Research on the Micro-business Information Dissemination in Complex Network

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Abstract: Based on the WeChat platform, Micro-business is a new mode of electricity providers as a set of mobile and social integration. In combination with the SEIR model of the social network, an ignorant-hidden-spreader-immune (IHSR) model was established which is suitable for the micro-business information dissemination (MBID). Using the characteristics of micro-business and circle of friends in the WeChat network, the mechanism of information dissemination and the influence of network parameters on the process of information dissemination are analyzed. By means of the interactive Markoff chain, the mean field equations are obtained, which reflect the change of MBID process with time. We study the similarities and differences of dynamic characteristics between homogeneous networks and inhomogeneous networks by analyzing the equilibria and their stability of the equations. The simulations show that the model basically reflect the trend of the information dissemination on the micro-business network.

Key–Words: Micro-business, information dissemination, IHSR, complex network, circle of friends

1 Introduction

With the development of the Internet and mobile terminals, people are moving into the real mobile information age. Taking our most familiar Taobao and Jingdong which are largest shopping sites in China as examples, their businesses are also transforming from the traditional large-scale platform to the small personal trading platforms. As an important mobile terminal, the mobile phone is closely related to people's life and learning and gradually becomes an irreplaceable items. WeChat is the most popular and widely used social service software in China. Micro-business are the individuals who disseminate product information (including to text or picture and so on) to their circle of friends through WeChat, and ultimately aim at achieving the effective information dissemination and making a profit. Micro-business are more flexible than the traditional Internet sales model due to removability and flexibility. Micro-business possess the characteristics of a simple operating, low investment, stable customers as well as wide and rapid propagation so that it is becoming an extremely popular occupation. Chinese consumers prefer shopping in the micro-business within friends circle with higher credits. In recent years, the number of micro-business is growing rapidly up to 1 million so far.

Although China's micro-business is booming, there are still some problems such as lack of personal contact, lack of protected transactions and data privacy. Typically, when people login the circle of friends in order to acquire the latest dynamics around friends, or to browse some new things and share them, if the micro-business occasionally disseminate one or two messages, you will feel it fresh and interesting. Once micro-business refresh repeatedly the product information in the circle of friends or the quality of the product is poor, which will cause most people's feelings of disgust, then people will block the message or exit the circle of friends[1].

At present, the information dissemination models of micro-business are based mainly on the model of infectious disease and social network transmission. Many models of wireless networks have been investigated in ([2]–[4]). However, most of them are not directly applied in complex networks due to assuming the transmission path between the two nodes is stable. DK and MK models have been widely used in the quantitative study of the spread of rumors([5]–[11]). But these models are unsuitable to describe the spreading mechanism of large-scale social network rumors for lacking the topological features of social networks[12]. After that, based on the epidemic dynamics theory and ISR models, the information dissemination models were established. Wang et al.[4] studied a rumor propagation model in complex social networks by an approach similar to the dynamics of epidemic. Nekovee et al.[13] introduced an SIR's rumor propagation model with the complex social network and discussed the corresponding thresholds in homogeneous and inhomogeneous networks, respectively. Zhang et al. in[14] proposed an online social network model including node degree and epidemiology. Zhang et al.[15] proposed an improved SI model to reduce the information propagation in online social networks by involving the topological relation. Wu et al. [16] proposed a basic and an extended model to evaluate the performance of information by supposing that information spreads among any node whether they are friends or not. In[17], the topological structure of the WeChat network was analyzed and the statistical properties such as degree distribution, clustering coefficient, average path length and so on were researched.

Modeling of the information dissemination is often predicated on the assumption that the user are always online, which is difficult to achieve in the reality. Even though considerably more attention has been given to the information dissemination models in social network, the study on the micro-business model of information dissemination is relatively rarely. In the micro-business network, the state of online or offline determined by the user's work, life and other habits, has an important influence on the dissemination of information. On the other hand, a time delay is incorporated into the process of information dissemination, which is referred to an incubation period. Therefore, the model with time delay is more suitable for describing Micro-business network. Inspired by [18] and the social network information dissemination models, we introduce the hidden node (H) representing the offline users into MBID model.

Hence an ignorant-hidden-spreader-immune (which is abbreviated as IHSR) model is proposed to describe the behavior characteristics of the users in the micro-business network. Furthermore, due to the existence of the degree-degree correlation function in the dynamics evolution equations of information dissemination, we study the similarities and differences of dynamic characteristics between homogeneous networks and inhomogeneous networks by analyzing the equilibria and their stabilities, which bridges the gaps of the information dissemination in the MBN.

Article structure is as follows. The second part describes the complex networks and MBID mechanism. In the third part, the mean field equation of the MBID is obtained through the interactive Markov chain. The fourth part is the dynamics analysis of a homogeneous network. The fifth part is the numerical simulation. The sixth part is the conclusion.

2 Micro-business Information Dissemination model in complex networks

2.1 Complex networks

Complex networks are a new and rapid development subject, which reveals the reality of network system, In particular, proposition of the small world network model (WS) and the scale-free networks (SF) has set off an upsurge of academic research on complex network. Additionally micro-business network is a kind of SF. In this paper, we consider a micro-business network with invariable total nodes N. Then an undirected graph $G = (V, E)$ is utilized to express the relationship of the nodes in micro-business network, where V is a set of nodes and E is a set of edges. Here the nodes denote the businessman and his friends in the friends circle, and edges denote the contact between two nodes.

2.2 Information dissemination mechanism

According to Maki and Thompson [19], the premise of rumor spreading is that people must contact with the spreader directly. The mechanism of MBID is consistent with the mechanism of rumor spreading. In order to introduce the MBID mechanism more clearly, suppose that the nodes N in micro-business network is divided into four compartments with $N = I(t) + H(t) + S(t) + R(t)$. Here $I(t)$ is the ignorant class in which the nodes do not receive the information. $H(t)$ is the hidden class in which the nodes receive but yet not forward the information. $S(t)$ is the spreader class in which the nodes forward the information. $R(t)$ is the immune class in which the nodes do not forward the information. When the micro-business disseminate the information of product which would be bought by his circle of friends, only a fractional friends can receive information due to others without the permission to browse their friends' circle. When the user logins WeChat, he will see the information and then determine whether to forward the information according to his preferences and other factors. The user who receives but does not forward the information will transform from the ignorant I into the hidden H. If the information is forwarded, the node transforms from the hidden H into the spreader S. If the user gives up forwarding information, the hidden H becomes the immune R. In the process of forwarding information, if the spreader S contacts with an immune R, then he may become the immune R at a certain probability. The nodes may block the message or exit the WeChat circle of friends due to the information repeated or defective product recommended by the spreader. In addition, the spreader will terminate the dissemination behavior in a certain period of time and turn into the immune because of the lethe or disinterest for the information

According to the above MBID process, the rule of an IHSR model is described as follows.

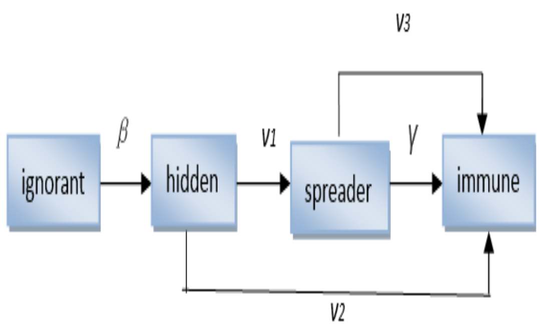


Figure 1: The transfer diagram for an IHSR model.

The following assumptions for the IHSR model are made (Fig. 1):

(H1) When a spreader contacts with an ignorant, the ignorant becomes a hidden at a rate $\beta(0 < \beta < 1)$.

(H2) The hidden either becomes a spreader who starts to forward information at a rate υ_1 or becomes an immune node who does not forward information at a rate υ_2.

(H3) The spreader becomes the immune at a rate γ when they contact an immune. On the other hand, the spreader does not forward information and eventually becomes the immune at the rate υ_3.

3 Mean Field Equations of Interactive Markov chain

We can describe the above model in the dynamic network by using the interactive Markov chains (IMC). The IMC was originally introduced as a way to simulate the social processes involving many interacting actors (or agents)[20]. According to the properties of the internal Markov chain, IMC is composed of N interactive nodes and in which each node has the state of time evolution. Different from the traditional Markoff chain, the corresponding internal transition probability is not only dependent on the state of the current node, but also relate to the state of nodes connected with it. The whole system evolve according to the global Markov chain whose state space dimension is the description of the state of each node. When dealing with large networks, the exponential growth of the state space is very difficult to find the numerical solution by IMC, so we have to adopt other ways to deal with it. In the MBID model, each node can be in one of the four states:ignorant, hidden, spreader, immune. In this case, we derive a set of probability equations using the mean-field method of the interactive Markov chain.

For a node m in the MBID network, the state may change among I, H, S and R in $[t, t + \triangle t]$. The probabilities of state transition of node m are defined by Table 1.

Table 1: One-step transition probability of the state

p^m_{II}	The probability that node m remains in an ignorant state
p^m_{IH}	The probability that node m transfers from the ignorant to the hidden
p^m_{HH}	The probability that node m remains in a hidden state
p^m_{HS}	The probability that node m transfers from the hidden to the spreader
p^m_{HR}	The probability that node m transfers from the hidden to the immune
p^m_{SS}	The probability that node m remains in a spreader state
p^m_{SR}	The probability that node m transfers from the spreader to the immune

3.1 Computation of probabilities

Case 1. The node m is in the ignorant state at time t. Let $g = g(t)$ represent the number of spreader nodes in the neighbor node of m at time t. Then

$$p^m_{II} = (1 - \Delta t\beta)^g. \tag{1}$$

Assume that node m has k edges and g is a random variable subjected to the following binomial distribution,

$$\prod(g,t) = \begin{pmatrix} k \\ g \end{pmatrix} \theta(k,t)^g (1-\theta(k,t))^{k-g}. \tag{2}$$

Here $\theta(k,t)$ is the probability that the ignorant node with k edges connects to a spreader node at time t,

$$\begin{aligned} \theta(k,t) &= \sum_{k'} p(k'|k) p(S_{k'}|I_k) \\ &\approx \textstyle\sum_{k'} p(k'|k) p^s(k',t), \end{aligned} \tag{3}$$

where $p(k'|k)$ is the degree-degree correlation function, which denotes the conditional probability of a node with degree k adjacent to a node of degree k'. $p(S_{k'}|I_k)$denotes the probability that a node with k' edge is in the spreader state when it is connected to a ignorant node with degree k. $p^s(k',t)$ denotes the density of the spreader nodes whose degree are k' at time t. The node with degree k maintains in the ignorant state in $[t, t+\triangle t]$ by the average probability

$$\begin{aligned} \overline{p_{II}(k,t)} &= \textstyle\sum_{g=0}^{k} \begin{pmatrix} k \\ g \end{pmatrix} (1-\Delta t\beta)^g \theta(k,t)^g \\ &\times \left(1-\theta(k,t)\right)^{k-g}) \\ &= (1 - \Delta t\beta \textstyle\sum_{k'} p(k'|k) p^s(k',t))^k. \end{aligned} \tag{4}$$

Owing to

$$p^m_{II} + p^m_{IH} = 1, \tag{5}$$

the average probability of the node with degree k transfers from the ignorant state to the hidden one.

$$\begin{aligned} \overline{p_{IH}(k,t)} &= 1 - \overline{p_{II}(k,t)} \\ &= 1 - (1 - \Delta t\beta \textstyle\sum_{k'} p(k'|k) p^s(k',t))^k. \end{aligned} \tag{6}$$

Case 2. The node m is in the hidden state at time t. Then,

$$p^m_{HH} + p^m_{HS} + p^m_{HR} = 1, \tag{7}$$

$$\begin{cases} p^m_{HS} = \Delta t v_1, \\ p^m_{HR} = \Delta t v_2. \end{cases} \tag{8}$$

Case 3. The node m is in the spreader state at time t. Then,

$$p^m_{SS} + p^m_{SR} = 1, \tag{9}$$

where

$$p^m_{SS} = (1 - \Delta t\gamma)^s (1 - v_3 \Delta t). \tag{10}$$

Assume that node m has k edges and $s = s(t)$ represents the number of immune nodes in the neighbor node of m at time t. Then s is a random variable which is subject to binomial distribution,

$$\prod(s,t) = \begin{pmatrix} k \\ s \end{pmatrix} \theta_1(k,t)^s (1-\theta_1(k,t))^{k-s}, \tag{11}$$

$\theta_1(k,t)$ is the probability that the spreader node with k edges connects to an immune node at time t,

$$\begin{aligned} \theta_1(k,t) &= \textstyle\sum_{k'} p(k'|k) p(R_{k'}|I_k) \\ &\approx \textstyle\sum_{k'} p(k'|k) p^r(k',t), \end{aligned} \tag{12}$$

where $p(R_{k'}|I_k)$ denotes the probability that a node with k' edge is in the immune state when it is connected to an immune node with degree k and $p^r(k',t)$ denotes the density of the immune nodes whose degree are k' at time t. The node with degree k maintains the average probability of spreader state in $[t, t+\triangle t]$.

$$\begin{aligned} \overline{p_{SS}(k,t)} &= \textstyle\sum_{s=0}^{k} (1-\Delta t\gamma)^s (1 - v_3\Delta t) \theta_1(k,t)^s \\ &\times (1-\theta_1(k,t))^{k-s} \\ &= (1 - \gamma\Delta t \textstyle\sum_{k'} p(k'|k) p^r(k',t))^k \\ &\times (1 - v_3 \Delta t) \end{aligned} \tag{13}$$

Therefore, the average probability of the node with degree k transfers from the spreader state to the immune state,

$$\begin{aligned}\overline{p_{SR}(k,t)} &= 1-\overline{p_{SS}(k,t)} \\ &= 1-(1-\gamma\Delta t\sum_{k'}p(k'|k)p^r(k',t))^k \\ &\times(1-v_3\Delta t)\end{aligned} \tag{14}$$

3.2 Variation of the number of nodes in $[t, t+\triangle t]$

At time t, suppose that the total number of nodes with degree k in the micro-business Network is $N(k,t)$. $I(k,t), H(k,t), S(k,t), R(k,t)$ respectively denote the number of ignorant, hidden, spreader and immune nodes with degree k. Then $I(k,t)+H(k,t)+S(k,t)+R(k,t)=N(k,t)$. In $[t, t+\triangle t]$, the number of various nodes varies as follows:

1) Ignorant node:

$$\begin{aligned}I(k,t+\Delta t) &= I(k,t)-I(k,t)(1-\overline{p_{II}(k,t)}) \\ &= I(k,t)-I(k,t) \\ &\times\left(1-(1-\Delta t\beta\textstyle\sum_{k'}p(k'|k)p^s(k',t))^k\right).\end{aligned} \tag{15}$$

2) Hidden node:

$$\begin{aligned}H(k,t+\Delta t) &= H(k,t)+I(k,t)(1-\overline{p_{II}(k,t)}) \\ &-H(k,t)(p_{HR}+p_{HS}) \\ &= H(k,t)+I(k,t) \\ &\times\left(1-(1-\Delta t\beta\textstyle\sum_{k'}p(k'|k)p^s(k',t))^k\right) \\ &-H(k,t)(\Delta tv_1+\Delta tv_2).\end{aligned} \tag{16}$$

3) Spreader node:

$$\begin{aligned}S(k,t+\Delta t) &= S(k,t)+H(k,t)\overline{p_{HS}(k,t)} \\ &-S(k,t)\overline{p_{SR}(k,t)} \\ &= S(k,t)+H(k,t)\Delta tv_1-S(k,t) \\ &\times(1-(1-\gamma\Delta t\textstyle\sum_{k'}p(k'|k)p^r(k',t))^k \\ &\times(1-v_3\Delta t)).\end{aligned} \tag{17}$$

4) Immune node:

$$\begin{aligned}R(k,t+\Delta t) &= R(k,t)+H(k,t)\overline{p_{HR}(k,t)} \\ &+S(k,t)\overline{p_{SR}(k,t)} \\ &= R(k,t)+H(k,t)\Delta tv_2+S(k,t) \\ &\times(1-(1-\gamma\Delta t\textstyle\sum_{k'}p(k'|k)p^r(k',t))^k \\ &\times(1-v_3\Delta t)).\end{aligned} \tag{18}$$

In the above equations, $p^i(k,t)$, $p^h(k,t)$, $p^s(k,t)$ and $p^r(k,t)$ represent the proportions of nodes which are in the ignorant, hidden, spreader and immune states, respectively, namely

$$\begin{cases} p^i(k,t)=\dfrac{I(k,t)}{N(k,t)}, \\ p^h(k,t)=\dfrac{H(k,t)}{N(k,t)}, \\ p^s(k,t)=\dfrac{S(k,t)}{N(k,t)}, \\ p^r(k,t)=\dfrac{R(k,t)}{N(k,t)}. \end{cases} \tag{19}$$

And they fulfill the normalization conditions

$$p^i(k,t)+p^h(k,t)+p^s(k,t)+p^r(k,t)=1.$$

According to (19), the new mean field equations of four types of nodes can be formed,

$$\begin{cases} \dot{p}^i(k,t) &= -k\beta p^i(k,t)\sum_{k'}p(k'|k)p^s(k',t), \\ \dot{p}^h(k,t) &= k\beta p^i(k,t)\sum_{k'}p(k'|k) \\ & \times p^s(k',t)-p^h(k,t)(v_1+v_2), \\ \dot{p}^s(k,t) &= v_1p^h(k,t)-k\gamma p^s(k,t)\sum_{k'}p(k'|k) \\ & \times p^r(k',t)-v_3p^s(k,t), \\ \dot{p}^r(k,t) &= v_2p^h(k,t)+k\gamma p^s(k,t)\sum_{k'}p(k'|k) \\ & \times p^r(k',t)+v_3p^s(k,t). \end{cases} \tag{20}$$

The dynamics evolution equations of information dissemination are used to characterize the change of the density of ignorant nodes, hidden nodes, spreader nodes and immune nodes over time. The dynamics dissemination process is influenced by the network topology structure and dissemination mechanism. By the above formula, we notice that these equations are expressed by the degree-degree correlation function, so we need to perform a numerical analysis of the expression $p(k'|k)$.

Generally, the MBID network is inhomogeneous. Here we consider a scale-free (SF) networks. In which the degree-degree correlations can be written as

$$p(k'|k) = \frac{k'p(k')}{< k >}, \tag{21}$$

where $p(k')$ is the degree distribution function and $< k >$ is the average degree of nodes [21]. It is assumed that the degree distribution of the SF network is subject to the power distribution ([22]–[23]).

$$p(k) = \begin{cases} Ak^{-\delta} & k_{min} < k, \\ 0 & otherwise, \end{cases} \tag{22}$$

where $2 < \delta \leq 3$, k_{min} is the minimum degree of the networks and A is a normalization constant.

4 Dynamics analysis of a homogeneous network

For convenience, we consider a homogeneous network. In which the fluctuations of the node degrees are very small and there are no correlations of these degrees, namely, $p(k'|k) = \bar{k}$, here $\bar{k}$ represents the average degree of the network. Then the MBID equations become:

$$\begin{cases} \dot{p^i}(t) = -\bar{k}\beta p^i(t)p^s(t), \\ \dot{p^h}(t) = \bar{k}\beta p^i(t)p^s(t) - p^h(t)(v_1 + v_2), \\ \dot{p^s}(t) = v_1 p^h(t) - \bar{k}\gamma p^s(t)p^r(t) - v_3 p^s(t), \\ \dot{p^r}(t) = v_2 p^h(t) + \bar{k}\gamma p^s(t)p^r(t) + v_3 p^s(t). \end{cases} \tag{23}$$

Next, the equilibria and their stability are investigated.

4.1 The existence of equilibria

1) If $p^i(t) = 0$, then $p^h(t) = 0$ and $-p^s(v_3 + \bar{k}\gamma)p^r(t) = 0$. The non-negative equilibrium $x_1(0, 0, 0, 1)$ is figured out.

2)If $p^i(t) \neq 0$ and $p^s(t) = 0$, then non-negative equilibrium $x_2(p^{i*}, 0, 0, 1 - p^{i*})$ is obtained, where $0 < p^{i*} < 1$.

4.2 Stability

Now let us compute the Jacobian matrix of (23) around the equilibrium $x_i(p^s, p^h, p^i, p^r)$:

$$J(x_i) = \begin{bmatrix} -\bar{k}\beta p^s & 0 & -\bar{k}\beta p^i & 0 \\ \bar{k}\beta p^s & -(v_1 + v_2) & \bar{k}\beta p^i & 0 \\ 0 & v_1 & -v_3 - \bar{k}\gamma p^r & -\bar{k}\gamma p^s \\ 0 & v_2 & v_3 + \bar{k}\gamma p^r & \bar{k}\gamma p^s \end{bmatrix}.$$

Then the corresponding characteristic equation is:

$$\big|\lambda E - J(x_i)\big| =$$

$$\begin{vmatrix} \lambda + \bar{k}\beta p^s & 0 & \bar{k}\beta p^i & 0 \\ -\bar{k}\beta p^s & \lambda + v_1 + v_2 & -\bar{k}\beta p^i & 0 \\ 0 & -v_1 & \lambda + v_3 + \bar{k}\gamma p^r & \bar{k}\gamma p^s \\ 0 & -v_2 & -v_3 - \bar{k}\gamma p^r & \lambda - \bar{k}\gamma p^s \end{vmatrix}.$$

Theorem 1. *The equilibrium $x_1(0, 0, 0, 1)$ is locally stable.*

Proof. From

$$|\lambda E - J(x_1)| = \begin{vmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda + v_1 + v_2 & 0 & 0 \\ 0 & -v_1 & \lambda + v_3 + \bar{k}\gamma & 0 \\ 0 & -v_2 & -v_3 - \bar{k}\gamma & \lambda \end{vmatrix}$$

$$= \lambda^2(\lambda + v_1 + v_2)(\lambda + v_3 + \bar{k}\gamma) = 0,$$

we have $\lambda_1 = 0, \lambda_2 = -v_1 - v_2 < 0$ and $\lambda_3 = -v_3 - \bar{k}\gamma < 0$. Then the equilibrium x_1 is locally stable. The proof is completed. □

Theorem 2. *The equilibrium $x_2(p^{i*}, 0, 0, 1 - p^{i*})$ is locally stable if*

$$1 > p^{i*} > \frac{v_1 + v_2}{\beta + v_1 + v_2}[1 + \frac{v_3}{\gamma\bar{k}}] \triangleq \Gamma \tag{24}$$

holds. If the reverse inequality of (24) establishes, then the equilibrium x_2 is unstable.

Proof. From

$$|\lambda E - J(x_2)| = \begin{vmatrix} \lambda & 0 & \bar{k}\beta p^{i*} & 0 \\ 0 & \lambda + v_1 + v_2 & -\bar{k}\beta p^{i*} & 0 \\ 0 & -v_1 & \lambda + v_3 + \bar{k}\gamma(1 - p^{i*}) & 0 \\ 0 & -v_2 & -v_3 - \bar{k}\gamma(1 - p^{i*}) & \lambda \end{vmatrix}$$

$$= \lambda^2\{(\lambda + v_1 + v_2)(\lambda + v_3 + \bar{k}\gamma(1 - p^{i*})) - \bar{k}\gamma\beta p^{i*}\} \triangleq \lambda^2 h(\lambda) = 0,$$

we have $\lambda_1 = \lambda_2 = 0$ and

$$h(\lambda) = \lambda^2 + \lambda(v_1 + v_2 + v_3 + \bar{k}\gamma(1 - p^{i*})) + (v_1 + v_2)(\bar{k}\gamma(1 - p^{i*}) + v_3) - \bar{k}\gamma\beta p^{i*} = 0. \tag{25}$$

Again, the roots of $h(\lambda) = 0$ meet

$$\lambda_3 + \lambda_4 = -[v_1 + v_2 + v_3 + \bar{k}\gamma(1 - p^{i*})] < 0$$

and

$$\lambda_3\lambda_4 = (v_1 + v_2)[\bar{k}\gamma(1 - p^{i*}) + v_3] - \bar{k}\gamma\beta p^{i*}.$$

So, in order to ensure the stability of the equilibrium x_2, it is obtained that (24) holds. The proof is complete. □

Remark *From the above discussion, we see that the equilibria x_1 and x_2 always exist and x_2 is multi-valued. Γ is a vital threshold which determines the stability of x_2. Theorem 2 indicates that whether the ignorant dies out or not depends on the proportion of the initial ignorant.*

5 Numerical simulations

Our focus so far has been on mean field equations of interactive Markov chain and dynamics analysis of a homogeneous network. To facilitate the interpretation of our mathematical results, we proceed to investigate it by numerical simulations.

5.1 Comparisons of homogeneous and inhomogeneous networks

First, let us investigate the evolutions of four kinds of nodes tending to equilibrium $(0, 0, 0, 1)$ in homogeneous and inhomogeneous networks, respectively.

Assume that the initial state are

$$p^s(0) = 0.0001, p^i(0) = 0.9999, p^h(0) = 0, p^r(0) = 0. \qquad (26)$$

Choose

$$\delta = 3, \gamma = 1, \beta = 5, kmin = 4, \bar{k} = 7,$$
$$v_1 = 0.25(Times/Hour), v_2 = 0.3(Times/Hour),$$
$$v_3 = 0.05(Times/Hour). \qquad (27)$$

As shown in Fig.2 (a) and (b), the equilibrium $(0, 0, 0, 1)$ is stable in the two types networks, which represents that the ignorant (the black line), hidden (the green line) and spreader (the blue line) nodes are free and eventually all nodes become immune (the red line). However, their evolution laws are apparent difference which are demonstrated as follows.

(1) In homogeneous network, the ignorant nodes tend to zero rapidly at about the 4th hour, which means every node gets information. Rather, in inhomogeneous network nodes go to zero slowly at about the 18th hour.

(2) In homogeneous network the hidden nodes reach the peak 0.5 at the 4th hour, whereas, in inhomogeneous network the hidden nodes culminate the peak 0.25 at the 15th hour. In contrast, the curve of the hidden nodes in homogeneous network is more steeper than in inhomogeneous one.

(3) The trend of the spreader nodes is analogous to the one of the hidden nodes. Obviously, the level of the spreader nodes in inhomogeneous network are higher than in homogeneous one, as indicates that the accumulated amount of the spreader nodes are large.

(4) The immune nodes increase rapidly after the 4th hour in homogeneous network, but, they grows fast after the 10th hour in inhomogeneous network.

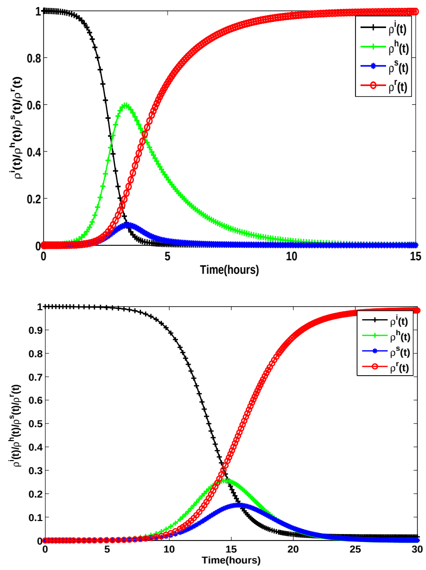
(a)
(b)


Figure 2: The evolutions of the ignorant, hidden, spreader and immune nodes for the equilibrium $(0, 0, 0, 1)$. (a) Homogeneous network. (b) Inhomogeneous network. Here the initial states are list in (25) and the parameters are shown in (26).

In sum, compared with homogeneous network, inhomogeneous network delays the speed of the information disseminate, and the spreader nodes in the inhomogeneous network are larger.

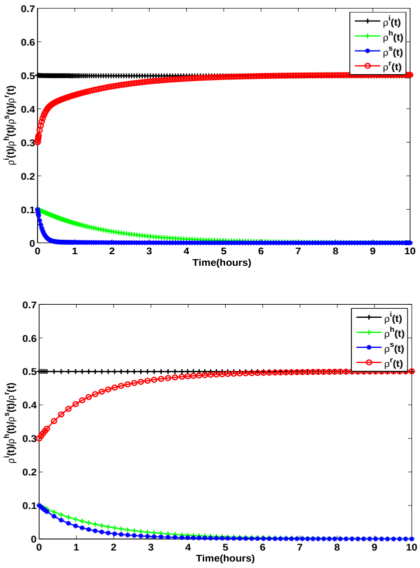
Next, let us investigate the evolutions of four kinds of nodes tending to equilibrium $(p^{i*}, 0, 0, 1 - p^{i*})$ in homogeneous and inhomogeneous networks, respectively.

Assume that the initial state are

$$p^s(0) = 0.1, p^i(0) = 0.5, p^h(0) = 0.1, p^r(0) = 0.3 \qquad (28)$$

Choose

$$\delta = 3, \gamma = 1, \beta = 0.01, kmin = 4, \bar{k} = 7,$$
$$v_1 = 0.25(Times/Hour), v_2 = 0.3(Times/Hour),$$
$$v_3 = 1(Times/Hour). \tag{29}$$



(a)

(b)

Figure 3: The evolutions of the ignorant, hidden, spreader and immune nodes for the equilibrium $(p^{i*}, 0, 0, 1-p^{i*})$. (a) Homogeneous network. (b) Inhomogeneous network. Here the initial states are list in (27) and the parameters are shown in (28).

As shown in Fig.3 (a) and (b), the equilibrium $(p^{i*}, 0, 0, 1-p^{i*})$ is stable in the two types networks, in which the four lines represent the trend of the four nodes. However, their evolution rules have small difference which are demonstrated as follows.

(1) In homogeneous network, the spreader nodes go to zero promptly at about the 0.5th hour. On the contrary, in inhomogeneous network nodes go to zero slowly at about the 4th hour.

(2) In homogeneous network, the immune nodes increase quickly and then reach the stable value before the 0.5th hour. Whereas, in inhomogeneous network the immune nodes increase slowly and then reach the stable value before the 4th hour.

In conclusion, compared with homogeneous network, inhomogeneous network delays the speed of the information disseminate.

5.2 Effects of parameters on homogeneous and inhomogeneous networks

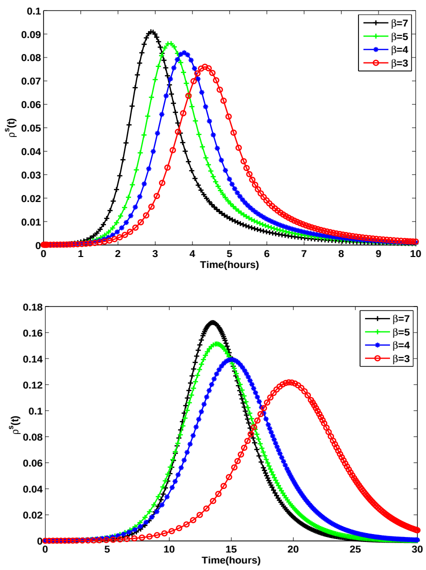
1) Firstly, for the equilibrium $(0, 0, 0, 1)$ the effects of parameter β on the dissemination process in the homogeneous and inhomogeneous networks are shown in Fig.4 (a) and (b), in which it is taken by $3, 4, 5$ and 7, respectively. It is easy to see the evolution laws of the spreader nodes are apparent difference which are demonstrated as follows.

(1) In homogeneous network, the spreader nodes reach the peaks which are less than 0.1 before the 5th hour for four different β. Rather, in inhomogeneous network, the spreader nodes reach the peaks which are larger than 0.12 after the 10th hours.

(2) The accumulated amounts of the spreader nodes in inhomogeneous network are larger than those in homogeneous for four different β. Furthermore, the duration of dissemination information in inhomogeneous network is longer.

(3) A smaller increase in β will result in a greater increase in the spreader nodes, which implies that β is critical to the information dissemination.

Therefore, in order to disseminate the information widely and quickly in the network, we should effectively improve the propagating rate of the information by all means.



(a)

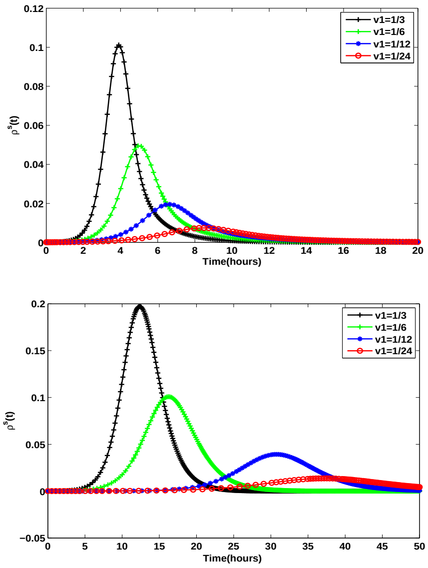
(b)

Figure 4: The impact of β on the spreader nodes for the equilibrium $(0, 0, 0, 1)$. (a) Homogeneous network. (b) Inhomogeneous network. Here the initial states are list in (26), the parameters are γ=1, $v_1 = 0.25(Times/Hour), v_2 = 0.3(Times/Hour), v_3 = 0.05(Times/Hour), kmin = 4$, $\bar{k}$=7 and the value of β is 3, 4, 5, 7.

Secondly, for the equilibrium $(0, 0, 0, 1)$ the effects of parameter v_1 on the dissemination process in the homogeneous and inhomogeneous networks are displayed in Fig.5 (a) and (b). Here v_1 is taken by $1/3(Times/Hour)$, $1/6(Times/Hour)$, $1/12(Times/Hour)$ and $1/24(Times/Hour)$

which show that nodes log on wechat 8, 4, 2 and 1 times a day, respectively.

Obviously, the effects of parameter v_1 on the dissemination process in the homogeneous and inhomogeneous networks are analogous to those of parameter β. Even so, the frequency of logging on WeChat has a significant impact on the time and scope of information dissemination. The lower the logging frequency, the slower the speed of information dissemination, the smaller the scope of the dissemination. On the contrary, the faster the dissemination of information, the greater the scope of the dissemination. But the nodes's logging frequency is his own behavioral characteristics, so it is difficult to control from the perspective of dissemination.



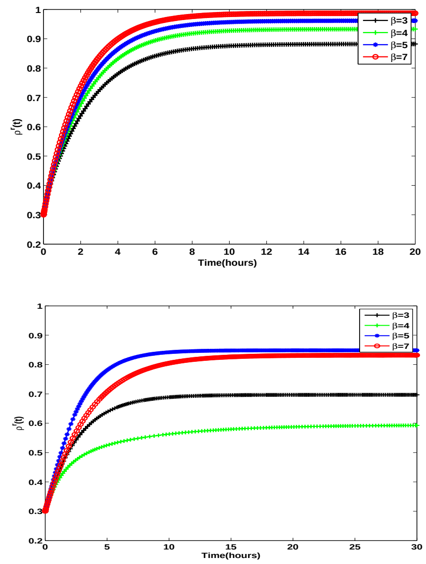
(a) (b)

Figure 5: The impact of v_1 on the spreader nodes for the equilibrium $(0, 0, 0, 1)$. (a) Homogeneous network. (b) Inhomogeneous network. Here the initial states are list in (26) , the parameters are $v_2 = 0.3(Times/Hour), v_3 = 0.05(Times/Hour), kmin = 4$, γ=1, β=5, $\bar{k}$=7 and the value of v_1 is 1/3, 1/6, 1/12, 1/24.

2) Firstly, for the equilibrium $(p^{i*}, 0, 0, 1 - p^{i*})$ the effects of parameter β on the dissemination process in the homogeneous and inhomogeneous networks are shown in Fig.6 (a) and (b), in which it is taken by $3, 4, 5$ and 7, respectively. Because the spreader nodes's changes are not obvious, we study the effects of parameters on immune nodes. It is easy to see the evolution laws of the immune nodes are apparent different which are demonstrated as follows.

(1) In homogeneous network, a smaller increase in β will result in a larger increase in the speed of dissemination and the number of the immune nodes, but inhomogeneous network doesn't have such a rule.

(2) The accumulated amounts of the immune nodes in homogeneous network are larger than those in inhomogeneous for four different β.



(a) (b)

Figure 6: The impact of β on the immune nodes for the equilibrium $(p^{i*}, 0, 0, 1 - p^{i*})$. (a) Homogeneous network. (b) Inhomogeneous network. Here the initial states are list in (27), the parameters are γ=1, $v_1 = 0.25(Times/Hour), v_2 = 0.3(Times/Hour), v_3 = 1(Times/Hour), kmin = 4$, $\bar{k}$=7 and the value of β is 3, 4, 5, 7.

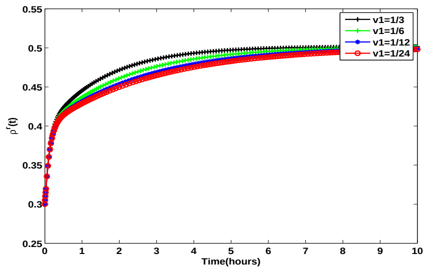
Secondly, for the equilibrium $(p^{i*}, 0, 0, 1 - p^{i*})$ the effects of parameter v_1 on the dissemination process in the homogeneous and inhomogeneous networks are displayed in Fig.7 (a) and (b). Here v_1 is taken by $1/3(Times/Hour)$, $1/6(Times/Hour)$, $1/12(Times/Hour)$ and $1/24(Times/Hour)$.

Apparently, comparing with homogeneous network, inhomogeneous network delays the speed of the information disseminate and the impact of v_1 on the immune nodes is not great.

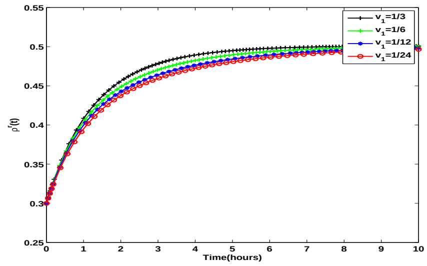
5.3 Comparison of SIR with IHSR models

The IHSR model is derived from the SIR model. In the existing information dissemination model, the IHSR model is the most similar to the SIR model. Fig.8 shows the comparison of the two models in the homogeneous and inhomogeneous networks for the equilibrium $(0, 0, 0, 1)$, where $v_1 = 1/6$(times/hour) in the IHSR model and the other parameter settings unchanged. The similarities and differences of these two models are as follows.

(1) The trend of the two graphs is the same.

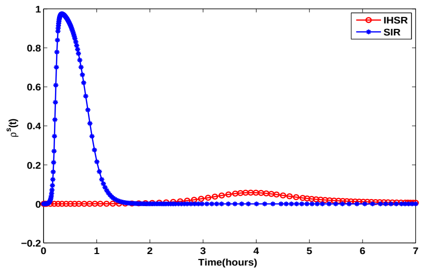


(a)

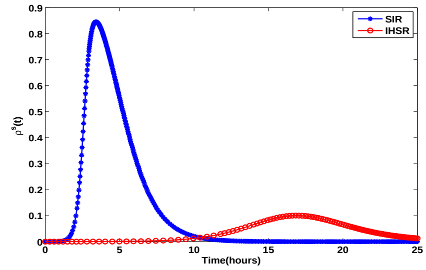


(b)

Figure 7: The impact of v_1 on the immune nodes for the equilibrium $(p^{i*}, 0, 0, 1 - p^{i*})$. (a) Homogeneous network. (b) Inhomogeneous network. Here the initial states are list in (27) , the parameters are $v_2 = 0.3(Times/Hour), v_3 = 1(Times/Hour), kmin = 4$, γ=1, β=5, $\bar{k}$=7 and the value of v_1 is 1/3, 1/6, 1/12, 1/24.



(a)



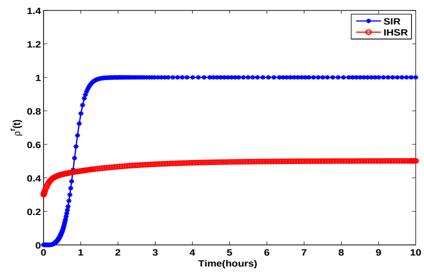
(b)

Figure 8: The comparison of the two models on the spreader nodes for the equilibrium $(0, 0, 0, 1)$. (a) Homogeneous network. (b) Inhomogeneous network. Here the initial states are list in (26) , the parameters are $v_1 = 1/6(Times/Hour)$,$v_2 = 0.3(Times/Hour), v_3 = 0.05(Times/Hour), kmin = 4$, γ=1, β=5, $\bar{k}$=7.

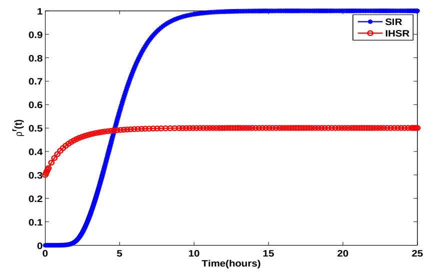
(2) In homogeneous network, the duration of the spreader nodes in dissemination information are less than those in the inhomogeneous network.

(3) In two networks, the dissemination of information in the IHSR model both in time and scope are obviously at a disadvantage. The IHSR model also takes longer time to spread the information.

Obviously, the comparison of the two models in the homogeneous and inhomogeneous networks for the equilibrium $(p^{i*}, 0, 0, 1 - p^{i*})$ are analogous to those for the equilibrium $(0, 0, 0, 1)$ in Fig9.



(a)



(b)

Figure 9: The comparison of the two models on the immune nodes for the equilibrium $(p^{i*}, 0, 0, 1 - p^{i*})$. (a) Homogeneous network. (b) Inhomogeneous network. Here the initial states are list in (28) , the parameters are $v_1 = 1/6(Times/Hour)$, $v_2 = 0.3(Times/Hour), v_3 = 1(Times/Hour), kmin = 4$, γ=1, β=0.01, $\bar{k}$=7.

In reality, most of the users are in offline state and offline behavior hinders the dissemination of information. In fact, the phenomenon that the vast majority of users receive information has not yet appeared in the micro-business network. Though the most popular products, the percentage of the users who receive the information is also very low because of the presence of hidden nodes. Therefore, the IHSR model is more closer to the actual situation than the SIR model which is an ideal model.

6 Discussion

In this paper, on the basis of the model of infectious disease and information dissemination in social network, the hidden node which describe the user's offline behavior was introduced into the SIR model. The IHSR model of the MBID and the mean field equations of four kinds of nodes density based on time evolution are obtained. We calculate the equilibria and analyze their stability of the homogeneous network. By comparing the similarities and differences of dynamic characteristics and the influence of network parameters on the process of information dissemination in homogeneous and inhomogeneous networks, the simulation results show that the IHSR model can better reflect the actual situation of information dissemination on Micro-business network.

To achieve a better results, there is still much work to do:

1)More characteristics of WeChat and micro-business should be considered. For example, thumb up and comments etc.

2)The networks that we considered are static. In reality, these kinds of networks are highly dynamic. Because the connection between the previous moment and the next moment is different and the network topology is changing at all times, so simulating the dissemination process in this dynamic network is challenge.

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