Common fixed point theorems for Kannan - Type multiplicative contractive maps in fuzzy G-partial metric spaces

KANAYO STELLA EKE
Department of Mathematics
Covenant University
KM 10 Idiroko Road, P. M. B. 1023, Ota, Ogun State
Nigeria
kanayo.eke@covenantuniversity.edu.ng

BIJAN DAVVAZ
Department of Mathematics
Yazd University
P. O. Box 89195-741,Yazd
Iran
davvaz"@ as you know".yazd.ac.ir

JIMEVWO GODWIN OGHONYON
Department of Mathematics
Covenant University
KM 10 Idiroko Road, P. M. B. 1023, Ota, Ogun State
Nigeria
godwin.oghonyon@covenantuniversity.edu.ng

Abstract: - In this paper, we introduce the classes of Kannan-type multiplicative contractive mappings into fuzzy G-partial metric spaces. Thereafter, we prove some common fixed point theorems for Kannan - type multiplicative contractive maps satisfying sequential convergent conditions in fuzzy G-partial metric space. Our results extend and generalize some well known results in the literature.

Key-Words: - fixed points, Kannan- type multiplicative contractive maps, sequential convergence, fuzzy set, fuzzy G-partial metric space.

1 Introduction

Metrical fixed point theory deals with existence and uniqueness of the solution of functional equations in analysis and applied mathematics. The existence and uniqueness of fixed point of Banach contraction principle in metric space were first proved by Banach [1] in 1922. This result was generalized by many authors in the literature. In particular, Kannan [2] introduced a contractive mapping and proved the existence and uniqueness of the fixed point of the map without the continuity of the map. Moradi and Alimohammadi [3] proved the existence and uniqueness of the fixed point of Kannan -type contractive mappings in complete metric spaces by employing the concept of sequential convergence. In the extension of the result of Moradi and Alimohammmadi[3], Malceski et al. [4] proved the common fixed points of Kannan and Chatterjea type contractive mappings satisfying a sequential convergent condition in complete metric spaces. Eke [5] proved some common fixed point results for weakly compatible mappings in uniform spaces by employing the concept of A-distance and E-distance as well as comparison functions.

Recently, Ozavsar and Cevikel [6] generalized Banach contraction map by introducing multiplicative contraction mappings . He et al. [7] proved the existence and uniqueness of common fixed point of two pairs of multiplicative contractive mappings satisfying a commutative condition in multplicative metric spaces. For the definition and topology of multiplicative metric spaces, we refer the reader to [6].

Metric space has several generalizations. Recently, Eke et al. [8] introduced the concept of fuzzy G-partial metric space by assigning every quadruplet of an arbitrary set to [0, 1]. In the same paper, the topology of fuzzy G-partial metric is established and the existence and uniqueness of fixed point of Hardy and Rogers contractive maps in this space are proved. Motivated by the results of Moradi and Alimohammadi [3] and He et al. [7], we established our results.

The aim of this paper, is to prove the existence and uniqueness of the common fixed point of a pair of Kannan -type multiplicative contractive mappings satisfying the sequential convergent condition in fuzzy G-partial metric spaces. Our results are extensions of some results in the literature.

2 Fuzzy G-partial Metric Spaces

According to Eke and Davvaz [7], we give the definition of fuzzy G-partial metric space and its motivations.

Definition 2.1 : A 3-tuple $\left(X,\ G_{pf},\ *\right)$ is called a fuzzy G-partial metric space if X is an arbitrary (nonempty) set, * is a continuous t-norm, and G_{pf} is fuzzy set on $X^3\times(0,\infty)$, satisfying the following conditions for each $x,\ y,\ z,\ a\ \in X$ and $t,\ s\ >\ 0$;

G_{pf1} $G_{pf}\left(x,\ y,\ z,\ t\right)=G_{pf}\left(x,\ x,\ y,\ t\right)$ $=G_{pf}\left(y,\ y,\ z,\ t\right)=G_{pf}\left(\ z,\ z,\ x,\ t\right)$ if and only if $x=y=z$,

G_{pf2} $G_{pf}\left(x,\ y,\ z,\ t\right)\geq\ G_{pf}\left(x,\ x,\ y,\ t\right)$ for all $x,\ y,\ z\ \in X$ and $y\neq\ z$,

G_{pf3} $G_{pf}\left(x,\ y,\ z,\ t\right)=G_{pf}\left(z,\ x,\ y,\ t\right)$ $=G_{pf}\left(\ y,\ z,\ x,\ t\right)$,

G_{pf4} $G_{pf}\left(x,\ y,\ z,\ max\{\ t,\ s\}\right)\leq G_{pf}\left(x,\ a,\ a,\ t\right)$ $*\ G_{pf}\left(a,\ y,\ z,\ s\right)\quad G_{pf}\left(a,\ a,\ a,\ max\ \{t,\ s\}\right)$

G_{pf5} $G_{pf}\left(x,\ y,\ z,\ .\ \right)\ :\ \left(0,\ \infty\right)\rightarrow\left[0,\ 1\right]$ is continuous.

Example 2.2: Let X be a nonempty set and G_p a G-partial metric on X . Denote $a*b.c=\dfrac{ab}{c}$ for all $a,\ b,\ c\ \in\left[0,\ 1\right]$ for each $t\ >0$, $x,\ y,\ z\ \in X$,

$$G_{pf}\ \left(x,\ y,\ z,\ t\right)\ =\ \frac{t}{t+\max\{x,y,z\}}.$$

Then $\left(X,\ G_{pf},\ *\right)$ is a fuzzy G-partial metric space.

Example 2.3: Let $\left(X,\ G_p\right)$ be a G-partial metric space and $G_{pf}:\ X^3\times\ \left(0,\ \infty\right)\rightarrow\ \left[0,\ 1\right]$ be a mapping defined as

$$G_{pf}\left(x,\ y,\ z,\ t\right)\ =exp\left(-\frac{G_p(x,y,z)}{t}\right).$$

If $a*b.c=\dfrac{ab}{c}$ for all $a,\ b,\ c\ \in\left[0,\ 1\right]$ then G_{pf} is a fuzzy G-partial metric.

Definition 2.4 : Let $\left(X,\ G_{pf},\ *\right)$ be a fuzzy G-partial metric space. For $t>0$, the open ball B(x, r, t) with centre $x\in X$ and radius $0<\ r<\ 1$ is defined by

$$B\left(x,\ r,\ t\right)\ =\ \left\{y\in\ X:\ G_{pf}\left(x,\ y,\ y,\ t\right)>1-\ r\right\}$$

A subset A of X is called an open set if for each $x\in A$ there exist $t>0$ and $0<\ r<\ 1$ such that $B\left(x,\ r,\ t\right)\subseteq\ A$.

A sequence $\{x_n\}$ in X converges to x if and only if $G_{pf}\left(x_n,\ x_n,\ x,\ t\right)\rightarrow 1$ as $n\rightarrow\infty$ for each $t>0$. It is called Cauchy sequence if for each $0<\ \varepsilon<\ 1$ and $t>0$, there exists $n_0\in N$ such that $G_{pf}\left(x_m,\ x_n,\ x_l,\ t\right)>\ 1\ -\varepsilon$ for each $l,\ m,\ n\geq n_0$. The fuzzy G-partial metric space is called complete if every Cauchy sequence is convergent.

Example 2.5 : Let $X=\ \left[0,\ \infty\right)$ and $G_{pf}\ \left(x,\ y,\ z,\ t\right)\ =\ \dfrac{t}{t+\max\{x,y,z\}}$ then $\left(X,\ G_{pf},\ *\right)$ is a fuzzy G-partial metric space where $a*b.c=\dfrac{ab}{c}$ for all $a,\ b,\ c\ \in\left[0,\ 1\right]$. Let $\{x_n\}=\left\{\dfrac{1}{n}\right\}$. Then the sequence is convergent and we have

$$\lim_{n\to\infty} G_{pf}\left(x_n,\, x_n,\, x,\, t\right) = G_{pf}\left(x,\, x,\, x,\, t\right).$$

The sequence is also Cauchy. Therefore we obtain

$$\lim_{n\to\infty} G_{pf}\left(x_n,\, x_n,\, x,\, t\right) = G_{pf}\left(x,\, x,\, x,\, t\right) = \lim_{n\to\infty} G_{pf}\left(x_n,\, x_m,\, x_m,\, t\right).$$

Definition 2.6 : Let $\left(X,\, G_{pf},\, *\right)$ be a fuzzy G-partial metric space. G_{pf} is said to be continuous function on $X^3 \times (0,\infty)$ if for every $x,\, y,\, z \in X$ and $t > 0$,

$$\lim_{n\to\infty} G_{pf}\left(x_n,\, y_n,\, z_n,\, t_n\right) = G_{pf}\left(x,\, y,\, z,\, t\right)$$

Whenever , $\lim_{n\to\infty} x_n = x$, $\lim_{n\to\infty} y_n = y$, $\lim_{n\to\infty} z_n = z$ and

$$\lim_{n\to\infty} G_{pf}\left(x,\, y,\, z,\, t_n\right) = G_{pf}\left(x,\, y,\, z,\, t\right)$$

Lemma 2.7 : If $\left(X,\, G_{pf},\, *\right)$ is a fuzzy G-partial metric space then $G_{pf}\left(x,\, y,\, z,\, t\right)$ is non-increasing with respect to t for all $x,\, y,\, z \in X$.

Proof: By G_{pf4}, let $a = x$ and for $t > 0$ we obtain

$$G_{pf}\left(x,\, y,\, z,\, max\{t,\, s\}\right) \le G_{pf}\left(x,\, x,\, x,\, t\right) * G_{pf}\left(x,\, y,\, z,\, s\right) \quad G_{pf}\left(x,\, x,\, x,\, max\{t,\, s\}\right)$$

Let $t > s$ then we have

$$G_{pf}\left(x,\, y,\, z,\, t\right) \le G_{pf}\left(x,\, x,\, x,\, t\right) * G_{pf}\left(x,\, y,\, z,\, s\right) \quad G_{pf}\left(x,\, x,\, x,\, t\right)$$

This implies,

$$G_{pf}\left(x,\, y,\, z,\, t\right) \le G_{pf}\left(x,\, y,\, z,\, s\right).$$

This shows that $G_{pf}\left(x,\, y,\, z,\, t\right)$ is non-increasing with the assumption that $\lim_{n\to\infty} G_{pf}\left(x,\, y,\, z,\, t\right) = 1$ and that $n \in N$ is the set of all natural numbers.

Lemma 2.8 : Let $\left(X,\, G_{pf},\, *\right)$ be a fuzzy G-partial metric space. Then G_{pf} is a continuous function on $X^3 \times (0,\infty)$.

Proof: Since $\lim_{n\to\infty} x_n = x$, $\lim_{n\to\infty} y_n = y$, $\lim_{n\to\infty} z_n = z$ and

$$\lim_{n\to\infty} G_{pf}\left(x,\, y,\, z,\, t_n\right) = G_{pf}\left(x,\, y,\, z,\, t\right)$$

then there is $n_0 \in N$ such that $|t - t_n| < \delta$ for $n \ge n_0$ and $\delta < \frac{t}{2}$.

From Lemma 2.7 , we prove that $G_{pf}\left(x,\, y,\, z,\, t\right)$ is non-increasing with respect to t , hence we have

$$G_{pf}\left(x_n,\, y_n,\, z_n,\, t_n\right) \le G_{pf}\left(x_n,\, x,\, x,\frac{\delta}{3}\right) * G_{pf}\left(x,\, y_n,\, z_n,\, t+\frac{4\delta}{3}\right) \quad G_{pf}\left(x,\, x,\, x,\frac{\delta}{3}\right)$$

$$\le G_{pf}\left(x_n,\, x,\, x,\frac{\delta}{3}\right) * G_{pf}\left(y_n,\, y,\, y,\frac{\delta}{3}\right) * G_{pf}\left(x,\, y,\, z_n,\, t+\frac{5\delta}{3}\right) \cdot G_{pf}\left(x,\, x,\, x,\frac{\delta}{3}\right) \quad G_{pf}\left(y,\, y,\, y,\frac{\delta}{3}\right)$$

$$\le G_{pf}\left(x_n,\, x,\, x,\frac{\delta}{3}\right) * G_{pf}\left(y_n,\, y,\, y,\frac{\delta}{3}\right) * G_{pf}\left(z_n,\, z,\, z,\frac{\delta}{3}\right) * G_{pf}\left(x,\, y,\, z,\, t+2\delta\right) \cdot G_{pf}\left(x,\, x,\, x,\frac{\delta}{3}\right) . G_{pf}\left(y,\, y,\, y,\frac{\delta}{3}\right) . G_{pf}\left(z,\, z,\, z,\frac{\delta}{3}\right)$$

and

$$G_{pf}\left(x,\, y,\, z,\, t - 2\delta\right) \le G_{pf}\left(x,\, y,\, z,\, t_n - \delta\right)$$

$$\le G_{pf}\left(x,\, x_n,\, x_n,\frac{\delta}{3}\right) * G_{pf}\left(x_n,\, y,\, z,\, t_n - \frac{2\delta}{3}\right) \quad G_{pf}\left(x_n,\, x_n,\, x_n,\frac{\delta}{3}\right)$$

$$\le G_{pf}\left(x,\, x_n,\, x_n,\frac{\delta}{3}\right) * G_{pf}\left(y,\, y_n,\, y_n,\frac{\delta}{3}\right) * G_{pf}\left(x_n,\, y_n,\, z,\, t_n - \frac{\delta}{3}\right) . G_{pf}\left(x_n,\, x_n,\, x_n,\frac{\delta}{3}\right) \quad G_{pf}\left(y_n,\, y_n,\, y_n,\frac{\delta}{3}\right)$$

$$\leq G_{pf}\left(x,\, x_n,\, x_n, \frac{\delta}{3}\right) * G_{pf}\left(y,\, y_n,\, y_n, \frac{\delta}{3}\right)$$
$$* G_{pf}\left(z,\, z_n,\, z_n, \frac{\delta}{3}\right) * G_{pf}\left(x_n,\, y_n,\, z_n,\, t_n\right).$$
$$G_{pf}\left(x_n,\, x_n, x_n,\, \frac{\delta}{3}\right). G_{pf}\left(y_n,\, y_n,\, y_n, \frac{\delta}{3}\right).$$
$$G_{pf}\left(z_n,\, z_n,\, z_n,\, \frac{\delta}{3}\right)$$

Let $n \to \infty$ and by continuity of the function G_{pf} with respect to t,
$$G_{pf}(x,\, y,\, z,\, t+2\delta) \geq G_{pf}(x,\, y,\, z,\, t)$$
$$\geq G_{pf}(x,\, y,\, z,\, t-\delta).$$
Thus G_{pf} is a continuous function on $X^3 \times (0, \infty)$.

Definition 2.9 [9] : Let $(X,\, d)$ be a metric space. A mapping $T : X \to X$ be continuous, injection and sequentially convergent if, for each sequence $\{y_n\}$ the following holds true: if $\{Ty_n\}$ converges, then $\{y_n\}$ also converges.

Recently, Ozavsar and Cevikel [6] introduced multiplicative contractive mappings as a generalization of the Banach contraction Principle. The multiplicative contraction mapping is given in the setting of fuzzy G-partial metric spaces.

Definition 2.10: Let $(X,\, G_{pf},\, *)$ be a fuzzy G-partial metric space. A mapping $T : X \to X$ is called multiplicative contraction if there exists a real constant $\alpha \in [0,1)$ such that
$G_{pf}(Tx,\, Ty,\, Ty,\, t) \leq G_{pf}(x,\, y,\, y,\, t)^{\alpha}$ for all $x,\, y \in X$.

3 Main Results

In this section, some common fixed point theorems for Kannan-Type multiplicative contraction maps in fuzzy G-partial metric spaces are established.

Theorem 3.1: Let $(X,\, G_{pf},\, *)$ be a complete fuzzy G-partial metric space. If $T : X \to X$ is continuous, injection and sequentially convergent multiplicative contractive mapping and $S_1,\, S_2 : X \to X$. If there exist $\alpha \in \left[0, \frac{1}{2}\right)$, $\beta \in [0,\, 1)$ such that
$$G_{pf}(TS_1x,\, TS_2y,\, TS_2y,\, t) \leq$$
$$\left\{ G_{pf}(Tx,\, TS_1x,\, TS_1x,\, t) * G_{pf}(Ty,\, TS_2y,\, TS_2y,\, t)\right\}^{\alpha}$$
$$* G_{pf}(Tx,\, Ty,\, Ty,\, t)^{\beta} \tag{1}$$

for all $x,\, y \in X$. Then S_1 and S_2 have a unique common fixed point. For any $x \in X$, iterative sequence $(T^n x)$ converges to the fixed point.

Proof : Let $x_0 \in X$ be arbitrarily chosen. We define a sequence $\{x_n\}$ as follows:
$$x_{2n+1} = S_1 x_{2n},\; x_{2n+2} = S_2 x_{2n+1},$$
for $n = 0,\, 1,\, 2, \cdots$.
If the sequence exists for $n \geq 0$, such that $x_n = x_{n+1} = x_{n+2}$, then it is clearly proven that x_n is the common fixed point for S_1 and S_2. If there are no three consecutive congruent terms of the sequence $\{x_n\}$, then using inequality (1), we prove that for each $n \geq 1$, the following holds when $x = x_{2n}$ and $y = x_{2n+1}$ in (1).
$$G_{pf}(Tx_{2n+1},\, Tx_{2n+2},\, Tx_{2n+2},\, t)$$
$$= G_{pf}(TS_1x_{2n},\, TS_2x_{2n+1},\, TS_2x_{2n+1}, t)$$
$$\leq \begin{Bmatrix} G_{pf}(Tx_{2n},\, TS_1x_{2n},\, TS_1x_{2n},\, t) \\ * G_{pf}(Tx_{2n+1},\, TS_2x_{2n+1}, TS_2x_{2n+1},\, t) \end{Bmatrix}^{\alpha}$$
$$* G_{pf}(Tx_{2n},\, Tx_{2n+1},\, Tx_{2n+1},\, t)^{\beta}$$
$$= \begin{Bmatrix} G_{pf}(Tx_{2n},\, Tx_{2n+1},\, Tx_{2n+1},\, t) \\ * G_{pf}(Tx_{2n+1},\, Tx_{2n+2}, Tx_{2n+2},\, t) \end{Bmatrix}^{\alpha}$$
$$* G_{pf}(Tx_{2n},\, Tx_{2n+1},\, Tx_{2n+1},\, t)^{\beta}$$
$$G_{pf}(Tx_{2n+1},\, Tx_{2n+2},\, Tx_{2n+2},\, t)^{(1-\alpha)}$$
$$\leq G_{pf}(Tx_{2n},\, Tx_{2n+1},\, Tx_{2n+1},\, t)^{\alpha+\beta}$$
$$G_{pf}(Tx_{2n+1},\, Tx_{2n+2},\, Tx_{2n+2},\, t)$$
$$\leq G_{pf}(Tx_{2n},\, Tx_{2n+1},\, Tx_{2n+1},\, t)^{\frac{\alpha+\beta}{1-\alpha}}$$
$$= G_{pf}(Tx_{2n},\, Tx_{2n+1},\, Tx_{2n+1},\, t)^{h},$$

Where $h = \dfrac{\alpha+\beta}{1-\alpha}$.

For $n > m$ we have

$$G_{pf}(Tx_n, Tx_m, Tx_m, t) \le G_{pf}(Tx_n, Tx_{n+1}, Tx_{n+1}, t) * G_{pf}(Tx_{n+1}, Tx_m, Tx_m, t) \cdot G_{pf}(Tx_{n+1}, Tx_{n+1}, Tx_{n+1}, t)$$

$$\le G_{pf}(Tx_n, Tx_{n+1}, Tx_{n+1}, t) * G_{pf}(Tx_{n+1}, Tx_m, Tx_m, t)$$

$$\le G_{pf}(Tx_n, Tx_{n+1}, Tx_{n+1}, t) * G_{pf}(Tx_{n+1}, Tx_{n+2}, Tx_{n+2}, t) * \cdots * G_{pf}(Tx_{m-1}, Tx_m, Tx_m, t).$$

$$G_{pf}(Tx_{n+2}, Tx_{n+2}, Tx_{n+2}, t) \cdot \cdots \; G_{pf}(Tx_{m-1}, Tx_{m-1}, Tx_{m-1}, t)$$

$$\le G_{pf}(Tx_n, Tx_{n+1}, Tx_{n+1}, t) * G_{pf}(Tx_{n+1}\, Tx_{n+2}, Tx_{n+2}, t) * \cdots * G_{pf}(Tx_{m-1}, Tx_m, Tx_m, t)$$

$$\le G_{pf}(Tx_0, Tx_1, Tx_1, t)^{h^n + h^{n+1} + \cdots + h^{m-1}}$$

$$\le G_{pf}(Tx_0, Tx_1, Tx_1, t)^{\frac{h^m}{1-h}}$$

This shows that $G_{pf}(Tx_n, Tx_m, Tx_m, t) \to 1$. Thus the sequence is Cauchy. Since X is complete then the sequence $\{Tx_n\}$ is convergent. Also,

$$\lim_{n\to\infty} G_{pf}(Tx_n, Tx_n, Tx_n, t) = G_{pf}(Tx, Tx, Tx, t) = \lim_{n\to\infty} G_{pf}(Tx_n, Tx_m, Tx_m, t) = 1$$

. Furthermore, the mapping $T: X \to X$ is sequentially convergent with $\{Tx_n\}$ convergent. Let $\{x_n\}$ converge to u. By the continuity of T, Tx_n converges to Tu. Hence,

$$G_{pf}(Tu, TS_1u, TS_1u, t) \le G_{pf}(Tu, Tx_{2n+1}, Tx_{2n+1}, t) * G_{pf}(Tx_{2n+1}, TS_1u, TS_1u, t) \cdot G_{pf}(Tx_{2n+1}, Tx_{2n+1}, Tx_{2n+1}, t)$$

$$\le G_{pf}(Tu, Tx_{2n+1}, Tx_{2n+1}, t) * G_{pf}(Tx_{2n+1}, TS_1u, TS_1u, t)$$

$$= G_{pf}(Tu, Tx_{2n+1}, Tx_{2n+1}, t) * G_{pf}(TS_2x_{2n+1}, TS_1u, TS_1u, t)$$

$$\le G_{pf}(Tu, Tx_{2n+1}, Tx_{2n+1}, t) \begin{Bmatrix} *G_{pf}(Tx_{2n+1}, TS_2x_{2n+1}, TS_2x_{2n+1}, t) \\ *G_{pf}(Tu, TS_1u, TS_1u, t) \end{Bmatrix}^{\alpha} * G_{pf}(Tx_{2n}, Tu, Tu, t)^{\beta}$$

$$= G_{pf}(Tu, Tx_{2n+1}, Tx_{2n+1}, t) * \begin{Bmatrix} G_{pf}(Tx_{2n+1}, Tx_{2n+2}, Tx_{2n+2}, t) \\ *G_{pf}(Tu, TS_1u, TS_1u, t) \end{Bmatrix}^{\alpha} * G_{pf}(Tx_{2n}, Tu, Tu, t)^{\beta}$$

$$\le \begin{pmatrix} G_{pf}(Tu, Tx_{2n+1}, Tx_{2n+1}, t) \\ * G_{pf}(Tx_{2n+1}, Tx_{2n+2}, Tx_{2n+2}, t)^{\alpha} \\ * G_{pf}(Tx_{2n}, Tu, Tu, t)^{\beta} \end{pmatrix}^{\frac{1}{1-\alpha}}.$$

As $n \to \infty$ we get

$$G_{pf}(Tu, TS_1u, TS_1u, t) \le \begin{pmatrix} G_{pf}(Tu, Tu, Tu, t) \\ * G_{pf}(Tu, Tu, Tu\, t)^{\alpha} \\ * G_{pf}(Tu\, Tu, Tu, t)^{\beta} \end{pmatrix}^{\frac{1}{1-\alpha}} \to 1.$$

Since $G_{pf}(Tu, TS_1u, TS_1u, t) = 1$ then we have $Tu = TS_1u$. By the injectivity of T we have $u = S_1u$. Thus u is the fixed point of S_1. Analogously, u is the fixed point of S_2. We shall prove that S_1 and S_2 have a unique common fixed point. Let $v \in X$ be a different fixed point of S_1. That is, $v = S_1v$. Hence

$$G_{pf}(Tu, Tv, Tv, t) = G_{pf}(TS_1u, TS_2v, TS_2v, t)$$
$$\leq \begin{Bmatrix} G_{pf}(Tu, TS_1u, TS_1u, t) \\ * G_{pf}(Tv, TS_2v, TS_2v, t) \end{Bmatrix}^{\alpha}$$
$$* G_{pf}(Tu, Tv, Tv, t)^{\beta}$$
$$= G_{pf}(Tu, Tv, Tv, t)^{\beta}$$

This is a contradiction. Hence, $Tu = Tv$. By the injectivity of T, we have $u = v$.

If $\beta = 0$ in Theorem 3.1 we have the following result.

Corollary 3.2 : Let $(X, G_{pf}, *)$ be a complete fuzzy G-partial metric space. If $T : X \to X$ is continuous, injection and sequentially convergent multiplicative contractive mapping and $S_1, S_2 : X \to X$. If there exist $\alpha \in \left[0, \frac{1}{2}\right)$, such that

$$G_{pf}(TS_1x, TS_2y, TS_2y, t) \leq$$
$$\{G_{pf}(Tx, TS_1x, TS_1x, t) * G_{pf}(Ty, TS_2y, TS_2y, t)\}^{\alpha} \tag{2}$$

for all $x, y \in X$. Then S_1 and S_2 have a unique common fixed point. For any $x \in X$, iterative sequence $(T^n x)$ converges to the fixed point.

If $S_1 = S_2 = 1$ in Corollary 3.2 , then we have the following Corollary.

Corollary 3.3 : Let $(X, G_{pf}, *)$ be a complete fuzzy G-partial metric space. If $T : X \to X$ is continuous, injection and sequentially convergent multiplicative contractive mapping. If there exist $\alpha \in \left[0, \frac{1}{2}\right)$, such that

$$G_{pf}(Tx, Ty, Ty, t)$$
$$\leq \{G_{pf}(x, Tx, Tx, t) * G_{pf}(y, Ty, Ty, t)\}^{\alpha} \tag{3}$$

for all $x, y \in X$. Then T has a unique fixed point. For any $x \in X$, the iterative sequence $(T^n x)$ converges to the fixed point.

4 Conclusion

The study established the existence and uniqueness of common fixed point of multiplicative contractive mappings in Fuzzy G-partial metric spaces. The result is achieved by employing the concept of sequential convergence of one of the maps. It shows that the existence of contractive maps in Fuzzy G-partial metric space is possible.

5 Acknowledgements

The authors thanked Covenant University for supporting this research work.

References:

[1] Banach S., Sur les operations dans les ensembles abstraits et leur applications aux equations integrals, *Fundam. Math.*, Vol.3, 1922, pp.133-181.

[2] Kannan R., Some results on fixed points, *Bull. Calcutta Math. Soc.*, Vol.10, 1968, pp.71-76.

[3] Moradi S. and Alimohammadi D., New extensions of Kannan fixed -point generalized metric spaces, *International Journal of Mathematical Analysis*, Vol.5, No. 47, 2011, pp. 2313-2320.

[4] Malceski R., Malceski A., Anevska K. and Malceski S., Common fixed points of Kannan and Chatterjea Types of mappings in a complete metric space, *British Journal of Mathematics and Computer Science*, Vol.18, No. 2, 2016, pp.1-11.

[5] Eke K. S., Common fixed point theorems for generalized contraction mappings on uniform spaces, *Far East Journal of Mathematical Sciences*, Vol. 99, No.11, 2016, pp.1753-1760.

[6] Ozavsar M. and Cevikel A., Fixed points of multiplicative contraction mappings on multiplicative metric spaces, *Math. GM.*, 2012, http://arxiv.org/abs/1205.5131v1.

[7] He X., Song M. and Chen D., Common fixed points for weak commutative mappings on a multiplicative metric spaces, *Fixed Point Theory and Appl.*, Vol.2014, Article 48, 2014.

[8] Eke K. S., Oghonyon J. G. and Davvaz B., Some fixed point theorems for contractive maps in fuzzy G-partial metric spaces, *International Journal of Mechanical Engineering and Technology*, Vol. 9, No. 8, 2018, pp.635-645.

[9] Branciari A., A fixed point theorem of Banach-Caccippoli type on a class of generalized metric

spaces, *Publi. Math. Debrecen*, Vol. 57, No.1-2, 2000, pp.45-48.