Double-parameter ridge-type Kalman filter based on signal-to-noise ratio test

HAO LI, YONGWEI GU, SHUMEI GUO, GUOCHAO ZHANG
Basic research Department
Information Engineering University
Henan Zhengzhou 450001
CHINA
lihao6618@163.com

Abstract: In this paper, the ill-conditioning diagnosis and processing of Kalman filter are combined. First, the ill-conditioning of Kalman filter and the disadvantage of ridge-type Kalman filter are analyzed. Then t he signal-to-noise ratio(SNR) statistic is introduced to measure how much each parameter suffers from the ill-conditioning. Accordingly, all parameters are divided into two parts, named involved parameters and n on-involved parameters respectively. Then, the two parts of parameters are corrected with two ridge para meters of different size. This method is named double-parameter ridge-type Kalman filter and can reduce the bias introduced in ridge-type Kalman filter while reducing the variance of the state parameter estimati on. Combined with the idea of generalized ridge estimation, the selection method of two ridge parameters are given. Finally, the example illustrates the new algorithm can effectively overcome the influence of th e ill-condition on K alman filter and the reduce the bias in ridge-type Kalman filter, which improves the a ccuracy of the estimates of parameters.

Key-Words: Kalman filter; ill-conditioning; double-parameter ridge estimation; signal-to-noise ratio

Kalman filter is one of the most commonly used methods in dynamic data processing. It has been studied and widely used in geodesy, satellite navigation and satellite orbit [1-5]. In order to overcome the influence of the observation matrix ill-conditioning and improve the accuracy of the parameter estimation, many scholars have given the improved algorithms [6-10]. At present, some scholars have proposed some methods from the perspective of biased estimation to solve the ill-conditioned problems in the discrete dynamic system. Tan [8] proposed biased Kalman filter by combining biased estimations with Kalman filter. Han Songhui et al. [10] combined the ridge regression with Kalman filter to overcome the adverse effects of the observation matrix ill-conditioning on the filtered values by correcting the gain matrix. Li Yongming [11] proposed biased Kalman filter and ridge-type Kalman filter as well as their algorithms by combining biased estimation and ridge regression with Kalman filter, and also gave the selection methods of the compression coefficient and the ridge parameter.

One of the common shortcomings of the above methods is that the diagnosis and processing of the discrete system ill-conditioning is not combined together. It is not considered that different parameters suffer differently from the ill-conditioning. The actual experience shows, the harm of the observation matrix ill-conditioning to each parameter is different. The size of this harm is related to the size of the parameter itself, but also to the degree the corresponding observation matrix data column involved in the collinearity [1]. In this paper,the processing of Kalman filter ill-conditioning is combined with the harm measurement. The signal-to-noise ratio statistic is used to measure how much parameters suffering from ill-conditioning. According to the measured results, the corresponding measure is adopted to improve the ridge Kalman filter algorithm, which further reduces the effect of ill-conditioning on estimation.

1 Kalman filter algorithm and ill-conditioning analysis

1.1. Discrete dynamic systems and Kalman filter basic equations

Consider the dynamic system described by the following state space model [1].

$$X_{k+1}=\boldsymbol{\Phi}_{k+1,k}X_k+\boldsymbol{W}_k \tag{1}$$

$$Y_k=H_kX_k+\boldsymbol{V}_k \tag{2}$$

In the formula, k is the discrete time, $X_k \in R^n$ is the state of the system in the time t_k; $Y_k \in R^m$ is the corresponding observed signal; $\boldsymbol{\Phi}_{k+1,k}$, the $p \times p$ dimension non-singular matrix, is the one-step transition matrix from time t_k to t_{k+1}; $\boldsymbol{H}_k$ is the observation matrix; $W_k \in R^r$ is the input white noise; formula (1) is the state equation, and formula (2) is the observation equation.

W_k and V_k meet

$$\begin{aligned}&E(W_k)=0,\mathrm{Cov}(W_k,W_j)=E(W_kW_j^{\mathrm{T}})=Q_k\delta_{kj}\\&E(V_k)=0,\mathrm{Cov}(V_k,V_j)=E(V_kV_j^{\mathrm{T}})=R_k\delta_{kj}\\&\mathrm{Cov}(W_k,V_j)=E(W_kV_j^{\mathrm{T}})=0\end{aligned} \tag{3}$$

In the formula, the variance matrix of the input noise Q_k is assumed to be a nonnegative matrix, and the variance matrix of the observed noise R_k is assumed to be a positive definite matrix. It can be seen from Eq. (3) that W_k and V_k are uncorrelated white noises with zero mean, and that Q_k and R_k are variance matrixes of W_k and V_k.

Basic equations of Kalman filter are as bellows:

One-step state prediction:

$$\hat{\hat{X}}_{k+1/k}=\boldsymbol{\Phi}_{k+1,k}X_k \tag{4}$$

Covariance matrix of one-step prediction:

$$P_{k+1/k}=\boldsymbol{\Phi}_{k+1,k}P_k\boldsymbol{\Phi}_{k+1,k}^{\mathrm{T}}+Q_k \tag{5}$$

Filter gain matrix:

$$K_{k+1}=P_{k+1/k}H_{k+1}^{\mathrm{T}}[H_{k+1}P_{k+1/k}H_{k+1}^{\mathrm{T}}+R_{k+1}]^{-1} \tag{6}$$

Status update:

$$\hat{\hat{\hat{X}}}_{k+1}=X_{k+1/k}+K_{k+1}(Y_{k+1}-H_{k+1}X_{k+1/k}) \tag{7}$$

Covariance matrix of state estimation:

$$P_{k+1}=(I-K_{k+1}H_{k+1})P_{k+1/k}(I-K_{k+1}H_{k+1})^{\mathrm{T}}+K_{k+1}R_{k+1}K_{k+1}^{\mathrm{T}} \tag{8}$$

(4)~ (8) are the basic equations of recursive Kalman filter. Given the initial value $\hat{X}_0$, P_0, and the observation at time t_{k+1}, the state estimation can be recursively calculated.

1.2. The influence of observation matrix

ill-conditioning on Kalman filter state estimation

By Kalman filter basic equation [12], the state estimation of the moment t_{k+1} can also be expressed as:

$$\hat{X}_{k+1} = (H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} H_{k+1} + P_{k+1/k}^{-1})^{-1} (H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} Y_{k+1} + P_{k+1/k}^{-1} \hat{X}_{k+1/k}) \tag{9}$$

It is the solution of Eq. (10).

$$(H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} H_{k+1} + P_{k+1/k}^{-1}) \hat{X}_{k+1} = (H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} Y_{k+1} + P_{k+1/k}^{-1} \hat{X}_{k+1/k})$$

(10)

Among them, $N_{k+1} = H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} H_{k+1} + P_{k+1/k}^{-1}$,

$l_{k+1} = H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} Y_{k+1} + P_{k+1/k}^{-1} \hat{X}_{k+1/k}$. N_{k+1} is called the normal matrix of Kalman filter, and can be proved as a nonnegative matrix [1]. The eigen-decomposition of N_{k+1} is as below:

$$N_{k+1} = U_{k+1} \Lambda_{k+1} U_{k+1}^{\mathrm{T}} \tag{11}$$

Then

$$\begin{aligned}\hat{X}_{k+1} &= (H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} H_{k+1} + P_{k+1/k}^{-1})^{-1} [H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} Y_{k+1} + P_{k+1/k}^{-1} X_{k+1/k}] \\ & U_{k+1} \Lambda_{k+1}^{-1} U_{k+1}^{\mathrm{T}} \; H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} Y_{k+1} + P_{k+1/k}^{-1} \hat{X}_{k+1/k}] \\ &= \sum_{i=1}^{t} u_{k+1}^{i} (u_{k+1}^{i})^{\mathrm{T}} \frac{1}{\sigma_{k+1}^{i}} [H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} Y_{k+1} + P_{k+1/k}^{-1} \hat{X}_{k+1/k}]\end{aligned} \tag{12}$$

among them,

$U_{k+1} = (u_{k+1}^1, u_{k+1}^2, ..., u_{k+1}^t); U_{k+1}^{\mathrm{T}} U_{k+1} = I_t$,

$\sigma_{k+1}^1 \geq \sigma_{k+1}^2 \geq ... \geq \sigma_{k+1}^t > 0$

$\Lambda_{k+1} = \mathrm{diag}(\sigma_{k+1}^1, \sigma_{k+1}^2, ..., \sigma_{k+1}^t)$.

If the observation matrix is ill-conditioned, then the combined effect of H_{k+1} and $P_{k+1/k}^{-1}$ is likely to make N_{k+1} also ill-conditioned. Actual work shows that the ill-conditioning of the observation matrix is weekly controlled by $P_{k+1/k}^{-1}$, and its adverse effect on the state estimation cannot be eliminated by $P_{k+1/k}^{-1}$ [11]. So, if N_{k+1} has one or more small eigenvalues, and there is a small observation error or deviation, the reciprocal of the small eigenvalue in equation (12) will amplify the error or deviation, so that the evaluation deviates from the true value far.

2 Double-parameter ridge-type Kalman filter based on SNR test

2.1. Ridge-type Kalman filter

The complete algorithm of ridge-type Kalman filter (RTKF) is given in [11]. The ridge-type Kalman filter state estimation can be expressed as:

$$\begin{aligned}\hat{X}_{k+1} &= (H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} H_{k+1} + P_{k+1/k}^{-1} + \alpha_{k+1} I)^{-1} \left(H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} Y_{k+1} + P_{k+1/k}^{-1} X_{k+1/k}\right) \\ &= \left(N_{k+1} + \alpha_{k+1} I\right)^{-1} \left(H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} Y_{k+1} + P_{k+1/k}^{-1} \hat{X}_{k+1/k}\right) \\ &= \sum_{i=1}^{t} u_{k+1}^{i} (u_{k+1}^{i})^{\mathrm{T}} \frac{1}{\sigma_{k+1}^{i} + \alpha_{k+1}} \left(H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} Y_{k+1} + P_{k+1/k}^{-1} \hat{X}_{k+1/k}\right)\end{aligned} \tag{13}$$

α_{k+1} is the ridge parameter. Ridge-type Kalman filter is to use the ridge parameter to suppress the small eigenvalues and reduce the estimated variance, thus weakening the amplification of the observation error.

Ridge-type Kalman filter has two defects. One is not using pathological information and making the same correction for all parameters, resulting in blindness of the correction for parameters; The other is the introduction of deviation by the ridge parameter. The deviation may be amplified in the continuous recursive process, thus affecting the accuracy of the estimation. Therefore, in order to improve the accuracy of the estimation, the introduction of bias should be minimized. Based on the above two aspects, double-parameter ridge-type Kalman filter based on SNR test is proposed. The idea is to divide the state parameters into two parts according to the ill-conditioned information and correct them in different intensities. By such fine processing, the introduction of bias is reduced while effectively reducing the influence of ill-conditioning.

2.2. Duoble-parameter ridge-type Kalman filter based on SNR test

From the above analysis, the state parameters of time t_{k+1} are estimated as the solution of the equation (10). If the normal matrix N_{k+1} is ill-conditioned, the state estimation of t_{k+1} time will become extremely unstable. This is how the ill-conditioning of the observation matrix affect the state estimation. The reason for the existence of ill-conditioning in normal matrix N_{k+1} is that there is a linear relationship between the data columns, which leads to the state estimation of t_{k+1} time is not so accurate. But not all the state estimations are not satisfactory. Study found that the ill-conditioning in normal matrix has a large effect on the estimation of the state parameters corresponding to the data columns involved in collinearity, and a small effect on the estimation of the state parameters corresponding to the data columns not involved in collinearity.[13].

As is shown in equation (14), by calculating the SNR statistic of each state estimation component, the estimated effect of each parameter is distinguished.

$$F_{k+1}^{i} = (\hat{X}_{k+1}^{i})^{2} / \mathrm{Var}(\hat{X}_{k+1}^{i}) = (\hat{X}_{k+1}^{i})^{2} * \frac{1}{\mathrm{Var}(\hat{X}_{k+1}^{i})} \tag{14}$$

$\hat{X}_{k+1}^{i}$ is the Kalman filter estimation of the i_{th} parameter at time t_{k+1}. F_{k+1}^{i} obeys the non-central distribution $\chi^2_{1,\tau}$, and $\tau = X_{k+1}^{i} / \mathrm{Var}(\hat{X}_{k+1}^{i})$ is the non-center parameter.

Equation (14) is called the SNR statistic of parameter i. And then use the test rule in [13]: when $F_i \le \chi^2_{1,\tau}(\omega)$, it is considered that the corresponding parameter is influenced more seriously by the collinearity, and its estimation effect is not good; When $F_i > \chi^2_{1,\tau}(\omega)$, it is considered that the collinearity has little harm to the corresponding parameters, and its estimation is good. ω is the significance level; $\chi^2_{1,\tau}(\omega)$ is the upper ω quantile of the non-central distribution $\chi^2_{1,\tau}$. In practice, the selection of the threshold can be determined flexibly according to the specific situation, and it is not necessary to stick to the quantile determined by the significance level. The specific selection method can be found in the literature [13].

By calculating the SNR statistic, the state parameters of time t_{k+1} can be divided into two parts $X = (X_1^{\mathrm{T}}, X_2^{\mathrm{T}})^{\mathrm{T}}$. The parameters with small SNR statistics are X_1 and they are more harmful by the collinearity, called as involved parameters; Other parameters with lager SNR statistics are X_2 and they are less harmful by the multicollinearity, known as non-involved parameters. For X_1, due to its poor estimation, this part of parameters is more greatly modified; For X_2, it is modified relatively slightly. Accordingly, the correction matrix is structured as below:

$$
\begin{array}{cl}
Z_{k+1} = \begin{bmatrix} \alpha_{k+1}^1 & 0 & & \cdots & 0 & 0 \\ 0 & \ddots & & & & 0 \\ & & \alpha_{k+1}^1 & & & \\ \vdots & & & \alpha_{k+1}^2 & & \vdots \\ 0 & & & & \ddots & 0 \\ 0 & 0 & & \cdots & 0 & \alpha_{k+1}^2 \end{bmatrix} & \begin{array}{l} \\ \text{the } s_{th} \text{ line} \\ \text{the } (s+1)_{th} \\ \text{line} \\ \\ \end{array}
\end{array} \tag{15}
$$

α_{k+1}^1 and α_{k+2}^2 are both ridge parameters, and $\alpha_{k+1}^2 > \alpha_{k+2}^2$ .Then the state estimation of double-parameter ridge-type Kalman filter based on SNR test (DPRTKF) is given:

$$
\begin{aligned} \hat{X}_{k+1} = & (H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} H_{k+1} + P_{k+1/k}^{-1} + Z_{k+1})^{-1} \\ & (H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} Y_{k+1} + P_{k+1/k}^{-1} \hat{X}_{k+1/k}) \end{aligned} \tag{16}
$$

2.3. The selection method of two ridge parameters

It is a very important problem in application to determine the ridge parameters α_{k+1}^1 and α_{k+1}^2 of time t_{k+1} reasonably. Noting that double-parameter ridge estimation is a special case of generalized ridge estimation, and referencing the idea of Hoerl-Kennard method for determining the ridge parameters, the following method is proposed to determine α_{k+1}^1 and α_{k+1}^2 [14-16].

Let $\boldsymbol{\theta}_{k+1} = (\boldsymbol{\theta}_{k+1}^1, \boldsymbol{\theta}_{k+1}^2, \cdots, \boldsymbol{\theta}_{k+1}^n)^{\mathrm{T}} = U_{k+1}^{\mathrm{T}} X_{k+1}$,

$\boldsymbol{\theta}_{k+1}$ is called the normal parameter [17], whose least square estimation is

$$
\begin{aligned} \hat{\boldsymbol{\theta}}_{k+1} & = (\boldsymbol{\theta}_{k+1}^1, \boldsymbol{\theta}_{k+1}^2, \cdots, \boldsymbol{\theta}_{k+1}^n)^{\mathrm{T}} \\ & = \boldsymbol{\Lambda}_{k+1}^{-1} U_{k+1}^{\mathrm{T}} (H_{k+1}^{\mathrm{T}} R_{k+1}^{-1} Y_{k+1} + P_{k+1/k}^{-1} \hat{X}_{k+1/k}) \end{aligned} \tag{17}
$$

Two ridge parameters α_{k+1}^1 and α_{k+1}^2 are taken as:

$$
\alpha_{k+1}^1 = \frac{1}{(\hat{\theta}_{k+1}^i)_{\max}^2}, \quad \alpha_{k+1}^2 = c * \alpha_{k+1}^1, \quad 0 < c < 1 \tag{18}
$$

In summary, the complete algorithm of double-parameter Kalman filter based on SNR test is as follows:

Step 1, initialize: give the initial value of state parameter $\hat{X}_0$ and its mean square error $\hat{P}_0$

Step 2, time update

$$
\hat{X}_{k+1/k} = \boldsymbol{\phi} X_{k/k} \tag{19}
$$

$$
P_{k+1/k} = \boldsymbol{\phi} P_{k/k} \boldsymbol{\phi}^{\mathrm{T}} + \boldsymbol{\Gamma} Q \boldsymbol{\Gamma}^{\mathrm{T}} \tag{20}
$$

Step 3, status update

$$
\hat{X}_{k+1} = X_{k+1/k} + K_{k+1}(Y_{k+1} - H_{k+1} X_{k+1/k}) \tag{21}
$$

$$
K_{k+1} = P_{k+1/k} H_{k+1}^{\mathrm{T}} [H_{k+1} P_{k+1/k} H_{k+1}^{\mathrm{T}} + R_{k+1}]^{-1} \tag{22}
$$

$$
\begin{aligned} & P_{k+1} \\ & = (I - K_{k+1} H_{k+1}) P_{k+1/k} (I - K_{k+1} H_{k+1})^{\mathrm{T}} \\ & + K_{k+1} R_{k+1} K_{k+1}^{\mathrm{T}} \end{aligned} \tag{23}
$$

Step 4, determine whether the number of conditions in the matrix is greater than 500, if more than 500, then proceed Step 5, otherwise return to step 2.

Step 5, use the signal to noise ratio test to determine the involved parameters and non-involved parameters.

Step 6, determine the two ridge parameters α_{k+1}^1 and α_{k+1}^2.

Step 7, use the DPRTKF estimation to correct the state estimation, and calculate the mean square error

$$\hat{X}_{k+1}^{\mathrm{DPRTKF}} = (H_{k+1}^{\mathrm{T}}R_{k+1}^{-1}H_{k+1} + P_{k+1/k}^{-1} + Z_{k+1})^{-1}(H_{k+1}^{\mathrm{T}}R_{k+1}^{-1}Y_{k+1} + P_{k+1/k}^{-1}\hat{X}_{k+1/k}) \tag{24}$$

$$\begin{aligned}\hat{P}_{k+1}^{\mathrm{DPRTKF}} = {} & M_{k+1}^{-1}(N_{k+1}X_{k+1}X_{k+1}^{\mathrm{T}} + I)N_{k+1}M_{k+1}^{-1} \\ & -M_{k+1}^{-1}N_{k+1}X_{k+1}X_{k+1}^{\mathrm{T}} \\ & -X_{k+1}X_{k+1}^{\mathrm{T}}N_{k+1}M_{k+1}^{-1} + X_{k+1}X_{k+1}^{\mathrm{T}}\end{aligned} \tag{25}$$

among them

$$M_{k+1} = H_{k+1}^{\mathrm{T}}R_{k+1}^{-1}H_{k+1} + P_{k+1/k}^{-1} + Z_{k+1} \tag{26}$$

Step 8, let $\hat{X}_k = X_{k+1}^{\mathrm{DPRTKF}}$ $P_k = P_{k+1}^{\mathrm{DPRTKF}}$ and return to step 2, then use Kalman filter to enter the next time state parameter estimation.

3 Simulation and analysis

Computer simulations are used to verify the validity of the new algorithm described in the previous sections. We consider a d iscrete linear system described by the state equation (1) and observation equation (2), where state $X_k \in R^n$ is estimated. The state transition matrix $\boldsymbol{\Phi}_{k+1,k}$, observation matrix H_k, system noise covariance Q_k and observation noise covariance R_k are set as follows:

$$\boldsymbol{\Phi}_{k+1,k} = \begin{pmatrix} 1.3108 & 0.1503 & 0.9499 & 0.2050 & -0.1128 \\ -0.3095 & -0.2044 & -0.5043 & -0.4275 & 0.7652 \\ -0.5322 & 0.0568 & 0.0425 & 0.2323 & -0.2351 \\ -0.2435 & -0.0473 & 0.4017 & -0.5181 & 0.0249 \\ -0.0572 & 0.3914 & 0.1991 & -0.7387 & -0.2515 \end{pmatrix}$$

$\boldsymbol{a}_{k1} = [15.57 \quad 44.02 \quad 20.42 \quad 18.74 \quad 49.20$
$44.92 \quad 55.48 \quad 59.28 \quad 94.39 \quad 128.02 \quad 96.00$
$131.42 \quad 127.21 \quad 252.90 \quad 409.20 \quad 463.70 \quad 510.22]^T$

$\boldsymbol{a}_{k2} = [2643 \quad 2048 \quad 3940 \quad 6505 \quad 5723$
$11520 \quad 5779 \quad 5969 \quad 8461 \quad 20106 \quad 11113$
$10771 \quad 45543 \quad 36194 \quad 34703 \quad 39204 \quad 86533]^T$

$\boldsymbol{a}_{k4} = [18.0 \quad 9.5 \quad 12.8 \quad 36.7 \quad 35.7$
$24.0 \quad 43.3 \quad 46.7 \quad 76.7 \quad 180.5 \quad 60.9$
$103.7 \quad 126.8 \quad 157.7 \quad 169.4 \quad 331.4 \quad 371.6]^T$

$\boldsymbol{a}_{k5} = [4.45 \quad 6.92 \quad 4.28 \quad 3.90 \quad 5.50$
$4.60 \quad 5.62 \quad 5.15 \quad 6.18 \quad 6.15 \quad 5.88$
$4.68 \quad 4.88 \quad 5.57 \quad 10.78 \quad 7.05 \quad 6.35]^T$

$a_{k3} = 2a_{k1} + 0.5a_{k4} + e_k$, $e_k \sim N_{17}(0, 0.05^2 I)$

$A_k = [a_{k1}\ a_{k2}\ a_{k3}\ a_{k4}\ a_{k5}]$, $Q_k = I_5$,

$R_k = 0.5^2 \times I_{17}$

The initial value is $\hat{\boldsymbol{x}}_0 = \boldsymbol{x}_0 + 0.01 \times [1\ 1\ 1\ 1\ 1\ 1]^{\mathrm{T}}$, where $\boldsymbol{x}_0 = [200 \quad 15 \quad 35 \quad 16 \quad -2.8 \quad 6]^{\mathrm{T}}$ and the initial error covariance matrix is $\hat{P}_0 = I_5$ .The condition number of the normal matrix $H_{k+1}^{\mathrm{T}}R_{k+1}^{-1}H_{k+1}$ is 4.05×10^{11}, which means that normal equation is ill-conditioned seriously. Choosing the ridge parameters α_{k+1}^1 and α_{k+2}^2 by means of method (18) and compared the new algorithm proposed in this paper with Kalman filter and Ridge-Type Kalman Filter, the results are described as Fig.1-Fig.3.

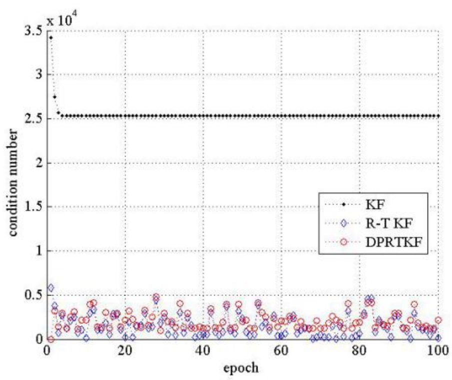

Fig. 1: Comparison of condition number

between Kalman filter , ridge-type Kalman filter and DPRTKF filter.

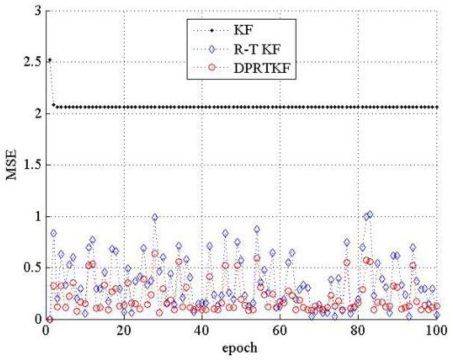

Fig. 2: Comparison of between MSE Kalman, ridge-type Kalman filter and DPRTKF .

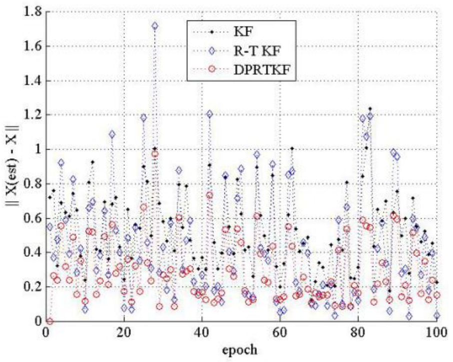

Fig. 3: Comparison of Euclidean distance between Kalman filter , ridge-type Kalman filter and DPRTKF.

It can be concluded from Fig.1-Fig.3 that:

(1) The condition number of $H_{k+1}^{\mathrm{T}}R_{k+1}^{-1}H_{k+1}+P_{k+1/k}^{-1}$ is reduced by $P_{k+1/k}^{-1}$ to some extent, compared with that of $H_{k+1}^{\mathrm{T}}R_{k+1}^{-1}H_{k+1}$ .However, it still has strong ill-condition, and both R-T KF algorithm and DPRTKF algorithm can reduce the condition number of

$H_{k+1}^{\mathrm{T}}R_{k+1}^{-1}H_{k+1}+P_{k+1/k}^{-1}$ effectively.

(2)The DPRTKF algorithm weakens the ill-condition of normal equation all the time and it works better than Kalman filter and ridge-type Kalman filter in the sense of the MSE.

(3)Compared to the Kalman filter algorithm, the solution of R-T KF algorithm has a larger Euclidean distance with the true value because of the bias brought by the ridge parameter. However, the DPRTKF algorithm improves R-T KF algorithm by reducing both the variance of the state estimation and the deviation of the R-T KF estimation. So, the solution of DPRTKF algorithm has a shorter Euclidean distance with the true value than RTKF algorithm.

4 Conclusion

In this paper, the DPRTKF algorithm is used to combine the ill-conditioning diagnosis with the ridge-type Kalman filter. According to the SNR statistic of each parameter, all the parameters are divided into two parts, the involved parameters and the non-involved parameters, and the two parts of parameters are corrected with ridge parameters of different size. For the involved parameters, the corresponding ridge parameter is relatively large, and for the non-involved parameters, the corresponding ridge parameter is relatively small. This meticulous correction of the proposed method reduces the variation of the deviation in the ridge Kalman filter while reducing the variance of the state estimation in Kalman filter.

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References:

[1] Y. Y. Qin and H. Y. Zhang and S. H. Wang, *Theory of Kalman Filter and Integrated Navigation Principles*, Northwestern Polytechnical University Press, 2012.

[2] D. E. Catlin, Estimation, Control, and the Discrete Kalman Filter, Springer, 1989.

[3] C. X. Zhang and J. J Yue, Application of an improved adaptive chaos prediction model in aero-engine performance parameters, WSEAS Trans-actions on M athematics. Vol.11, 2012, pp. 114-124.

[4] A. A. Keller, l_p-Norm Minimization Method for Solving Nonlinear Systems of Equations, WSEAS Transactions on Mathematics. Vol.13, 2014, pp. 654-665.

[5] N. A. Nechval and G. Berzins and M. Pur-gailis, Improved Estimation of State of Stochas-tic Systems via Invariant Embedding Technique, WSEAS Transactionson Mathematics. Vol.7,2008, pp. 141-159.

[6] Kaipio J and Somersalo E, Nonstationary Inverse Problems and State Estimation, Journal of Inverse Ill-Posed Problems, Vol.7, 1998, pp. 273-282.

[7] Baroudi D and Kaipio J and somersalo E, Dynamical Electric Wire Tomography: Time Series Approach. Inverse Problems, Vol.14, 1998, pp. 799-813.

[8] Tan Jiajia and Li Dan and Zhang Jianqing, Biased Kalman Filter, International Conference on S ensing Technology, Palmerson North, 2011.

[9] Ou Jikun and Ding Wenwu and Liu Jihua, An Improved Algorithm for Autonomous Orbit Determination of Navigation Satellite Constellation, Survey Review, Vol.43, 2011, pp.361-369.

[10] Han Song-hui and DU Lan and Gui Qing-ming and GU Yong-wei, Characteristics of Diagnostic Complex Multicollinearity and Its Application in GEO Orbit Determination, Acta Metallurgica Sinica, Vol.1, 2013, pp.19-26.

[11] Li Yongming and Gui Qingming and Gu Yongwei, Ridge-Type Kalman Filter and Its Algorithm,WSEAS Transactions on Mathematics, Vol. 13, 2014, pp. 852-862.

[12] Yang Yuanxi, Adaptive Navigation and Kinematic Positioning, Surveying and Mapping Press , 2006.

[13] Gu Yongwei, Regularization methods based on multicollinearity diagnosis and their applications to geodesy , Information Engineering University, 2010.

[14] Han Songhui and Gui Qingming and Li Jianwen, Ridge—Type EKF of Distributed Autonomous Orbit Determination, Geomatics and Information Science of Wuhan University, Vol. 38, 2013, pp. 399-402.

[15] Hoerl AE and Kennard R W, Ridge Regression: Applications to Non-Orthogonal Problems, Technometrics, Vol. 12, 19 70, pp. 69-82.

[16] Han Songhui and Du Lan and Gui Qingming, Characteristics Analysis Approach for Multicollinearity Diagnosis and Its Applications in Orbit Determination of GEO Satellites, Acta Geodaetica et Cartographica Sinica, Vol. 42,2013, pp.19-26.

[17] Wang Songgui, The Theory of Linear Model and Its Application, Anhui Education Press, 1987.