Natural Decomposition Method for Analytical solutions of Linear and Nonlinear Newell-Whitehead-Segel Model

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Abstract: - In this paper, natural transform combined with decomposition method is applied to the Newell-Whitehead-Segel model for analytical solutions. For the purpose of illustration, examples on linear and nonlinear are considered. The results show efficiency, reliability, and simplicity of the proposed method. Hence, it is recommended for highly nonlinear differential models and systems.

Key-Words: - Analytical solutions; Decomposition Method; Newell-Whitehead-Segel model

1 Introduction

Modeling real life situations deals with the applications of differential equations either in the form of linear, nonlinear, ordinary, or partial. In most cases, these nonlinear versions are hard to solve in terms of analytical or exact solutions (despite their immense roles). As such, there is need for reliable and efficient approximate or semi-analytical methods or schemes such as the decomposition, and integral transform methods [1-15]. In this present work, consideration will be on one of the vital models referred to as Newell-Whitehead-Segel model equation (NWSME) whose general form is given as:

$$\begin{cases} \psi_t(x,t) = \begin{Bmatrix} a\psi_{xx}(x,t) + b\psi(x,t) \\ -c\psi^{\kappa}(x,t), \end{Bmatrix} \\ \psi(x,0) = h(x), \end{cases} \quad (1)$$

where $b,c \in \mathbb{R}$, and $a,\kappa \in \mathbb{Z}^+$.

The NWSME has found applications in different areas of pure and applied sciences: nonlinear optics, biological systems, Rayleigh-Benard convection, and so on. In recent times, the NWSME in (1) has attracted the attention of many researchers in terms of solution methods [16-21]. Other methods for solving differential models include of [22-25]. This present work aims at combining natural transform method with decomposition method for analytical solutions of Newell-Whitehead-Segel Model of any form (linear and nonlinear).

2 Natural Transform and Natural Decomposition Method

The preliminaries of Natural Transform (NT), and its basic properties are given in this section [26-28]. Let H be a class of functions such that:

$$H = \left\{ g(t) : \exists\, c, k_1, k_2 > 0 \ni \left| g(t) \right| < ce^{|t| k_i} \right\},$$

then, the natural transform of $g(t)$ is defined and denoted as:

$$\mathrm{N}\left[g(t) \right] = Q(s,\varphi) = \int_0^{\infty} g(\varphi t) e^{-st} dt,\ t \in [0,\infty) \quad (2)$$

provided the integral in the left hand side exists. As a consequence, the Inverse Natural Transform (INT) associated with (3) is defined and denoted as:

$$\mathrm{N}^{-1}\left\{ \mathrm{N}\left[g(t) \right] \right\} = \mathrm{N}^{-1}\left\{ Q(s,\varphi) \right\} = g(t). \quad (3)$$

2.1.1 Properties of Natural Transform

Considering $\mathrm{N}[\cdot]$, some of the properties associated with the NT are noted in [P1-P6] as follows:

P1 $\mathrm{N}[1] = \dfrac{1}{s}$

P2 $\mathrm{N}[t]=\dfrac{\varphi}{s^2}$

P3 $\mathrm{N}\left[t^n\right]=\dfrac{n!\varphi^n}{s^{n+1}},\ n\geq 0$

P4 $\mathrm{N}\left[e^{\vartheta t}\right]=\dfrac{1}{s-\vartheta\varphi}$

P5 $\mathrm{N}\left[\cos(\vartheta t)\right]=\dfrac{s}{s^2+(\vartheta\varphi)^2}$

P6 $\mathrm{N}\left[\sin(\vartheta t)\right]=\dfrac{\vartheta\varphi}{s^2+(\vartheta\varphi)^2}$

2.1.2 Natural Transform of Derivatives

For a continuous function, $g(x,t)$ in A as defined earlier, we have the following:

D1 $$\begin{cases}\mathrm{N}[g_t]=\dfrac{s}{\varphi}Q(s,\varphi)-\dfrac{g(x,0)}{\varphi}\\ \mathrm{N}\left[\dfrac{\partial^n g}{\partial t^n}\right]=\dfrac{s^n}{\varphi^n}Q(s,\varphi)-\displaystyle\sum_{j=0}^{n-1}\frac{s^{n-j-1}}{\varphi^{n-j}}g^{(j)}(x,0)\end{cases}$$

D2 $$\mathrm{N}\left[\frac{\partial^n g}{\partial x^n}\right]=\frac{\partial^n}{\partial x^n}\left[Q(s,\varphi)\right]$$

2.1.3. Natural Decomposition Method

Let a general nonlinear nonhomogeneous partial differential equation be defined as:

$$\begin{cases}Dp(x,t)+Rp(x,t)+\Xi p(x,t)=h(x,t)\\ p(x,0)=m(x)\end{cases}\tag{4}$$

where D is a first order differential operator in t, R is the remaining part of the linear differential operator, Ξ and $h(x,t)$ are nonlinear differential operator and source term respectively.

So taking the natural transform of (4) gives:

$$N\left[Dp(x,t)\right]=\begin{Bmatrix}N\left[h(x,t)\right]\\ -N\left[\begin{pmatrix}Rp(x,t)\\ +Np(x,t)\end{pmatrix}\right]\end{Bmatrix}.\tag{5}$$

$$\Rightarrow\ \frac{s}{\varphi}Q(s,\varphi)-\frac{p(x,0)}{\varphi}=\begin{Bmatrix}N\left[h(x,t)\right]\\ -N\left[\begin{pmatrix}Rp(x,t)\\ +\Xi p(x,t)\end{pmatrix}\right]\end{Bmatrix}\tag{6}$$

Showing that:

$$Q(s,\varphi)=\begin{pmatrix}\dfrac{m(x)}{s}+\dfrac{\varphi}{s}N\left[h(x,t)\right]\\ -\dfrac{\varphi}{s}N\left[\left(Rp(x,t)+\Xi p(x,t)\right)\right]\end{pmatrix}.\tag{7}$$

So, taking the Inverse Natural Transform (INT) $L_t^{-1}(\cdot)$ on both sides of (7) gives:

$$\begin{cases}p(x,t)=G(x,t)-N^{-1}\left\{\dfrac{\varphi}{s}N\left[\begin{pmatrix}Rp(x,t)\\ +\Xi p(x,t)\end{pmatrix}\right]\right\},\\ G(x,t)=N^{-1}\left\{\dfrac{m(x)}{s}+\dfrac{\varphi}{s}N\left[h(x,t)\right]\right\}.\end{cases}\tag{8}$$

Suppose the solution and the nonlinear term are expressed as follows according to Adomian and its polynomial:

$$\begin{cases}p(x,t)=\displaystyle\sum_{n=0}^{\infty}p_n(x,t),\\ \Xi p(x,t)=\displaystyle\sum_{n=0}^{\infty}A_n.\end{cases}\tag{9}$$

and A_n defined as:

$$A_n=\frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}\left[N\left(\sum_{i=0}^{n}\lambda^i p_i\right)\right]_{\lambda=0}.\tag{10}$$

Thus, (8) becomes:

$$\sum_{n=0}^{\infty}p_n(x,t)=\begin{Bmatrix}G(x,t)\}\\ -N^{-1}\left\{\dfrac{\varphi}{s}N\left[\begin{pmatrix}R\left(\displaystyle\sum_{n=0}^{\infty}p_n(x,t)\right)\\ +\left(\displaystyle\sum_{n=0}^{\infty}A_n\right)\end{pmatrix}\right]\right\}\end{Bmatrix}.\tag{11}$$

Therefore, the solution $p(x,t)$ is determined via the recursive relation:

$$\begin{cases}p_0=\widetilde{N}^{-1}\left\{\dfrac{m(x)}{s}+\dfrac{\varphi}{s}\widetilde{N}\left[h(x,t)\right]\right\}\\ p_{n+1}=-\widetilde{N}^{-1}\left\{\dfrac{\varphi}{s}\widetilde{N}\left[\left(R(p_n)+A_n\right)\right]\right\},n\geq 0.\end{cases}\tag{12}$$

Whence, $p(x,t)$ is finalized as:

$$p(x,t)=\lim_{N\to\infty}\sum_{n=0}^{N}p_n.\tag{13}$$

3 Examples

Case 1: Let the following linear NWSE [1, 21] be considered:

$$\begin{cases} \psi_t - \psi_{xx} + 3\psi = 0, \\ \psi(x,0) = e^{2x}, \end{cases} \tag{14}$$

with an exact solution:

$$\psi(x,t) = e^{2x+t}. \tag{15}$$

Procedure w.r.t Case 1:

By applying the N-transform to (14), we have:

$$N[\psi_t] = N[\psi_{xx} - 3\psi]. \tag{16}$$

$$\Rightarrow \frac{s}{\omega} R(s,\omega) - \frac{\psi(s,0)}{\omega} = N[\psi_{xx} - 3\psi]. \tag{17}$$

$$\therefore\ R(s,\omega) = \frac{\psi(s,0)}{s} + \frac{\omega}{s} N[\psi_{xx} - 3\psi]. \tag{18}$$

Applying the N-inverse, $N^{-1}[\cdot]$ and the initial condition to (18) gives:

$$\begin{aligned} N^{-1}\{R(s,\omega)\} &= N^{-1}\left\{\frac{e^{2x}}{s}\right\} + N^{-1}\left\{\frac{\omega}{s} N[\psi_{xx} - 3\psi]\right\} \\ &= e^{2x} N^{-1}\left\{\frac{1}{s}\right\} + N^{-1}\left\{\frac{\omega}{s} N[\psi_{xx} - 3\psi]\right\}. \end{aligned} \tag{19}$$

That implies that:

$$\psi(x,t) = e^{2x} + \widetilde{N}^{-1}\left\{\frac{\omega}{s} \widetilde{N}[\psi_{xx} - 3\psi]\right\}. \tag{20}$$

Expressing the solution in series (Adomian) form:

$$\psi(x,t) = \psi = \sum_{n=0}^{\infty} \psi_n \tag{21}$$

applied to (19) gives:

$$\sum_{n=0}^{\infty} \psi_n = e^{2x} + N^{-1}\left\{\frac{\omega}{s} N\left[\sum_{n=0}^{\infty} (\psi_n)_{xx} - 3\sum_{n=0}^{\infty} (\psi_n)\right]\right\}. \tag{22}$$

Hence, the recursive relation:

$$\begin{cases} \psi_0 = e^{2x} \\ \psi_{n+1} = \widetilde{N}^{-1}\left\{\frac{\omega}{s} \widetilde{N}\left[(\psi_n)_{xx} - 3(\psi_n)\right]\right\}. \end{cases} \tag{23}$$

As such, for $n \geq 1$, we have the following:

$$\begin{aligned} \psi_1 &= \widetilde{N}^{-1}\left\{\frac{\omega}{s} \widetilde{N}\left[(\psi_0)_{xx} - 3(\psi_0)\right]\right\} \\ &= \widetilde{N}^{-1}\left\{\frac{\omega}{s} \widetilde{N}\left[\left(e^{2x}\right)_{xx} - 3\left(e^{2x}\right)\right]\right\} \\ &= \widetilde{N}^{-1}\left\{\frac{\omega}{s} \widetilde{N}\left[e^{2x}\right]\right\} \\ &= e^{2x} \widetilde{N}^{-1}\left\{\frac{\omega}{s} \widetilde{N}[1]\right\} \\ &= e^{2x} \widetilde{N}^{-1}\left\{\frac{\omega}{s^2}\right\} = te^{2x} \end{aligned}$$

$$\begin{aligned} \psi_2 &= \widetilde{N}^{-1}\left\{\frac{\omega}{s} \widetilde{N}\left[(\psi_1)_{xx} - 3(\psi_1)\right]\right\} \\ &= \widetilde{N}^{-1}\left\{\frac{\omega}{s} \widetilde{N}\left[\left(te^{2x}\right)_{xx} - 3\left(te^{2x}\right)\right]\right\} \\ &= e^{2x} \widetilde{N}^{-1}\left\{\frac{\omega}{s} \widetilde{N}[t]\right\} \\ &= e^{2x} \widetilde{N}^{-1}\left\{\omega^2 s^{-3}\right\} \\ &= \frac{t^2}{2!} e^{2x} \end{aligned}$$

$$\begin{aligned} \psi_3 &= \widetilde{N}^{-1}\left\{\frac{\omega}{s} \widetilde{N}\left[(\psi_2)_{xx} - 3(\psi_2)\right]\right\} \\ &= e^{2x} \widetilde{N}^{-1}\left\{\omega^3 s^{-4}\right\} \\ &= \frac{t^3}{3!} e^{2x} \end{aligned}$$

$$\begin{aligned} \psi_4 &= \widetilde{N}^{-1}\left\{\frac{\omega}{s} \widetilde{N}\left[(\psi_3)_{xx} - 3(\psi_3)\right]\right\} \\ &= e^{2x} \widetilde{N}^{-1}\left\{\omega^4 s^{-5}\right\} \\ &= \frac{t^4}{4!} e^{2x} \\ &\vdots \end{aligned}$$

$$\begin{aligned} \psi_k &= \widetilde{N}^{-1}\left\{\frac{\omega}{s} \widetilde{N}\left[(\psi_{k-1})_{xx} - 3(\psi_{k-1})\right]\right\} \\ &= e^{2x} \widetilde{N}^{-1}\left\{\omega^k s^{k-1}\right\} \\ &= \frac{t^k}{k!} e^{2x}. \end{aligned}$$

Hence,

$$\psi(x,t) = \lim_{M\to\infty}\sum_{n=0}^{M}\psi_n = e^{2x}\sum_{n=0}^{\infty}\frac{t^k}{k!} = e^{2x+t}. \tag{24}$$

Equation (24) corresponds to the exact solution of the classical NWSE obtained in [1, 21].

Case 2: Consider the following nonlinear NWSE [1, 21]:

$$\begin{cases}\psi_t = 5\psi_{xx} + 2\psi + \psi^2,\\ \psi(x,0) = \lambda,\end{cases} \tag{25}$$

Procedure w.r.t Case 2:

By applying the N-transform to (25), we have:

$$N[\psi_t] = N\left[5\psi_{xx} + 2\psi + \psi^2\right]. \tag{26}$$

$$\Rightarrow \frac{s}{\omega}R(s,\omega) - \frac{\psi(s,0)}{\omega} = \begin{Bmatrix}5N[\psi_{xx}]\\ +2N[\psi] + N\left[\psi^2\right]\end{Bmatrix}. \tag{27}$$

$$\Rightarrow \underbrace{R(s,\omega)}_{N[\psi]} = \frac{\lambda}{s} + \frac{\omega}{s}\left\{5N[\psi_{xx}] + 2N[\psi] + N\left[\psi^2\right]\right\}. \tag{28}$$

$$\Rightarrow N[\psi] - \frac{2\omega}{s}N[\psi] = \frac{\lambda}{s} + \frac{\omega}{s}\left\{5N[\psi_{xx}] + N\left[\psi^2\right]\right\}. \tag{29}$$

Thus,

$$\left(\frac{s-2\omega}{s}\right)N[\psi] = \frac{\lambda}{s} + \frac{\omega}{s}\left\{5N[\psi_{xx}] + N\left[\psi^2\right]\right\}. \tag{30}$$

$$\therefore\ N[\psi] = \left(\frac{1}{s-2\omega}\right)\left\{\lambda + \omega\left(5N[\psi_{xx}] + N\left[\psi^2\right]\right)\right\}. \tag{31}$$

Applying the N-inverse, $N^{-1}[\cdot]$ to (31) gives:

$$N^{-1}\{N[\psi]\} = \left\{\begin{matrix}\lambda N^{-1}\left\{\dfrac{1}{s-2\omega}\right\}\\ +N^{-1}\left\{\left(\begin{matrix}\dfrac{5\omega}{s-2\omega}N[\psi_{xx}]\\ +\dfrac{\omega}{s-2\omega}N\left[\psi^2\right]\end{matrix}\right)\right\}\end{matrix}\right\}. \tag{32}$$

Expressing the solution in series via (21), produces (32) as:

$$\begin{cases}\displaystyle\sum_{n=0}^{\infty}\psi_n = \lambda e^{2t} + N^{-1}\left\{\left(\begin{matrix}\dfrac{5\omega}{s-2\omega}N\left[\displaystyle\sum_{n=0}^{\infty}(\psi_n)_{xx}\right]\\ +\dfrac{\omega}{s-2\omega}N\left[\displaystyle\sum_{n=0}^{\infty}A_n\right]\end{matrix}\right)\right\},\\ \psi = \displaystyle\sum_{n=0}^{\infty}\psi_n,\ \psi^2 = \sum_{n=0}^{\infty}A_n\ .\end{cases} \tag{33}$$

Thus, the corresponding recursive relation is:

$$\begin{cases}\psi_0 = \lambda e^{2t}\\ \psi_{n+1} = \widetilde{N}^{-1}\left\{\left(\begin{matrix}\dfrac{5\omega}{s-2\omega}\widetilde{N}\left[(\psi_n)_{xx}\right]\\ +\dfrac{\omega}{s-2\omega}\widetilde{N}[A_n]\end{matrix}\right)\right\}, n\ge 1,\end{cases} \tag{34}$$

where the first few Adomian polynomials are:

$$\begin{cases}A_0 = \psi_1^2\ ,\\ A_1 = 2\psi_0\psi_1\ ,\\ A_2 = 2\psi_0\psi_2 + \psi_1^2\ ,\\ A_3 = 2\psi_0\psi_3 + 2\psi_1\psi_2\ ,\\ \vdots\end{cases}$$

As such,

$$\begin{aligned}\psi_1 &= N^{-1}\left\{\left(\frac{5\omega}{s-2\omega}N\left[(\psi_0)_{xx}\right] + \frac{\omega}{s-2\omega}N[A_0]\right)\right\}\\ &= \lambda^2 N^{-1}\left\{\left(\frac{\omega}{s-2\omega}N\left[e^{4t}\right]\right)\right\}\\ &= \lambda^2 N^{-1}\left\{\left(\frac{\omega}{(s-2\omega)(s-4\omega)}\right)\right\}\\ &= \frac{\lambda^2}{2}N^{-1}\left\{\left(\frac{1}{(s-4\omega)} - \frac{1}{(s-2\omega)}\right)\right\}\\ &= \frac{\lambda^2 e^{2t}}{2}\left(e^{2t}-1\right)\end{aligned}$$

$$\psi_2 = N^{-1}\left\{\left(\frac{5\omega}{s-2\omega}N\left[(\psi_1)_{xx}\right]+\frac{\omega}{s-2\omega}N\left[A_1\right]\right)\right\}$$
$$= N^{-1}\left\{\left(\frac{\omega}{s-2\omega}N\left[\left(2\left(\lambda e^{2t}\right)\left(\frac{\lambda^2 e^{2t}}{2}\left(e^{2t}-1\right)\right)\right)\right]\right)\right\}$$
$$= \lambda^3 N^{-1}\left\{\left(\frac{\omega}{s-2\omega}N\left[e^{6t}-e^{4t}\right]\right)\right\}$$
$$= \lambda^3 N^{-1}\left\{\left(\frac{\omega}{s-2\omega}\left(\frac{1}{s-6\omega}-\frac{1}{s-4\omega}\right)\right)\right\}$$
$$= 2\lambda^3 N^{-1}\left\{\left(\frac{\omega^2}{(s-2\omega)(s-4\omega)(s-6\omega)}\right)\right\}$$
$$= \frac{1}{4}\lambda^3\left(e^{6t}+e^{2t}-2e^{4t}\right)$$
$$= \frac{1}{4}\lambda^3\left(e^{2t}-1\right)^2 e^{2t}$$

In a similar manner,

$$\psi_3 = \frac{1}{8}\lambda^4\left(e^{2t}-1\right)^3 e^{2t},$$
$$\psi_4 = \frac{1}{16}\lambda^5\left(e^{2t}-1\right)^4 e^{2t},$$
$$\psi_5 = \frac{1}{32}\lambda^6\left(e^{2t}-1\right)^5 e^{2t},$$
$$\vdots$$

In general, we have:

$$\psi_k = \frac{\lambda^{k+1}}{(2)^k}\left(e^{2t}-1\right)^k e^{2t},\ k \geq 0. \tag{35}$$

$$\therefore\ \psi(x,t) = \lim_{M\to\infty}\sum_{k=0}^{M}\psi_k$$
$$= \left(\lambda + \frac{\lambda^2}{2}\left(e^{2t}-1\right) + \frac{\lambda^3}{4}\left(e^{2t}-1\right)^2 + \cdots\right)e^{2t}. \tag{36}$$

The bracketed expression in (36) is a geometric series with first term, $a=\lambda$ and a common ratio, $r=\frac{\lambda}{2}\left(e^{2t}-1\right)$. Hence,

$$\psi(x,t) = \left(\frac{2\lambda e^{2t}}{2+\lambda\left(1-e^{2t}\right)}\right). \tag{37}$$

The graphical solutions are presented in Fig. 1 and Fig.2 for *Case* 1 and *Case* 2 respectively.

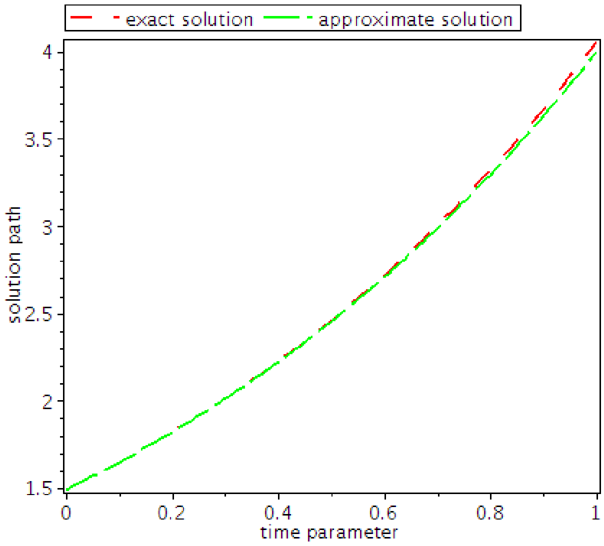

Figure1: NDM Exact and Approximate solution of Case 1

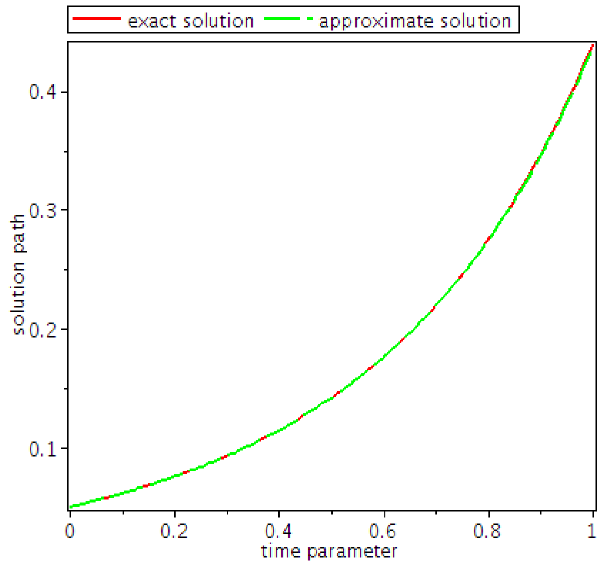

Figure 2: NDM Exact and Approximate solutions of *Case* 2 at $\eta = 0.5$

4 Concluding Remarks

This work has successfully applied Natural Decomposition Method (NDM) to the Newell-Whitehead-Segel Model for analytical solutions by considering both linear and nonlinear forms of the NWSME. The results showed that the NDM is efficient, effective and reliable. The solutions were expressed in closed form with less computational time involvement. The NDM can be extended to highly nonlinear NWSME and other related differential models.

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