bmatrix}\left\{(C_5\chi_2^2C_0+\mu(\chi_3^2E_0+2\chi_3e^{-\chi_3 d})\right\}\\ -\lambda k\left\{C_0C_1+\mu(E_0+\dfrac{2}{\chi_3}e^{-\chi_3 d})\right\}\end{bmatrix}^{\eta=k-\lambda}\bar{h}(\lambda)d\lambda$$

$$A_0=\frac{4\mu e^{-\chi_3 d}\left[C_5\chi_2\sec^2\chi_2H_2+2\mu\chi_3\tan\chi_2H_2\right]\cos\chi_2H_2}{E(k)\cos\chi_1H_2},$$

$$B_0=-\frac{4\mu e^{-\chi_3 d}\tan(H_1+H_2)\chi_1\left[C_5\chi_2\sec^2\chi_2H_2+2\mu\chi_3\tan\chi_2H_2\right]\cos\chi_2H_2}{E(k)\cos\chi_1H_2},$$

$$C_0=\frac{4\mu e^{-\chi_3 d}\begin{bmatrix}\mu\chi_1\tan\chi_2H_2\left(\tan\chi_1H_2-\tan\chi_1(H_1+H_2)\right)\\ +C_5\chi_2\left(1+\tan\chi_1H_2\tan\chi_1(H_1+H_2)\right)\end{bmatrix}}{E(k)},$$

$$D_0=\frac{4\mu e^{-\chi_3 d}\begin{bmatrix}\mu\chi_1\left(\tan\chi_1H_2-\tan\chi_1(H_1+H_2)\right)\\ -C_5\chi_2\tan\chi_2H_2\left(1+\tan\chi_1H_2\tan\chi_1(H_1+H_2)\right)\end{bmatrix}}{E(k)},$$

$$E_0=\frac{2e^{-\chi_3 d}}{\chi_3E(k)}\begin{bmatrix}\begin{Bmatrix}\mu\chi_1\left(\tan\chi_1H_2-\tan\chi_1(H_1+H_2)\right)\\ \left(\mu\chi_3\tan\chi_2H_2-C_5\chi_2\right)\end{Bmatrix}\\ +\begin{Bmatrix}C_5\chi_2\left(1+\tan\chi_1H_2\tan\chi_1(H_1+H_2)\right)\\ \left(C_5\chi_2\tan\chi_2H_2+\mu\chi_3\right)\end{Bmatrix}\end{bmatrix},$$

$$A_1=\frac{(R_2-\mu\chi_3R_1)\cos\chi_2H_2}{E(k)\cos\chi_1H_2}\begin{bmatrix}\left(C_5\chi_2+\mu\chi_3\tan\chi_2H_2\right)\\ +\tan\chi_2H_2\left(C_5\chi_2\tan\chi_2H_2-\mu\chi_3\right)\end{bmatrix},$$

$$B_1=-\frac{\begin{bmatrix}\begin{pmatrix}(R_2-\mu\chi_3R_1)\\ .\tan\chi_1(H_1+H_2)\\ .\cos\chi_2H_2\end{pmatrix}\begin{pmatrix}\left(C_5\chi_2+\mu\chi_3\tan\chi_2H_2\right)\\ +\tan\chi_2H_2\\ .\left(C_5\chi_2\tan\chi_2H_2-\mu\chi_3\right)\end{pmatrix}\end{bmatrix}}{E(k)\cos\chi_1H_2},$$

$$C_1=\frac{(R_2-\mu\chi_3R_1)\begin{bmatrix}C_5\chi_2\left(1+\tan\chi_1H_2\tan\chi_1(H_1+H_2)\right)\\ -\mu\chi_1\tan\chi_2H_2\begin{pmatrix}\tan\chi_1H_2\\ -\tan\chi_1(H_1+H_2)\end{pmatrix}\end{bmatrix}}{E(k)},$$

$$D_1=\frac{(R_2-\mu\chi_3R_1)}{E(k)}\begin{bmatrix}\mu\chi_1\left(\tan\chi_1H_2-\tan\chi_1(H_1+H_2)\right)\\ -C_5\chi_2\tan\chi_2H_2\\ .\left(1+\tan\chi_1H_2\tan\chi_1(H_1+H_2)\right)\end{bmatrix},$$

$$E_1=\frac{C_5\chi_2}{E(k)}\begin{bmatrix}(R_2-C_5\chi_2R_1)\tan\chi_2H_2\begin{pmatrix}1+\tan\chi_1H_2\\ .\tan\chi_1(H_1+H_2)\end{pmatrix}\\ +\mu\chi_1R_1\left(\tan\chi_1H_2-\tan\chi_1(H_1+H_2)\right)\end{bmatrix},$$

where

$$E(k)=\begin{bmatrix}\mu\chi_1\begin{Bmatrix}\left(\tan\chi_1H_2-\tan\chi_1(H_1+H_2)\right)\\ \left(\mu\chi_3\tan\chi_2H_2+C_5\chi_2\right)\end{Bmatrix}\\ -C_5\chi_2\begin{Bmatrix}\left(1+\tan\chi_1H_2\tan\chi_1(H_1+H_2)\right)\\ \left(C_5\chi_2\tan\chi_2H_2-\mu\chi_3\right)\end{Bmatrix}\end{bmatrix}$$

$$B_2=\mu\chi_1\begin{bmatrix}\left(\tan\chi_1H_2-\tan\chi_1(H_1+H_2)\right)\\ \left(\sin\chi_2z-\tan\chi_2H_2\cos\chi_2z\right)\end{bmatrix}$$

$$B_3=C_5\chi_2\begin{bmatrix}\left(1+\tan\chi_1H_2\tan\chi_1(H_1+H_2)\right)\\ \left(\cos\chi_2z-\tan\chi_1(H_1+H_2)\right)\sin\chi_2z\end{bmatrix}$$

$$\phi(k-\lambda)=A_2+A_3,$$

$$A_2=C_5\chi_2^2C_0-\mu\chi_3\left(\chi_2D_0-4e^{-\chi_3 d}\right)$$

$$A_3=-\lambda k\left[C_1C_0+\mu\left(E_0+\frac{2}{\chi_3}e^{-\chi_3 d}\right)\right]$$

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