Theta – ω – Mappings in Topological Spaces

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Abstract: - In 2017 S. Ghour and B. Irshedat defined the $\theta_\omega-$closure operator as a new topological operator and introduced $\theta_\omega-$open sets as a new class of sets and proved that this class of sets is strictly between the class of open sets and the class of $\theta-$open sets. In this paper we introduce $\theta_\omega-$continuous, $\theta_\omega-$irresolute, $\theta_\omega-$open, $\theta_\omega-$closed, pre$-\theta_\omega-$open, pre$-\theta_\omega-$closed, contra $\theta_\omega-$continuous and almost contra $\theta_\omega-$continuous mappings and investigate properties and characterizations of these new types of mappings in topological spaces.

Key-Words:- $\theta_\omega-$open, $\theta_\omega-$continuous, $\theta_\omega-$irresolute, $\theta_\omega-$closed, pre$-\theta_\omega-$open, pre$-\theta_\omega-$closed, contra $\theta_\omega-$continuous, almost contra $\theta_\omega-$continuous.

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1 Introduction

The notions of $\theta-$open subsets, $\theta-$closed subsets and $\theta-$closure were introduced by Velicko [39] for the purpose of studying the important class of H-closed spaces in terms of arbitrary filterbases. Dickman and Porter [8,9], Joseph [20] and Jankovic [18,19] continued the work of Velicko. Recently Noiri and Jafari [33] and Jafari [17] have also obtained several new and interesting results related to these sets. In what follows (X,τ) $(or\ X)$ denotes topological spaces on which no separation axioms are assumed unless explicitly stated. We denote the interior and the closure of a subset A of X by $Int(A)$ and $Cl(A)$, respectively. A point $x \in X$ is called a $\theta-$adherent point of A [10], if $A \cap Cl(A) \neq \phi$ for every open set V containing x. The set of all $\theta-$adherent points of A is called the $\theta-$closure of A and is denoted by $A\ Cl_\theta(A)$. A subset A of X is called $\theta-$closed if $A = Cl_\theta(A)$. Dontchev and Maki [[10], Lemma 3.9] have shown that if A and B are subsets of a space (X,τ), then $Cl_\theta(A \cup B) = Cl_\theta(A) \cup Cl_\theta(B)$ and $Cl_\theta(A \cap B) = Cl_\theta(A) \cap Cl_\theta(B)$. Note also that the $\theta-$closure of a given set need not be a $\theta-$closed set. But it is always closed. The complement of a $\theta-$closed set is called a $\theta-$open set. The $\theta-$interior of set A in X, written $Int_\theta(A)$, consists of those points x of A such that for some open set U containing x, $Cl(U) \subseteq A$. A set A is $\theta-$open if and only if $A = Int_\theta(A)$, or equivalently, $X - A$ is $\theta-$closed. The collection of all $\theta-$open sets in a topological space (X,τ) forms a topology τ_θ on X, coarser than τ and $\tau_\theta = \tau$ if and only if (X,τ) is regular.

Several authors continued the study of $\theta-$closure operator, $\theta-$open sets and their related topological concepts. Recently some authors have studied several generalizations of $\theta-$open sets. A set A is $\omega-$open set in (X,τ) if for each $x \in A$, there is $U \in \tau$ and a countable set $C \subseteq X$ such that $x \in U - C \subseteq A$. The family of all $\omega-$open sets in (X,τ) is denoted by τ_ω. It is well known that τ_ω forms a topology on X finer than τ. $\omega-$open sets played a vital role in general topology research. Al Ghour used $\omega-$open sets to define $\omega-$regularity as a generalization of regularity as follows. A topological space (X,τ) is $\omega-$regular if for each closed set F in (X,τ) and $x \in X - F$, there exist $U \in \tau$ and $V \in \tau$ such that $x \in U$ and $F \subseteq V$ with $U \cap V = \phi$. The closure of A in the topological space (X,τ_ω) is called the $\omega-$closure of A in (X,τ) and is denoted by $Cl_\omega(A)$. In 2017 Al Ghour used the $\omega-$closure operator to define the $\theta_\omega-$closure operator in a similar way to that used in the definition of the $\omega-$closure operator. A point $x \in X$ is in $\theta_\omega-$closure of A $\left(x \in Cl_{\theta_\omega}(A)\right)$ if $Cl_\omega(A) \cap A \neq \phi$ for any $U \in \tau$ with $x \in U$. A set A is called $\theta_\omega-$closed if $Cl_{\theta_\omega}(A) = A$. The complement of a $\theta_\omega-$closed set is called a $\theta_\omega-$open set. The family of all $\theta_\omega-$open sets in (X,τ) denoted by τ_{θ_ω} forms a topology on X which is strictly between τ_θ and τ. In this paper we introduce $\theta_\omega-$continuous, $\theta_\omega-$irresolute, $\theta_\omega-$open, $\theta_\omega-$closed, pre$-\theta_\omega-$open, pre$-\theta_\omega-$closed, contra $\theta_\omega-$continuous and almost contra $\theta_\omega-$continuous and investigate properties and characterizations of these new types of mappings.

2 Preliminaries

Definition 2.1. ([39]) Let (X,τ) be a topological space and let $A \subseteq X$.
(a). A point x in X is in the $\theta-$closure of A $\left(x \in Cl_\theta(A)\right)$ if $Cl(U) \cap A \neq \phi$ for any $U \in \tau$ and $x \in U$.
(b). A is $\theta-$closed if $Cl_\theta(A) = A$.
(c). A is $\theta-$open if the complement of A is $\theta-$closed.
(d). The family of all $\theta-$open sets in (X,τ) is denoted by τ_θ.

Theorem 2.2. ([39]) Let (X,τ) be a topological space. Then (a). τ_θ forms a topology on X.
(b). $\tau_\theta \subseteq \tau$ and $\tau_\theta \neq \tau$ in general.

Definition 2.3. ([16]) Let (X,τ) be a topological space and let $A \subseteq X$.
(a). A point x in X is a condensation point of A if for each $U \in \tau$ with $x \in U$, the set $U \cap A$ is uncountable.
(b). A set A is $\omega-$closed if it contains all its condensation points.
(c). A set A is $\omega-$open if the complement of A is $\omega-$closed.
The family of all $\omega-$open sets in a topological space (X,τ) is denoted by τ_ω. For a subset A of a topological space (X,τ), it is known that $A \in \tau_\omega$ if and only if for each $x \in A$, there exists $U \in \tau$ such that $x \in U$ and $U - A$ is countable.

Theorem 2.4. ([3]) Let (X,τ) be a topological space. Then the following statements are true.
(a). τ_ω is a topology on X.
(b). $\tau \subseteq \tau_\omega$ and $\tau_\omega \neq \tau$ in general.

Theorem 2.5. Let (X,τ) be a topological space and let $A \subseteq X$. Then $Cl_\omega(A) \subseteq Cl(A)$ and $Cl_\omega(A) \neq Cl(A)$ in general.

Definition 2.6. ([1]) Let (X,τ) be a topological space and let $A \subseteq X$.

(a). A point x in X is in the $\theta_\omega-$closure of A $\left(x\in Cl_{\theta_\omega}(A)\right)$ if $Cl_\omega(U)\cap A\neq\phi$ for any $U\in\tau$ with $x\in U$.
(b). A set A is called $\theta_\omega-$closed if $Cl_{\theta_\omega}(A)=A$.
(c). A set A is called $\theta_\omega-$open if the complement of A is $\theta_\omega-$closed.
(d). The family of all $\theta_\omega-$open sets in (X,τ) is denoted by τ_{θ_ω} $\left(or\ \theta_\omega O(X)=\theta_\omega O(X,\tau)\right)$.
(e). The family of all $\theta_\omega-$closed sets in (X,τ) is denoted by $\theta_\omega C(X)=\theta_\omega C(X,\tau)$.

Theorem 2.7. $([1])$ Let (X,τ) be a topological space and let $A\subseteq X$. Then
(a). $Cl(A)\subseteq Cl_{\theta_\omega}(A)\subseteq Cl_\theta(A)$.
(b). If A is $\theta-$closed, then A is $\theta_\omega-$closed,
(c). If A is $\theta_\omega-$closed, then A is closed.

Theorem 2.8. $([1])$ Let (X,τ) be a topological space. Then $\tau_\theta\subseteq\tau_{\omega_\theta}\subseteq\tau$.

Theorem 2.9. $([1])$ Let (X,τ) be a topological space.
(a). If $A\subseteq B\subseteq X$, then $Cl_{\theta_\omega}(A)\subseteq Cl_{\theta_\omega}(B)$.
(b). For each subsets $A,B\subseteq X$, $Cl_{\theta_\omega}(A\cup B)=Cl_{\theta_\omega}(A)\cup Cl_{\theta_\omega}(B)$.
(c). For each subset $A\subseteq X$, $Cl_{\theta_\omega}(A)$ is closed in
(X,τ).
(d). For each $A\in\tau_{\theta_\omega}$, $Cl_{\theta_\omega}(A)=Cl(A)$.
(e). For each $A\in\tau$, $Cl_\theta(A)=Cl_{\theta_\omega}(A)=Cl(A)$.

Theorem 2.10. $([1])$ Let (X,τ) be a topological space. Then
(a). ϕ and X are $\theta_\omega-$closed sets.
(b). Finite union of $\theta_\omega-$closed sets is $\theta_\omega-$closed.
(c). Arbitrary intersection of $\theta_\omega-$closed sets is $\theta_\omega-$closed.

Theorem 2.11. $([1])$ Let (X,τ) be a topological space. Then τ_{θ_ω} is a topology on X.

Theorem 2.12. $([1])$ Let (X,τ) be a topological space and $A\subseteq X$. Then $A\in\tau_{\theta_\omega}$ if and only if for each $x\in A$, there exists $U\in\tau$ such that $x\in U\subseteq Cl_\omega(U)\subseteq A$.

Corollary 2.13. Every open $\omega-$closed set in a topological space (X,τ) is $\theta_\omega-$open.

Corollary 2.14. Every countable open set in a topological space (X,τ) is $\theta_\omega-$open.

The following example shows that $\theta_\omega-$open are strictly between $\theta-$open sets and open sets.

Example 2.15. $([1])$ Let $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Q}^c$, and $\mathbb{N}$ denote, respectively the set of real numbers, the set of rational numbers, the set of irrational numbers and the set of natural numbers.
Consider (X,τ) where $\tau=\left\{\phi,\mathbb{R},\mathbb{N},\mathbb{Q}^c,\mathbb{N}\cup\mathbb{Q}^c\right\}$.
Then $\tau_{\theta_\omega}=\{\phi,\mathbb{R},\mathbb{N}\}$ and $\tau_\theta=\{\phi,\mathbb{R}\}$.

Definition 2.16. Let A be a subset of a topological space (X,τ). Then the Kernel of A, denoted by $Ker(A)$, is the intersection of all open supersets of A.

Lemma 2.17. Let A and B be subsets of a topological space (X,τ), then the following properties hold:

(i). $x\in Ker(A)$ if and only if $A\cap F\neq\phi$ for every closed set F in (X,τ) containing x.

(ii). $A\subseteq Ker(A)$ and if A is open in (X,τ), then $A=Ker(A)$.

(iii). If $A\subseteq B$, then $Ker(A)\subseteq Ker(B)$.

3 $\theta_\omega-$Continuous Mappings

The purpose of this section is to investigate properties and characterizations of $\theta_\omega-$continuous functions.

Definition 3.1. A function $f:(X,\tau)\to(Y,\sigma)$ is said to be $\theta_\omega-$continuous if $f^{-1}(V)\in\tau_{\theta_\omega}$ for every $V\in\sigma$.

Theorem 3.2. Let $f:(X,\tau)\to(Y,\sigma)$ be a function. Then the following are equivalent:

(1) f is $\theta_\omega-$continuous;

(2) The inverse image of each closed set in Y is a $\theta_\omega-$closed set in X;

(3) $Cl_{\theta_\omega}\left[f^{-1}(V)\right]\subseteq f^{-1}\left[Cl(V)\right]$, for every $V\subseteq Y$;

(4) $f\left[Cl_{\theta_\omega}(U)\right]\subseteq Cl\left[f(U)\right]$, for every $U\subseteq X$;

(5) For any point $x\in X$ and any open set V of Y containing $f(x)$, there exists $U\in\tau_{\theta_\omega}$ such that $x\in U$ and $f(U)\subseteq V$;

(6) $Bd_{\theta_\omega}\left[f^{-1}(V)\right]\subseteq f^{-1}\left[Bd(V)\right]$, for every $V\subseteq Y$;

(7) $f\left[D_{\theta_\omega}(U)\right]\subseteq Cl\left[f(U)\right]$, for every $U\subseteq X$;

(8) $f^{-1}\left[Int(V)\right]\subseteq Int_{\theta_\omega}\left[f^{-1}(V)\right]$, for every $V\subseteq Y$;

Proof. $(1)\Rightarrow(2)$: Let $F\subseteq Y$ be closed. Since f is $\theta_\omega-$continuous, $f^{-1}(Y-F)=X-f^{-1}(F)$ is $\theta_\omega-$open. Therefore, $f^{-1}(F)$ is $\theta_\omega-$closed in X.

$(2)\Rightarrow(3)$: Since $Cl(V)$ is closed for every $V\subseteq Y$, then $f^{-1}\left[Cl(V)\right]$ is $\theta_\omega-$closed. Therefore $f^{-1}\left[Cl(V)\right]=Cl_{\theta_\omega}\left[f^{-1}(Cl(V))\right]\supseteq Cl_{\theta_\omega}\left[f^{-1}(V)\right]$.

$(3)\Rightarrow(4)$: Let $U\subseteq X$ and $f(U)=V$. Then $Cl_{\theta_\omega}\left[f^{-1}(V)\right]\subseteq f^{-1}\left[Cl(V)\right]$. Thus $Cl_{\theta_\omega}(U)\subseteq Cl_{\theta_\omega}\left[f^{-1}(f(U))\right]\subseteq f^{-1}\left[Cl(f(U))\right]$ and $f\left[Cl_{\theta_\omega}(U)\right]\subseteq Cl\left[f(U)\right]$.

$(4)\Rightarrow(2)$: Let $W\subseteq Y$ be a closed set, and $U=f^{-1}(W)$. Then $f\left[Cl_{\theta_\omega}(U)\right]\subseteq Cl\left[f(U)\right]$ $=Cl\left[f\left(f^{-1}(W)\right)\right]\subseteq Cl(W)=W$. Thus $Cl_{\theta_\omega}(U)\subseteq f^{-1}\left[f\left(Cl_{\theta_\omega}(U)\right)\right]\subseteq f^{-1}(W)=U$. So U is $\theta_\omega-$closed.

$(2)\Rightarrow(1)$: Let $V\subseteq Y$ be an open set. Then $Y-V$ is closed. Then $f^{-1}(Y-V)=X-f^{-1}(V)$ is $\theta_\omega-$closed in X and hence $f^{-1}(V)$ is $\theta_\omega-$open in X.

$(1)\Rightarrow(5)$: Let $f:(X,\tau)\to(Y,\sigma)$ be $\theta_\omega-$continuous. For any $x\in X$ and any open set V of Y containing $f(x)$, $U=f^{-1}(V)\in\tau_{\theta_\omega}$, and $f(U)=f\left[f^{-1}(V)\right]\subseteq V$.

$(5)\Rightarrow(1)$: Let $V\in\sigma$. We prove $f^{-1}(V)\in\tau_{\theta_\omega}$. Let $x\in f^{-1}(V)$. Then $f(x)\in V$ and there exists $U\in\tau_{\theta_\omega}$ such that $x\in U$ and $f(x)\in f(U)\subseteq V$. Hence $x\in U\subseteq f^{-1}\left[f(U)\right]\subseteq f^{-1}(V)$. It shows that $f^{-1}(V)$ is a $\theta_\omega-$neighbourhood of each of its points. Therefore $f^{-1}(V)\in\tau_{\theta_\omega}$.

$(6)\Rightarrow(8)$: Let $V\subseteq Y$. Then by hypothesis, $Bd_{\theta_\omega}\left[f^{-1}(V)\right]\subseteq f^{-1}\left[Bd(V)\right]$

$\Rightarrow f^{-1}(V)-Int_{\theta_\omega}\left[f^{-1}(V)\right]\subseteq f^{-1}\left[V-Int(V)\right]$ $=f^{-1}(V)-f^{-1}\left[Int(V)\right]$ $\Rightarrow$

$f^{-1}\left[Int(V)\right]\subseteq Int_{\theta_\omega}\left[f^{-1}(V)\right]$.

$(8)\Rightarrow(6)$: Let $V\subseteq Y$. Then by hypothesis, $f^{-1}\left[Int(V)\right]\subseteq Int_{\theta_\omega}\left[f^{-1}(V)\right]$ $f^{-1}(V)-Int_{\theta_\omega}\left[f^{-1}(V)\right]$ $\subseteq f^{-1}(V)-f^{-1}\left[Int(V)\right]=f^{-1}\left[V-Int(V)\right]$

$\Rightarrow Bd_{\theta_\omega}\left[f^{-1}(V)\right]\subseteq f^{-1}\left[Bd(V)\right]$.

$(1)\Rightarrow(7)$: It is obvious, since f is $\theta_\omega-$continuous and by (4) $f\left[Cl_{\theta_\omega}(U)\right]\subseteq Cl\left[f(U)\right]$ for each $U\subseteq X$. So $f\left[D_{\theta_\omega}(U)\right]\subseteq Cl\left[f(U)\right]$.

$(7)\Rightarrow(1)$: Let $U\subseteq Y$ be an open set, $V=Y-U$ and $f^{-1}(V)=W$. Then by hypothesis

$f\left[D_{\theta_\omega}(W)\right]\subseteq Cl\left[f(W)\right]$. Thus $f\left[D_{\theta_\omega}\left(f^{-1}(V)\right)\right]\subseteq Cl\left[f\left(f^{-1}(V)\right)\right]\subseteq Cl(V)=V$.

Then $D_{\theta_\omega}\left[f^{-1}(V)\right]\subseteq f^{-1}(V)$ and $f^{-1}(V)$ is $\theta_\omega-$closed. Therefore, f is $\theta_\omega-$continuous.

$(1)\Rightarrow(8)$: Let $V\subseteq Y$. Then $f^{-1}\left[Int(V)\right]$ is $\theta_\omega-$open in X. Thus $f^{-1}\left[Int(V)\right]=$

$Int_{\theta_\omega}\left[f^{-1}\left(Int(V)\right)\right]\subseteq Int_{\theta_\omega}\left[f^{-1}(V)\right]$. Therefore $f^{-1}\left[Int(V)\right]\subseteq Int_{\theta_\omega}\left[f^{-1}(V)\right]$.

$(8)\Rightarrow(1)$: Let $V\subseteq Y$ be an open set. Then $f^{-1}(V)=f^{-1}\left[Int(V)\right]\subseteq Int_{\theta_\omega}\left[f^{-1}(V)\right]$. Therefore, $f^{-1}(V)$ is $\theta_\omega-$open in X. Hence f is $\theta_\omega-$continuous.

In the next Theorem, $\#\theta_\omega-c.$ denotes the set of points x of X for which a function $f:(X,\tau)\rightarrow(Y,\sigma)$ is not $\theta_\omega-$continuous.

Theorem 3.3. $\#\theta_\omega-c.$ is identical with the union of the $\theta_\omega-$frontier of the inverse images of $\theta_\omega-$open sets containing $f(x)$.

Proof. Suppose that f is not $\theta_\omega-$continuous at a point x of X. Then there exists an open set $V\subseteq Y$ containing $f(x)$ such that $f(U)$ is not a subset of V for every $U\in\tau_{\theta_\omega}$ containing x. Hence, we have $U\cap f^{-1}\left(X-f^{-1}(V)\right)\neq\phi$ for every $U\in\tau_{\theta_\omega}$ containing x. It follows that $x\in Cl_{\theta_\omega}\left[X-f^{-1}(V)\right]$. We also have $x\in f^{-1}(V)\subseteq Cl_{\theta_\omega}\left[f^{-1}(V)\right]$. This means that $x\in Fr_{\theta_\omega}\left[f^{-1}(V)\right]$. Now, let f be $\theta_\omega-$continuous at $x\in X$ and $V\subseteq Y$ any open set containing $f(x)$. Then, $x\in f^{-1}(V)$ is a $\theta_\omega-$open set of X. Thus. $x\in Int_{\theta_\omega}\left[f^{-1}(V)\right]$ and therefore $x\notin Fr_{\theta_\omega}\left[f^{-1}(V)\right]$ for every open set V containing $f(x)$.

Remarks 3.4. (1) Every $\theta_\omega-$continuous function is continuous but the converse may not be true.

(2) If a function $f:(X,\tau)\rightarrow(Y,\sigma)$ is $\theta_\omega-$continuous and a function $g:(Y,\sigma)\rightarrow(Z,\vartheta)$ is $\theta_\omega-$continuous, then $gof:(X,\tau)\rightarrow(Z,\vartheta)$ is $\theta_\omega-$continuous.

(3) If a function $f:(X,\tau)\rightarrow(Y,\sigma)$ is $\theta_\omega-$continuous and a function $g:(Y,\sigma)\rightarrow(Z,\vartheta)$ is continuous, then $gof:(X,\tau)\rightarrow(Z,\vartheta)$ is $\theta_\omega-$continuous.

(4) Let (X,τ) and (Y,σ) be topological spaces. If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a function, and one of the following

(a) $f^{-1}\left[Int(B)\right]\subseteq Int_{\theta_\omega}\left[f^{-1}(B)\right]$ for each $B\subseteq Y$.

(b) $Cl_{\theta_\omega}\left[f^{-1}(B)\right]\subseteq f^{-1}\left[Cl(B)\right]$ for each $B\subseteq Y$.

(c) $f\left[Cl_{\theta_\omega}(A)\right]\subseteq Cl\left[f(A)\right]$ for each $A\subseteq X$.

holds, then f is continuous.

Lemma 3.5. Let $A\subseteq Y\subseteq X$, Y is $\theta_\omega-$open in X and A is $\theta_\omega-$open in Y. Then A is $\theta_\omega-$open in X.

Proof. Since A is $\theta_\omega-$open in Y, there exists a $\theta_\omega-$open set $U\subseteq X$ such that $A=Y\cap U$. Thus A being the intersection of two $\theta_\omega-$open sets in X, is $\theta_\omega-$open in X.

Theorem 3.6. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a mapping and $\{U_i:i\in I\}$ be a cover of X such that $U_i\in\tau_{\theta_\omega}$ for each $i\in I$. Then prove that f is $\theta_\omega-$continuous.

Proof. Let $V\subseteq Y$ be an open set, then $\left(f|U_i\right)^{-1}(V)$ is $\theta_\omega-$open in U_i for each $i\in I$. Since U_i is $\theta_\omega-$open in X for each $i\in I$. So by Lemma 3.5, $\left(f|U_i\right)^{-1}(V)$ is $\theta_\omega-$open in X for each $i \in I$. But, $f^{-1}(V) = \cup\left\{\left(f|U_i\right)^{-1}(V) : i \in I\right\}$, then $f^{-1}(V) \in \tau_{\theta_\omega}$ because τ_{θ_ω} is a topology on X. This implies that f is $\theta_\omega -$continuous.

4 θ_ω – Irresolute Mappings

In this section, the functions to be considered are those for which inverses of $\theta_\omega -$open sets are $\theta_\omega -$open. We investigate some properties and characterizations of such functions.

Definition 4.1. Let (X, τ) and (Y, σ) be topological spaces. A function $f : (X, \tau) \to (Y, \sigma)$ is called $\theta_\omega -$irresolute if the inverse image of each $\theta_\omega -$open set of Y is a $\theta_\omega -$open set in X.

Theorem 4.2. Let $f : (X, \tau) \to (Y, \sigma)$ be a function between topological spaces. Then the following are equivalent:

(1) f is $\theta_\omega -$irresolute.

(2) the inverse image of each $\theta_\omega -$closed set in Y is a $\theta_\omega -$closed set in X;

(3) $Cl_{\theta_\omega}\left[f^{-1}(V)\right] \subseteq f^{-1}\left[Cl_{\theta_\omega}(V)\right]$ for every $V \subseteq Y$;

(4) $f\left[Cl_{\theta_\omega}(U)\right] \subseteq Cl_{\theta_\omega}\left[f(U)\right]$ for every $U \subseteq X$;

(5) $f^{-1}\left[Int_{\theta_\omega}(B)\right] \subseteq Int_{\theta_\omega}\left[f^{-1}(B)\right]$ for every $B \subseteq Y$.

Theorem 4.3. Prove that a function $f : (X, \tau) \to (Y, \sigma)$ is $\theta_\omega -$irresolute if and only if for each point p in X and each $\theta_\omega -$open set B in Y with $f(p) \in B$, there is a $\theta_\omega -$open set A in X such that $p \in A$, $f(A) \subseteq B$.

Proof. Necessity. Let $p \in X$ and $B \in \sigma_{\theta_\omega}$ such that $f(p) \in B$. Let $A = f^{-1}(B)$. Since f is $\theta_\omega -$irresolute, A is $\theta_\omega -$open in X. Also $p \in f^{-1}(B) = A$ as $f(p) \in B$. Thus we have $f(A) = f\left[f^{-1}(B)\right] \subseteq B$.

Sufficiency. Let $B \in \sigma_{\theta_\omega}$, let $A = f^{-1}(B)$. We show that A is $\theta_\omega -$open in X. For this let $x \in A$. It implies that $f(x) \in B$. Then by hypothesis, there exists $A_x \in \tau_{\theta_\omega}$ such that $x \in A_x$ and $f(A_x) \subseteq B$. Then $A_x \subseteq f^{-1}\left[f(A_x)\right] \subseteq f^{-1}(B) = A$. Thus $A = \cup\{A_x : x \in A\}$. It follows that A is $\theta_\omega -$open in X. Hence f is $\theta_\omega -$irresolute.

Definition 4.4. Let (X, τ) be a topological space. Let $x \in X$ and $N \subseteq X$. We say that N is a $\theta_\omega -$neighbourhood of x if there exists a $\theta_\omega -$open set M of X such that $x \in M \subseteq N$.

Theorem 4.5. Prove that a function $f : (X, \tau) \to (Y, \sigma)$ is $\theta_\omega -$irresolute if and only if for each x in X, the inverse image of every $\theta_\omega -$neighbourhood of $f(x)$, is a $\theta_\omega -$neighbourhood of x.

Proof. Necessity. Let $x \in X$ and let B be a $\theta_\omega -$neighbourhood of $f(x)$. Then there exists $U \in \sigma_{\theta_\omega}$ such that $f(x) \in U \subseteq B$. This implies that $x \in f^{-1}(U) \subseteq f^{-1}(B)$. Since f is $\theta_\omega -$irresolute, so $f^{-1}(U) \in \tau_{\theta_\omega}$. Hence $f^{-1}(B)$ is a $\theta_\omega -$neighbourhood of x.

Sufficiency. Let $B \in \sigma_{\theta_\omega}$. Put $A = f^{-1}(B)$. Let $x \in A$. Then $f(x) \in B$. But then, B being $\theta_\omega -$open set, is a $\theta_\omega -$neighbourhood of $f(x)$. So by hypothesis, $A = f^{-1}(B)$ is a $\theta_\omega -$neighbourhood of x. Hence by definition, there exists $A_x \in \tau_{\theta_\omega}$ such that $x \in A_x \subseteq A$. Thus $A = \cup\{A_x : x \in A\}$. It follows that A is a $\theta_\omega -$open set in X. Therefore f is $\theta_\omega -$irresolute.

Theorem 4.6. Prove that a function $f : (X, \tau) \to (Y, \sigma)$ is $\theta_\omega -$irresolute if and only if for each x in X. and each $\theta_\omega -$neighbourhood U of $f(x)$, there is a $\theta_\omega -$neighbourhood V of x such that $f(V) \subseteq U$.

Proof. **Necessity.** Let $x \in X$ and let U be a $\theta_\omega-$neighbourhood of $f(x)$. Then there exists $O_{f(x)} \in \sigma_{\theta_\omega}$ such that $f(x) \in O_{f(x)} \subseteq U$. It follows that $x \in f^{-1}\left[O_{f(x)}\right] \subseteq f^{-1}(U)$. By hypothesis, $f^{-1}\left[O_{f(x)}\right] \in \tau_{\theta_\omega}$. Let $V = f^{-1}(U)$. Then it follows that V is a $\theta_\omega-$neighbourhood of x and $f(V) = f\left[f^{-1}(U)\right] \subseteq U$.

Sufficiency. Let $B \in \sigma_{\theta_\omega}$. Put $O = f^{-1}(B)$. Let $x \in O$. Then $f(x) \in B$. Thus B is a $\theta_\omega-$neighbourhood of $f(x)$. So by hypothesis, there exists a $\theta_\omega-$neighbourhood V_x of x such that $f(V_x) \subseteq B$. Thus it follows that $x \in V_x \subseteq f^{-1}\left[f(V_x)\right] \subseteq f^{-1}(B) = O$. Since V_x is a $\theta_\omega-$neighbourhood of x, so there exists an $O_x \in \tau_{\theta_\omega}$ such that $x \in O_x \subseteq V_x$. Hence $x \in O_x \subseteq O$, $O_x \in \tau_{\theta_\omega}$. Thus $O = \cup\{O_x : x \in O\}$. It follows that O is $\theta_\omega-$open in X. Therefore, f is $\theta_\omega-$irresolute.

Theorem 4.7. Prove that a function $f:(X,\tau) \to (Y,\sigma)$ is $\theta_\omega-$irresolute if and only if $f\left[D_{\theta_\omega}(A)\right] \subseteq f(A) \cup D_{\theta_\omega}\left[f(A)\right]$, for all $A \subseteq X$.

Proof. **Necessity.** Let $f:(X,\tau) \to (Y,\sigma)$ be $\theta_\omega-$irresolute. Let $A \subseteq X$, and $a_0 \in D_{\theta_\omega}(A)$. Assume that $f(a_0) \notin f(A)$ and let V denote a $\theta_\omega-$neighbourhood of $f(a_0)$. Since f is $\theta_\omega-$irresolute, so by Theorem 4.6, there exists a $\theta_\omega-$neighbourhood U of a_0 such that $f(U) \subseteq V$. From $a_0 \in D_{\theta_\omega}(A)$, it follows that $U \cap A \neq \phi$; there exists, therefore, at least one element $a \in U \cap A$ such that $f(a) \in f(A)$ and $f(a) \in f(V)$. Since $f(a_0) \notin f(A)$, we have $f(a) \neq f(a_0)$. Thus every $\theta_\omega-$neighborhood of $f(a_0)$ contains an element of $f(A)$ different from $f(a_0)$, consequently, $f(a_0) \in D_{\theta_\omega}\left[f(A)\right]$. This proves necessity of the condition.

Sufficiency. Assume that f is not $\theta_\omega-$irresolute Then by Theorem 4.6, there exists $a_0 \in X$ and a $\theta_\omega-$neighborhood V of $f(a_0)$ such that every $\theta_\omega-$neighborhood U of a_0 contains at least one element $a \in U$ for which $f(a) \notin V$. Put $A = \{a \in X : f(a) \notin V\}$. Then $a_0 \notin A$ since $f(a_0) \in V$, and therefore $f(a_0) \notin A$; also $f(a_0) \notin D_{\theta_\omega}\left[f(A)\right]$ since $V \cap \left(V - \{f(a_0)\}\right) = \phi$. So $f(a_0) \in f\left[D_{\theta_\omega}(A)\right] - \left[f(A) \cup D_{\theta_\omega}\left(f(A)\right)\right] \neq \phi$, which is a contradiction to the given condition. The condition of the Theorem is therefore sufficient and the theorem is proved.

Theorem 4.8. Let $f:(X,\tau) \to (Y,\sigma)$ be a one-to-one function. Then f is $\theta_\omega-$irresolute. if and only if $f\left[D_{\theta_\omega}(A)\right] \subseteq D_{\theta_\omega}\left[f(A)\right]$, for all $A \subseteq X$.

Proof. **Necessity.** Let f be $\theta_\omega-$irresolute. Let $A \subseteq X$, $a_0 \in D_{\theta_\omega}(A)$ and V be a $\theta_\omega-$neighborhood of $f(a_0)$. Since f is $\theta_\omega-$irresolute, so by Theorem 4.6, there exists a $\theta_\omega-$neighborhood U of a_0 such that $f(U) \subseteq V$. But $a_0 \in D_{\theta_\omega}(A)$; hence there exists an element $a \in U \cap A$ such that $a \neq a_0$; then $f(a) \in f(A)$ and, since f is one to one, $f(a) \neq f(a_0)$. Thus every $\theta_\omega-$neighborhood V of $f(a_0)$ contains an element of $f(A)$ different from $f(a_0)$; consequently $f(a_0) \in D_{\theta_\omega}\left[f(A)\right]$. We have therefore $f\left[D_{\theta_\omega}(A)\right] \subseteq D_{\theta_\omega}\left[f(A)\right]$.

Sufficiency. Follows from Theorem 4.7.

5 θ_ω – Open Mappings

The purpose of this section is to investigate some characterizations of $\theta_\omega-$open mappings.

Definition 5.1. Let (X,τ) and (Y,σ) be topological spaces. A function $f:(X,\tau) \to (Y,\sigma)$ is called $\theta_\omega-$open if for every open set G in X, $f(G)$ is a $\theta_\omega-$open set in Y.

Theorem 5.2. Prove that a mapping $f:(X,\tau)\to(Y,\sigma)$ is $\theta_\omega-$open if and only if for each $x\in X$, and θ_ω $U\in\tau_{\theta_\omega}$ such that $x\in U$, there exists a $\theta_\omega-$open set $W\subseteq Y$ containing $f(x)$ such that $W\subseteq f(U)$.

Proof. Follows immediately from Definition 5.1.

Theorem 5.3. Let $f:(X,\tau)\to(Y,\sigma)$ be $\theta_\omega-$open. If $W\subseteq Y$ and $F\subseteq X$ is a closed set containing $f^{-1}(W)$, then there exists a $\theta_\omega-$closed. $H\subseteq Y$ containing W such that $f^{-1}(H)\subseteq F$.

Proof. Let $H=Y-f(Y-F)$. Since $f^{-1}(W)\subseteq F$, we have $f^{-1}(Y-F)\subseteq(Y-W)$. Since f is $\theta_\omega-$open, then H is $\theta_\omega-$closed and $f^{-1}(H)=X-f^{-1}\left[f(X-F)\right]\subseteq X-(X-F)=F$.

Theorem 5.4. Let $f:(X,\tau)\to(Y,\sigma)$ be a $\theta_\omega-$open. function and let $B\subseteq Y$. Then $f^{-1}\left[Cl_{\theta_\omega}\left(Int_{\theta_\omega}\left(Cl_{\theta_\omega}(B)\right)\right)\right]\subseteq Cl\left[f^{-1}(B)\right]$.

Proof. $Cl\left[f^{-1}(B)\right]$ is closed in X containing $f^{-1}(B)$. By Theorem 5.3, there exists a $\theta_\omega-$closed set $B\subseteq H\subseteq Y$ such that $f^{-1}(H)\subseteq Cl\left[f^{-1}(B)\right]$. Therefore, we obtain $f^{-1}\left[Cl_{\theta_\omega}\left(Int_{\theta_\omega}\left(Cl_{\theta_\omega}(B)\right)\right)\right]\subseteq$
$f^{-1}\left[Cl_{\theta_\omega}\left(Int_{\theta_\omega}\left(Cl_{\theta_\omega}(H)\right)\right)\right]\subseteq f^{-1}[H]\subseteq Cl\left[f^{-1}(B)\right]$.

Theorem 5.5. Prove that a function $f:(X,\tau)\to(Y,\sigma)$ is $\theta_\omega-$open if and only if $f\left[Int(A)\right]\subseteq Int_{\theta_\omega}\left[f(A)\right]$, for all $A\subseteq X$.

Proof. Necessity. Let $A\subseteq X$. Let $x\in Int(A)$. Then there exists $U_x\in\tau$ such that $x\in U_x\subseteq A$. So $f(x)\in f(U_x)\subseteq f(A)$. and by hypothesis, $f(U_x)\in\sigma_{\theta_\omega}$. Hence $f(x)\in Int_{\theta_\omega}\left[f(A)\right]$. Thus $f\left[Int(A)\right]\subseteq Int_{\theta_\omega}\left[f(A)\right]$.

Sufficiency. Let $U\in\tau$. Then by hypothesis, $f\left[Int(U)\right]\subseteq Int_{\theta_\omega}\left[f(U)\right]$. Since $Int(U)=U$ as U is open. Also $Int_{\theta_\omega}\left[f(U)\right]\subseteq f(U)$.Hence $f(U)=Int_{\theta_\omega}\left[f(U)\right]$. Thus $f(U)$ is $\theta_\omega-$open open in Y. So f is $\theta_\omega-$open.

Remark 5.6. The equality may not hold in the preceding Theorem.

Theorem 5.7. Prove that a function $f:(X,\tau)\to(Y,\sigma)$ is $\theta_\omega-$open if and only if $Int\left[f^{-1}(B)\right]\subseteq f^{-1}\left[Int_{\theta_\omega}(B)\right]$, for all $B\subseteq Y$.

Proof. Necessity. Let $B\subseteq Y$. Since $Int\left[f^{-1}(B)\right]$ is open in X and f is $\theta_\omega-$open, $f\left[Int\left(f^{-1}(B)\right)\right]$ is $\theta_\omega-$open in Y. Also we have $f\left[Int\left(f^{-1}(B)\right)\right]\subseteq f\left[f^{-1}(B)\right]\subseteq B$. Hence, we have $f\left[Int\left(f^{-1}(B)\right)\right]\subseteq Int_{\theta_\omega}(B)$. Therefore, we obtain $Int\left(f^{-1}(B)\right)\subseteq f^{-1}\left[Int_{\theta_\omega}(B)\right]$.

Sufficiency. Let $A\subseteq X$. Then $f(A)\subseteq Y$. Hence by hypothesis, we obtain $Int(A)\subseteq Int\left[f^{-1}\left(f(A)\right)\right]\subseteq f^{-1}\left[Int_{\theta_\omega}\left(f(A)\right)\right]$. Thus $f\left[Int(A)\right]\subseteq Int_{\theta_\omega}\left[f(A)\right]$, for all $A\subseteq X$. Hence, by Theorem 5.5, f is $\theta_\omega-$open.

Theorem 5.8. Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping. Then a necessary and sufficient condition for f to be $\theta_\omega-$open is that $f^{-1}\left[Cl_{\theta_\omega}(B)\right]\subseteq Cl\left[f^{-1}(B)\right]$ for every subset B of Y.

Proof. Necessity. Assume f is $\theta_\omega-$open Let $B\subseteq Y$. Let $x\in f^{-1}\left[Cl_{\theta_\omega}(B)\right]$. Then $f(x)\in Cl_{\theta_\omega}(B)$. Let $U\in\tau$ such that $x\in U$. Since f is $\theta_\omega-$open, then $f(U)$ is a $\theta_\omega-$open set in Y. Therefore, $B\cap f(U)\neq\phi$. Then $U\cap f^{-1}(B)\neq\phi$. Hence $x\in Cl\left[f^{-1}(B)\right]$. We conclude that $f^{-1}\left[Cl_{\theta_\omega}(B)\right]\subseteq Cl\left[f^{-1}(B)\right]$.

Sufficiency. Let $B \subseteq Y$. Then $(Y-B) \subseteq Y$. By hypothesis, $f^{-1}\left[Cl_{\theta_\omega}(Y-B)\right] \subseteq Cl\left[f^{-1}(Y-B)\right]$. $X - Cl\left[f^{-1}(Y-B)\right] \subseteq X - f^{-1}\left[Cl_{\theta_\omega}(Y-B)\right]$. Thus $X - Cl\left[X - f^{-1}(B)\right] \subseteq f^{-1}\left[Y - Cl_{\theta_\omega}(Y-B)\right]$. By applying a well-known result, it implies that $Int\left[f^{-1}(B)\right] \subseteq f^{-1}\left[Int_{\theta_\omega}(B)\right]$. Now form Theorem 5.7, it follows that f is $\theta_\omega -$open.

6 θ_ω – Closed Mappings

In this section we introduce $\theta_\omega -$closed functions and study certain properties and characterizations of this type of functions.

Definition 6.1. A mapping $f:(X,\tau) \to (Y,\sigma)$ is called $\theta_\omega -$closed if the image of each closed set in X is a $\theta_\omega -$closed set in Y.

Theorem 6.2. Prove that a mapping $f:(X,\tau) \to (Y,\sigma)$ is $\theta_\omega -$closed if and only if $Cl_{\theta_\omega}\left[f(A)\right] \subseteq f\left[Cl(A)\right]$ for each $A \subseteq X$.

Proof. Necessity. Let f be $\theta_\omega -$closed and let $A \subseteq X$. Then $f(A) \subseteq f\left[Cl(A)\right]$ and $f\left[Cl(A)\right]$ is a $\theta_\omega -$closed set in Y. Thus $Cl_{\theta_\omega}\left[f(A)\right] \subseteq f\left[Cl(A)\right]$.

Sufficiency. Suppose that $Cl_{\theta_\omega}\left[f(A)\right] \subseteq f\left[Cl(A)\right]$, for each $A \subseteq X$. Let $A \subseteq X$ be a closed set. Then $Cl_{\theta_\omega}\left[f(A)\right] \subseteq f\left[Cl(A)\right] = f(A)$. This shows that $f(A)$ is a $\theta_\omega -$closed set. Hence f is $\theta_\omega -$closed.

Theorem 6.3. Let $f:(X,\tau) \to (Y,\sigma)$ be $\theta_\omega -$closed. If $V \subseteq Y$ and $E \subseteq X$ is an open set containing $f^{-1}(V)$, then there exists a $\theta_\omega -$open set $G \subseteq Y$ containing V such that $f^{-1}(G) \subseteq E$.

Proof. Let $G = Y - f(X-E)$. Since $f^{-1}(V) \subseteq E$, we have $f(X-E) \subseteq Y - V$. Since f is $\theta_\omega -$closed, then G is a $\theta_\omega -$open set and $f^{-1}(G) = X - f^{-1}\left[f(X-E)\right] \subseteq X - (X-E) = E$.

Theorem 6.4. Suppose that $f:(X,\tau) \to (Y,\sigma)$ is a $\theta_\omega -$closed mapping. Then $Int_{\theta_\omega}\left[Cl_{\theta_\omega}(f(A))\right] \subseteq f\left[Cl(A)\right]$ for every subset A of X.

Proof. Suppose f is a $\theta_\omega -$closed mapping and A is an arbitrary subset of X. Then $f\left[Cl(A)\right]$ is $\theta_\omega -$closed in Y. Then $Int_{\theta_\omega}\left[Cl_{\theta_\omega}(f(Cl(A)))\right] \subseteq f\left[Cl(A)\right]$. But also $Int_{\theta_\omega}\left[Cl_{\theta_\omega}(f(A))\right] \subseteq Int_{\theta_\omega}\left[Cl_{\theta_\omega}(f(Cl(A)))\right]$. Hence $Int_{\theta_\omega}\left[Cl_{\theta_\omega}(f(A))\right] \subseteq f\left[Cl(A)\right]$.

Theorem 6.5. Let $f:(X,\tau) \to (Y,\sigma)$ be a $\theta_\omega -$closed function, and $B, C \subseteq Y$.

Proof. (1) If U is an open neighborhood of $f^{-1}(B)$, then there exists a $\theta_\omega -$open neighborhood V of B such that $f^{-1}(B) \subseteq f^{-1}(V) \subseteq U$.

(2) If f is also onto, then if $f^{-1}(B)$ and $f^{-1}(C)$ have disjoint open neighborhoods, so have B and C.

Proof. (1) Let $V = Y - f(X-U)$. Then $V^c = Y - V = f(U^c)$. Since f is $\theta_\omega -$closed, so V is a $\theta_\omega -$open set. Since $f^{-1}(B) \subseteq U$, we have $V^c = f(U^c) \subseteq f\left[f^{-1}(B^c)\right] \subseteq B^c$. Hence, $B \subseteq V$, and thus V is a $\theta_\omega -$open neighborhood of B. Further $U^c \subseteq f^{-1}\left[f(U^c)\right] = f^{-1}(V^c) = \left[f^{-1}(V)\right]^c$. This proves that $f^{-1}(V) \subseteq U$.

(2) If $f^{-1}(B)$ and $f^{-1}(C)$ have disjoint open neighborhoods M and N, then by (1), we have $\theta_\omega -$open neighborhoods U and V of B and C respectively such that $f^{-1}(B) \subseteq f^{-1}(U) \subseteq Int_{\theta_\omega}(M)$ and $f^{-1}(C) \subseteq f^{-1}(V) \subseteq Int_{\theta_\omega}(N)$. Since M and N are disjoint, so are $Int_{\theta_\omega}(M)$ and $Int_{\theta_\omega}(N)$, hence so $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint as well. It follows that U and V are disjoint too as f is onto.

Theorem 6.6. Prove that a surjective mapping $f:(X,\tau)\to(Y,\sigma)$ is $\theta_\omega-$closed if and only if for each subset B of Y and each open set U in X containing $f^{-1}(B)$, there exists a $\theta_\omega-$open set V in Y containing B such that $f^{-1}(V)\subseteq U$.

Proof. Necessity. This follows from (1) of Theorem 6.5.

Sufficiency. Suppose F is an arbitrary closed set in X. Let y be an arbitrary point in $Y-f(F)$. Then $f^{-1}(y)\subseteq X-f^{-1}[f(F)]\subseteq(X-F)$ and $(X-F)$ is open in X. Hence by hypothesis, there exists a $\theta_\omega-$open set V_y containing y such that $f^{-1}(V_y)\subseteq(X-F)$. This implies that $y\in V_y\subseteq[Y-f(F)]$. Thus we obtain $Y-f(F)=\bigcup\{V_y:y\in Y-f(F)\}$. So $Y-f(F)$ being a union of $\theta_\omega-$open sets, is $\theta_\omega-$open Thus its complement $f(F)$ is $\theta_\omega-$closed. This shows that f is $\theta_\omega-$closed.

Theorem 6.7. Let $f:(X,\tau)\to(Y,\sigma)$ be a bijection. Then the following are equivalent:

(a) f is $\theta_\omega-$closed.

(b) f is $\theta_\omega-$open.

(c) f^{-1} is $\theta_\omega-$ocontinuous.

Proof. $(\mathrm{a})\Rightarrow(\mathrm{b})$: Let $U\in\tau$. Then $X-U$ is closed in X. By (a), $f(X-U)$ is $\theta_\omega-$closed in Y. But $f(X-U)=f(X)-f(U)=Y-f(U)$. Thus $f(U)$ is $\theta_\omega-$open in Y. This shows that f is $\theta_\omega-$open.

$(\mathrm{b})\Rightarrow(\mathrm{c})$: Let $U\subseteq X$. be an open set. Since f is $\theta_\omega-$open. So $f(U)=\left(f^{-1}\right)^{-1}(U)$ is $\theta_\omega-$open in Y. Hence f^{-1} is $\theta_\omega-$ocontinuous.

$(\mathrm{c})\Rightarrow(\mathrm{a})$: Let A be an arbitrary closed set in X. Then $X-A$ is open in X. Since f^{-1} is $\theta_\omega-$ocontinuous, $\left(f^{-1}\right)^{-1}(X-A)$ is $\theta_\omega-$open in Y. But $\left(f^{-1}\right)^{-1}(X-A)=f(X-A)=Y-f(A)$. Thus $f(A)$ is $\theta_\omega-$closed in Y. This shows that f is $\theta_\omega-$closed.

Remark 6.8. A bijection $f:(X,\tau)\to(Y,\sigma)$ may be open and closed but neither $\theta_\omega-$open nor $\theta_\omega-$closed.

7 Pre $-\theta_\omega-$ Open Mappings

The purpose of this section is to introduce and discuss certain properties and characterizations of pre $-\theta_\omega-$open functions.

Definition 7.1. Let (X,τ) and (Y,σ) be topological spaces. Then a function $f:(X,\tau)\to(Y,\sigma)$ is said to be pre $-\theta_\omega-$open if and only if for each $A\in\tau_{\theta_\omega}$, $f(A)\in\sigma_{\theta_\omega}$.

Theorem 7.2. Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\mu)$ be any two pre $-\theta_\omega-$open functions. Then the composition function $gof:(X,\tau)\to(Z,\mu)$ is a pre $-\theta_\omega-$open function.

Proof. Let $U\in\tau_{\theta_\omega}$. Then $f(U)\in\sigma_{\theta_\omega}$. Since f is pre $-\theta_\omega-$open But then $g(f(U))\in\mu_{\theta_\omega}$ as g is pre $-\theta_\omega-$open. Hence, gof is pre $-\theta_\omega-$open.

Theorem 7.3. Prove that a mapping $f:(X,\tau)\to(Y,\sigma)$ is pre $-\theta_\omega-$open if and only if for each $x\in X$ and for any $U\in\tau_{\theta_\omega}$ such that $x\in U$, there exists $V\in\sigma_{\theta_\omega}$ such that $f(x)\in V$ and $V\subseteq f(U)$.

Proof. Routine.

Theorem 7.4. Prove that a mapping $f:(X,\tau)\to(Y,\sigma)$ is pre $-\theta_\omega-$open if and only if for each $x\in X$ and for any $\theta_\omega-$neighbourhood U

of x in X, there exists a $\theta_\omega-$neighbourhood V of $f(x)$ in Y such that $V \subseteq f(U)$.

Proof. Necessity. Let $x \in X$ and let U be a $\theta_\omega-$neighbourhood of x. Then there exists $W \in \tau_{\theta_\omega}$ such that $x \in W \subseteq U$. Then $f(x) \in f(W) \subseteq f(U)$. But $f(W) \in \sigma_{\theta_\omega}$ as f is pre$-\theta_\omega-$open Hence $V = f(W)$ is a $\theta_\omega-$neighbourhood of $f(x)$ and $V \subseteq f(U)$.

Sufficiency. Let $U \in \tau_{\theta_\omega}$. Let $x \in U$. Then U is a $\theta_\omega-$neighbourhood of x. So by hypothesis, there exists a $\theta_\omega-$neighbourhood $V_{f(x)}$ of $f(x)$ such that $f(x) \in V_{f(x)} \subseteq f(U)$. It follows at once that $f(U)$ is a $\theta_\omega-$neighbourhood of each of its points. Therefore $f(U)$ is $\theta_\omega-$open. Hence f is pre$-\theta_\omega-$open.

Theorem 7.5. Prove that a function $f:(X,\tau) \to (Y,\sigma)$ is pre$-\theta_\omega-$open if and only if $f\left[Int_{\theta_\omega}(A)\right] \subseteq Int_{\theta_\omega}\left[f(A)\right]$, for all $A \subseteq X$.

Proof. Necessity. Let $A \subseteq X$. Let $x \in Int_{\theta_\omega}(A)$. Then there exists $U_x \in \tau_{\theta_\omega}$ such that $x \in U_x \subseteq A$. So $f(x) \in f(U_x) \subseteq f(A)$ and by hypothesis, $f(U_x) \in \sigma_{\theta_\omega}$. Hence $f(x) \in Int_{\theta_\omega}\left[f(A)\right]$. Thus $f\left[Int_{\theta_\omega}(A)\right] \subseteq Int_{\theta_\omega}\left[f(A)\right]$.

Sufficiency. Let $U \in \tau_{\theta_\omega}$. Then by hypothesis, $f\left[Int_{\theta_\omega}(U)\right] \subseteq Int_{\theta_\omega}\left[f(U)\right]$. Since $Int_{\theta_\omega}(U) = U$ as U is $\theta_\omega-$open. Also $Int_{\theta_\omega}\left[f(U)\right] \subseteq f(U)$. Hence $f(U) = Int_{\theta_\omega}\left[f(U)\right]$. Thus $f(U)$ is $\theta_\omega-$open in Y. So f is pre$-\theta_\omega-$open.

We remark that the equality does not hold in Theorem 7.5 as the following example shows.

Example 7.6. Let $X = Y = R$. suppose X be with topology $\tau = \{\phi, ˜ , ¥ , ¤^c, ¥ \text{ U} ¤^c\}$. Then $\tau_{\theta_\omega} = \{\phi, ˜ , ¥\}$. Let Y be with discrete topology $\tau_D = \{A : A \subseteq X\} = P(X)$. Let $f = Id : X \to Y$ be an identity function defined as $f(x) = x$, for each $x \in X$. Let $A = ¤^c$. Then $\phi = f\left[Int_{\theta_\omega}(A)\right] \neq Int_{\theta_\omega}\left[f(A)\right] = ¤^c$.

Theorem 7.7. Prove that a function $f:(X,\tau) \to (Y,\sigma)$ is pre$-\theta_\omega-$open if and only if $Int_{\theta_\omega}\left[f^{-1}(B)\right] \subseteq f^{-1}\left[Int_{\theta_\omega}(B)\right]$, for all $B \subseteq Y$.

Proof. Necessity. Let $B \subseteq Y$. Since $Int_{\theta_\omega}\left[f^{-1}(B)\right]$ is $\theta_\omega-$open in X and f is pre$-\theta_\omega-$open, $f\left[Int_{\theta_\omega}\left(f^{-1}(B)\right)\right]$ is $\theta_\omega-$open in Y. Also we have $f\left[Int_{\theta_\omega}\left(f^{-1}(B)\right)\right] \subseteq f\left[f^{-1}(B)\right] \subseteq B$. Hence, $f\left[Int_{\theta_\omega}\left(f^{-1}(B)\right)\right] \subseteq Int_{\theta_\omega}(B)$. Therefore $Int_{\theta_\omega}\left[f^{-1}(B)\right] \subseteq f^{-1}\left[Int_{\theta_\omega}(B)\right]$.

Sufficiency. Let $A \subseteq X$. Then $f(A) \subseteq Y$. Hence by hypothesis, we obtain $Int_{\theta_\omega}(A) \subseteq Int_{\theta_\omega}\left[f^{-1}\left(f(A)\right)\right] \subseteq f^{-1}\left[Int_{\theta_\omega}\left(f(A)\right)\right]$. This implies that $f\left[Int_{\theta_\omega}(A)\right] \subseteq f\left[f^{-1}\left(Int_{\theta_\omega}\left(f(A)\right)\right)\right] \subseteq Int_{\theta_\omega}\left[f(A)\right]$. Thus $f\left[Int_{\theta_\omega}(A)\right] \subseteq Int_{\theta_\omega}\left[f(A)\right]$, for all $A \subseteq X$. Hence, by Theorem 7.5, f is pre$-\theta_\omega-$open.

Theorem 7.8. Prove that a mapping $f:(X,\tau) \to (Y,\sigma)$ is pre$-\theta_\omega-$open if and only if $f^{-1}\left[Cl_{\theta_\omega}(B)\right] \subseteq Cl_{\theta_\omega}\left[f^{-1}(B)\right]$, for every subset B of Y.

Proof. Necessity. Let $B \subseteq Y$. Let $x \in f^{-1}\left[Cl_{\theta_\omega}(B)\right]$. Then $f(x) \in Cl_{\theta_\omega}(B)$. Let $U \in \tau_{\theta_\omega}$ such that $x \in U$. By hypothesis, $f(U) \in \sigma_{\theta_\omega}$ and $f(x) \in f(U)$. Thus $f(U) \text{ I } B \neq \phi$. Hence $U \text{ I } f^{-1}(B) \neq \phi$. Therefore, $x \in Cl_{\theta_\omega}\left[f^{-1}(B)\right]$, So we obtain $f^{-1}\left[Cl_{\theta_\omega}(B)\right] \subseteq Cl_{\theta_\omega}\left[f^{-1}(B)\right]$.

Sufficiency. Let $B \subseteq Y$. Then $(Y-B) \subseteq Y$. By hypothesis, $f^{-1}\left[Cl_{\theta_\omega}(Y-B)\right] \subseteq Cl_{\theta_\omega}\left[f^{-1}(Y-B)\right]$. So $X - Cl_{\theta_\omega}\left[f^{-1}(Y-B)\right] \subseteq X - f^{-1}\left[Cl_{\theta_\omega}(Y-B)\right]$.

So $X - Cl_{\theta_\omega}\left[X - f^{-1}(B)\right] \subseteq f^{-1}\left[Y - Cl_{\theta_\omega}(Y - B)\right]$. By a well-known result, it follows that $Int_{\theta_\omega}\left[f^{-1}(B)\right] \subseteq f^{-1}\left[Int_{\theta_\omega}(B)\right]$. Now by Theorem 7.7, it follows that f is pre$-\theta_\omega-$open.

Theorem 7.9. Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\mu)$ be two mappings such that $gof:(X,\tau)\to(Z,\mu)$ is $\theta_\omega-$irresolute. Then

(1) If g is a pre$-\theta_\omega-$open injection, then f is $\theta_\omega-$irresolute.

(2) If f is a pre$-\theta_\omega-$open surjection, then g is $\theta_\omega-$irresolute.

Proof. (1) Let $U \in \sigma_{\theta_\omega}$. Then $g(U) \in \mu_{\theta_\omega}$ since g is pre$-\theta_\omega-$open. Also gof is $\theta_\omega-$irresolute. Therefore, we have $(gof)^{-1}\left[g(U)\right] \in \tau_{\theta_\omega}$. Since g is an injection, so we have : $(gof)^{-1}\left[g(U)\right] = \left(f^{-1}og^{-1}\right)\left[g(U)\right] = f^{-1}\left[g^{-1}\left(g(U)\right)\right] = f^{-1}(U)$. Consequently $f^{-1}(U)$ is $\theta_\omega-$open in X. This proves that f is $\theta_\omega-$irresolute.

(2) Let $V \in \mu_{\theta_\omega}$. Then $(gof)^{-1}(V) \in \tau_{\theta_\omega}$ since gof is $\theta_\omega-$irresolute. Also f is pre$-\theta_\omega-$open $\theta_\omega-$open $f\left[(gof)^{-1}(V)\right]$ is $\theta_\omega-$open in Y. Since f is surjective, we note that $f\left[(gof)^{-1}(V)\right] = \left[fo(gof)^{-1}\right](V) = \left[fo\left(f^{-1}og^{-1}\right)\right](V) = \left[\left(fof^{-1}\right)og^{-1}(V)\right] = g^{-1}(V)$. Hence g is $\theta_\omega-$irresolute.

8 Pre $-\theta_\omega-$ Closed Mappings

In this last section, we introduce and explore several properties and characterizations of pre$-\theta_\omega-$closed functions.

Definition 8.1. A function $f:(X,\tau)\to(Y,\sigma)$ is said to be pre$-\theta_\omega-$closed if and only if the image set $f(A)$ is $\theta_\omega-$closed for each $\theta_\omega-$closed subset A of X.

Theorem 8.2. The composition of two pre$-\theta_\omega-$closed mappings is a pre$-\theta_\omega-$closed mapping.

Proof. The straight forward proof is omitted.

Theorem 8.3. Prove that a mapping $f:(X,\tau)\to(Y,\sigma)$ is pre$-\theta_\omega-$closed if and only if $Cl_{\theta_\omega}\left[f(A)\right] \subseteq f\left[Cl_{\theta_\omega}(A)\right]$ for every subset A of X.

Proof. Necessity. Suppose f is a pre$-\theta_\omega-$closed mapping and A is an arbitrary subset of X. Then $f\left[Cl_{\theta_\omega}(A)\right]$ is $\theta_\omega-$closed in Y. Since $f(A) \subseteq f\left[Cl_{\theta_\omega}(A)\right]$, we obtain $Cl_{\theta_\omega}\left[f(A)\right] \subseteq f\left[Cl_{\theta_\omega}(A)\right]$.

Sufficiency. Suppose F is an arbitrary $\theta_\omega-$closed set in X. By hypothesis, we obtain $f(F) \subseteq Cl_{\theta_\omega}\left[f(F)\right] \subseteq f\left[Cl_{\theta_\omega}(F)\right] = f(F)$. Hence $f(F) = Cl_{\theta_\omega}\left[f(F)\right]$. Thus $f(F)$ is $\theta_\omega-$closed in Y. It follows that f is pre$-\theta_\omega-$closed.

Theorem 8.4. Let $f:(X,\tau)\to(Y,\sigma)$ be a pre$-\theta_\omega-$closed function, and $B,C \subseteq Y$.

(1) If U is a $\theta_\omega-$open neighborhood of $f^{-1}(B)$, then there exists a $\theta_\omega-$open neighborhood V of B such that $f^{-1}(B) \subseteq f^{-1}(V) \subseteq U$.

(2) If f is also onto, then if $f^{-1}(B)$ and $f^{-1}(C)$ have disjoint $\theta_\omega-$open neighborhoods, so have B and C.

Proof. (1) Let $V = Y - f(X - U)$. Then $V^c = Y - V = f(U^c)$. Since f is pre$-\theta_\omega-$closed, so V is $\theta_\omega-$open. Since $f^{-1}(B) \subseteq U$, we have $V^c = f(U^c) \subseteq f\left[f^{-1}(B^c)\right] \subseteq B^c$. Hence, $B \subseteq V$, and thus V is a $\theta_\omega-$open neighborhood of B.

Further $U^c \subseteq f^{-1}\left[f\left(U^c\right)\right] = f^{-1}\left(V^c\right) = \left[f^{-1}(V)\right]^c$. This proves that $f^{-1}(V) \subseteq U$.

(2) If $f^{-1}(B)$ and $f^{-1}(C)$ have disjoint $\theta_\omega -$open neighborhoods M and N, then by (1), we have $\theta_\omega -$open neighborhoods U and V of B and C respectively such that $f^{-1}(B) \subseteq f^{-1}(U) \subseteq Int_{\theta_\omega}(M)$ and $f^{-1}(C) \subseteq f^{-1}(V) \subseteq Int_{\theta_\omega}(N)$. Since M and N are disjoint, so are $Int_{\theta_\omega}(M)$ and $Int_{\theta_\omega}(N)$, and hence so $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint as well. It follows that U and V are disjoint too as f is onto.

Theorem 8.5. Prove that a surjective mapping $f:(X,\tau) \to (Y,\sigma)$ is pre$-\theta_\omega -$closed if and only if for each subset B of Y and each $\theta_\omega -$open set U in X containing $f^{-1}(B)$, there exists a $\theta_\omega -$open set V in Y containing B such that $f^{-1}(V) \subseteq U$.

Proof. Necessity. This follows from (1) of Theorem 8.4.

Sufficiency. Suppose F is an arbitrary $\theta_\omega -$closed set in X. Let y be an arbitrary point in $Y - f(F)$. Then $f^{-1}(y) \subseteq X - f^{-1}\left[f(F)\right] \subseteq (X - F)$ and $(X - F)$ is $\theta_\omega -$open in X. Hence by hypothesis, there exists a $\theta_\omega -$open set V_y containing y such that $f^{-1}(V_y) \subseteq (X - F)$. This implies that $y \in V_y \subseteq \left[Y - f(F)\right]$. Thus $Y - f(F) = \cup\left\{V_y \middle| y \in Y - f(F)\right\}$. Hence $Y - f(F)$, being a union of $\theta_\omega -$open sets is $\theta_\omega -$open. Thus its complement $f(F)$ is $\theta_\omega -$closed. This shows that f is $\theta_\omega -$closed.

Theorem 8.6. Let $f:(X,\tau) \to (Y,\sigma)$ be a bijection. Then the following are equivalent:

(1) f is pre$-\theta_\omega -$closed.

(2) f is pre$-\theta_\omega -$open.

(3) f^{-1} is $\theta_\omega -$irresolute.

Proof. $(1) \Rightarrow (2)$: Let $U \in \tau_{\theta_\omega}$. Then $X - U$ is $\theta_\omega -$closed in X. By (1), $f(X - U)$ is $\theta_\omega -$closed in Y. But $f(X - U) = f(X) - f(U) = Y - f(U)$. Thus $f(U)$ is $\theta_\omega -$open in Y.This shows that f is pre$-\theta_\omega -$open.

$(2) \Rightarrow (3)$: Let $A \subseteq X$. Since f is pre$-\theta_\omega -$open, so by Theorem 7.8, $f^{-1}\left[Cl_{\theta_\omega}\left(f(A)\right)\right] \subseteq Cl_{\theta_\omega}\left[f^{-1}\left(f(A)\right)\right]$. It implies that $Cl_{\theta_\omega}\left[f(A)\right] \subseteq f\left[Cl_{\theta_\omega}(A)\right]$.

Thus $Cl_{\theta_\omega}\left[\left(f^{-1}\right)^{-1}(A)\right] \subseteq \left(f^{-1}\right)^{-1}\left[Cl_{\theta_\omega}(A)\right]$, for all $A \subseteq X$. Then by Theorem 4.2, it follows that f^{-1} is $\theta_\omega -$irresolute.

$(3) \Rightarrow (1)$: Let A be an arbitrary $\theta_\omega -$closed set in X. Then $X - A$ is $\theta_\omega -$open in X. Since f^{-1} is $\theta_\omega -$irresolute. $\left(f^{-1}\right)^{-1}(X - A)$ is $\theta_\omega -$open in Y. But $\left(f^{-1}\right)^{-1}(X - A) = f(X - A) = Y - f(A)$. Thus $f(A)$ is $\theta_\omega -$closed in Y. This shows that f is pre$-\theta_\omega -$closed.

9 Contra θ_ω – Continuous Mappings

We introduce the definition of contra $\theta_\omega -$continuous functions in topological spaces and study some of their properties in this section.

Definition 9.1. A function $f:(X,\tau) \to (Y,\sigma)$ is said to be contra $\theta_\omega -$continuous if $f^{-1}(V)$ is $\theta_\omega -$closed in (X,τ) for each open set V of (Y,σ).

Observe that if Observe that if X is a countable set, then every function $f:(X,\tau) \to (Y,\sigma)$ is contra $\theta_\omega -$continuous.

Theorem 9.2. Let $f:(X,\tau) \to (Y,\sigma)$ be a function. Then the following are equivalent.

(1) f is contra $\theta_\omega -$continuous.

(2) $f^{-1}(F)$ is $\theta_\omega-$open in (X,τ) for every closed subset F of (Y,σ).

(3) For each $x\in X$ and each closed set F in (Y,σ) containing $f(x)$, there exists a $\theta_\omega-$open. set U in (X,τ) containing x such that $f(U)\subseteq F$.

(4) $f\left[Cl_{\theta_\omega}(A)\right]\subseteq Ker\left[f(A)\right]$ for ever subset A of (X,τ).

(5) $Cl_{\theta_\omega}\left[f^{-1}(B)\right]\subseteq f^{-1}\left[Ker(B)\right]$ for ever subset B of (Y,σ).

Proof. $(1)\Rightarrow(2)$: Let F be any closed set of Y. Then $Y-F$ is open. Hence by hypothesis $f^{-1}(Y-F)$ is $\theta_\omega-$closed. Thus $f^{-1}(Y-F)=Cl_{\theta_\omega}\left[f^{-1}(Y-F)\right]$. We can obtain $X-f^{-1}(F)=X-Int_{\theta_\omega}\left[f^{-1}(F)\right]$. Therefore, we have $f^{-1}(F)=Int_{\theta_\omega}\left[f^{-1}(F)\right]$. Thus $f^{-1}(F)$ is $\theta_\omega-$open in X.

$(2)\Rightarrow(3)$: Let $x\in X$ and F be a closed set of Y containing $f(x)$. By (2), $x\in Int_{\theta_\omega}\left[f^{-1}(F)\right]$. Hence there exists $U\in\theta_\omega(X)$ containing x such that $x\in U\subseteq f^{-1}(F)$. Then, $x\in U$ and $f(U)\subseteq F$.

$(3)\Rightarrow(4)$: Let A be any subset of X. Let $x\in Cl_{\theta_\omega}(A)$ and F be a closed set of Y containing $f(x)$. Then by (3) there exists $U\in\theta_\omega O(X)$ containing x such that $f(U)\subseteq F;$ hence $x\in U\subseteq f(F)$. Since $x\in Cl_{\theta_\omega}(A)$, so $U\cap A\neq\phi$ and hence it follows that $\phi\neq f(U\cap A)\subseteq f(U)\cap f(A)\subseteq F\cap f(A)$. Then by Lemma 2.15, we have $f(x)\in Ker\left[f(A)\right]$ and hence we obtain $f\left[Cl_{\theta_\omega}(A)\right]\subseteq Ker\left[f(A)\right]$.

$(4)\Rightarrow(5)$: Let B be any subset of Y. By (4), $f\left[Cl_{\theta_\omega}\left(f^{-1}(B)\right)\right]\subseteq Ker\left[f\left(f^{-1}(B)\right)\right]\subseteq Ker(B)$ and hence $Cl_{\theta_\omega}\left[f^{-1}(B)\right]\subseteq f^{-1}\left[Ker(B)\right]$.

$(5)\Rightarrow(1)$: Let V be any open set of Y. Then by (5) and Lemma 2.15 we obtain $Cl_{\theta_\omega}\left[f^{-1}(V)\right]\subseteq f^{-1}\left[Ker(V)\right]=f^{-1}(V)$. Thus $Cl_{\theta_\omega}\left[f^{-1}(V)\right]=f^{-1}(V)$. Hence $f^{-1}(V)$ is $\theta_\omega-$closed in X. This shows that f is contra $\theta_\omega-$continuous.

Proposition 9.3. Let $f:(X,\tau)\to(Y,\sigma)$ be contra $\theta_\omega-$continuous. If one of the following conditions holds, then f is $\theta_\omega-$continuous.

(1) (Y,σ) is regular,

(2) $Int_{\theta_\omega}\left[f^{-1}(Cl(V))\right]\subseteq f^{-1}(V)$ for each open set V in (Y,σ).

Proof. (1) Let $x\in X$ and V be an open set of (Y,σ) containing $f(x)$. Since (Y,σ) is regular, there exists an open set W in (Y,σ) containing $f(x)$ such that $Cl(W)\subseteq V$. Since f is contra $\theta_\omega-$continuous, so by Theorem 9.2, there exists a $\theta_\omega-$open set U in (X,τ) containing x such that $f(U)\subseteq Cl(W);$ hence $f(U)\subseteq V$. Therefore f is $\theta_\omega-$continuous.

(2) Let V be an open set of (Y,σ). Since f is contra $\theta_\omega-$continuous and $Cl(V)$ is closed, by Theorem 9.2, $f^{-1}\left[Cl(V)\right]$ is $\theta_\omega-$open set in (X,τ) and hence by (2), it implies $f^{-1}\left[Cl(V)\right]\subseteq Int_{\theta_\omega}\left[f^{-1}(Cl(V))\right]\subseteq f^{-1}(V)$. So, we obtain $f^{-1}(V)=Int_{\theta_\omega}\left[f^{-1}(Cl(V))\right]$ and consequently $f^{-1}(V)$ is $\theta_\omega-$open in (X,τ). So f is a $\theta_\omega-$continuous function.

Recall that for a function $f:(X,\tau)\to(Y,\sigma)$, the subset $\{(x,f(x)):x\in X\}\subseteq X\times Y$ is called the graph of f and is denoted by $G(f)$.

Theorem 9.4. Let $f:(X,\tau)\to(Y,\sigma)$ be a function and $g:(X,\tau)\to(X\times Y,\tau\times\sigma)$ the graph function of f, defined by $g(x)=(x,f(x))$ for every $x\in X$. If g is contra $\theta_\omega-$continuous, then f is contra $\theta_\omega-$continuous.

Proof. Let U be an open set in (Y,σ), then $X\times U$ is an open set in $(X\times Y,\tau\times\sigma)$. Since g is contra $\theta_\omega-$continuous, $g^{-1}(X\times U)=f^{-1}(U)$ is $\theta_\omega-$closed in (X,τ). This shows that f is contra $\theta_\omega-$continuous.

Definition 9.5. A subset A of a topological space (X,τ) is said to be $\theta_\omega-$dense in X if $Cl_{\theta_\omega}(A)=X$.

Definition 9.6. A topological space (X,τ) is said to be a Urysohn space if for any two distinct points $x,y\in X$, there exist open subsets U and V of (X,τ) such that $x\in U$, $y\in V$ and $Cl(U)\cap Cl(V)=\phi$.

Theorem 9.7. Let $f,g:(X,\tau)\to(Y,\sigma)$ be two contra $\theta_\omega-$continuous functions. If (Y,σ) is Urysohn, the following properties hold:

(1) The set $E=\{x\in X: f(x)=g(x)\}$ is $\theta_\omega-$closed in (X,τ).

(2) $f=g$ on (X,τ) whenever $f=g$ on a $\theta_\omega-$dense set $A\subseteq X$.

Proof. (1) Let $x\in X-E$. Then $f(x)\neq g(x)$. By assumption on the space (Y,σ), there exist open sets V and W in (Y,σ) such that $f(x)\in V$, $g(x)\in W$ and $Cl(V)\cap Cl(W)=\phi$. Since f and g are contra $\theta_\omega-$continuous, $f^{-1}[Cl(V)]$ and $g^{-1}[Cl(W)]$ are $\theta_\omega-$open sets in (X,τ) containing x. Let $U=f^{-1}[Cl(V)]$ and $G=g^{-1}[Cl(W)]$ and set $A=U\cap G$. Then A is $\theta_\omega-$open set in (X,τ) containing x. Now, $f(A)\cap g(A)=f(U\cap G)\cap g(U\cap G)\subseteq$ $f(U)\cap g(G)\subseteq Cl(V)\cap Cl(W)=\phi$. This implies that $A\cap E=\phi$, where A is $\theta_\omega-$open in (X,τ). Hence $x\notin Cl_{\theta_\omega}(E)$. So E $\theta_\omega-$closed in (X,τ).

(2) Let $E=\{x\in X: f(x)=g(x)\}$. Since f and g are contra $\theta_\omega-$continuous and (Y,σ) is Urysohn, by the previous part, E is $\theta_\omega-$closed in (X,τ). By assumption, we have $f=g$ on A, where A is $\theta_\omega-$dense in (X,τ). Since $A\subseteq E$, A is $\theta_\omega-$dense and E is $\theta_\omega-$closed in (X,τ), so $X=Cl_{\theta_\omega}(A)\subseteq Cl_{\theta_\omega}(E)=E$. Hence $f=g$ on (X,τ).

Theorem 9.8. Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\mu)$ be functions, then the following properties hold:

(1) gof is $\theta_\omega-$continuous, if f is contra $\theta_\omega-$continuous and g is contra$-$continuous.

(2) gof is contra $\theta_\omega-$continuous, if f is contra $\theta_\omega-$continuous and g is continuous.

((3) gof is contra $\theta_\omega-$continuous, if f is $\theta_\omega-$irresolute and g is contra $\theta_\omega-$continuous.

Theorem 9.9. let $f:(X,\tau)\to(Y,\sigma)$ be a surjective $\theta_\omega-$irresolute and pre$-\theta_\omega-$open function and $g:(Y,\sigma)\to(Z,\mu)$ be any function. Then $gof:(X,\tau)\to(Z,\mu)$ is contra $\theta_\omega-$continuous if and only if g is contra $\theta_\omega-$continuous.

Proof. Suppose $gof:(X,\tau)\to(Z,\mu)$ is contra $\theta_\omega-$continuous. Let F be a closed set in (Z,μ). Then $f^{-1}[g^{-1}(F)]=(gof)^{-1}(F)$ is $\theta_\omega-$open in (X,τ). Since f is pre$-\theta_\omega-$open and surjective, $g^{-1}(F)=f[f^{-1}(g^{-1}(F))]$ is $\theta_\omega-$open in (Y,σ) and we obtain that g is contra $\theta_\omega-$continuous..

For the converse, suppose g is contra $\theta_\omega-$continuous. Let V be a closed set in (Z,μ). Then $g^{-1}(V)$ is $\theta_\omega-$open in (Y,σ). Since f is $\theta_\omega-$irresolute, $f^{-1}[g^{-1}(V)]=(gof)^{-1}(V)$ is $\theta_\omega-$open in (X,τ) and so gof is a contra $\theta_\omega-$continuous.

Definition 9.10. A space topological (X,τ) is said to be Strongly $\mathrm{S}-$closed if every closed cover of X has a finite cover.

Definition 9.11. A space topological (X,τ) is said to be $\theta_\omega-$compact if every $\theta_\omega-$open cover of X has a finite cover.

Definition 9.12. A subset A of a space (X,τ) is said to be $\theta_\omega-$compact relative to X if for any cover $\{V_\alpha:\alpha\in\mathrm{V}\}$ of A by $\theta_\omega-$open sets of X, there exists a finite subset V_0 of V such that $A\subseteq\bigcup\{V_\alpha:\alpha\in\mathrm{V}_0\}$.

Theorem 9.13. Let $f:(X,\tau)\to(Y,\sigma)$ be contra $\theta_\omega-$continuous surjection.

(1) If A is $\theta_\omega-$compact relative to (X,τ), then $f(A)$ is strongly $\mathrm{S}-$closed in (Y,σ).

(2) If (X,τ) is strongly $S-$closed, then (Y,σ) is compact.

Proof. (1) Let $\{V_\alpha : \alpha \in \nabla\}$ be any cover of $f(A)$ by closed sets of the subspace $f(A)$. For $\alpha \in \nabla$, there exists a closed set A_α of (Y,σ) such that $V_\alpha = A_\alpha \cap f(A)$. For each $x \in A$, there exists $\alpha_x \in \nabla$ such that $f(x) \in A_{\alpha_x}$.
Now by hypothesis f is contra $\theta_\omega-$continuous and hence by Theorem 9.2, there exists a $\theta_\omega-$open set U_x in (X,τ) such that $x \in U$ and $f(U_x) \subseteq A_{\alpha_x}$. Since the family $\{U_x : x \in A\}$ is a cover of A by $\theta_\omega-$open sets of (X,τ), there exists a finite subset A_0 of A such that $A \subseteq \cup\{U_x : x \in A_0\}$. Therefore, $f(A) \subseteq \cup\{f(U_x) : x \in A_0\} \subseteq \cup\{A_{\alpha_x} : x \in A_0\}$. Thus $f(A) = \cup\{V_{\alpha_x} : x \in A_0\}$ and hence $f(A)$ is strongly $S-$closed.
(2) Let $\{V_\alpha : \alpha \in \nabla\}$ be any open cover of Y. Since f is contra $\theta_\omega-$continuous, $\{f^{-1}(V_\alpha) : \alpha \in \nabla\}$ is a $\theta_\omega-$closed cover of the strongly $S-$closed space (X,τ). We have $X = \cup\{f^{-1}(V_\alpha) : \alpha \in \nabla_0\}$ for some finite subset ∇_0 of ∇. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in \nabla_0\}$. This shows that (Y,σ) is compact.

Theorem 9.14. Let $\{(X_\alpha,\tau_\alpha) : \alpha \in \Lambda\}$ be any family of topological spaces. If a function $f : X \to \prod_{\alpha\in\nabla} X_\alpha$ is contra $\theta_\omega-$continuous, then $\pi_\alpha o f : X \to X_\alpha$ is contra $\theta_\omega-$continuous. for each $\alpha \in \Lambda$, where π_α is the projection of $\prod_{\alpha\in\nabla} X_\alpha$ onto X_α.

Proof. For a fixed $\alpha \in \Lambda$, let V_α be any open subset of X_α. Since π_α is continuous, $\pi_\alpha^{-1}(V_\alpha)$ is open in $\prod_{\alpha\in\nabla} X_\alpha$ Since f is contra $\theta_\omega-$continuous, $f^{-1}\left[\pi_\alpha^{-1}(V_\alpha)\right] = (\pi_\alpha o f)^{-1}(V_\alpha)$ is $\theta_\omega-$closed in X. Therefore, $\pi_\alpha o f$ is contra $\theta_\omega-$continuous, for each $\alpha \in \Lambda$.

Definition 9.15. Let (X,τ) be a topological space. Then the $\theta_\omega-$frontier of a subset A of X, denoted by $Fr_{\theta_\omega}(A)$, is defined as $Fr_{\theta_\omega}(A) = \left[Cl_{\theta_\omega}(A)\right] \cap \left[Cl_{\theta_\omega}(X-A)\right]$
$= \left[Cl_{\theta_\omega}(A)\right] - \left[Int_{\theta_\omega}(A)\right]$.

Theorem 9.16. The set of all points x of X at which $f : (X,\tau) \to (Y,\sigma)$ is not contra $\theta_\omega-$continuous is identical with the union of $\theta_\omega-$frontier of the inverse images of closed sets of Y containing $f(x)$.

Proof. ***Necessity:*** Let f be not contra $\theta_\omega-$continuous at a point $x \in X$. Then by Theorem 9.2, there exists a closed set F of Y containing $f(x)$ such that $f(U) \cap (Y-F) \neq \phi$ for every $U \in \theta_\omega O(X,x)$, which implies that $U \cap f^{-1}(Y-F) \neq \phi$. Thus $x \in Cl_{\theta_\omega}\left[f^{-1}(Y-F)\right]$ $= Cl_{\theta_\omega}\left[X - f^{-1}(F)\right]$. Again, since $x \in f^{-1}(F)$, we get $x \in Cl_{\theta_\omega}\left[f^{-1}(F)\right]$ and so it follows that $x \in Fr_{\theta_\omega}\left[f^{-1}(F)\right]$.

Sufficiency: Suppose that $x \in \left(Fr_{\theta_\omega}\left[f^{-1}(F)\right]\right)$ for some closed set F of Y containing $f(x)$ and f is contra $\theta_\omega-$continuous at x. Then there exists $U \in \theta_\omega O(X,x)$ such that $f(U) \subseteq F$. Therefore $x \in U \subseteq f^{-1}(F)$ and hence it follows that $x \in Int_{\theta_\omega}\left[f^{-1}(F)\right] \subseteq X - \left(Fr_{\theta_\omega}\left[f^{-1}(F)\right]\right)$. But this is a contradiction. So f is not contra $\theta_\omega-$continuous at x.

Definition 9.17. A function $f : (X,\tau) \to (Y,\sigma)$ is called almost weakly $\theta_\omega-$continuous, if, for each $x \in X$ and for each open set V of Y containing

$f(x)$, there exists $U \in \theta_\omega O(X,x)$ such that $f(U) \subseteq Cl(V)$.

Theorem 9.18. Suppose that a function $f:(X,\tau) \to (Y,\sigma)$ is contra $\theta_\omega-$continuous. Then f is almost weakly $\theta_\omega-$continuous.

Proof. For any open set V of Y, $Cl(V)$ is closed in Y. Since f is contra $\theta_\omega-$continuous, $f^{-1}[Cl(V)]$ is $\theta_\omega-$open set in X. We take $U = f^{-1}[Cl(V)]$, then $f(U) \subseteq Cl(V)$. Hence f is almost weakly $\theta_\omega-$continuous.

Definition 9.19. A space (X,τ) is said to be $\theta_\omega-$connected provided that X is not the union of two disjoint nonempty $\theta_\omega-$open sets.

Proposition 9.20. Let $f:(X,\tau) \to (Y,\sigma)$ be surjective and contra $\theta_\omega-$continuous. If (X,τ) is $\theta_\omega-$connected, then (Y,σ) is connected.

Proof. Assume that (Y,σ) is not connected. Then, there exist nonempty open sets V_1, V_2 of (Y,σ) such that $V_1 \cap V_2 = \phi$ and $V_1 \cup V_2 = Y$. Hence we have $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$ and $f^{-1}(V_1) \cup f^{-1}(V_2) = X$. Since f is surjective, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are nonempty sets. Since f is contra $\theta_\omega-$continuous and V_1, V_2 are open sets. Hence $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are $\theta_\omega-$open sets in (X,τ). Therefore, (X,τ) is not $\theta_\omega-$connected.

Theorem 9.21. If every contra $\theta_\omega-$continuous function from a space (X,τ) into any T_0-space (Y,σ) is constant, then (X,τ) is $\theta_\omega-$connected.

Proof. Suppose that (X,τ) is not $\theta_\omega-$connected and every contra $\theta_\omega-$continuous function from (X,τ) into any T_0-space (Y,σ) is constant. Since (X,τ) is not $\theta_\omega-$connected, there exists a proper nonempty $\theta_\omega-$open subset A of (X,τ). Let $Y = \{a,b\}$ and $\sigma = \{\phi, Y, \{a\}, \{b\}\}$ be a topology for Y. Let $f:(X,\tau) \to (Y,\sigma)$ be a function such that $f(A) = \{a\}$ and $f(X-A) = \{b\}$.
Then f is not constant and contra $\theta_\omega-$continuous such that (Y,σ) is T_0-space. This is a contradiction. Hence (X,τ) must be $\theta_\omega-$connected.

Definition 9.22. A topological space (X,τ) is said to be $\theta_\omega-T_2$ if for each two distinct points $x, y \in X$, there exist $\theta_\omega-$open sets U and V in (X,τ) such that $x \in U$, $y \in V$ and $U \cap V = \phi$.

Definition 9.23. A topological space (X,τ) is said to be weakly Hausdorff if each element of X is an intersection of regular closed sets.

Definition 9.24. A topological space (X,τ) is said to be ultra Hausdorff if every two distinct points of X can be separated by disjoint clopen sets.

Definition 9.25. A topological space (X,τ) is said to be ultra normal $(\text{resp. } \theta_\omega-\text{normal})$ if each pair of non-empty disjoint closed sets can be separated by disjoint clopen $(\text{resp. } \theta_\omega-\text{open})$ sets.

Theorem 9.26. Let $f:(X,\tau) \to (Y,\sigma)$ be a contra $\theta_\omega-$continuous injection, then the following properties hold:

(1) (X,τ) is $\theta_\omega-T_1$ if (Y,σ) is weakly Hausdorff.

(2) (X,τ) is $\theta_\omega-T_2$ if (Y,σ) is a Urysohn space or ultra Hausdorff.

(3) (X,τ) is $\theta_\omega-$normal if (Y,σ) is ultra normal and f is closed.

Proof. (1) Suppose that (Y,σ) is weakly Hausdorff. For any distinct points x and y in (X,τ), there exist regular closed sets A, B in (Y,σ) such that $f(x) \in A$, $f(y) \notin A$, $f(x) \notin B$ and $f(y) \in B$. Since f is contra $\theta_\omega-$continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are $\theta_\omega-$open sets in (X,τ) such that $x \in f^{-1}(A)$, $y \notin f^{-1}(A)$, $x \notin f^{-1}(B)$ and $y \in f^{-1}(B)$. This shows that (X,τ) is $\theta_\omega-T_1$.

(2) Let x_1 and x_2 be any distinct points in X. Then, since f is injective, $f(x_1) \neq f(x_2)$. Moreover, since (Y,σ) is ultra-Hausdorff, there exist clopen sets V_1, V_2 such that $f(x_1) \in V_1$, $f(x_2) \in V_2$ and $V_1 \cap V_2 = \phi$. Since f is contra $\theta_\omega-$continuous. So there exists $U_i \in \theta_\omega O(X,\tau)$ containing x_i such that $f(U_i) \subseteq V_i$ for $i = 1,2$. Clearly, we obtain $U_1 \cap U_2 = \phi$. Thus (X,τ) is $\theta_\omega - T_2$.

In case (Y,σ) is Urysohn space, there here exist open sets U_1, U_2 such that $f(x_1) \in U_1$, $f(x_2) \in U_2$ and $Cl(U_1) \cap Cl(U_2) = \phi$. Let $G = f^{-1}[Cl(U_1)]$ and $H = f^{-1}[Cl(U_2)]$. Then $x_1 \in G$, $x_2 \in H$ and $G \cap H = \phi$. Since f is contra $\theta_\omega-$continuous. Therefore G and H are $\theta_\omega-$open sets in (X,τ). Thus (X,τ) is $\theta_\omega - T_2$.

(3) Let F_1 and F_2 be disjoint closed subsets of (Y,σ). Since f is closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of (Y,σ). Since (Y,σ) is ultra normal, $f(F_1)$ and $f(F_2)$ are separated by disjoint clopen sets V_1 and V_2, respectively. Since f is contra $\theta_\omega-$continuous, $F_i \subseteq f^{-1}(V_i)$ and $f^{-1}(V_i)$ is $\theta_\omega-$open in (X,τ) for $i = 1,2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus (X,τ) is $\theta_\omega-$normal.

Theorem 9.27. Let (X,τ) be a topological space. If for each pair of distinct points x_1 and x_2 in X there exists a function f of (X,τ) into a Urysohn space (Y,σ) such that $f(x_1) \neq f(x_2)$ and f is contra $\theta_\omega-$continuous at x_1 and x_2, then (X,τ) is $\theta_\omega - T_2$.

Proof. Let x and y be any two distinct points of X. Then by the hypothesis, there exist a Urysohn space (Y,σ) and a function $f:(X,\tau) \to (Y,\sigma)$ which satisfies the condition of the theorem. Let $y_i = f(x_i)$ for $i = 1,2$. Then $y_1 \neq y_2$. Since Y is Urysohn, there exist open sets U and V containing y_1 and y_2, respectively, such that $Cl(U) \cap Cl(V) = \phi$. Since f is contra $\theta_\omega-$continuous at x_1 and x_2, so there exists $\theta_\omega-$open sets G and H in (X,τ) containing x_1 and x_2, respectively, such that $f(G) \subseteq Cl(U)$ and $f(H) \subseteq Cl(V)$. Hence we obtain $G \cap H = \phi$. Therefore, (X,τ) is $\theta_\omega - T_2$.

Definition 9.28. .A function $f:(X,\tau) \to (Y,\sigma)$ is called almost contra $\theta_\omega-$continuous if $f^{-1}(V)$ is $\theta_\omega-$closed for every regular open set V of Y.

Theorem 9.29. Let $f:(X,\tau) \to (Y,\sigma)$ be a function. Then the following statements are equivalent:

(a) f is almost contra $\theta_\omega-$continuous

(b) $f^{-1}(F)$ is $\theta_\omega-$open in X for every regular closed set F of Y.

(c) for each $x \in X$ and each regular open set F of Y containing $f(x)$, there exists $U \in \theta_\omega O(X)$ such that $x \in U$ and $f(U) \subseteq F$.

(d) for each $x \in X$ and each regular open set V of Y non-containing $f(x)$, there exists a $\theta_\omega-$closed set K of X non-containing x such that $f^{-1}(V) \subseteq K$.

Proof. $(a) \Leftrightarrow (b)$: Let F be any regular closed set of Y. Then $(Y-F)$ is regular open and therefore $f^{-1}(Y-F) = X - f^{-1}(F) \in \theta_\omega C(X)$. Hence, $f^{-1}(F) \in \theta_\omega O(X)$. The converse part is obvious.

$(b) \Rightarrow (c)$: Let F be any regular closed set of Y containing $f(x)$. Then $f^{-1}(F) \in \theta_\omega O(X)$ and $x \in f^{-1}(F)$. Taking $U = f^{-1}(F)$ we get $f(U) \subseteq F$.

$(c) \Rightarrow (b)$: Let F be any regular closed set of Y and $x \in f^{-1}(F)$. Then, there exists

$U_x \in \theta_\omega O(X, x)$ such that $f(U_x) \subseteq F$ and so $U_x \subseteq f^{-1}(F)$. Also, we have $f^{-1}(F) = \bigcup_{x \in f^{-1}(F)} U_x$. Hence $f^{-1}(F) \in \theta_\omega O(X)$.

$(c) \Rightarrow (d)$: Let V be any regular open set of Y non-containing $f(x)$. Then $(Y - V)$ is regular closed set of Y containing $f(x)$. Hence by (c), there exists $U \in \theta_\omega O(X, x)$ such that $f(U) \subseteq (Y - V)$. Hence, we obtain $U \subseteq f^{-1}(Y - V) \subseteq X - f^{-1}(V)$ and so $f^{-1}(V) \subseteq (X - U)$. Now, since $U \in \theta_\omega O(X)$, $(X - U)$ is $\theta_\omega -$closed set of X not containing x. The converse part is obvious.

Theorem 9.30. Let $f:(X,\tau) \to (Y,\sigma)$ be almost contra $\theta_\omega -$continuous. Then f is almost weakly $\theta_\omega -$continuous.

Proof. For $x \in X$, let H be any open set of Y containing $f(x)$. Then $Cl(H)$ is a regular closed set of Y containing $f(x)$. Then by Theorem 9.29, there exists $G \in \theta_\omega O(X, x)$ such that $f(G) \subseteq Cl(H)$. So f is almost weakly $\theta_\omega -$continuous.

Theorem 9.31. Let $f:(X,\tau) \to (Y,\sigma)$ be an almost contra $\theta_\omega -$continuous injection and Y is weakly Hausdorff. Then X is $\theta_\omega - T_1$.

Proof. Since Y is weakly Hausdorff, for distinct points x, y of Y, there exist regular closed sets U and V such that $f(x) \in U$, $f(y) \notin U$ and $f(y) \in V$, $f(x) \notin V$. Now, f being almost contra $\theta_\omega -$continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are $\theta_\omega -$open subsets of X such that $x \in f^{-1}(U)$, $y \notin f^{-1}(U)$ and $y \in f^{-1}(V)$, $x \notin f^{-1}(V)$. This shows that X is $\theta_\omega - T_1$.

Corollary 9.32. If $f:(X,\tau) \to (Y,\sigma)$ is a contra $\theta_\omega -$continuous injection and Y is weakly Hausdorff, then X is $Bc - T_1$.

Theorem 9.33. Let $f:(X,\tau) \to (Y,\sigma)$ be an almost contra $\theta_\omega -$continuous surjection and X be $\theta_\omega -$connected. Then Y is connected.

Proof. If possible, suppose that Y is not connected. Then there exist disjoint non-empty open sets U and V of Y such that $Y = U \cup V$. Since U and V are clopen sets in Y, they are regular open sets of Y. Again, since f is almost contra $\theta_\omega -$continuous surjection, $f^{-1}(U)$ and $f^{-1}(V)$ are $\theta_\omega -$open sets of X and $X = f^{-1}(U) \cup f^{-1}(V)$. This shows that X is not $\theta_\omega -$connected. But this is a contradiction. Hence Y is connected.

Definition 9.34. A topological space (X,τ) is said to be countably $\theta_\omega -$compact if every countable cover of X by $\theta_\omega -$open sets has a finite subcover.

Definition 9.35. A topological space (X, τ) is said to be $\theta_\omega -$Lindelof if every $\theta_\omega -$open cover of X has a countable subcover.

Theorem 9.36. Let $f:(X,\tau) \to (Y,\sigma)$ be an almost contra $\theta_\omega -$continuous surjection. Then the following statements hold:

(a) If X is $\theta_\omega -$compact, then Y is $S -$closed.

(b) If X is $\theta_\omega -$Lindelof, then Y is $S -$Lindelof.

(c) If X is countably $\theta_\omega -$compact, then Y is countably $S -$closed.

Proof. (a): Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Since f is almost contra $\theta_\omega-$continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a $\theta_\omega-$open cover of X. Again, since X is $\theta_\omega-$compact, there exist a finite subset I_0 of I such that $X = \bigcup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$ and hence $Y = \{V_\alpha : \alpha \in I_0\}$. Therefore, Y is $S-$closed.

The proofs of (b) and (c) are being similar to (a): omitted.

Definition 9.37. A topological space (X, τ) is said to be $\theta_\omega-$closed compact if every $\theta_\omega-$closed cover of X has a finite subcover.

Definition 9.38. A topological space (X, τ) is said to be countably $\theta_\omega-$closed if every countable cover of X by $\theta_\omega-$closed sets has a finite subcover.

Definition 9.39. A topological space (X, τ) is said to be $\theta_\omega-$closed Lindelof if every $\theta_\omega-$closed cover of X has a countable subcover.

Theorem 9.40. Let $f : (X, \tau) \to (Y, \sigma)$ be an almost contra $\theta_\omega-$continuous surjection. Then the following statements hold:

(a) If X is $\theta_\omega-$closed compact, then Y is nearly compact.

(b) If X is $\theta_\omega-$closed Lindelof, then Y is nearly Lindeloff.

(c) If X is countably $\theta_\omega-$closed compact, then Y is nearly countable compact.

Proof. (a): Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is almost contra $\theta_\omega-$continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a $\theta_\omega-$closed cover of X. Again, since X is $\theta_\omega-$closed compact, there exists a finite subset I_0 of I such that $X = \bigcup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$ and hence $Y = \{V_\alpha : \alpha \in I_0\}$. Therefore, Y is nearly compact.

The proofs of (b) and (c) are being similar to (a): omitted.

10 Conclusion

Sets and functions in topological spaces are developed and used in many engineering problems, information systems and computational topology. By researching generalizations of closed sets, some new separation axioms and compact spaces have founded and are turned to be useful in the study of digital topology. In this paper we have introduced $\theta_\omega-$continuous, $\theta_\omega-$irresolute, $\theta_\omega-$open, $\theta_\omega-$closed, pre$-\theta_\omega-$open, pre$-\theta_\omega-$closed, contra $\theta_\omega-$continuous and almost contra $\theta_\omega-$ mappings and have investigated properties and characterizations of these new types of mappings in topological spaces. We have studied new types of functions using $\theta_\omega-$open sets and these functions will have many possibilities of applications in computer graphics and digital topology.

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