Generalized Trapezoidal Intuitionistic Fuzzy Number for Finding Radial Displacement of a Solid Disk

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Abstract: - The present paper describes the method of finding the radial displacement of a solid disk which uses the concept of a generalized trapezoidal intuitionistic fuzzy number.

Key-Words: - Cauchy-Euler fuzzy differential equations, Intuitionistic fuzzy number, Trapezoidal fuzzy number, Hukuhara Differentiability.

1 Introduction

There are different types of uncertainties involved in almost everywhere in day to day real life problems. Fuzzy set theory, introduced by Lofti Zadeh [1] in 1965, is a natural way for dealing such kind of uncertain environment. The fuzzy set theory is further generalized by Atanassov ([2]-[3]), who introduced the concept of intuitionistic fuzzy set (IFS). An intuitionistic fuzzy set is characterized by degrees of membership and non-membership. Intuitionistic fuzzy set theory is more powerful tool to solve real world problems. In past few years many authors have applied IFS theory to solve several problems in different application areas [4-6].

Differential equations have potential to model the real life physical phenomenon. A differential equation involves one or more parameters, which plays a significant role to represent real world problems. But in many real life situations, we encounter many uncertainties. In 1972, Chang and Zadeh [7] introduced the concept of fuzzy derivative. Fuzzy derivative was further studied and extended by many other researchers (see, [8]-[11]). In 1987, Kaleva [12] first introduced the concept of fuzzy differential equations and gave the existence and uniqueness theorem for a solution to a fuzzy differential equation. In 1987, Seikkala [13] studied the notion of fuzzy initial value problem by using extension principle and the use of extremal solutions of deterministic initial value problems. Friedman and Kandel [14] proposed a numerical algorithm for solving fuzzy ordinary differential equation. Thereafter this topic has attracted widespread attention to many researchers and has started work in this direction (see for instance, [15]-[19]). Millani and Chandi ([20]-[21]) discussed the ordinary and partial differential equations under intuitionistic fuzzy environments. Abbasbandy and Allahviranloo

[22] proposed a numerical solution of fuzzy differential equation by Runge-kutta method with intuitionistic fuzzy treatment.

Many authors (see, [23]-[24]) have used the concept of fuzzy Laplace transform for solving various differential equations and their applications. Thus, the intuitionistic fuzzy differential equations model real life problems more precisely. In 2015, Mondal and Roy [15] described the generalized intuitionistic fuzzy Laplace transform method for solving first order generalized intuitionistic fuzzy differential equations. Mondal and Roy [16] solved second order intuitionistic fuzzy differential equation using generalized trapezoidal intuitionistic fuzzy number.

Cauchy-Euler equation is a special form of a linear ordinary differential equation with variable coefficients. The second order Cauchy–Euler equations are used in time-harmonic vibrations of a thin elastic rod, problems on annual and solid disc, wave mechanics, etc. This paper deals with the Cauchy-Euler homogeneous second order linear differential equation with intuitionistic fuzzy boundary conditions. In addition, we apply this approach to solve problem of solid disk whose differential equation is the second order linear ordinary differential equation with variable coefficients.

2 Preliminaries

Definition 2.1 Fuzzy Set [1]: Let X be a fixed set. A fuzzy set A in X is a set of ordered pairs defined as $A = \{(x, \mu_A(x)) : x \in X, \mu_A(x) \in [0,1]\}$.

Definition 2.2. Intuitionistic Fuzzy Set [2]: Let X be any fixed set. An intuitionistic fuzzy set is defined as the set of the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X, \mu_A(x) \in [0,1], \nu_A(x) \in [0,1] \ \& \ 0 \le \mu_A(x) + \nu_A(x) \le 1\}$. Here, $\mu_A(x), \nu_A(x) : X \to [0,1]$ define the degree of membership and degree of non-membership respectively, of the element $x \in X$ to the set A.

Definition 2.3. α–cut on Fuzzy set [26]: Let A be a fuzzy interval in R and $\alpha \in (0,1]$. The α -cut of A is the crisp set $[A]^\alpha$ that contains all elements with membership degree in A greater than or equal to α, i.e., $[A]^\alpha = \{x \in R | A(x) \ge \alpha\}$.

Definition 2.4. Convex Fuzzy set [26]: A fuzzy set A on a real Euclidean space X is said to be convex fuzzy set iff the $\alpha -$ level set of A, denoted by $[A]^\alpha$ is a convex subset of X. If $= R^n$, with R being the set of real numbers, then the fuzzy set A is convex iff the following condition holds: Given any two different points x and y in $[A]^\alpha$, then for any $a \in [0,1]$, $ax + (1-a)y \in [A]^\alpha$.

Definition 2.5. Normal Fuzzy set [25]: A fuzzy set A defined on fixed set X is said to be normal if and only if $\sup_{x \in X} \mu_A(x) = 1$.

Definition 2.6. Fuzzy Number [26]: A fuzzy number is a fuzzy set on the real line that satisfies the conditions of normality and convexity.

Definition 2.7. Trapezoidal Fuzzy Number [27]: A Trapezoidal fuzzy number $A = (a_1, a_2, a_3, a_4)$ is a fuzzy set, where a_1, a_2, a_3 and a_4 are real numbers and $a_1 \le a_2 \le a_3 \le a_4$ with membership function defined as

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & for\ a_1 \le x < a_2 \\ 1, & for\ a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & for\ a_3 < x \le a_4 \\ 0, & otherwise \end{cases}$$

Definition 2.8. Trapezoidal Intuitionistic Fuzzy Number [16]: A Trapezoidal intuitionistic fuzzy number is denoted by $A = ((a_1, a_2, a_3, a_4), (a_1', a_2, a_3, a_4'))$ where $a_1' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4'$ with membership and non-membership function defined as

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x < a_2 \\ 1, & a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}, & a_3 < x \le a_4 \\ 0, & otherwise \end{cases},$$

$$\nu_A(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'}, & a_1' \le x < a_2 \\ 0, & a_2 \le x \le a_3 \\ \frac{x-a_3}{a_4'-a_3}, & a_3 < x \le a_4' \\ 1, & otherwise \end{cases}$$

with $a_1' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4'$.

Definition 2.9. Generalized Trapezoidal Intuitionistic Fuzzy Number [16]: A generalized

Trapezoidal intuitionistic fuzzy number is denoted by $A = \big((a_1, a_2,\ a_3,\ a_4; w_A), (a_1', a_2,\ a_3,\ a_4'; u_A)\big)$ with membership and non-membership function defined as

$$\mu_A(x) = \begin{cases} w_A\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ w_A, & a_2 \le x \le a_3 \\ w_A\left(\frac{a_4-x}{a_4-a_3}\right), & a_3 \le x \le a_4 \\ 0\,, & otherwise \end{cases},$$

$$\nu_A(x) = \begin{cases} u_A\left(\dfrac{a_2 - x}{a_2 - a_1'}\right), & a_1' \le x < a_2 \\ u_A, & a_2 \le x \le a_3 \\ u_A\left(\dfrac{x - a_3}{a_4' - a_3}\right), & a_3 < x \le a_4' \\ 0\,, & otherwise \end{cases}$$

with $a_1' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4'$.

Definition 2.10. (α,β) - cuts on Intuitionistic fuzzy number [16]: Let $[A_1(\alpha), A_2(\alpha), A_1{}'(\beta), A_2{}'(\beta)]$ be the (α, β)-cuts of a trapezoidal intuitionistic fuzzy number A and ω, σ be the gradation of membership and non-membership function respectively then the intuitionistic fuzzy number is given by

$$A = ((A_1(\alpha = 0), A_1(\alpha = \omega), A_2(\alpha = \omega), A_2(\alpha = 0); \omega); (A_1{}'(\beta = \sigma), A_1{}'(\beta = 0), A_2{}'(\beta = 0), A_2{}'(\beta = \sigma); \sigma)$$

Definition 2.11. Generalized Hukuhara derivative for first order [23]: The generalized Hukuhara derivative of a fuzzy valued function $f: (a, b) \to R_F$ at t_0 is defined as $f'(t_0) = lim_{h\to 0} \frac{f(t_0+h)\Theta_{\text{gH}} f(t_0)}{h}$.

If $f'(t_0) \in R_F$ exists, we say that f is generalized Hukuhara differentiable at t_0.

Also we say that $f(t)$ is (i) $-gH$ differentiable at t_0 if

$$\left|f'(t_0)\right|^{\alpha} = \left|f_1'(t_0, \alpha), f_2'(t_0, \alpha)\right|$$

and $f(t)$ is (ii) - gH differentiable at t_0 if

$$\left|f'(t_0)\right|^{\alpha} = \left|f_2'(t_0, \alpha), f_1'(t_0, \alpha)\right|.$$

Definition 2.12. Generalized Hukuhara derivative for second order [23]: The second order generalized Hukuhara derivative of a fuzzy valued function $f: (a, b) \to R_F$ at t_0 is defined as

$$f''(t_0) = lim_{h\to 0} \frac{f'(t_0+h)\Theta_{\text{gH}} f'(t_0)}{h}.$$

If $f''(t_0) \in R_F$, we say that $f'(t_0)$ is generalized Hukuhara at t_0.

Also we say that $f'(t_0)$ is (i) $-gH$ differentiable at t_0 if

$$f''(t_0, \alpha) = \begin{cases} \big(f_1'(t_0, \alpha), f_2'(t_0, \alpha)\big)\ if \\ f\ is\ (i) - gH\ differentiable\ on\ (a, b) \\ \big(f_2'(t_0, \alpha), f_1'(t_0, \alpha)\big)\ if \\ f\ is\ (ii) - gH\ differentiable\ on\ (a, b) \end{cases}$$

for all $\alpha \in [0,1]$, and that $f'(t_0)$ is (ii) – gH differentiable at t_0 if

$$f''(t_0, \alpha) = \begin{cases} \big(f_2'(t_0, \alpha), f_1'(t_0, \alpha)\big) \\ if\ f\ is\ (i) - gH\ differentiable\ on\ (a, b) \\ \big(f_1'(t_0, \alpha), f_2'(t_0, \alpha)\big) \\ if\ f\ is\ (ii) - gH\ differentiable\ on\ (a, b) \end{cases}$$

for all $\alpha \in [0,1]$.

3 Finding Radial Displacement of a Solid Disk under Intuitionistic Fuzzy Environment

Here, the problem of a solid disk is taken. The differential equation is of the form of second order Cauchy-Euler where the dependent variable is radial displacement (u) and boundary condition is defined at $r = 0$ and $r = L$.

Let the radial displacement u in a rotating disc at a distance r from the axis is given by

$$r^2 \frac{d^2u}{dr^2} + r\frac{du}{dr} - u = 0 \tag{1}$$

with Boundary conditions: $u = 0$, when $r = 0$ and $r = L$.

Considering the above differential equation $r^2 \frac{d^2u}{dr^2} + r\frac{du}{dr} - u = 0$, with boundary conditions $u(0) = \tilde{a}$ and $u(L) = \tilde{b}$, where $\tilde{a}, \tilde{b}$ are generalized trapezoidal intuitionistic fuzzy number.

Let $\tilde{a} = \big((a_1, a_2, a_3, a_4;\ w_1), (a_1', a_2, a_3, a_4';\ \sigma_1)\big)$

and $\tilde{b} = \big((b_1, b_2, b_3, b_4;\ w_2), (b_1', b_2, b_3, b_4';\ \sigma_2)\big)$.

The four cases arises depending on (i) and (ii) – gH differentiability

Case 1: When $u(r)$ is (i) –gH differentiable and $\frac{du(r)}{dr}$ is (i) –gH differentiable.

For membership function, we have

$$r^2\frac{d^2u_1(r,\alpha)}{dr^2}+r\frac{du_1(r,\alpha)}{dr}-u_1(r,\alpha)=0$$

$$r^2\frac{d^2u_2(r,\alpha)}{dr^2}+r\frac{du_2(r,\alpha)}{dr}-u_2(r,\alpha)=0$$

For non-membership, it is written as

$$r^2\frac{d^2u_1'(r,\beta)}{dr^2}+r\frac{du_1'(r,\beta)}{dr}-u_1'(r,\beta)=0$$

$$r^2\frac{d^2u_2'(r,\beta)}{dr^2}+r\frac{du_2'(r,\beta)}{dr}-u_2'(r,\beta)=0$$

with boundary conditions,

$u_1(0,\alpha)=a_1+\frac{\alpha}{w}l_{\tilde{a}}$, $u_2(0,\alpha)=a_4-\frac{\alpha}{w}r_{\tilde{a}}$, $u_1(L,\alpha)=b_1+\frac{\alpha}{w}l_{\tilde{b}}$, $u_2(L,\alpha)=b_4-\frac{\alpha}{w}r_{\tilde{b}}$, $u_1'(0,\beta)=a_2-\frac{\beta}{\sigma}l_{\tilde{a}}$, $u_2'(0,\beta)=a_3+\frac{\beta}{\sigma}r'_{\tilde{a}}$, $u_1'(L,\beta)=b_2-\frac{\beta}{\sigma}l_{\tilde{b}}$, $u_2'(L,\beta)=b_3+\frac{\beta}{\sigma}r'_{\tilde{b}}$, where $l_{\tilde{a}}=a_2-a_1$, $r_{\tilde{a}}=a_4-a_3$, $l_{\tilde{b}}=b_2-b_1$, $r_{\tilde{b}}=b_4-b_3$, $l'_{\tilde{a}}=a_2-a_1'$, , $r'_{\tilde{a}}=a_4'-a_3$, $l'_{\tilde{b}}=b_2-b_1'$, $r'_{\tilde{b}}=b_4'-b_3$ and $w=\min\{w_1,w_2\}$, $\sigma=\min\{\sigma_1,\sigma_2\}$.

The general solution of equation (1) is $u(r)=C_1r+\frac{C_2}{r}$.

Using the initial condition $u_1(0,\alpha)=a_1+\frac{\alpha}{w}l_{\tilde{a}}$, we get $C_2=0$.

Using the condition at the boundary $u_1(L,\alpha)=b_1+\frac{\alpha}{w}l_{\tilde{b}}$, we get $C_1=\frac{b_1+\frac{\alpha}{w}(b_2-b_1)}{L}$.

Thus, $u_1(r,\alpha)=\frac{\left[b_1+\frac{\alpha}{w}(b_2-b_1)\right]}{L}.r$

Similarly, using $u_2(0,\alpha)=a_4-\frac{\alpha}{w}(a_4-a_3)$, $u_2(L,\alpha)=b_4-\frac{\alpha}{w}(b_4-b_3)$.

we get, $u_2(r,\alpha)=\frac{1}{L}\left[b_4-\frac{\alpha}{w}(b_4-b_3)\right].r$

Using, other conditions such as $u_1'(0,\beta)=a_2-\frac{\beta}{\sigma}l_{\tilde{a}}$ and $u_1'(L,\beta)=b_2-\frac{\beta}{\sigma}l_{\tilde{b}}$, we get $u_1'(r,\beta)=\frac{1}{L}\left[b_2-\frac{\beta}{\sigma}(b_2-b_1')\right].r$ and using conditions $u_2'(0,\beta)=a_3+\frac{\beta}{\sigma}r'_{\tilde{a}}$ and $u_2'(L,\beta)=b_3+\frac{\beta}{\sigma}r'_{\tilde{b}}$, we get $u_2'(r,\beta)=\frac{1}{L}\left[b_3+\frac{\beta}{\sigma}(b_4'-b_3)\right].r$

Case 2: When $u(r)$ is (ii) –gH differentiable and $\frac{du(r)}{dr}$ is (i) –gH differentiable.

For membership function, we have

$$r^2\frac{d^2u_2(r,\alpha)}{dr^2}+r\frac{du_2(r,\alpha)}{dr}=u_1(r,\alpha)$$

$$r^2\frac{d^2u_1(r,\alpha)}{dr^2}+r\frac{du_1(r,\alpha)}{dr}=u_2(r,\alpha)$$

For non-membership, it is written as

$$r^2\frac{d^2u_2'(r,\beta)}{dr^2}+r\frac{du_2'(r,\beta)}{dr}=u_1'(r,\beta)$$

$$r^2\frac{d^2u_1'(r,\beta)}{dr^2}+r\frac{du_1'(r,\beta)}{dr}=u_2'(r,\beta)$$

On solving above differential equation, we get

$$u_1(r,\alpha)=C_1r+\frac{C_2}{r}+rC_3\cos(\log r)+rC_4\sin(\log r)\ .$$

Similarly other equations can be derived.

$$u_2(r,\alpha)=C_1r+\frac{C_2}{r}-rC_3\cos(\log r)-rC_4\sin(\log r)\ ,$$

$$u_1'(r,\beta)=d_1r+\frac{d_2}{r}+d_3\cos(\log r)+d_4\sin(\log r)\ ,$$

$$u_2'(r,\beta)=d_1r+\frac{d_2}{r}-d_3\cos(\log r)-d_4\sin(\log r).$$

Using the initial conditions $u_1(0,\alpha)=a_1+\frac{\alpha}{w}l_{\tilde{a}}$, $u_2(0,\alpha)=a_4-\frac{\alpha}{w}r_{\tilde{a}}$, we get $C_2=0$. From the conditions $u_1(L,\alpha)=b_1+\frac{\alpha}{w}l_{\tilde{b}}$ and $u_2(L,\alpha)=b_4-\frac{\alpha}{w}r_{\tilde{b}}$, we get

$$\mathrm{L}.u_1(L,\alpha)=C_1.L^2+L.C_3\cos(\log L)+L.C_4\sin(\log L)$$

$$\mathrm{L}.u_2(L,\alpha)=C_1.L^2-L.[C_3\cos(\log L)+C_4\sin(\log L)].$$

Adding above two equations, we get $C_1 = \frac{u_1(L,\alpha)+u_2(L,\alpha)}{2L}$.

As it is not possible to get the values of other two constants, we will get the general solution, but not particular solution for case 2.

Case 3: When $u(r)$ is (i) –gH differentiable and $\frac{du(r)}{dr}$ is (ii) –gH differentiable. For membership function, we have

$$r^2 \frac{d^2u_2(r,\alpha)}{dr^2} + r\frac{du_2(r,\alpha)}{dr} - u_1(r,\alpha) = 0$$

$$r^2 \frac{d^2u_1(r,\alpha)}{dr^2} + r\frac{du_1(r,\alpha)}{dr} - u_2(r,\alpha) = 0$$

For non-membership, it is written as

$$r^2 \frac{d^2u_1'(r,\beta)}{dr^2} + r\frac{du_1'(r,\beta)}{dr} - u_1'(r,\beta) = 0$$

$$r^2 \frac{d^2u_2'(r,\beta)}{dr^2} + r\frac{du_2'(r,\beta)}{dr} - u_2'(r,\beta) = 0$$

The boundary conditions will remain same as per case 1 and 2.

The case 3 is same as that of case 2.

Case 4: When $u(r)$ is (ii) –gH differentiable and $\frac{du(r)}{dr}$ is (ii) –gH differentiable.

For membership function, we have

$$r^2 \frac{d^2u_1(r,\alpha)}{dr^2} + r\frac{du_1(r,\alpha)}{dr} - u_1(r,\alpha) = 0$$

$$r^2 \frac{d^2u_2(r,\alpha)}{dr^2} + r\frac{du_2(r,\alpha)}{dr} - u_2(r,\alpha) = 0$$

For non-membership, it is written as

$$r^2 \frac{d^2u_1'(r,\beta)}{dr^2} + r\frac{du_1'(r,\beta)}{dr} - u_1'(r,\beta) = 0$$

$$r^2 \frac{d^2u_2'(r,\beta)}{dr^2} + r\frac{du_2'(r,\beta)}{dr} - u_2'(r,\beta) = 0 .$$

Case 4 is same as that of case 1 and having the same boundary conditions. The solutions for cases 1 and 4 are same.

To illustrate the above example, we take particular values of the above trapezoidal intuitionistic fuzzy numbers for case 1.

Let $u(0) = \tilde{a} = \big((10,15,20,25;\ 0.6\ ),(8,15,20,28;\ 0.3)\big)$ and $u(5) = \tilde{b} = \big((30,35,40,45;\ 0.7\ ),(29,35,40,48;\ 0.3)\big)$. Here $L = 5$.

Here $w = min(0.6,0.7)$ = 0.6 and $\sigma = min(0.3,0.3) = 0.3$. And $\alpha \in [0,w]$ and $\beta \in [\sigma,1]$. Thus $\alpha \in [0,0.6]$ and $\beta \in [0.3,1]$.

Table 1: Solutions for $r = 1$

α	$u_1(r,\alpha)$	$u_2(r,\alpha)$	β	$u_1'(r,\beta)$	$u_2'(r,\beta)$
0	6	9	0.3	5.8	9.6
0.1	6.1667	8.8333	0.4	5.4	10.1333
0.2	6.3333	8.6667	0.5	5	10.6667
0.3	6.5	8.5	0.6	4.6	11.2
0.4	6.6667	8.3333	0.7	4.2	11.7333
0.5	6.8333	8.1667	0.8	3.8	12.2667
0.6	7	8	0.9	3.4	12.8
			1.0	3	13.3333

Table 2: Solutions for $r = 4$

α	$u_1(r,\alpha)$	$u_2(r,\alpha)$	β	$u_1'(r,\beta)$	$u_2'(r,\beta)$
0	24.0000	36	0.3	23.2	38.4
0.1	24.6667	35.3333	0.4	21.6	40.533
0.2	25.3333	34.6667	0.5	20	42.667
0.3	26.0000	34.0000	0.6	18.4	44.8
0.4	26.6667	33.3333	0.7	16.8	46.933
0.5	27.3333	32.6667	0.8	15.2	49.067
0.6	28	32	0.9	13.6	51.2
			1.0	12	53.333

The tables are drawn for radii at $r = 1$ and $r = 4$. It is observed that with the increase in value of α, $u_1(r,\alpha)$ increases and $u_2(r,\alpha)$ decreases, whereas with the increase in β, $u_1{}'(r,\beta)$ decreases and $u_2{}'(r,\beta)$ increases.

4 Conclusion

In this paper, we have considered the boundary value problem of a solid disk under intuitionistic fuzzy environment. The problem has modeled in the form of fuzzy linear Cauchy Euler second order ordinary differential equation. The radial displacement at different radii is calculated using trapezoidal intuitionistic fuzzy numbers and the results are shown in the form of tables for different radii.

References:

[1] Zadeh, L.A., Fuzzy Sets, *Information and Control*, Vol. 8, 1965, pp. 338-353.

[2] Atanassov K.T., Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, Vol. 20, 1986, pp. 87-96.

[3] Atanassov K.T., More on Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, Vol. 33, No. 1, 1989, pp. 37-46.

[4] Khatibi V., Montazer G. A., Intuitionistic fuzzy set vs. fuzzy set application in medical pattern recognition, *Artificial Intelligence in Medicine*, Vol. 47, No.1, 2009, pp. 43-52.

[5] Rencova M., An example of applications of Intuitionistic fuzzy sets to sociometry, Bulgarian Academy of Sciences, *Cybernetics and Information Technologies*, Vol. 9, No. 2, 2009, pp. 43-45.

[6] Davvaz B., Sadrabadi E.H., An application of intuitionistic fuzzy sets in medicine, *International Journal of Biomathematics*, Vol. 9, No. 3, 2016, pp.1-15.

[7] Chang, S. L. and Zadeh, L.A., On fuzzy mapping and control, *IEEE Trans, Systems Man Cybernet.*, Vol. 2, 1972, pp. 30-34.

[8] Kandel A. and Byatt, W. J., Fuzzy differential equations, *Proc. Intern. Conf. Cybernet. & Soc.*, Tokyo, Japan, 1978, pp.1213–1216.

[9] Dubois, D. and Prade, H., Towards fuzzy differential calculus: Part 3, differentiation, *Fuzzy Sets and Systems*, Vol. 8, 1982, pp. 225-233

[10] Puri, M. L. and Ralescu, D., Differential for fuzzy function, *J. Math. Anal. Appl.*, Vol. 91, 1983, pp.552-558.

[11] Voxman, R. G., Elementary fuzzy calculus. *Fuzzy sets and systems*, Vol. 18, No. 1, 1986, pp. 31-43.

[12] Kaleva, O., Fuzzy differential Equations, Fuzzy Sets and Systems, Vol. 24, No. 3, 1987, pp. 301-317.

[13] Seikkala S., On the fuzzy initial value problem, Fuzzy Sets Systems, Vol. 24, 1987, pp. 319–330.

[14] Friedman M., Ming M., Kandel A., Solutions to fuzzy integral equations with arbitrary kernels, *International Journal of Approxiamte Reasoning*, Vol. 20, No. 3, 1999, pp. 249-262.

[15] Mondal S.P. and Roy T.K., First order homogeneous ordinary differential equation with initial value as triangular intuitionistic fuzzy number, *Journal of Uncertainty in Mathematics Science*, Vol. 2014, 2014, pp. 1-17.

[16] Mondal S. P. and Roy T.K., Second order linear differential equations with generalized trapezoidal intuitionistic fuzzy boundary value, *Journal of Linear and Topological Algebra*, Vol. 4, No. 2, 2015, pp.115-129.

[17] Mondal S.P. and Roy T. K., Generalized intuitionistic fuzzy Laplace transform and its application in electrical circuit, TWMS *J. Appl. Eng. Math.*, Vol. 5, No. 1, 2015, pp. 30-45.

[18] Biswas S., Banerjee S. and Roy T.K., Solving intuitionistic fuzzy differential equations with linear differential operator by Adomian decomposition method, *3rd Int. IFS Conf. 29th Aug. 1 Sep. 2016, Mersin, Turkey*, Vol. 22, No. 4, 2016, pp. 25-41.

[19] Shapique M. and Jesuraj C., Solutions to Fuzzy differential equations using pentagonal intuitionistic fuzzy numbers, *MAYFEB Journal of Mathematics*, Vol. 2, 2017, pp.8-20.

[20] Melliani S., Chadli L.S., Introduction to intuitionistic fuzzy differential equations, *Notes on IFS*, Vol. 6, No.2, 2000, pp. 31-41.

[21] Melliani S., Chadli L.S., Introduction to intuitionistic fuzzy partial differential equations, *Notes on IFS*, Vol. 7, No. 3, 2001, pp. 39-42.

[22] Abbasbandy S., Viranloo T.A., Numerical solution of Fuzzy differential equations by Runge-kutta method, *Mathematical and Computational Applications*, Vol. 11, No. 1, 2004, pp. 117-129.

[23] Ramazannia Tolouti, S. J. and Barkhordary Ahmadi, M., Fuzzy Laplace Transform on Two Order Derivative and Solving Fuzzy Two Order Differential Equation, *Int. J. Industrial Mathematics*, Vol. 2, No. 4, 2010, pp. 279-293.

[24] Ahmad, N., Mamat, M., Kumar, J. K. and Amir Hamzah, N. S., Solving Fuzzy Duffing's Equation by the Laplace Transform

Decomposition, *Applied Mathematical Sciences*, Vol. 6, No. 59, 2012, pp. 2935-2944.

[25] Saade S.S., Mapping convex and normal fuzzy sets, *Fuzzy Sets and Systems*, Vol. 81, 1996, pp. 251-256.

[26] Nasseri H., Fuzzy Numbers: Positive and Nonnegative, *International Mathematical Forum*, Vol. 3, No. 36, 2008, pp.1777-1780.

[27] Allahviranloo, T. and Barkhordari Ahmadi, M., Fuzzy Laplace transforms, *Soft Computing*, Vol. 14, 2010, pp. 235-243.