Comparison of Newton-Raphson and Kang's Method with newly developed Fuzzified He's Iterative method for solving nonlinear equations of one variable

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Abstract: - Iterative schemes are the important tool for solving nonlinear equations arising in many real life problems. Our literature is rich with lots of iterative schemes, which are useful for solving nonlinear equations of one or more variables. Among them, Newton-Raphson method is the simplest and highly convergent with second order convergence. It is vastly used by researchers, applied mathematicians and engineers. Problems arising in our day to day life cannot be easily describe by crisp values, because in real life situations, always some uncertainty is involved and in those situations we receive some fuzzy values instead of crisp values. So it is immensely important to develop some iterative schemes, which can easily tackle this kind of fuzzy environment. The intent of this paper is to show advantage of using newly developed fuzzified He's iterative method over N-R method and Kang method for solving nonlinear equations of one variable arising in the fuzzy environment. Some numerical examples are illustrated for depicting the efficiency of new fuzzified He's iterative scheme.

Key-Words: - He's iterative method; Nonlinear algebraic equations; Transcendental equations; Newton-Raphson method; Fuzzified iterative scheme; Kang Iterative method.

1 Introduction

Most of the real-life problems, arising in engineering, physics, biology, chemistry or any stream can be easily represented by means of nonlinear algebraic or transcendental equation of the form T(x) = 0. The analytical methods available in the literature are difficult to use or not possible for higher order algebraic equations. When it becomes impossible or extremely difficult to obtain the solution of any nonlinear problem analytically, numerical methods plays impotant role as they provide at least approximate solution to those problems. Many numerical methods are discussed and proposed in literature, allow us to solve most of the nonlinear equations in their usual forms. But due to uncertain environment, these numerical schemes fail to tackle real life situations. Because, the nonlinear equations formed in real life situations cannot describe the problems exactly what they are. Hence in general for any practical situations, if we collect data or information to measure something, then in most of the cases that data is not accurate and if on the basis of that data, we model the problem through some non linear equation then that nonlinear equation also not describes the situation exactly and correctly. Zadeh [1] provides a solution, known as fuzzy set theory to deal with these kinds of situations. On the basis of this fuzzy set theory, many researchers [2-10] studied the fuzzy nonlinear equations in which the parameters used in the equation are fuzzified. Standard numerical techniques dealing with usual nonlinear equation are not suitable in the case of fuzzy nonlinear equation. Therefore it is needed to develop new numerical schemes which deal with these kinds of problems. There are many schemes that are available in numerical analysis by which it is easy to solve nonlinear equations, such as bisection method [2], secant method [3], method of false position [4], fixed point iteration [5], Newton Raphson method [6-7], etc. To deal with fuzzy nonlinear equation, some of the above iterative schemes are fuzzified by several authors [8-10]. Out of these iterative schemes N-R method is the most widely used method [11-12], because it provides faster convergence, but it is also very sensitive to the initial guess and hence this method fails when function derivative becomes zero. Thus to deal with this problems, many modifications [13-23] are proposed in the literature, out of which one is known as He's iteration [13] formula. Kang et al. [14] has proposed a new second order iteration method. Prasad and Sahni [15] has developed a new variant of Newton-Raphson and discussed its convergence. For solving transcendental and non-linear algebraic equations, many other modifications were proposed by researchers, which was based on fixed point iterative method [16] and Newton-Raphson method [17-18]. Saqib et al. [20] has developed a new multi-step iterative scheme to increase the convergence rate. A higher convergence rate of third order was developed by Saqib et al. [21] for solving non-linear equations. Recently in 2018 based on variational iteration technique a new iterative method was developed by Nawaz et al. [22]. Also in recent work of Shah and Sahni [23], one can find a way to calculate optimum number of iterations for fixed point iteration method.

The concept of fuzzy number was introduced by Dubois and Parde [24] which well incorporates the vagueness in the data. Many extensions of fuzzy number were also introduced, such as triangular fuzzy number, trapezoidal fuzzy number and many more. The use of triangular fuzzy number was shown in many research papers [25-26].

Since then a lot of work has been done for the development of fuzzy numbers and fuzzy theory. Triangular fuzzy number [25], which is an extension of fuzzy number, gives better result as compared to fuzzy number. Triangular fuzzy numbers are used in many applications such as student assessment [25], collaborative filtering recommendation to measure users' similarity [26] and many more.

In the present work we have shown advantage of the fuzzified He's iteration method over N-R method and Kang's method by taking several examples containing polynomials and fuzzy transcendental equations.

2 Basic Method and Fuzzification of He's Iteration

In this section, we will define some basic definitions such as fuzzy sets, fuzzy number, triangular fuzzy number and its algebra.

Definition 2.1. (Fuzzy Sets) [1]: Let us consider a non-empty set X. A fuzzy set A defined on the elements of the set X having the membership value $\mu_A(x)$, defined as

$$A = \{< x, \mu_A(x) >: x \in X, \mu_A(x) \in [0,1]\}.$$

Definition 2.2. (Fuzzy Number) [24]: A map $T_a: R \rightarrow [0, l]$ such that

$$T_a(x) = 1 \Leftrightarrow x = a$$

and $\lim(|x| \rightarrow \infty), T(x) = 0$ will be simply called a fuzzy number.

Definition 2.3. Triangular Fuzzy Number [6]: A fuzzy number A is called triangular, if the 3-tuples of real numbers $[a,\ b,\ c]$ with $a \leq b \leq c$ and having membership function are defined as follows:

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a}, & for\ a \leq x \leq b \\ \dfrac{c-x}{c-b}, & for\ b \leq x \leq c \\ 0 & for\ x < a\ and\ x > c \end{cases}$$

Any set containing triangular fuzzy number is called triangular fuzzy set.

Let us suppose $A = [A_1\ \ B_1\ \ C_1]$ and $B = [A_2\ \ B_2\ \ C_2]$ are two triangular fuzzy numbers and either $T(A) > 0, T(B) < 0$ or $T(A) < 0$, $T(B) > 0$, then the root of fuzzy nonlinear equation exists within these two triangular fuzzy numbers. The membership functions of these two triangular fuzzy numbers A and B are defined as:

$$\mu_A(x) = \begin{cases} \dfrac{x - A_1}{B_1 - A_1}, & for\ A_1 \le x \le B_1 \\ \dfrac{C_1 - x}{C_1 - B_1}, & for\ B_1 \le x \le C_1 \\ 0 & for\ x < A_1\ and\ x > C_1 \end{cases}$$

and

$$\mu_B(x) = \begin{cases} \dfrac{x - A_2}{B_2 - A_2}, & for\ A_2 \le x \le B_2 \\ \dfrac{C_2 - x}{C_2 - B_2}, & for\ B_2 \le x \le C_2 \\ 0 & for\ x < A_2\ and\ x > C_2 \end{cases}$$

Definition 2.4. α –cut on Fuzzy set [6]: Let A be any fuzzy interval defined in R and $\alpha \in (0,1]$. The α -cut of A is the crisp set, denoted by $[A]^\alpha$ containing all the elements having some membership degree in A greater than or equal to α, i.e.

$$[A]^\alpha = \{x \in R | A(x) \ge \alpha\}.$$

The α −cuts with respect to triangular fuzzy numbers is defined as

$$[A]^\alpha = [A_1 + \alpha(B_1 - A_1), C_1 + \alpha(B_1 - C_1)]$$

and

$$[B]^\alpha = [A_2 + \alpha(B_2 - A_2), C_2 + \alpha(B_2 - C_2)]$$

Definition 2.5. Addition of two triangular fuzzy numbers [6]:

The addition of two triangular fuzzy numbers $A = [A_1\ \ B_1\ \ C_1]$ and $B = [A_2\ \ B_2\ \ C_2]$ are defined as:

$$[A_1\ \ B_1\ \ C_1] + [A_2\ \ B_2\ \ C_2] = [A_1 + A_2\ \ B_1 + B_2\ \ C_1 + C_2].$$

Definition 2.6. Subtraction of two triangular fuzzy numbers [6]:

The subtraction of two triangular fuzzy numbers $A = [A_1\ \ B_1\ \ C_1]$ and $B = [A_2\ \ B_2\ \ C_2]$ are given as:

$$[A_1\ \ B_1\ \ C_1] - [A_2\ \ B_2\ \ C_2] = [A_1 - C_2\ \ B_1 - B_2\ \ C_1 - A_2].$$

Definition 2.7. Multiplication of two triangular fuzzy numbers [6]:

The multiplication of two triangular fuzzy numbers $A = [A_1\ \ B_1\ \ C_1]$ and $B = [A_2\ \ B_2\ \ C_2]$ is:

$$[A_1\ \ B_1\ \ C_1] \times [A_2\ \ B_2\ \ C_2] = [A_1A_2\ \ B_1B_2\ \ C_1C_2].$$

Definition 2.8. Division of two triangular fuzzy numbers [6]:

The division of two triangular fuzzy numbers $A = [A_1\ \ B_1\ \ C_1]$ and $B = [A_2\ \ B_2\ \ C_2]$ are defined as

$$[A_1\ \ B_1\ \ C_1] / [A_2\ \ B_2\ \ C_2] = [A_1/C_2\ \ B_1/B_2\ \ C_1/A_2].$$

Definition 2.9. Scalar Multiplication of triangular fuzzy numbers [6]:

Let λ be any scalar and $A = [A_1\ \ B_1\ \ C_1]$ be any triangular fuzzy number. Then their scalar multiplication is defined as

$$\lambda[A_1\ \ B_1\ \ C_1] = [\lambda A_1\ \ \lambda B_1\ \ \lambda C_1], \quad \text{where } \lambda > 0$$

$$\lambda[A_1\ \ B_1\ \ C_1] = [\lambda C_1\ \ \lambda B_1\ \ \lambda A_1], \quad \text{where } \lambda < 0$$

Definition 2.10. Square root of a Triangular fuzzy number [25]:

The square root of a triangular fuzzy number is defined as

$$\sqrt{[A_1\ \ B_1\ \ C_1]} = [\sqrt{A_1}\ \ \sqrt{B_1}\ \ \sqrt{C_1}]$$

Definition 2.11. Nth root of a Triangular fuzzy number [25]:

The Nth root of a triangular fuzzy number is defined as

$$\sqrt[n]{[A_1\ \ B_1\ \ C_1]} = [\sqrt[n]{A_1}\ \ \sqrt[n]{B_1}\ \ \sqrt[n]{C_1}]$$

Definition 2.12. Intermediate Value Theorem [6]

Let $I = [A, B]$ be any real valued interval and $T: I \to R$ be any continuous function defined on R. If w be any real number between $T(a) < 0$ and $T(b) > 0$, or $T(a) > 0$ and $T(b) < 0$, then there exists is at least one $c \in (a, b)$ such that $T(c) = w$.

Newton's method also known as Newton-Raphson method is named on great scientist Issac Newton and Joseph Raphson. This is used to find the real and complex roots of a real valued function. It is better than other methods such as bisection, method of false position, fixed point iterative method because of its convergence rate. This iterative method requires only one initial approximation to start an iterative process.

Let the algebraic – linear/ non-linear or transcendental equations in general is represented as $f(x) = 0$. The general formula for Newton Raphson method is as follows:

Definition 2.13. Newton Raphson Method [7]:

Let x_0 be an initial approximation, then $x_1, x_2, \ldots$ is calculated from following general formula:

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, where $n \geq 0$ and is an integer.

Definition 2.14. He's Iterative Formula [13]:

The general iterative formula of He's is given in the following form:

$x_{n+1} = x_n - \frac{T(x_n)}{T'(x_n)+\alpha T(x_n)}$, where $n \geq 0$ and is an integer. Here, α, is a free parameter, known as the control parameter, which can be adjusted according to the need and helps to increase the convergence of the iterative scheme.

Definition 2.15. Kang's Method [15]:

Let in general the given equation is represented as $T(x) = 0$, $x \in R$. The general equation can be converted into the form $x = g(x)$.

For a given x_0, we calculate the approximate solution x_{n+1}, by the iteration scheme

$x_{n+1} = -\frac{g'(x_n)x_n+g(x_n)}{1-g'(x_n)}$, $g'(x_n) \neq 1$.

3 Fuzzified He's Iteration Methodology

We initiate with a nonlinear equation $T(x) = 0$, which may be transcendental, linear or non-linear algebraic equation. The intermediate value theorem (IVT) helps to find the interval of convergence, where function changes its sign, that means, if the value of any continuous function $T(x)$ at $x = A$ is negative and at $x = B$ positive or vice-versa, then the root lies between those two points A and B. In this research paper, we will use triangular fuzzy number for the solution of both non-linear algebraic and transcendental equations. Here He's iterative scheme is fuzzified and the results are compared with original He's iteration method, Newton-Raphson method and the Kang method [15].

To start any iterative scheme, we have to choose some initial value so that the first initial value to the root of the fuzzy nonlinear equation $T(x) = 0$ is either $x_0 = A$ or $x_0 = B$.

According to He's iteration, we have

$x_{n+1} = x_n - \frac{T(x_n)}{T'(x_n)+\alpha T(x_n)}$.

In fuzzy environment, the above scheme reduces for $n = 0$, as

$$x_1 = x_0 - \frac{T(x_0)}{T'(x_0) + \alpha T(x_0)} = [x_1^{(1)}, x_1^{(2)}, x_1^{(3)}]$$

with membership function

$$\mu_{x_1}(x) = \begin{cases} \dfrac{x - x_1^{(1)}}{x_1^{(2)} - x_1^{(1)}} & for\ x_1^{(1)} \leq x \leq x_1^{(2)} \\ \dfrac{x - x_1^{(3)}}{x_1^{(2)} - x_1^{(3)}} & for\ x_1^{(2)} \leq x \leq x_1^{(3)} \\ 0 & for\ x < x_1^{(1)}\ and\ x > x_1^{(3)} \end{cases}$$

and

$$[x_1]^\alpha =$$

$$\left[x_1^{(1)} + \alpha\left(x_1^{(2)} - x_1^{(1)}\right), x_1^{(3)} + \alpha\left(x_1^{(2)} - x_1^{(3)}\right)\right]$$

Similarly,

$$x_2 = x_1 - \frac{T(x_1)}{T'(x_1) + \alpha T(x_1)} = [x_2^{(1)}, x_2^{(2)}, x_2^{(3)}]$$

with membership function

$$\mu_{x_2}(x) = \begin{cases} \frac{x - x_2^{(1)}}{x_2^{(2)} - x_2^{(1)}} & for\ x_2^{(1)} \le x \le x_2^{(2)} \\ \frac{x - x_2^{(3)}}{x_2^{(2)} - x_2^{(3)}} & for\ x_2^{(2)} \le x \le x_2^{(3)} \\ 0 & for\ x < x_2^{(1)}\ and\ x > x_2^{(3)} \end{cases}$$

and $[x_2]^\alpha =$

$$\left[x_2^{(1)} + \alpha\left(x_2^{(2)} - x_2^{(1)}\right), x_2^{(3)} + \alpha\left(x_2^{(2)} - x_2^{(3)}\right)\right]$$

and so on.

4 Fuzzified He's Iteration Methodology

We have taken following examples, whose solutions are given step by step to illustrate the effectiveness of this fuzzy iterative scheme.

Example 1: $T(x) = x^{10} - 1, T^{'}(x) = 9x^5$

Solution

Let $A = [0.49,\ \ 0.5,\ \ 0.51]$,

$B = [1.59,\ \ 1.6,\ \ 1.61]$

be two fuzzy triangular numbers. We see that $T(A) = [-0.9992, -0.9990,\ -0.9988]$,

which is negative and

$T(B) = [102.2693, 108.9512,\ 116.0196]$, which is positive, hence the root lies between two fuzzy triangular numbers A and B using intermediate value theorem.

Now using fuzzified iteration formula for $\alpha = -2$, $x_{n+1} = x_n - \frac{T(x_n)}{T'(x_n) - 2T(x_n)}$,

$x_{n+1} = x_n - \frac{T(x_n)}{T'(x_n) - 2T(x_n)}$, $x_0 = [0.49, 0.5, 0.51]$

$$x_1 = x_0 - \frac{T(x_0)}{T'(x_0) - T(x_0)}$$

$x_1 = [0.49, 0.5, 0.51] -$

$$\frac{[-.9992, -.9990,\ -.9988]}{[.0163, .0195, .0233] - 2[-.9992, -.9990, -.9988]}$$

$x_1 = [0.986, 0.9952, 1.0042]$,

Crisp value = 0.9951.

The crisp value is calculated by taking the mean value of the triangular fuzzy number.

$$x_2 = x_1 - \frac{T(x_1)}{T'(x_1) - 2T(x_1)}$$

$x_2 = [0.986, 0.9952, 1.0042] -$

$$\frac{[-.1315,\ -.0470,\ .0428]}{[8.8083,\ 9.5762,\ 10.3844] - 2[-.1315, -.0470,\ .0428]}$$

$x_2 = [0.9909, 1.0001, 1.0001]$,

Crisp value = 1.0025

Example 2:

$T(x) = e^{sinx} - x,\ T'(x) = \cos x. e^{sinx}\ - 1.$

Solution

For $\alpha = -2$,

$x_{n+1} = x_n - \frac{T(x_n)}{T'(x_n) - 2T(x_n)}$,
$x_0 = [0.99, 1.00, 1.01]$

$$x_1 = x_0 - \frac{T(x_0)}{T'(x_0) - 2T(x_0)}$$

$x_1 = [0.99, 1.00, 1.01] -$

$$\frac{[1.31718, 1.31978,\ 1.32225]}{[.26593,\ .25338,\ .24043] - 2[1.31718, 1.31978,\ 1.32225]}$$

$x_1 = [1.5459, 1.55309, 1.56022]$,

Crisp value = 1.55307.

$x_2 = [1.5459, 1.55309, 1.56022] -$

$$\frac{[1.17154,\ 1.16477,\ 1.15791]}{[-.93235,\ -.95188, -.97125] - 2[1.17154, 1.16477,\ 1.15791]}$$

$x_2 = [1.90238, 1.90805, 1.9137]$,

Crisp value = 1.90804

$x_3 = [1.90238, 1.90805, 1.9137] -$

$-\frac{[0.67179, 0.66134, 0.65085]}{[-1.83799, -1.85020, -1.86226] - 2[.67179, .66134, .65085]}$

$x_3 = [2.10968, 2.11649, 2.12325]$,

Crisp value = 2.11647

$x_4 = [2.10968, 2.11649, 2.12325] -$

$\frac{[0.24942, 0.23434, 0.21930]}{[-2.21064, \ -2.22010, \ -2.22932] - 2[.24942, .23434, .21930]}$

$x_4 = [2.19246, 2.20364, 2.21467]$,

Crisp value = 2.20359.

$x_5 = [2.19246, 2.20364, 2.21467] -$

$\frac{[0.06200, 0.03608, 0.01037]}{[-2.31297, \ -2.32466, \ -2.33569] - 2[.06200, .03608, .01037]}$

$x_5 = [2.1969, 2.21869, 2.23988]$,

Crisp value = 2.21849.

Example 3: $T(x) = x - \cos(x)$,

$T'(x) = 1 + sin(x)$

Solution

Let $A = [-0.01, \ 0, \ 0.01]$, $B = [0.99, \ 1.0, \ 1.01]$,

$T(A) = [-1.00995, -1, -0.99995]$,

which is negative and

$T(B) = [0.44131, 0.45970, 0.47814]$, which is positive. Hence root lies between A and B. The root lies near B, so we choose $x_0 = B$.

For $\alpha = 1.0$, $x_{n+1} = x_n - \frac{T(x_n)}{T'(x_n) + T(x_n)}$,

$x_0 = [0.99, \quad 1.0, \quad 1.01]$

$$x_1 = x_0 - \frac{T(x_0)}{T'(x_0) + T(x_0)}$$

$x_1 = [0.99 \ \ 1.0 \ \ 1.01] -$

$\frac{[0.44131 \ \ 0.45970 \ \ 0.47814]}{[1.8363 \ \ 1.84147 \ \ 1.84683] + [0.44131 \ 0.45970 \ \ 0.47814]}$

$x_1 = [0.78004, 0.80023, 0.82019]$,

Crisp value = 0.80015.

$x_2 = [0.78004, 0.80023, 0.82019] -$

$\frac{[0.06915, 0.10369, \ 0.13811]}{[1.70331, \ 1.71752, \ 1.73128] - [.06915, .10369, \ .13811]}$

$x_2 = [0.6918, 0.73986, 0.77859]$,

Crisp value = 0.73675.

$x_3 = [0.6918, 0.73986, 0.77859] -$

$\frac{[-0.07830, -0.00130, \ 0.06669]}{[1.63792, \ 1.67419, \ 1.70228] + [0.07830, -0.00130, \ 0.06669]}$

$x_3 = [0.64904, 0.74064, 0.82285]$,

Crisp value = 0.73751.

Example 4: $T(x) = \sin x$,

$T'(x) = cos(x)$.

Solution

Let $A = [2.99, \ 3.0, \ 3.01]$, $B = [3.99, \ 4.0, \ 4.01]$,

$T(A) = [0.15101, 0.14112, 0.13121]$, which is positive and

$T(B) = [-0.75023, -0.75680, -0.76330]$, which is negative. Hence root lies between A and B. The root lies near A, so we choose $x_0 = A = [2.99, \ 3.0, \ 3.01]$.

For $\alpha = -1.0$, $x_{n+1} = x_n - \frac{T(x_n)}{T'(x_n) - T(x_n)}$, $x_0 = [2.99, \ 3.0, \ 3.01]$

$$x_1 = x_0 - \frac{T(x_0)}{T'(x_0) - T(x_0)}$$

$x_1 = [2.99, \quad 3.0, \quad 3.01] -$

$\frac{[0.15101, 0.14112, \ 0.13121]}{[-0.98853, -0.98999, -0.99135] - [0.15101, 0.14112, \ 0.13121]}$

$x_1 = [3.10718, 3.12476, 3.14219]$,

Crisp value = 3.12471.

$$x_2 = [3.10718, 3.12476, 3.14219] - \frac{[0.03441, 0.01683, -0.00060]}{[-0.99941, -0.99986, -1.0] - [0.03441, 0.01683, -0.00060]}$$

$$x_2 = [3.10658, 3.14131, 3.17546],$$

Crisp value = 3.14112

Following Table I shows the comparison between Newton-Raphson method, He's iteration method, Kang method and fuzzified He's iteration method.

TABLE I
COMPARISON BETWEEN THE NEWTON RAPHSON METHOD, HE'S ITERATION METHOD, KANG METHOD AND THE FUZZIFIED HE'S ITERATION METHOD

Newton-Raphson Method			**He' iteration Method**			**Kang Method**			**Fuzzy He's iteration Method**		
Example	n	x_n	α	n	x_n	n	$\emptyset(x)$	x_n	α	n	x_n
Ex. 1	42	1.0	-1	8	1.0	-	$1/x^9$	Div.	-2	2	1.0025
Ex. 2	20	2.2191	-1	3	2.2191	-	$\sin^{-1}(\ln(x))$	Div.	-2	5	2.21849
Ex. 3	28	0.7391	1	9	0.7391	3	cos(x)	0.7390	1	3	0.73751
Ex. 4	7	3.1416	-1	6	3.1416	-	-		-1	2	3.14112

From Table I, it is observed that the new fuzzified He's method is very fast convergent with respect to the other given methods. One can easily see that the Newton-Raphson method and He's iteration method have slow convergence rate. For the Kang's method we have shown that in example 3, the method is highly convergent but fails to converge in first two examples, whereas for the fourth example there exists no fixed point function to apply the kang method. So, we conclude that the fuzzified He's iterative method is very effective method to apply for solving nonlinear algebraic or transcendental equations containing one variable. This is a new approach for solving fuzzified algebraic or transcendental equations.

4 Conclusion

In the present study we have solved nonlinear equations of one variable by fuzzyfying the He's iteration method using triangular fuzzy number, which is the modification of Newton-Raphson method. In this way we have provided a new fuzzified iterative method which can be used for solving various fuzzy nonlinear algebraic and transcendental equations. He's iteration scheme in a fuzzified form is not applied earlier as per our literature review. Earlier, fuzzified iterative schemes were also not applied on transcendental equations. Here we have also solved the transcendental equations using the fuzzified He's iterative scheme.

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