Soft topological spaces induced via soft relations

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Abstract: Soft relation is a basic mathematical model that can be related to several real-life data. Throughout many fields, soft relations are used to build soft topological structures. In addition, soft topological constructs are generalized methods to calculate similarity and dissimilarity of objects. Within this article, we present a new approach for directly producing a soft topology by soft relation without using base or subbase. This process is important technique for applications of soft topology. There is investigations into the relationship between soft set topologies and different relations and some of their properties are obtained.

Key–Words: Soft sets, soft topology, soft topological spaces, soft relations.

1 Introduction

In 1999, as a new approach to modeling uncertainties, Molodtsov [27] proposed the idea of a soft set and established bases of the corresponding theory. In solving several practical problems in finance, electronics, social science and medical research, he has given many applications of this theory. The goal of this notion was to give a certain effect of continual discretization of these fundamental mathematical principles and to provide a new method in real-life issues for mathematical analysis. A certain parameterisation of a specified set X was suggested to achieve this aim resulting in the definition of a soft structure over the set X. As expected, this latest insight concept attracted the interest of both pure mathematicians as well as applied mathematics researchers. The experts clearly consider a soft set definition well matched with current mathematical principles such as fuzzy sets, multisets, and vague sets. However, several researchers (see ([4]-[6]), ([12]-[26]), ([29]-[31]), [36], [38]) have implemented various variations on this principle in several areas, such as soft topologies, fuzzy soft topologies and soft rings and fuzzy soft rings. It should be noted that soft topology generation through soft relations and the representation of soft topology principles by soft relations can narrow the gap between topology and its applications. In 2016, using the principles of its subbase and base, Babitha, et al. [10] presented method for producing certain topology from soft binary relation. Similarity and dissimilarity of objects are studied in many branches of mathematics such as [1, 22, 32]. Recently, many topologists studied soft set theory in a view point of topology such as [2, 7, 8, 28, 37].

The main aim of this paper is to create soft topological spaces that are described over a soft relation without using subbase concepts or their basis, and some of its properties will be studied. We're offering some basic ideas about the soft sets and the already studied results. We then discuss certain basic properties of soft topological spaces and define open and closed soft sets. The soft closure of a soft set is established and in a broader sense is essentially a generalization of closure of a set. We may assume that a soft topological space on the initial universe provides a parameterized set of topologies, but the reverse is not valid. It is shown that for any soft relation the soft topology generated by the post-soft sets is the dual of the soft topology generated by pre-soft sets. Comparisons are made between our method and the Babitha model. Many examples are provided to illustrate the relationships between the topologies and relations of the soft set.

2 Preliminaries

In this section some basic definitions, results and notations as introduced by ([5], ([9]-[13]]), [24],[27],[38],[33]) are mentioned.

Definition 1 *Let U be a universe set, E be a set of parameters and $P(U)$ be the power set of U. For $A \subseteq E$, (F, A) is called a soft set over U (F for abbreviation), where $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of U i.e., for $a \in A$, $F(a)$ may be considered as the set of a-approximate elements of the soft set (F, A).*

Definition 2 *Let F and G be two soft sets over X. Then, we have*

(i) F is a null soft set, say, $\tilde{\phi}$, if $F(e) = \phi \; \forall \, e \in E$.

(ii) F is an absolute soft set, say, $\tilde{X}$, if $F(e) = X \; \forall \, e \in E$.

(iii) F is a soft subset of G, say, $F \tilde{\subseteq} G$, if $F(e) \subseteq G(e) \; \forall \, e \in E$.

(iv) F and G are soft equal, say, $F \tilde{=} G$, if $F \tilde{\subseteq} G$ and $G \tilde{\subseteq} F$.

(v) The soft union of F and G, say, $F \tilde{\cup} G$, is a soft over X and defined by $F \tilde{\cup} G : E \to P(X)$ s.t. $(F \tilde{\cup} G)(e) = F(e) \cup G(e) \; \forall \, e \in E$.

(vi) The soft intersection of F and G, say, $F \tilde{\cap} G$, is a soft over X and defined by $F \tilde{\cap} G : E \to P(X)$ s.t. $(F \tilde{\cap} G)(e) = F(e) \cap G(e) \; \forall \, e \in E$.

(vii) The soft complement $(\tilde{X} - F)$ of a soft set F, say, $F^{\tilde{c}}$ and defined by $F^{\tilde{c}} : E \to P(X)$ s.t. $F^{\tilde{c}}(e) = X \setminus F(e) \; \forall \, e \in E$.

Example 3 *Consider U is the set of all students under consideration and $E = \{$brilliant, average, healthy $\}$ is set of parameters. Thus the soft set (F, A) describes different types of students. Suppose that there are six students $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$, where $e_1 =$ stands for brilliant, $e_2 =$ stands for average and $e_3 =$ stands for healthy s.t. $e_1 = \{x_1, x_2, x_5\}$, $e_1 = \{x_3, x_4, x_6\}$ and $e_1 = \{x_1, x_4, x_5, x_6\}$. Thus, the soft set (F, A) is a parameterized family $(F, A) = \{F(e_i) : i = 1, 2, 3\} = \{(e_i, F(e_i)) : i = 1, 2, 3\}$ of subsets of U and gives us a collection of approximate descriptions of an object. Here note that for each $e \in E$, $F(e)$ is a crisp set. Thus the soft set (F, A) is called a standard soft set. In [38] Yang et al. defined a fuzzy soft set where $F(e)$ is a fuzzy subset of U, for each parameter "e".*

Definition 4 *Let $E = \{e_1, e_2, e_3, \cdots, e_n\}$ be a set of parameters. The NOT set of E denoted by $\neg E$ is defined by: $\neg E = \{e_1, e_2, e_3, \cdots, e_n\}$, where $\neg e_i =$ not e_i, $\forall \, i$.*

Definition 5 *Let (F, A) and (G, B) be two soft sets over U. Then the Cartesian product of (F, A) and (G, B) is defined by $(F, A) \times (G, B) = (H, A \times B)$, where $H : A \times B \to P(U \times U)$ and $H(a, b) = F(a) \times G(b)$, where $(a, b) \in A \times B$. i.e., $H(a, b) = \{(h_i, h_j)$, where $h_i \in F(a)$ and $h_j \in G(b)\}$.*

Example 6 *Consider the soft set (F, A) which describes the "cost of the houses" and the soft set (G, B) which describes the "attractiveness of the houses". Suppose that $= \{h_1, h_2, \cdots, h_{10}\}$, $A = \{$very costly, costly, cheap $\}$ and $B = \{$beautiful, in the green surroundings, cheap$\}$. Let F(very costly) $= \{h_2, h_4, h_7, h_8\}$, F(costly) $= \{h_1, h_3, h_5\}$, F(cheap) $= \{h_6, h_9, h_{10}\}$, G(beautiful) $= \{h_2, h_3, h_7\}$, G(in the green surroundings) $= \{h_5, h_6, h_8\}$, and G(cheap) $= \{h_6, h_9, h_{10}\}$. Now $(F, A) \times (G, B) = (H, A \times B)$ where a typical element will look like H (very costly, beautiful) $= \{h_2, h_4, h_7, h_8\} \times \{h_2, h_3, h_7\} = \{(h_2, h_2), (h_2, h_3), (h_2, h_7), (h_4, h_2), (h_4, h_3), (h_4, h_7), (h_7, h_2), (h_7, h_3), (h_7, h_7), (h_8, h_2), (h_2, h_3), (h_8, h_7)\}$.*

Definition 7 *Let (F, A) and (G, B) be two soft sets over U, then a soft relation from (F, A) to (G, B) is a soft subset of $(F, A) \times (G, B)$. In other words, a relation from (F, A) to (G, B) is of the form (H_1, R) where $R \subseteq A \times B$ and $H_1(a, b) = H(a, b)$, $\forall \, (a, b) \in R$ such that $(H, A \times B) = (F, A) \times (G, B)$. Any subset of $(F, A) \times (F, A)$ is called "a soft relation" on (F, A). In an equivalent way, we can define the relation R on the soft set (F, A) in the parameterized form as follows: If $(F, A) = \{F(a), F(b), \cdots\}$ then $F(a)RF(b)$ if and only if $F(a) \times F(b) \in R$*

Definition 8 *Let R be a soft relation on (F, A), then R is*

(i) Reflexive if $H(a, a) \in R$, $\forall \, a \in A$.

(ii) Symmetric if $H(a, b) \in R$, then $H(b, a) \in R \; \forall \, a, b \in A$.

(iii) Transitive if $H(a, b), (b, c) \in R$ then $H(a, c) \in R \; \forall \, a, b, c \in A$.
Where H is the function representing the soft set relation R on (F, A). If R is a reflexive and transitive soft set relation, then R is a pre order. Also, if R is a reflexive, symmetric and transitive soft set relation, then R is an equivalence.

Definition 9 *Let R be a soft set relation on F_A. The post-soft set of $x \in F_A$ in R is defined as $xR = \{y \in F_A : xRy\}$, and the pre-soft set of $x \in F_A$ is defined as $Rx = \{y \in F_A : yRx\}$.*

Definition 10 *If R is a soft set relation defined on F_A, then the post-class and pre-class are defined respectively by*

$P^* = \{xR : x \in F_A\}$, *and*
$P_* = \{Rx : x \in F_A\}$.

Definition 11 *Let $F_A \in S(U)$. Then, a soft topology on F_A, denoted by $\widetilde{\tau}$, is a collection of F_A having the following properties:*

(T1) $F_\phi, F_A \in \widetilde{\tau}$.

(T2) If $\{F_{A_i} \in F_A : i \in I \subseteq \mathbb{N}\} \subseteq \widetilde{\tau}$, *then* $\bigcup_i F_{A_i} \in \widetilde{\tau}$.

(T3) If $\{F_{A_i} \in F_A : l \leq i \leq n, n \in \mathbb{N}\} \subseteq \widetilde{\tau}$, *then* $\bigcap_i^n F_{A_i} \in \widetilde{\tau}$.
The pair $(F_A, \widetilde{\tau})$ is called a soft topological space.

Definition 12 *Let $(F_A, \widetilde{\tau})$ be a soft topological space. Then, every element of $\widetilde{\tau}$ is called a soft open set and its complement is called soft closed. The soft interior of $F_B \subseteq F_A$, denoted $\mathcal{S}int(F_B)$, is defined by as the soft union of all soft open subsets of F_B, that is the soft interior of F_B represent the largest soft open subset contained in F_B. The soft closure of $F_B \subseteq F_A$, denoted $\mathcal{S}cl(F_B)$, is defined as the soft intersection of all soft open sets which contain F_B, that is the soft closure of F_B represent the smallest soft closed set contains F_B.*

Definition 13 *If $\widetilde{\tau}$ is a soft topology on F_A and the collection $\widetilde{\tau}^c = \{F_B^c : F_A \in \widetilde{\tau}\}$, where F_B^c represents the complement of F_B, is also a soft topology on F_A and $\widetilde{\tau}^c$ is called dual of the soft topology $\widetilde{\tau}$ on F_A.*

Definition 14 *A soft topological space is called a quasi-discrete soft topological space if every soft open set is soft closed set and vice versa.*

Theorem 15 *Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B \subseteq F_A$. F_B is a soft open set if and only if $\mathcal{S}int(F_B) = F_B$.*

Definition 16 *Let $(F_A, \widetilde{\tau})$ be a soft topological space and $\mathcal{B} \subseteq \widetilde{\tau}$. Then $\mathcal{B}$ is called a soft basis for the soft topology $\widetilde{\tau}$ if every element of $\mathcal{B}$ can be written as the union of elements of $\mathcal{B}$. Each element of $\mathcal{B}$ is called a soft basis element.*

Theorem 17 *Let $(F_A, \widetilde{\tau})$ be a soft topological space and $\mathcal{B}$ be a soft basis for $\widetilde{\tau}$. Then $\widetilde{\tau}$ equals the collection of all soft unions of elements of $\mathcal{B}$.*

Theorem 18 *If R is a soft set relation on F_A, then the post-class P^* and pre-class P_* form a subbasis.*

Remark 19 *By using the above theorem, Babitha et al.[10], get two different topologies $\widetilde{\tau}_1$ and $\widetilde{\tau}_2$ induced respectively by the post-class P^* and pre-class P_*. For any arbitrary soft set relation, $\widetilde{\tau}_1$ is not the dual of $\widetilde{\tau}_2$.*

3 Soft topologies generated by soft relations

In this section, we obtain two different soft topologies via soft set relations. We use soft relation directly to generate topologies.

Definition 20 *If R is a soft set relation on a soft set F_A. We define two classes $\widetilde{\tau_R} = \{F_B(x) : \forall x \in F_B, xR \subseteq F_B\}$ and $\widetilde{\tau_R^*} = \{F_B(x) : \forall x \in F_B, Rx \subseteq F_B\}$.*

Theorem 21 *Let R is a soft set relation on a soft set F_A. Then each of $\widetilde{\tau_R}$ and $\widetilde{\tau_R^*}$ is a soft topology on F_A.*

Proof 22 *We proof the first statement and the other similarly.*

(T1) Clearly, F_A *and* $F_\phi \in \widetilde{\tau_R}$.

(T2) Let $\{F_{A_i} \subseteq F_A : i \in I \subseteq \mathbb{N}\} \subseteq \widetilde{\tau_R}$ *and let* $x \in \bigcup_i F_{A_i}$. *Then* $\exists\, i_0 \in I$ *such that* $x \in F_{A_{i_0}}$ *which implies* $xR \subseteq F_{A_{i_0}}$. *Thus* $xR \subseteq \bigcup_i F_{A_i}$ *and this means that* $\bigcup_i F_{A_i} \in \widetilde{\tau_R}$.

(T3) Let $\{F_{A_i} \subseteq F_A : l \leq i \leq n, n \in \mathbb{N}\} \subseteq \widetilde{\tau_R}$. *Then* $\forall\, xR \subseteq F_{A_i}$ *such that* $l \leq i \leq n, n \in \mathbb{N}$ *which this implies* $xR \subseteq \bigcap_i^n F_{A_i}$ *and this means that* $\bigcap_i^n F_{A_i} \in \widetilde{\tau_R}$.

Theorem 23 *The soft topology $\widetilde{\tau_R}$ is the dual topology of $\widetilde{\tau_R^*}$ and vice versa.*

Proof 24 *Firstly, if $F_B \in \widetilde{\tau_R}$, then $\forall\, x \in F_B$, $xR \subseteq F_B$. Now, let $x \in F_B^c$ and $Rx \cap F_B \neq \phi$. Thus $\exists\, z \in Rx$ and $z \in F_B$ and this implies zRx such that $z \in F_B$. Accordingly, $x \in F_B$ which is a contradiction to assumption that $x \in F_B^c$. Thus $Rx \cap F_B = \phi$ and then $Rx \subseteq F_B^c$, $\forall\, x \in F_B^c$ this implies $F_B^c \in \widetilde{\tau_R^*}$. By similar way, we can prove that if $F_B \in \widetilde{\tau_R^*}$, then $F_B \in \widetilde{\tau_R}$.*

Example 25 *Let* $F_A = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_4, h_7\}), (e_3, \{h_2, h_6\}), (e_4, \{h_3, h_7\})\}$ *and R be a soft set relation on F_A defined by:* $R = \{F_A(e_1)RF_A(e_2), F_A(e_1)RF_A(e_4), F_A(e_2)RF_A(e_2), F_A(e_2)RF_A(e_4), F_A(e_2)RF_A(e_3), F_A(e_3)RF_A(e_3), F_A(e_3)RF_A(e_4)\}$.
Then the post-soft set and pre-soft sets are given by
$F_A(e_1)R = \{F_A(e_2), F_A(e_4)\}$.
$F_A(e_2)R = \{F_A(e_2), F_A(e_3), F_A(e_4)\}$.
$F_A(e_3)R = \{F_A(e_3), F_A(e_4)\}$.
$F_A(e_4)R = \phi$. *And*
$RF_A(e_1) = \phi$. $RF_A(e_2) = \{F_A(e_1), F_A(e_2)\}$.
$RF_A(e_3) = \{F_A(e_2), F_A(e_3)\}$.

$RF_A(e_4) = \{F_A(e_1), F_A(e_2), F_A(e_3)\}$.
Thus, we get the soft topologies induced by the soft relation R by:
$\widetilde{\tau_R} = \{F_A, F_\phi, \{F_A(e_4)\}, \{F_A(e_3), F_A(e_4)\}, \{F_A(e_2), F_A(e_3), F_A(e_4)\}\}$*, and*
$\widetilde{\tau_R^*} = \{F_A, F_\phi, \{F_A(e_1)\}, \{F_A(e_1), F_A(e_2)\}, \{F_A(e_1), F_A(e_2), F_A(e_3)\}\}$.
Clearly, $\widetilde{\tau_R}$ represents the dual topology of $\widetilde{\tau_R^*}$.

Remark 26 *The relationship between the topologies generated by our method and Babitha et al. [10] method is given by:*

(i) Our soft topologies $\widetilde{\tau_R}$ and $\widetilde{\tau_R^*}$ are dual topologies in general case of Theorem 21, but the soft topologies $\widetilde{\tau}_1$ and $\widetilde{\tau}_1$ generated by Babitha et al.[10] method are not dual, in general.

(ii) The implication between topologies is given by $\widetilde{\tau_R} \Rightarrow \widetilde{\tau}_1$ and $\widetilde{\tau_R^*} \Rightarrow \widetilde{\tau}_2$. The converse may not be true, in general.

Example 27 *According to Example 25, the post-class and pre-class obtained from the post-soft set and pre-soft sets is:*
$P^* = \{\{F_A(e_2), F_A(e_4)\}, \{F_A(e_3), F_A(e_4)\}, \{F_A(e_2), F_A(e_3), F_A(e_4)\}\}$*, and*
$P_* = \{\{F_A(e_1), F_A(e_2)\}, \{F_A(e_2), F_A(e_3)\}, \{F_A(e_1), F_A(e_2), F_A(e_3)\}\}$.
The soft bases $\mathcal{B}^*$ and $\mathcal{B}_*$ generated by P^* and P_* respectively are given by:
$\mathcal{B}^* = \{\{F_A(e_4)\}, \{F_A(e_2), F_A(e_4)\}, \{F_A(e_3), F_A(e_4)\}, \{F_A(e_2), F_A(e_3), F_A(e_4)\}\}$, *and*
$\mathcal{B}_* = \{\{F_A(e_2)\}, \{F_A(e_1), F_A(e_2)\}, \{F_A(e_2), F_A(e_3)\}, \{F_A(e_1), F_A(e_2), F_A(e_3)\}\}$.
Then the topologies $\widetilde{\tau}_1$ and $\widetilde{\tau}_1$ generated by P^* and P_* respectively are given by:
$\widetilde{\tau}_1 = \{F_A, F_\phi, \{F_A(e_4)\}, \{F_A(e_2), F_A(e_4)\}, \{F_A(e_3), F_A(e_4)\}, \{F_A(e_2), F_A(e_3), F_A(e_4)\}\}$,
$\widetilde{\tau}_2 = \{F_A, F_\phi, \{F_A(e_1)\}, \{F_A(e_2), F_A(e_4)\}, \{F_A(e_1), F_A(e_2)\}, \{F_A(e_1), F_A(e_2), F_A(e_3)\}\}$.

Lemma 28 *If R is a symmetric soft set relation on a soft set F_A, then $\widetilde{\tau_R} = \widetilde{\tau_R^*}$.*

Proof 29 *Let R be a symmetric soft set relation on a soft set F_A. Then $\forall\, x, y \in F_A$ such that xRy iff yRx and this implies $P^* = P_*$. Thus $\widetilde{\tau_R} = \widetilde{\tau_R^*}$.*

Lemma 30 *If R is an equivalence soft set relation on a soft set F_A, then $\widetilde{\tau_R}$ and $\widetilde{\tau_R^*}$ are a quasi discrete soft topologies.*

Proof 31 *Obvious.*

Definition 32 *Let $\widetilde{\tau_R}$ and $\widetilde{\tau_R^*}$ are the soft topologies generated by the soft relation R on the soft set F_A. Then the soft interior and closure of any soft subset F_B of $\widetilde{\tau_R}$ and $\widetilde{\tau_R^*}$ of are given, respectively, by:*

(i) Soft interior operator:

$$\mathcal{S}int_{\widetilde{\tau_R}}(F_B) = \bigcup\{F_G \in \widetilde{\tau_R} : F_G \subseteq F_B\},$$

$$\mathcal{S}int_{\widetilde{\tau_R^*}}(F_B) = \bigcup\{F_G \in \widetilde{\tau_R^*} : F_G \subseteq F_B\}.$$

(ii) Soft closure operator:

$$\mathcal{S}cl_{\widetilde{\tau_R}}(F_B) = \bigcap\{F_H \in (\widetilde{\tau_R})^c = \widetilde{\tau_R^*} : F_B \subseteq F_G\},$$

$$\mathcal{S}cl_{\widetilde{\tau_R^*}}(F_B) = \bigcap\{F_H \in (\widetilde{\tau_R^*})^c = \widetilde{\tau_R} : F_B \subseteq F_G\}.$$

Definition 33 *Let $\widetilde{\tau_R}$ and $\widetilde{\tau_R^*}$ are the soft topologies generated by the soft relation R on the soft set F_A. Then the soft boundary, positive and negative region of any soft subset F_B of $\widetilde{\tau_R}$ and $\widetilde{\tau_R^*}$ are given respectively by:*

(i) For soft topology $\widetilde{\tau_R}$:

$$\mathcal{P}os_{\widetilde{\tau_R}}(F_B) = \mathcal{S}int_{\widetilde{\tau_R}}(F_B).$$
$$\mathcal{N}eg_{\widetilde{\tau_R}}(F_B) = F_A - \mathcal{S}cl_{\widetilde{\tau_R}}(F_B).$$
$$\mathcal{B}nd_{\widetilde{\tau_R}}(F_B) = \mathcal{S}cl_{\widetilde{\tau_R}}(F_B) - \mathcal{S}int_{\widetilde{\tau_R}}(F_B).$$

(ii) For soft topology $\widetilde{\tau_R^*}$:

$$\mathcal{P}os_{\widetilde{\tau_R^*}}(F_B) = \mathcal{S}int_{\widetilde{\tau_R^*}}(F_B).$$
$$\mathcal{N}eg_{\widetilde{\tau_R^*}}(F_B) = F_A - \mathcal{S}cl_{\widetilde{\tau_R^*}}(F_B).$$
$$\mathcal{B}nd_{\widetilde{\tau_R^*}}(F_B) = \mathcal{S}cl_{\widetilde{\tau_R^*}}(F_B) - \mathcal{S}int_{\widetilde{\tau_R^*}}(F_B).$$

Definition 34 *Let $\widetilde{\tau_R}$ and $\widetilde{\tau_R^*}$ are the soft topologies generated by the soft relation R on the soft set F_A. Then the soft accuracy of soft interior and closure operators of any soft subset F_B of $\widetilde{\tau_R}$ and $\widetilde{\tau_R^*}$ are given respectively by:*

(i) For soft topology $\widetilde{\tau_R}$:

$$\mu_{\widetilde{\tau_R}}(F_B) = \frac{|\mathcal{S}int_{\widetilde{\tau_R}}(F_B)|}{|\mathcal{S}cl_{\widetilde{\tau_R}}(F_B)|}.$$

(ii) For soft topology $\widetilde{\tau_R^*}$:

$$\mu_{\widetilde{\tau_R^*}}(F_B) = \frac{|\mathcal{S}int_{\widetilde{\tau_R^*}}(F_B)|}{|\mathcal{S}cl_{\widetilde{\tau_R^*}}(F_B)|}.$$

From Definitions 32, 33 and 34, we consider soft interior and closure operators as soft rough approximations. Now, we study the case of reflexivity of soft relation R. A method to compute soft interior operator of the soft topologies using the post-soft set directly from the relation without using topologies is given.

Lemma 35 *If R is a reflexive soft set relation on a soft set F_A, then the soft interior of the soft topology $\widetilde{\tau_R}$ and $\widetilde{\tau_R^*}$ are given, respectively, by*

$$\mathcal{S}int_{\widetilde{\tau_R}}(F_A) = \{F_B(x) : \forall F_B(x) \in F_A, F_B(x)R \subseteq F_A\}, \text{ and}$$

$$\mathcal{S}int_{\widetilde{\tau_R^*}}(F_A) = \{F_B(x) : \forall F_B(x) \in F_A, RF_B(x) \subseteq F_A\}.$$

Proof 36 *We prove the first statement and the other by similarly. Let $\mathcal{S}int_{\widetilde{\tau_R}}(F_B) = \bigcup\{F_G \in \widetilde{\tau_R} : F_G \subseteq F_B\} \subseteq \{F_B(x) : F_B(x)R \subseteq F_A\}$. Firstly, let $H_B = \bigcup\{F_G \in \widetilde{\tau_R} : F_G \subseteq F_B\}$. Then $H_B \in \widetilde{\tau_R}$ such that $H_B \subseteq F_B$ (i. e. H_B is the largest soft open set contained in F_B). Now, let $F_B(x) \in H_B$. Thus, $F_B(x)R \subseteq H_B$. Since R is a reflexive soft set relation, then $F_B(x) \subseteq F_B(x)R \subseteq F_A$ and this means that $F_B(x)R \subseteq F_A$ such that $F_B(x) \in F_A$. Hence, $F_B(x) \in \{F_B(x) : F_B(x)R \subseteq F_A\}$. Conversely, by the same way, we can prove that: $\{F_B(x) : F_B(x)R \subseteq F_A\} \subseteq \bigcup\{F_G \in \widetilde{\tau_R} : F_G \subseteq F_B\}$.*

Corollary 37 *If R is a reflexive soft set relation on a soft set F_A, then the soft closure of the soft topology $\widetilde{\tau_R}$ and $\widetilde{\tau_R^*}$ are given, respectively, by*

$$\mathcal{S}cl_{\widetilde{\tau_R}}(F_A) = \{F_B(x) : \forall F_B(x) \in F_A, F_B(x)R \cap F_A \neq \phi\}, \text{ and}$$

$$\mathcal{S}cl_{\widetilde{\tau_R^*}}(F_A) = \{F_B(x) : RF_B(x) \cap F_A \neq \phi\}.$$

Example 38 *Let $F_A = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_4, h_7\}), (e_3, \{h_2, h_6\})\}$ and R be a reflexive soft set relation on F_A defined by:*
$R = \{F_A(e_1)RF_A(e_1), F_A(e_1)RF_A(e_3), F_A(e_2)RF_A(e_2), F_A(e_3)RF_A(e_3), F_A(e_3)RF_A(e_2)\}$.
Then the post-soft set and pre-soft sets are given by
$F_A(e_1)R = \{F_A(e_1), F_A(e_3)\}$.
$F_A(e_2)R = \{F_A(e_2)\}$.
$F_A(e_3)R = \{F_A(e_2), F_A(e_3)\}$. *And*
$RF_A(e_1) = \{F_A(e_1)\}$.
$RF_A(e_2) = \{F_A(e_2), F_A(e_3)\}$.
$RF_A(e_3) = \{F_A(e_1), F_A(e_3)\}$.
Thus, we get the soft topologies induced by the soft relation R by:
$\widetilde{\tau_R} = \{F_A, F_\phi, \{F_A(e_2)\}, \{F_A(e_2), F_A(e_3)\}\}$, *and*
$\widetilde{\tau_R^*} = \{F_A, F_\phi, \{F_A(e_1)\}, \{F_A(e_1), F_A(e_3)\}\}$.
Consider F_B is a soft subset of F_A where $F_B = \{F_A(e_1), F_A(e_2)\}$. Thus, by using post-(pre-)soft sets, we get:
$\mathcal{S}int_{\widetilde{\tau_R}}(F_B) = \{F_A(e_2)\}$ *and*
$\mathcal{S}cl_{\widetilde{\tau_R}}(F_B) = \{F_A(e_1), F_A(e_2), F_A(e_3)\} = F_A$.
$\mathcal{S}int_{\widetilde{\tau_R^*}}(F_B) = \{F_A(e_1)\}$ *and*
$\mathcal{S}cl_{\widetilde{\tau_R^*}}(F_B) = \{F_A(e_1), F_A(e_2), F_A(e_3)\} = F_A$.
Which are identical with operators that computed using the soft topologies $\widetilde{\tau_R}$ and $\widetilde{\tau_R^*}$.

Remark 39 *From Lemma 35 and Corollary 37, we establish an application on soft rough set theory. In fact, we use the introduced operators to define new soft rough approximations with topological properties. Hence, we connect the soft topological structures with both of soft rough theory and fuzzy soft theory.*

4 A decision making for information system

A set valued information system is presented in Table 1, where $U = \{S_1, S_2, S_3, S_4, S_5, S_6\}$ of students. $E = \{e_1 =$ Food containing preservatives, $e_2 =$ Carbohydrate, $e_3 =$ Protein, $e_4 =$ Vitamins, $e_5 =$ Fat, $e_6 =$ Minerals, $e_7 =$ Junk food, $e_8 =$ Icecream $\}$, be a set of parameters which illustrate food nutrients for students. Consider the soft set (F, E) which describes the attractiveness of the students given by $(F, E) =$ Students eat food containing Food containing preservatives $= \phi$, Students eat food containing Carbohydrate $= \{S_1, S_2, S_3, S_4, S_5, S_6\}$, Students eat food containing Protein $= \{S_1, S_2, S_3, S_4, S_6\}$, Students eat food containing Vitamins $= \{S_1, S_2, S_3, S_4, S_5, , S_6\}$, Students eat food containing Fat $= \{S_1, S_3, S_6\}$, Students eat food containing Minerals $= \{S_1, S_2, S_6\}$, Students eat Junk food $= \{S_2, S_4, S_5\}$, Students eat Icecream $= \{S_1, S_3, S_6\}$. Suppose that, Mr.X is interested to buy food on the basis of his choice parameters $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ which constitute the subset $P = \{e_2 =$ Carbohydrate, $e_3 =$ Protein, $e_4 =$ Vitamins, $e_5 =$ Fat, $e_6 =$ Minerals$\}$ of the set E. That means, out of available food in U, he is to select that food which qualifies with all (or with maximum number of) parameters of the soft set. The problem is to select the food which is suitable the choice parameters set by Mr.X.

Let us first make a tabular representation of the problem. Consider the soft set (F, P) where P is the choice parameter of Mr.X as in Table 1. Here (F, P) is a soft subset of (F, E).

Table 1: Food Information System.

Students	e_2	e_3	e_4	e_5	e_6
S_1	1	1	1	1	1
S_2	1	1	1	0	1
S_3	1	1	1	1	0
S_4	1	1	1	0	0
S_5	1	0	1	0	0
S_6	1	1	1	1	1

Thus, the soft subset $(F,P) = \{F_A(e_2), F_A(e_3), F_A(e_4), F_A(e_5), F_A(e_6)\}$. Suppose the following soft reflexive relation on the soft subset (F,P) which represents a best feeding system of the students:
$R = \{F_A(e_2)RF_A(e_2), F_A(e_3)RF_A(e_3), F_A(e_4)RF_A(e_4), F_A(e_5)RF_A(e_5), F_A(e_6)RF_A(e_6), F_A(e_2)RF_A(e_3), F_A(e_2)RF_A(e_4), F_A(e_2)RF_A(e_6), F_A(e_3)RF_A(e_4), F_A(e_3)RF_A(e_6), F_A(e_5)RF_A(e_3), F_A(e_6)RF_A(e_4)\}$. Then the post-soft sets are given by
$F_A(e_2)R = \{F_A(e_2), F_A(e_3), F_A(e_4), F_A(e_6)\}$.
$F_A(e_3)R = \{F_A(e_3), F_A(e_4), F_A(e_6)\}$.
$F_A(e_4)R = \{F_A(e_4)\}$.
$F_A(e_5)R = \{F_A(e_3), F_A(e_5)\}$.
$F_A(e_6)R = \{F_A(e_4), F_A(e_6)\}$.
Thus, we get the soft topologies induced by the soft relation R by:
$\widetilde{\tau_R} = \{F_A, F_\phi, \{F_A(e_4)\}, \{F_A(e_4), F_A(e_6)\}, \{F_A(e_3), F_A(e_4), F_A(e_6)\}, \{F_A(e_2), F_A(e_3), F_A(e_4), F_A(e_6)\}, \{F_A(e_3), F_A(e_4), F_A(e_5), F_A(e_6)\}\}$, and
$\widetilde{\tau_R^*} = \{F_A, F_\phi, \{F_A(e_2)\}, \{F_A(e_5)\}, \{F_A(e_2), F_A(e_5)\}, \{F_A(e_2), F_A(e_3), F_A(e_5)\}, \{F_A(e_2), F_A(e_3), F_A(e_5), F_A(e_6)\}\}$.

Thus, we conclude that the topology $\widetilde{\tau_R^*}$ represents a best feeding system for students. Accordingly, using the topological properties of $\widetilde{\tau_R^*}$, we can study the properties of the feeding system of students and give the accuracy of decision making.
For example, assume that Mr.X wants to choice best feeding system that contains some of the basic elements in food and then according to the soft topologies he make their initial decision. For example, let Mr. X points out that $F_B = \{F_A(e_2), F_A(e_3), F_A(e_6)\}$ is the best choice. Then, we can make the following computations:

From Table 2, we conclude that soft topology $\widetilde{\tau_R^*}$, Mr.X is most likely (or must) to buy the food in $\mathcal{P}os_{\widetilde{\tau_R}}(F_B) = \{F_A(e_2)\}$ and he may also consider the food in $\mathcal{B}nd_{\widetilde{\tau_R}}(F_B) = \{F_A(e_2), F_A(e_3), F_A(e_4), F_A(e_6)\}$ as possible choices, but the food $\{F_A(e_5)\}$ would never be taken since $\mathcal{N}eg_{\widetilde{\tau_R^*}}(F_B) = \{F_A(e_5)\}$. Thus Mr. X's initial demand for food is analyzed and delineated by using the concept of soft topology $\widetilde{\tau_R^*}$ and related soft computing techniques. Moreover, if we use the soft topology $\widetilde{\tau_R}$, Mr.X can not to decide the best food or the food never be taken exactly. In addition, the accuracy in soft topology $\widetilde{\tau_R^*}$ is better than in soft topology $\widetilde{\tau_R}$.

5 Conclusion and future work

The present paper represents a starting point for the framework to generate soft topologies directly without the use of bases or subbases concepts through soft relations. The technique used can be considered as an easy tool for connecting topological structures and the other theories of uncertainty such as soft rough and fuzzy soft rough theory [15, 18, 31]. Soft topologies can also be extended through multi-granular soft rough covering sets as in [3, 35]. The association between soft set topologies and various forms of soft relations is explored and some of these soft set topologies'properties are established. For this reason, post-class and pre-class are implemented from the soft set relations, and some methods are obtained using soft set relations to produce soft set topology. The soft topology generated by the post-soft sets has been found to be the double of the soft topology generated by pre-soft sets. Moreover, we have introduced interesting results for defining soft interior and closure operators via a reflexive soft relation directly without using the soft topology. These operators are the soft operators that are given from soft topologies. Hence, using these operators, we connect between soft topological structures and soft rough theory. In our further studies, a suggested structure can be applied in many theories such as:
(i) Soft near open sets induced by soft relations in soft rough and soft fuzzy theory.
(ii) Soft functions and topology on hybrid systems on soft sets such as fuzzy soft sets, rough soft sets may

Table 2: Soft operators and accuracy of F_B by $\widetilde{\tau_R}$ and $\widetilde{\tau_R^*}$.

	For soft topology $\widetilde{\tau_R}$	For soft topology $\widetilde{\tau_R^*}$
Soft interior	ϕ	$\{F_A(e_2)\}$
Soft closure	$\{F_A(e_2), F_A(e_3), F_A(e_5), F_A(e_6)\}$	$\{F_A(e_2), F_A(e_3), F_A(e_4), F_A(e_6)\}$
Soft boundary	$\{F_A(e_2), F_A(e_3), F_A(e_5), F_A(e_6)\}$	$\{F_A(e_3), F_A(e_5), F_A(e_6)\}$
Soft positive	ϕ	$\{F_A(e_2)\}$
Soft negative	$\{F_A(e_4)\}$	$\{F_A(e_4)\}$
Soft accuracy	0	$\frac{1}{4}$

lead to more interesting applications in the field of Database Management and Flexible Querying.
(iii) Many real-life applications of these structures will study.

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