# Initial Imperfections of Steel and Steel-Concrete Composite Columns Subjected to Buckling Compression

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*Abstract:* - The paper deals with the problems of the load-carrying capacity of steel and steel-concrete composite members composed of high-strength materials, subjected to compression. The attention is mainly paid to the buckling resistance in the connection with member imperfections, which are usually covered by the equivalent initial geometrical imperfection expressed as the maximal initial member curving in the mid-length of the buckled member subjected to compression. The paper is oriented to the analytical solution of the initial eccentricity based on the conception of the buckling strength and to the possibilities how to verify the initial imperfection expresented by open HEA cross-section and the same cross-sections partially encased by concrete, and by steel circular tubes and steel tubes filled by concrete, in both cases applying normal-strength and high-strength concrete. Using test results of the specimens subjected to compression of actual values of initial imperfections with the corresponding theoretical values is presented here.

*Key-Words:* - Column buckling, compression, resistance, strength, imperfection, eccentricity, steel, concrete, HEA cross-section, circular cross-section, theoretical analysis, experimental verification, Southwell's line.

## **1** Introduction

According to the most of current normative rules (see e.g. [10], [11]), the buckling resistance of compression members can be usually determined based on two basic approaches: (i) the second order method considering the initially curved member subjected to the centric compression, that means the member is subjected to the centric compression force combined with the bending moment caused by member curving, or (ii) the method considering the reduction factor, so that the full plastic compression resistance is reduced applying the buckling factor expressing the influence of added bending moment.

In the case (i) it is necessary to know the initial member curving substituting all imperfections of the member, i.e. geometrical, structural and material imperfections. To substitute all essential member imperfections, almost of the standards recommend the equivalent geometrical imperfection, usually as a part of the member buckling length, but it is a question, whether this value of the imperfection sufficiently corresponds with the reality. In the case (ii) the buckling resistance is influenced by the relative initial eccentricity, which is one of the most important determining parameters. The relative initial eccentricity can be given using the absolute initial eccentricity, which can be determined from the tests, for example.

# 2 Initial Eccentricity of Columns

The conception of compression member buckling covered by the normative rules for the design of steel structures arises from the model of the real members with the equivalent initial curving, which is similar to the form of the stability lost of the ideal member – see, for example, [1], [2], [3].

## 2.1 Steel Columns

The maximal stress of the slender member with the cross-section area of A, subjected to the axial force N, is given by the known formula of

$$\sigma_{\max} = \sigma_0 \cdot \left[ 1 + m_0 / \left( 1 - \frac{\sigma_0}{\sigma_{cr}} \right) \right], \qquad (1)$$

where the axial stress  $\sigma_0$  is

$$\sigma_0 = \frac{N}{A},\tag{2}$$

the critical stress of the ideal member  $\sigma_{cr}$  is

$$\sigma_{cr} = \pi^2 \frac{E}{\lambda^2} \tag{3}$$

and the relative initial eccentricity  $m_0$  is

$$m_0 = \frac{e_0}{j},\tag{4}$$

if j = W / A and  $e_0$  is the maximal initial eccentricity in the mid-length of the member sinusoidal curved.

Defining the buckling strength  $\overline{\sigma_0} \equiv \sigma_0$  as axial stress, when the yield strength is reached in the most loaded edge of the cross-section of curved member, i.e.  $\sigma_{\text{max}} \equiv f_y$ , then the buckling strength according to (1) is explicitly given by the formula

$$\overline{\sigma_{0}} = \frac{1}{2} [f_{y} + (1 + m_{0}) \cdot \sigma_{cr}] - \sqrt{\frac{1}{4} [f_{y} + (1 + m_{0}) \cdot \sigma_{cr}]^{2} - f_{y} \cdot \sigma_{cr}}$$
(5)

Analogically to the condition for the structural members subjected to tension

$$\frac{N}{A} \le \frac{f_y}{\gamma_{\rm M}} = f_{yd} \tag{6}$$

with the partial safety factor for material  $\gamma_M$ , the condition for buckling strength of the compression member can be written in the form of

$$\frac{N}{A} \le \frac{\sigma_0}{\gamma_{\rm M}} = \overline{\sigma_0} \cdot \frac{f_{yd}}{f_y} = \chi \cdot f_{yd}, \qquad (7)$$

where the reduction buckling factor  $\chi$  is

$$\chi = \frac{\sigma_0}{f_y},\tag{8}$$

and using the equation (5) the reduction buckling factor  $\chi$  is expressed by the formula of

$$\chi = \frac{1}{2} \left[ 1 + (1 + m_0) \cdot \frac{\sigma_{cr}}{f_y} \right] - \sqrt{\frac{1}{4} \left[ 1 + (1 + m_0) \cdot \frac{\sigma_{cr}}{f_y} \right]^2 - \frac{\sigma_{cr}}{f_y}}$$
(9)

Within the accepted concept, the relative initial eccentricity  $m_0$  only is undetermined quantity. By the suitable choosing eccentricity  $m_0$ , it is possible to define the level of convention buckling strength in the structural design process, to respect the test results of real compression members. The general equation for the relative initial eccentricity covering

the influence of imperfections of the real member is efficient to choose in the form of

$$m_0 = \alpha_1 \cdot \frac{f_y}{\sigma_{cr}} = \alpha_1 \cdot \left(\frac{\lambda}{\lambda_{fy}}\right)^2 \tag{10}$$

where  $\alpha_1$  is the imperfection factor, and material slenderness characteristics  $\lambda_{fy}$  is given as

$$\lambda_{fy} = \pi \cdot \sqrt{\frac{E}{f_y}} = \pi \cdot \sqrt{\frac{E}{\gamma_M \cdot f_{yd}}} \,. \tag{11}$$

The formula for the reduction buckling factor  $\chi$  in dependence on the slenderness  $\lambda$  can be modified to the general form of

$$\chi = \frac{1}{2} \left[ 1 + \alpha_1 + \left( \frac{\pi}{\lambda} \cdot \sqrt{\frac{E}{f_y}} \right)^2 \right] - \sqrt{\frac{1}{4} \left[ 1 + \alpha_1 + \left( \frac{\pi}{\lambda} \cdot \sqrt{\frac{E}{f_y}} \right)^2 \right]^2 - \left( \frac{\pi}{\lambda} \cdot \sqrt{\frac{E}{f_y}} \right)^2} \right]^2} . (12)$$

For the determination of the reliable dependence  $\chi(\lambda)$ , the essential question is to choose the relative initial eccentricity  $m_0$ . Test result analysis can give significant knowledge for the verification of the rightness of the convention normative values of the equivalent initial curving covering the imperfection influence on the compression buckling capacity.

If the equations (4) and (10) are considered, then the following dependence must be valid

$$\frac{e_0}{j} = \alpha_1 \cdot \left(\frac{\lambda}{\lambda_{fy}}\right)^2, \tag{13}$$

from where the maximal initial eccentricity  $e_0$  in the middle of the member length is

$$e_0 = j \cdot \alpha_1 \cdot \left(\frac{\lambda}{\lambda_{fy}}\right)^2 = \alpha_1 \cdot \frac{f_y}{\pi^2 \cdot E} \cdot \frac{L_{cr}^2}{z}, \qquad (14)$$

where z is the distance of the cross-section edge from the gravity centre.

#### 2.2 Steel-Concrete Columns

Accepting the approach for steel slender columns described above, then for columns composed of two materials with different mechanical properties the concept of substitute (ideal) steel cross-section can be used, that means geometrical parameters, i.e. the area  $A_i$  and the second moment of area  $I_i$  of the substitute cross-section must be applied as follows:

$$A_i = A_a + \frac{A_c}{n},\tag{15}$$

$$I_i = I_a + \frac{I_c}{n}, \tag{16}$$

where n is the ratio of steel-to-concrete Young's modulus of elasticity

$$n = \frac{E_a}{E_c}.$$
 (17)

The maximal stress  $\sigma_{\text{max}}$  of steel-concrete slender column with the cross-section area consisting of steel part  $A_a$  and concrete part  $A_c$  and with yield strength  $f_y$  and concrete cylindrical strength  $f_c$  can be given by the formula (1), where the axial stress  $\sigma_0$  is

$$\sigma_0 = \frac{N}{A_i},\tag{18}$$

the critical stress of the ideal member  $\sigma_{cr}$  is

$$\sigma_{cr} = \pi^2 \frac{E_a}{\lambda^2},\tag{19}$$

the relative initial eccentricity  $m_0$  can be calculated

$$m_0 = \frac{e_0}{j_i} \tag{20}$$

and the buckling strength can be given by (5).

Analogically to the members in tension – see (6), the condition for buckling load-carrying capacity of steel-concrete compression column can be written as

$$\frac{N}{A_i} \le \frac{\sigma_0}{\gamma_{\rm M}} = \overline{\sigma_0} \cdot \frac{f_{yd}}{f_y} = \chi \cdot f_{yd}$$
(21)

with the reduction buckling factor according to (8), and subsequently according to (9).

Then, the relative initial eccentricity  $m_0$  can be considered according to the form of

$$m_0 = \alpha_1 \cdot \frac{f_y}{\sigma_{cr}} = \alpha_1 \cdot \left(\frac{\lambda_i}{\lambda_{fy,i}}\right)^2, \qquad (22)$$

where the characteristics of material slenderness is

$$\lambda_{fy,i} = \pi \cdot \sqrt{\frac{E_a}{f_y}} = \pi \cdot \sqrt{\frac{E_a}{\gamma_M \cdot f_{yd}}} \,. \tag{23}$$

The maximal initial eccentricity  $e_0$  in the middle of the member length can be given, as in the case of circular steel columns, by the formula (14).

The reduction buckling factor  $\chi$  in dependence on the slenderness  $\lambda$  can be written as

$$\chi = \frac{1}{2} \left[ 1 + \alpha_1 + \left( \frac{\pi}{\lambda_i} \cdot \sqrt{\frac{E_a}{f_y}} \right)^2 \right] - \sqrt{\frac{1}{4} \left[ 1 + \alpha_1 + \left( \frac{\pi}{\lambda_i} \cdot \sqrt{\frac{E_a}{f_y}} \right)^2 \right]^2 - \left( \frac{\pi}{\lambda_i} \cdot \sqrt{\frac{E_a}{f_y}} \right)^2} .$$
(24)

# 3 Initial Imperfections of Steel and Steel-Concrete Composite Columns with HEA Cross-Section

Let us have the open HEA cross-section with the section height of h and the width of b (flange width). Then, for that cross-sections with concrete encasement between flanges, in the case of the buckling in the flange direction (to the weaker axis), the distance z in (14) is a half of the flange width, i.e. b/2, so that the initial imperfection  $e_0$  is

$$e_0 = \alpha_1 \cdot \frac{f_y}{\pi^2 \cdot E} \cdot \frac{2 \cdot L_{cr}^2}{b}.$$
 (25)

For selected steel and steel-concrete columns based on HEA cross-section experimentally verified (see [4], [5], [6]), the actual values of initial imperfections have been derived from test results. The specimens of various geometrical and material parameters have been tested, to verify their actual behaviour, failure mechanism and buckling loadcarrying capacity, if subjected to compression. Using the formula (25) the theoretical values of initial imperfections have been calculated for the properties of tested columns. For the comparison, also recommended values of initial eccentricities have been calculated according to standards for the case if the second order theory is applied.

### **3.1 Actual Values of Initial Imperfection** Experimentally Verified

Experimental verification has been realized with the specimens of HEA cross-sections represented by steel and steel-concrete members composed of normal-strength and high-strength materials. The investigated columns were simply supported with the lengths of L = 3070 mm and L = 3000 mm, that buckling lengths were  $L_{cr} = 3070$  mm and  $L_{cr} = 3000$  mm. Various material properties including their combinations have been used: (i) steel grades of S 235 and S 355, (ii) concrete classes of C 20/25, C50/60, C70/85 and C 80/95. The overview of tested specimens, including geometrical parameters and measured mechanical properties, is in Table 1.

	steel		concrete	
Cross-section	$f_y$ [MPa]	E <sub>a</sub> [GPa]	f <sub>c</sub> [MPa]	E <sub>c</sub> [GPa]
HE 140A	456	not meas- ured		
HE 140A + NS1 concrete			27.0	32.0
HE 140A + HS1 concrete			87.1	49.0
HE 160A-I	258	202		
HE 160A-I + NS2 concrete			56.5	48.0
HE 160A-I + HS2 concrete			71.1	42.0
HE 160A-II	439	216		
HE 160A-II + NS2 concrete			56.5	48.0
HE 160A-II + HS2 concrete			71.1	42.0

Table 1 Overview of investigated columns: geometrical parameters and mechanical properties (mean values of measured ones)

NS concrete – normal-strength concrete; HS concrete – high-strength concrete; 3 test specimens performed for each group



Fig. 1 Illustration of test arrangement and realization: HEA cross-section columns

The actual values of the initial eccentricities have been derived using Southwell's lines, examples of which are illustrated by the graphs in Fig. 2. On the horizontal axis there are the values of the transverse deformation, i.e. deflection w in the mid-length of the member. On the vertical axis there are the values of the ratio of deflection w to axial force N that means w / N. Initial eccentricity  $e_0$ , that means the equivalent initial geometrical imperfection, is a value of the deflection for which w / N = 0.



Fig.2 Illustration of Southwell's lines: a) HEA 140A cross-section without concrete; b) HE 140A crosssections with partial encasement by normal-strength concrete NS1; c) HE 140A cross-sections with partial encasement by high-strength concrete NS2

## 3.2 Theoretical Values of Initial Imperfection Analytically Calculated

According to the formula (25), the theoretical value of equivalent initial imperfection  $e_0$ , also so-called equivalent geometrical imperfection, depends on the yield strength and Young's modulus of steel, and on buckling length and flange thickness, both in the case of steel columns and in the case of steel-concrete ones. It is because steel-concrete cross-section has been transformed to the substitute steel cross-section.

Table 2 Comparison of actual and theoretical values of initial imperfections: HEA cross-section columns

Cross-section	actual		theoretical	
	$e_0$ [mm]	e <sub>0</sub> / L [-]	$e_0$ [mm]	<i>e</i> <sub>0</sub> / <i>L</i> [-]
HE 140A	1.324 0.629 1.210	1/2319 1/4881 1/2537		
HE 140A + NS1 concrete	0.630 1.260 0.565	1/4873 1/2437 1/5434	14.515	$\frac{1}{212}$
HE 140A + HS1 concrete	0.768 1.892 1.318	1/3997 1/1623 1/2329		
HE 160A-I	1.513 0.722 0.652	1/1983 1/4155 1/4601		
HE 160A-I + NS2 concrete	0.613 1.350 0.633	1/4894 1/2222 1/4739	7.134	$\frac{1}{421}$
HE 160A-I + HS2 concrete	0.465 0.915 1.297	1/6452 1/3279 1/2313		
HE 160A-II	0.540 0.700 1.154	1/5556 1/4286 1/2600	5 721	1
HE 160A-II + NS2 concrete	1.100 0.585 0.817	1/2727 1/5128 1/3672	5.751	265

The important quantity for the calculation is the imperfection factor  $\alpha_1$  usually recommended in normative documents according to the buckling curve, which depends on the cross-section type and its sensitivity to the imperfection influence [2]. For the columns of open HEA cross-sections [10] and their buckling in the flanges direction, the buckling curve of *c* with the imperfection factor  $\alpha_1 = 0.49$  is

valid, as well as for HEA cross-sections partially encased by concrete [11]. Member imperfections are given in dependence on the member length as  $e_0 = L/200$  or  $e_0 / L = 1/200$  in the case of steel columns [10], and as  $e_0 = L/150$  or  $e_0 / L = 1/150$  in the case of steel-concrete columns [11].

# **3.3** Comparison of Theoretical and Actual Values of Initial Imperfections

In Table 2, the actual initial imperfections experimentally evaluated from test results are listed in comparison with the initial imperfections calculated analytically using the formula (25), including the ratios of  $e_0 / L$  to compare these ones with the value of 1 / 200 or 1 / 150, respectively, recommended in [10], [11].

# 4 Initial Imperfections of Steel and Steel-Concrete Composite Columns with Circular Cross-Section

For circular tubes, the distance z in equation (14) is a half of the diameter, i.e. d/2, so that

$$e_0 = \alpha_1 \cdot \frac{f_y}{\pi^2 \cdot E} \cdot \frac{2 \cdot L_{cr}^2}{d}.$$
 (26)

For the selected steel and steel-concrete circular columns, which have been experimentally verified (see [7], [8], [9]), the actual values of initial imperfections have been derived evaluating test results. The specimens of various geometrical and material parameters have been tested, to verify their actual behaviour, failure mechanism and buckling load-carrying capacity, if subjected to compression.

Using the formula (26) the theoretical values of initial imperfections have been calculated for the properties of tested columns. For the comparison, also the recommended values of initial eccentricities according to the rules given in standard documents, if the buckling resistance is verified applying the second order theory.

## 4.1 Actual Values of Initial Imperfection Experimentally Verified

The experimental verification has been realized with the specimens represented by steel circular tubes and steel circular tubes filled by concrete composed of normal-strength materials and high-strength ones.

The investigated columns were simply supported on both ends with the structural length of  $L = 3\ 070$ mm or  $L = 3\ 000$  mm, respectively, that the buckling length was  $L_{cr} = 3\ 070$  mm and  $L_{cr} = 3\ 000$  mm, respectively. Various material properties including their combinations in particular cross-sections have been used: (i) steel grade of S 235 and S 355, (ii) concrete class of C 20/25, C55/67 and C 80/95.

Table 3 Overview of investigated columns: geometrical parameters (nominal values) and mechanical properties (mean values)

	steel		concrete	
Cross-section	fy [MPa]	E <sub>a</sub> [GPa]	f <sub>c</sub> [MPa]	E <sub>c</sub> [GPa]
TR Ø152/4.5	356.4	210.0 <sup>2)</sup>		
TR Ø152/4.5 + NS1 concrete	356.4	210.0 <sup>2)</sup>	27.0	32.0
TR Ø152/4.5 + HS1 concrete	356.4	210.0 <sup>2)</sup>	87.1 <sup>1)</sup>	49.0
TR Ø159/4.5-I	344.4	209.2		
TR Ø159/4.5-I + NS2 concrete	344.4	209.2	70.9	36.2
TR Ø159/4.5-I + HS2 concrete	344.4	209.2	97.7	45.3
TR Ø159/4.5-II	475.4	199.8		
TR Ø159/4.5-II + NS2 concrete	475.4	199.8	70.9	36.2
TR Ø159/4.5-II + HS2 concrete	475.4	199.8	97.7	45.3

NS concrete – normal-strength concrete; HS concrete – high-strength concrete; 3 test specimens performed for each group excluding 1) – 1 test only; mechanical parameters measured excluding 2) – not measured



Fig. 3 Illustration of test arrangement and realization

The overview of the tested specimens, including their geometrical parameters and measured physical

mechanical properties, is listed in Table 3. Some illustrations presenting the test arrangement and realization are shown by photographs in Fig. 3.



Fig. 4 Illustration of Southwell's lines: a) circular tubes TR Ø152/4.5 without concrete filling; circular tubes TR Ø152/4.5 filled by normal-strength NS1 concrete; c) circular tubes TR Ø152/4.5 filled by high-strength HS1 concrete

0,00E+00 -2 0 2 4 6 8 10 12 14 16 18 20

c)

The actual values of the initial eccentricities have been derived using Southwell's lines, examples of which are illustrated by the graphs in Fig. 4. On the horizontal axis there are the values of the transverse deformation, i.e. deflection w in the mid-length of the member. On the vertical axis there are the values of the ratio of deflection w to axial force N, i.e. w / N. The initial eccentricity  $e_0$ , that means the equivalent initial geometrical imperfection, is such value of the deflection, for which w / N = 0.

Table 4 Comparison	of initial	imperfections:
actual values vs.	theoretica	l values

	actual		theoretical	
Cross-section	$e_0$ [mm]	<i>e</i> <sub>0</sub> / <i>L</i> [-]	$e_0$ [mm]	$e_0 / L$ [-]
TR Ø152/4.5	0.473 0.344 0.139	1/6490 1/8924 1/22086		
TR Ø152/4.5 + NS1 concrete	0.973 1.120 1.672	1/3155 1/2741 1/1836	4.478	$\frac{1}{686}$
TR Ø152/4.5 + HS1 concrete	0.070	1/43857		
TR Ø159/4.5-I	1.000 0.500 0.500	1/3000 1/6000 1/6000		
TR Ø159/4.5-I + NS2 concrete	0.100 0.100 0.100	1/30000 1/30000 1/30000	3.961	$\frac{1}{757}$
TR Ø159/4.5-I + HS2 concrete	0.100 0.400 0.600	1/30000 1/7500 1/5000		
TR Ø159/4.5-II	0.400 0.100 0.400	1/7500 1/30000 1/7500		
TR Ø159/4.5-II + NS2 concrete	0.100 0.500 0.300	1/30000 1/6000 1/10000	5.731	$\frac{1}{523}$
TR Ø159/4.5-II + HS2 concrete	0.100 0.100 0.100	1/30000 1/30000 1/30000		

## 4.2 Theoretical Values of Initial Imperfection Analytically Calculated

According to the formula (25), the theoretical value of equivalent initial imperfection  $e_0$ , also so-called equivalent geometrical imperfection, depends on yield strength and Young's modulus of steel, and on

buckling length and tube diameter, both in the case of steel tubes and in the case of steel-concrete tubes. It is because steel-concrete cross-section has been transformed to the substitute steel cross-section.

The important quantity for the calculation is the imperfection parameter  $\alpha_1$  usually recommended in normative documents according to the buckling curve, which depends on the cross-section type and its sensitivity to the imperfection influence (see [2], [6], for example). For circular hot-rolled tubes the buckling curve of *a* with the imperfection parameter  $\alpha_1 = 0.21$  is valid [10], as well as for circular tubes filled by concrete [11].

Member imperfections according to [10], [11] are considered in dependence on the member length as  $e_0 = L / 300$ , or in the form of  $e_0 / L = 1 / 300$ .

# 4.3 Comparison of Theoretical and Actual Values of Initial Imperfections

In Table 4, the actual initial imperfections derived experimentally evaluating test results are listed in comparison with the initial imperfections calculated analytically using the formula (26), including the ratios of  $e_0 / L$  to compare these ones with the value of 1 / 300 recommended in standards [10], [11].

# **5** Conclusion

Based on the examples of steel and steel-concrete composite columns with HEA cross-sections mentioned above, it is shown:

- The actual values of the initial imperfections obtained using test results are less than those ones calculated using analytically derived formula, approximately in the range from 5 to 25 times, both in the case of steel columns, and in the case of steel-concrete columns partially encased by normal- strength and high-strength concrete. Thus, the imperfection parameter, hereto given as  $\alpha_1 = 0.49$ , shows, that the imperfection importance for the buckling strength of this column type can be significantly lower than considered in standard rules.
- The values of initial imperfections recommended in the normative documents for the needs of the second order calculation, i.e. L / 200 for steel columns with open cross-section and L / 150 for steel columns with open cross-section partially encased by concrete between flanges (see above), are (i) the same or up to about 2 times larger than theoretical values calculated analytically in the case of steel columns mentioned above, and (ii) in the range from

about 1.4 to 2.8 times larger than the theoretical values calculated analytically in the case of steelconcrete columns mentioned, even with respect to the fact, that also this theoretical values are probably too high compared to the actual values of the initial imperfections.

Based on the examples of steel and steel-concrete composite columns with circular cross-section mentioned above, it is shown:

- The actual values of the initial imperfections obtained using test results are much less than those ones calculated using analytically derived formula, usually in the range from 10 to 50 times, both in the case of steel tubes, and in the case of steel-tubes, filled by normal-strength and high-strength concrete. Thus, the imperfection parameter, hereto given as  $\alpha_1 = 0.21$ , shows, that the imperfection importance for the buckling strength of this column type can be significantly lower than considered in standard rules.
- The values of initial imperfections recommended in the normative documents for the needs of the second order calculation, are about 2 times larger than the theoretical value calculated analytically, even respecting the fact, that also this theoretical value is probably too high compared to the actual values of the initial imperfections.

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