Tikhonov Regularized Kalman Filter and Its Applications in Autonomous Orbit Determination of BDS

YONGMING LI
Institute of Science,
Information Engineering University
450001,Zhengzhou
CHINA
keepming@163.com

QINGMING GUI
Institute of Science,
Information Engineering University
450001,Zhengzhou
CHINA
guiqingming@126.com

SONGHUI HAN
Institute of Science,
Information Engineering University
450001,Zhengzhou
CHINA
hansonghui@126.com

YONGWEI GU
Institute of Science,
Information Engineering University
450001,Zhengzhou
CHINA
gyw1019@sina.com

Abstract: Kalman filter is one of the most common ways to deal with dynamic data and has been widely used in project fields. However, the accuracy of Kalman filter for discrete dynamic system is poor when the observation matrix is ill-conditioned. Therefore, the method for overcoming the harmful effect caused by ill-conditioned observation matrix in discrete dynamic system is studied in this paper. Firstly, Tikhonov regularized Kalman filter (TRKF) and its algorithm are proposed by combining Tikhonov regularization method and Kalman filter. Meanwhile, some excellent properties of TRKF are proved. Secondly, the methods of choosing regularization parameter and regularization matrix in TRKF are given. Thirdly, simulated examples are designed to evaluate the performance of TRKF and comparisons between TRKF and Ordinary Ridge-type Kalman Filter (ORKF) are given. Finally, TRKF is applied in autonomous orbit determination of BeiDou Navigation Satellite System (BDS) with cross-link ranging observations and ground tracking observations so as to prevent filter divergent which is caused by ill-conditioned observation matrix. Simulations and applications illustrate that TRKF can overcome the harmful effect caused by ill-conditioned observation matrix in discrete dynamic system and the accuracy is improved effectively.

Key–Words: Discrete dynamic system, Kalman filter, Ill-conditioning, Tikhonov regularization, Regularization parameter, Regularization matrix, Autonomous orbit determination

1 Introduction

In 1960, R. E. Kalman published his famous paper describing a recursive solution to a dynamic system that involves random perturbations. More precisely, Kalman filter gives a linear, unbiased, and minimum error variance recursive algorithm to optimally estimate the unknown state of a dynamic system from noisy data taken at discrete real-time [1]. Due in large part to advances in digital computing, Kalman filter has become the subject of extensive research and application, particularly in the area of autonomous or assisted navigation [2].

Numerical analysts have been keenly aware of the phenomenon known as ill-conditioning in connection with matrix inversion for many years. In general, least squares usually give rise to the problems of ill-conditioned matrix inversion [3,4]. Considering that Kalman filter is simply a recursive solution about a certain weighted least squares problem, it is not surprising that Kalman filters tend to be instability for the ill-conditioning [5,6]. The ill-conditioning in discrete dynamic system does exist in actual matters, such as single photon emission computed tomography (SPECT), electrocardiography, satellite navigation and autonomous orbit determination and so on [7,8]. For example, satellite navigation system can deploy the ground observation stations, comprehensive use of the cross-link ranging observations and ground tracking observations to improve the accuracy of orbit determination in non-wartime. However, limitations of the ground-based observations often lead to poor observation geometry, and the observation matrix is ill-conditioned. There is few research on the ill-conditioning of the observation equation of the dynamic system. Baroudi et al. replaced the original observation equation of Kalman filter with a fictitious augmented observation equation to overcome the ill-conditioning. It is difficult to choose the augmented observation equation in this method [8,9]. Qranfal and Tanoh formulated a new dynamic system and incorporated a projection method to enforce a spatial regularization using Tikhonov and median approaches [10]. This method is too simple when choosing regularization parameters and there is no regularization matrix in their algorithm. Schulze directly incorporated an inversion of the observation matrix using the Tikhonov regularization method. This method can solve the ill-conditioned problem but there is too much premises need to satisfied [11].

Tikhonov regularization method is one of the most common ways to deal with ill-conditioned problems. It uses some prior information on specific issues to restrict the parameters to be estimated and gives an approximation of the original problem. Tikhonov regularization was widely used and developed for its stability and accuracy [12,13]. Tikhonov regularization method is introduced to Kalman filter to overcome the harmful effect caused by ill-conditioned observation matrix in discrete dynamic system.

The paper is organized as follows. The Kalman filter of discrete dynamic system is introduced and the impact of ill-conditioning of observation matrix on the Kalman filter is analyzed by means of perturbation analysis theory in Section 2. In Section 3, Tikhonov regularized Kalman filter (TRKF) and its algorithm are proposed by combining Tikhonov regularization method and Kalman filter. Meanwhile, some excellent properties of the new algorithm are proved. The methods of choosing regularization parameter and regularization matrix are given in Section 4. In Section 5, simulated examples are designed to evaluate the performance of TRKF and comparisons between TRKF and ordinary ridge-type Kalman filter (ORKF) are given. In Section 6, TRKF is applied in autonomous orbit determination of BDS with cross-link ranging observations and ground tracking observations to prevent filter divergence which is caused by ill-conditioned observation matrix. Finally, some brief conclusions are given in Section 7.

2 Kalman Filter of Discrete Dynamic System and Analysis of Ill-Conditioning

2.1 Discrete Dynamic System and Kalman Filter

Consider a discrete dynamic system with state equation and observation equation as follows:

$$X_k = \Phi_{k,k-1}X_{k-1} + \omega_{k-1}, \tag{1}$$

$$L_k = A_k X_k + \nu_k, \tag{2}$$

where X_k denotes a $p \times 1$ vector describing the state of the system at time t_k and L_k is the $n \times 1$ observation vector. $\Phi_{k,k-1}$ is the $p \times p$ state transition matrix at time at time t_k. A_k is the $n \times p$ observation matrix. The vectors ω_{k-1} and ν_k are the system noise vector and the observation noise vector, respectively.

It is assumed that

$$\begin{cases} E(\omega_k) = 0, \quad Cov(\omega_k, \omega_j) = Q_k \delta_{k,j} \\ E(\nu_k) = 0, \quad Cov(\nu_k, \nu_j) = R_k \delta_{k,j} \\ Cov(\omega_k, \nu_j) = 0 \end{cases} \tag{3}$$

where the covariance matrix Q_k of system noise vector is assumed to be non-negative definite and the covariance matrix R_k of observation noise vector is assumed to be positive definite. $\delta_{k,j}$ is the function of the Kronecher - δ.

The Kalman filter can be deduced by using a risk function

$$\varphi(\widehat{X}_k) = V_{\widehat{x}_k}^T R_k^{-1} V_{\widehat{x}_k} + V_{\overline{x}_k}^T P_{k/k-1}^{-1} V_{\overline{x}_k} \tag{4}$$

where

$$V_{\widehat{x}_k} = A_k \widehat{X}_k - L_k \tag{5}$$

and

$$V_{\overline{x}_k} = \widehat{X}_k - \widehat{X}_{k/k-1} \tag{6}$$

Letting $\frac{\partial \varphi(\widehat{X}_k)}{\partial \widehat{X}_k} = 0$,we can get the solution

$$\widehat{X}_k = (A_k^T R_k^{-1} A_k + P_{k/k-1}^{-1})^{-1} (A_k^T R_k^{-1} L_k + P_{k/k-1}^{-1} \widehat{X}_{k/k-1}) \tag{7}$$

Using the transformation matrix, we can get the general form of Kalman filter

$$\widehat{X}_{k/k-1} = \Phi_{k,k-1} \widehat{X}_{k-1} \tag{8}$$

$$P_{k/k-1} = \Phi_{k,k-1} P_{k-1} \Phi_{k,k-1}^T + Q_{k-1} \tag{9}$$

$$K_k = P_{k/k-1} A_k^T (A_k P_{k/k-1} A_k^T + R_k)^{-1} \tag{10}$$

$$\widehat{X}_k = \widehat{X}_{k/k-1} + K_k (L_k - A_k \widehat{X}_{k/k-1}) \tag{11}$$

$$P_k = (I - K_k A_k) P_{k/k-1} \tag{12}$$

where $P_{k/k-1}$ is the covariance matrix of $\widehat{X}_{k/k-1}$. P_k is the covariance matrix of $\widehat{X}_k$ and K_k is the filter gain matrix. The optimal estimator $\widehat{X}_k$ can be calculated by (8)-(12) if the initial estimator $\widehat{X}_0$ and $\widehat{P}_0$ are given.

2.2 Analysis of Ill-Conditioning in Discrete Dynamic System

Obviously, $\widehat{X}_k$ is the solution of equation

$$N_k X_k = b_k \tag{13}$$

where $N_k = A_k^T R_k^{-1} A_k + P_{k/k-1}^{-1}$ and $b_k = A_k^T R_k^{-1} L_k + P_{k/k-1}^{-1} \widehat{X}_{k/k-1}$. (13) is called the normal equation of Kalman filter and N_k is called the normal matrix.

Unlike Gauss-Markov model, the discrete dynamic system not only includes observation equation, but also includes the state equation. So, the harmful effect caused by the ill-conditioned observation matrix on Kalman filter must be subject to the state equation [6]. The Kalman filter cannot be decided by A_k or $A_k^T R_k^{-1} A_k$ separately, and the influence of $P_{k/k-1}^{-1}$ on N_k must be considered too. It is more reasonable to take $A_k^T R_k^{-1} A_k$ and $P_{k/k-1}^{-1}$ as a whole to carry out the perturbation analysis in the discrete dynamic system.

In equation (13), ΔN_k and Δb_k are disturbed when there were disturbances in the matrix N_k and b_k, and satisfy the following equation.

$$(N_k + \Delta N_k)(X_k + \Delta X_k) = b_k + \Delta b_k$$

The simplification of the equation above is

$$\Delta X_k = N_k^{-1}[-\Delta N_k (X_k + \Delta X_k) + \Delta b_k].$$

When the perturbation of matrix N_k is very small and satisfies $||N_k^{-1}|| \cdot ||\Delta N_k|| < 1$, we have

$$\frac{||\Delta X_k||}{||X_k||} \leq \frac{Cond(N_k)}{1 - Cond(N_k)\frac{||\Delta N_k||}{||N_k||}} \left(\frac{||\Delta N_k||}{||N_k||} + \frac{||\Delta b_k||}{||b_k||}\right)$$

where $Cond(N_k) = ||N_k|| \cdot ||N_k^{-1}||$ is the condition number of the normal matrix N_k.

From the equation above, we can conclude that small perturbations in the normal matrix N_k or in the observation vector L_k will be magnified by the condition number $Cond(N_k)$ when the condition number $Cond(N_k)$ is large, which leads to tremendous changes in estimator of X_k. This is the way how the ill-conditioning of the observation matrix influences the Kalman filter.

3 Tikhonov Regularized Kalman Filter and Its Properties

3.1 Tikhonov Regularized Kalman Filter and Its Algorithm

The objective function of TRKF is designed as follows

$$\Psi(\widehat{X}_k^{TRKF}) = V_{\widehat{x}_k^{TRKF}}^T R_k^{-1} V_{\widehat{x}_k^{TRKF}} + V_{\widehat{x}_k^{TRKF}}^T P_{k/k-1}^{-1} V_{\widehat{x}_k^{TRKF}} + \alpha_k Z_k^T H_k Z_k \tag{14}$$

where $\alpha_k > 0$ is the regularization parameter, $H_k > 0$ is the regularization matrix and $Z_k = \widehat{X}_k^{TRKF} - \widehat{X}_{k/k-1}$.

Using the weighted least square method, we can get

$$K_k^{TRKF} = (A_k^T R_k^{-1} A_k + P_{k/k-1}^{-1} + \alpha_k H_k)^{-1} A_k^T R_k^{-1} \tag{15}$$

$$\widehat{X}_k^{TRKF} = \widehat{X}_{k/k-1} + K_k^{TRKF}(L_k - A_k \widehat{X}_{k/k-1}) \tag{16}$$

$$P_k^{TRKF} = (I - K_k^{TRKF} A_k) P_{k/k-1} \tag{17}$$

In summary, the complete algorithm of TRKF is designed as follows:

Step 1: Initialization. Give the initial estimate $\widehat{X}_0$ and the covariance matrix $\widehat{P}_0$.

Step 2: Time update process.

$$\widehat{X}_{k/k-1} = \Phi_{k,k-1} \widehat{X}_{k-1}$$

$$P_{k/k-1} = \Phi_{k,k-1} P_{k-1} \Phi_{k,k-1}^T + Q_{k-1}$$

Step 3: Measurement update process.

$$K_k = P_{k/k-1} A_k^T (A_k P_{k/k-1} A_k^T + R_k)^{-1}$$

$$\widehat{X}_k = \widehat{X}_{k/k-1} + K_k (L_k - A_k \widehat{X}_{k/k-1})$$

$$P_k = (I - K_k A_k) P_{k/k-1}$$

Step 4: Compute the condition number N_k. If the condition number N_k is bigger than 100, the step 5 is implemented. Otherwise, the step 2 is carried on.

Step 5: Choosing the regularization parameter α_k and the regularization matrix H_k.

Step 6: Using TRKF to revise the optimal estimator $\widehat{X}_k$.

$$K_k^{TRKF} = (A_k^T R_k^{-1} A_k + P_{k/k-1}^{-1} + \alpha_k H_k)^{-1} A_k^T R_k^{-1}$$

$$\widehat{X}_k^{TRKF} = \widehat{X}_{k/k-1} + K_k^{TRKF}(L_k - A_k \widehat{X}_{k/k-1})$$

$$P_k^{TRKF} = (I - K_k^{TRKF} A_k) P_{k/k-1}$$

Step 7: Command $\widehat{X}_{k-1} = \widehat{X}_k^{TRKF}$, $P_{k-1} = P_k^{TRKF}$ and return step 2.

3.2 Properties of Tikhonov Regularized Kalman Filter

1° Supposing that the filter gain K_{k+1}^{TRKF} is obtained when TRKF is used at epoch t_k and Kalman filter is used at epoch t_{k+1}, the following equation is tenable.

$$K_{k+1}^{TRKF} < K_{k+1}$$

Proof: The covariance matrix $P_{k+1/k}^{TRKF}$ is smaller than $P_{k+1/k}$, because the TRKF is more accurate than Kalman filter. Since the filter gain is proportional to the covariance matrix of predicted estimator, therefore, $K_{k+1}^{TRKF} < K_{k+1}$ is tenable.

This property indicates that the influence of observation equation on Kalman filter is reduced but the influence of state equation on Kalman filter is increased when TRKF is used at epoch t_{k+1}.

2° Supposing that Kalman filter is used before epoch t_k and TRKF is used ant epoch t_k, we can get $MSEM(\delta\widehat{X}_k^{TRKF}) < MSEM(\delta\widehat{X}_k)$ if and only if

$$(X_k - \widehat{X}_{k/k-1})^T M_k (X_k - \widehat{X}_{k/k-1}) < 1$$

where $M_k = (K_k^{TRKF} - I)^T [K_k R_k K_k^T - K_k^{TRKF} R_k (K_k^{TRKF})^T] (K_k^{TRKF} - I)$ and $\delta\widehat{X}_k^{TRKF} = K_k^{TRKF}(L_k - A_k \widehat{X}_{k/k-1})$. MSEM denotes Mean Square Error Matrix.

Proof: The predicted estimator $\widehat{X}_{k/k-1}$ can be regarded as a known value because the estimator $\widehat{X}_{k-1}$ has been obtained at time t_k. It is obvious $E(\delta\widehat{X}_k) = X_k - \widehat{X}_{k/k-1}$. Therefore, the MSEM of $\delta\widehat{X}_k$ is

$$\begin{aligned} MSEM(\delta\widehat{X}_k) &= Cov(\delta\widehat{X}_k) + E[(\delta\widehat{X}_k) - (X_k \\ &\quad - \widehat{X}_{k/k-1})][E(\delta\widehat{X}_k) - (X_k - \widehat{X}_{k/k-1})]^T \\ &= K_k R_k K_k^T \end{aligned}$$

We can get

$$\begin{aligned} &MSEM(\delta\widehat{X}_k^{TRKF}) = Cov(\delta\widehat{X}_k^{TRKF}) + [E(\delta\widehat{X}_k^{TRKF}) \\ &-(X_k - \widehat{X}_{k/k-1})][E(\delta\widehat{X}_k^{TRKF}) - (X_k - \widehat{X}_{k/k-1})]^T \\ &= K_k^{TRKF} R_k (K_k^{TRKF})^T + (K_k^{TRKF} A_k - I)(X_k \\ &- \widehat{X}_{k/k-1})(X_k - \widehat{X}_{k/k-1})^T (K_k^{TRKF} A_k - I)^T \end{aligned}$$

for $E(\delta\widehat{X}_k^{TRKF}) = K_k^{TRKF} A_k (X_k - \widehat{X}_{k/k-1})$.

It can be concluded that

$$\begin{aligned} &MSEM(\delta\widehat{X}_k) - MSEM(\delta\widehat{X}_k^{TRKF}) = \\ &K_k R_k K_k^T - K_k^{TRKF} R_k (K_k^{TRKF})^T + (K_k^{TRKF} A_k - I) \\ &(X_k - \widehat{X}_{k/k-1})(X_k - \widehat{X}_{k/k-1})^T (K_k^{TRKF} A_k - I)^T \end{aligned}$$

It has been proved that $K_k^{TRKF} < K_k$ in property 1°. Therefore, $K_k R_k K_k^T - K_k^{TRKF} R_k (K_k^{TRKF})^T$ is a positive definite matrix. It can be obtained $MSEM(\delta\widehat{X}_k) - MSEM(\delta\widehat{X}_k^{TRKF}) > 0$ if and only if $(X_k - \widehat{X}_{k/k-1})^T M_k (X_k - \widehat{X}_{k/k-1}) < 1$ by the theorem in [14].

The property indicates that TRKF is better than Kalman filter in the sense of reduced MSE.

3° The last two terms of the objective function (14) are equal if H_k is equal to $P_{k/k-1}$ and α_k equals 1. However, their meanings are different, because $V_{\overline{X}_k^{TRKF}}^T P_{k/k-1}^{-1} V_{\overline{X}_k^{TRKF}}$ includes the information of state equation and historical information which are must be used in the Kalman filter to obtain the optimal estimator and $\alpha_k Z_k^T H_k Z_k$ is the constraint condition in order to make the optimal estimator stability. From further analysis, we can know that $V_{\overline{X}_k^{TRKF}}^T P_{k/k-1}^{-1} V_{\overline{X}_k^{TRKF}}$ also can be regarded as the constraint condition of observation equation when taking no account of the state equation. However, the estimator is calculated by the state equation and the observation equation. Therefore, $V_{\overline{X}_k^{TRKF}}^T P_{k/k-1}^{-1} V_{\overline{X}_k^{TRKF}}$ cannot overcome or weaken the ill-conditioning of the observation matrix but $\alpha_k Z_k^T H_k Z_k$ can do it.

4 Choosing Regularization Parameter and Regularization Matrix

4.1 Choosing Regularization Parameter

A lot of theoretical researches and practical works show that the L-curve method to choosing the regularization parameter is not only simpler, but also easier to implement [15-18].

However, the L-curve method is given based on Gauss-Markov model, and it is not suitable for discrete dynamic system. To this end, a method for choosing regularization parameter in TRKF is proposed as follows, which is suitable for discrete dynamic systems in this paper.

Because R_k^{-1} and $P_{k/k-1}^{-1}$ are positive definite matrices, equation (6) is equivalent to

$$\begin{aligned} \varphi(\widehat{X}_k) &= ||(R_k^{-1})^{1/2} V_{\widehat{X}_k}||^2 + ||(P_{k/k-1}^{-1})^{1/2} V_{\overline{X}_k}||^2 \\ &= ||F_k \widehat{X}_k - b_k||^2 \end{aligned} \tag{18}$$

where $F_k = [(R_k^{-1})^{1/2} A_k,\ (P_{k/k-1}^{-1})^{1/2}]^T$ and $b_k = [(R_k^{-1})^{1/2} L_k,\ (P_{k/k-1}^{-1})^{1/2} \widehat{X}_{k/k-1}]^T$.

The corresponding objective function is equivalent to equation (14) after adding constraints,

$$\Psi(\widehat{X}_k^{TRKF}) = ||F_k \widehat{X}_k^{TRKF} - b_k||^2 + \alpha_k ||Z_k||_{H_k}^2 \tag{19}$$

Choosing different values of , we can get many points $(||F_k\widehat{X}_k^{TRKF} - b_k||^2, \ \ ||Z_k||^2_{H_k})$. A curve in the plane is obtained after curve fitting and the point of maximum curvature can be determined. The corresponding value of α_k is determined as the regularization parameter.

4.2 Choosing Regularization Matrix

The method for choosing regularization matrix must make full use of the priori information of state parameter [19,20]. The matrix $P_{k/k-1}$ in the constraints Z_k is known and it is reasonable to choosing $P_{k/k-1}$ as the regularization matrix by the second term in objective function (14). Therefore, this paper determines $H_k = P^{-1}_{k/k-1}$ as the regularization matrix.

5 Simulations and Analysis

Simulations are implemented to illustrate the TRKF described above. A discrete linear system is described by state equation (1) and observation equation (2), where state parameter $X_k \in R^{4\times1}$ is estimated. The state transition matrix $\Phi_{k,k-1}$, observation matrix A_k, the covariance matrix of system noise Q_{k-1} and the covariance matrix of observation noise R_k are set as follows:

$$\Phi_{k,k-1} = I_4 \ ,$$

$$A_k = \begin{bmatrix} 1 & -1 & 1 & 0 \\ -2 & 4.2 & 0 & 1 \\ 3 & -2 & 4 & 8 \\ -1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 \end{bmatrix} \ ,$$

$$Q_{k-1} = 0.1^2 \times I_4 \ ,$$

$$R_k = 0.5^2 \times I_5 \ ,$$

The initial value is $\widehat{X}_0 = X_0 + 0.001 \times [1 \ 1 \ 1 \ 1]^T$ where $X_0 = [2 \ 4 \ 6 \ 8]^T$ and the initial error covariance matrix is $P_0 = I_4$.

Comparisons among Kalman filter, ORKF and TRKF are implemented in this paper. The criteria to evaluate the accuracy of the algorithms are precision (the European norm between the true-value and estimator of state parameter) and MSE of estimator. The calculation results are shown as Fig.1-3.

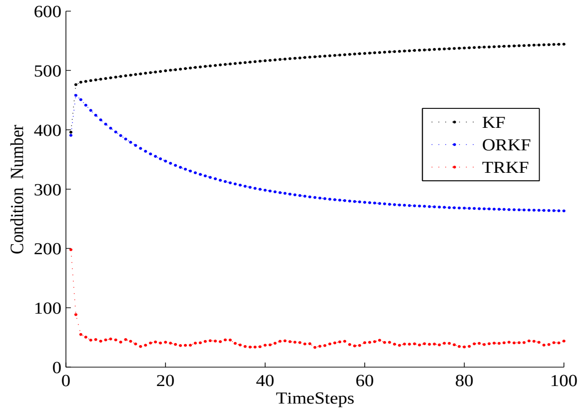

Fig. 1: Condition Number of KF, ORKF and TRKF

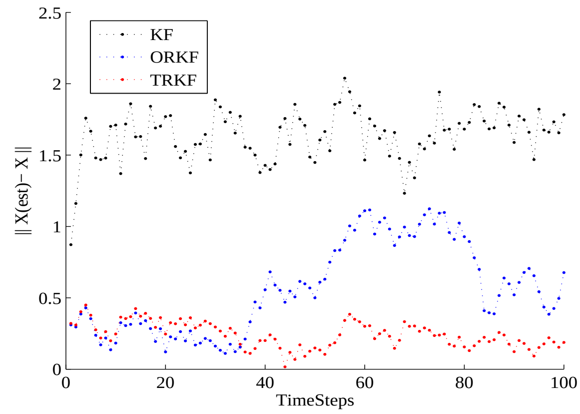

Fig. 2: Precision of KF, ORKF and TRKF

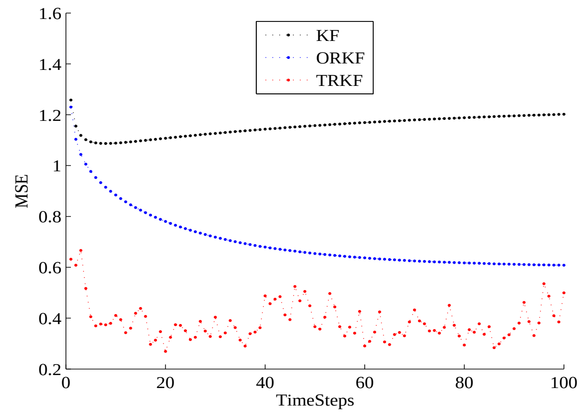

Fig. 3: MSE of KF, ORKF and TRKF

From Fig.1-3, conclusions can be obtained as follows.

1) The Kalman filter is seriously influenced by the ill-conditioning of observation matrix in discrete dynamic system.

2) TRKF can reduce the condition number of normal matrix greatly when the observation matrix is ill-conditioned.

3) Because the predicted estimator is used to restrict the optimal estimator, the regularization parameter and the regularization matrix are used to balance data fitting part and constraint conditions in TRKF. Therefore, the harmful effect caused by ill-conditioned observation matrix on Kalman filter is weakened and TRKF is more accurate and more stability than Kalman filter.

4) The filtering convergence speed of TRKF is faster and the precision is higher than that of ordinary ridge-type Kalman filter.

6 Applications of Tikhonov Kalman Filter in Autonomous Orbit Determination of BDS

Kalman filter is one of the most common ways for autonomous orbit determination. China has not built ground stations on global scale, and the satellite orbit determination which depends on global ground stations is not suitable for the BDS of China. Meanwhile, the autonomous orbit determination using inter-satellite measurements has been well researched and a conclusion has been obtained that the Earth Orientation Parameters (EOP) as one of the error sources cannot be determined. That is to say, the essence of autonomous orbit determination is rank deficient. The distributed autonomous orbit determination mode based on inter-satellite measurements combined with several ground stations is a feasible scheme for the actual situation of China. However, the accuracy of orbit products could be much poor if only regional ground monitoring stations are available, as the satellites are tracked only when they fly over the regional network, which consequently leads to a rather weak observing geometry. This makes the observation matrix ill-conditioned [21,22].

To be simple, only the J2 perturbation of the earth central body is taken into account. The fixed positions of ground stations are used, and the tropospheric delays and earth rotation parameters are not considered. J2000 is used as the spatial reference system. A mixed navigation constellation of 5GEO+3IGSO+24MEO is simulated by STK (Satellite Tool Kit). The five GEO satellites are positioned at longitude $59°, 87.5°, 110.5°, 142°$ and $163°$ respectively. The intersection of the three IGSO satellites is positioned at longitude $118°$, and the orbital inclination is $55°$. The 24 MEO satellites compose a Walker24/3/1 sub-constellation, and the GPS SPS (2001) standards are used to determine the parameters of altitude and orbital inclination of MEO satellites and they are respectively 26559.8km and $55°$. The wide-beam inter-satellite link is used, and the beam angle of transmitting antenna ranges from $20°$ to $60°$. The noise factor of dynamical model is 10^{-9}, the mean square deviation of observation noise of inter-satellite measurement is 0.15m, and the sampling period of filter is 15min. All data are processed on satellites for distributed autonomous orbit determination, and thus two-way measurements between satellites and one-way measurements between ground stations and satellites are used. The altitude angle of ground station ranges from $5°$ to $90°$. The prior mean square deviations of each satellite are: $\delta a = 100m$, $\delta e = 1.0 \times 10^{-5}$, $\delta\theta = 1.0 \times 10^{-5}$, $\delta\Omega = 1.0 \times 10^{-10}$, $\delta\omega = 1.0 \times 10^{-10}$ and $\delta M = 1.0 \times 10^{-5}$, where a, e, θ, Ω, ω and M are the semi-major axis, eccentricity, orbit inclination, longitude ascending node, argument of perigee, and mean anomaly respectively. The initial location errors of each satellite are 20m.

The inter-satellite observations are combined with ground station observations from Xian, Shanghai, Kunming, Guangzhou Hainan, and Kashgar.

The User Range Error (URE) [23] and the Root Mean Square (RMS) of a constellation are used as the indexes to evaluate the accuracy of orbit determination.

The calculation formula of URE is

$$URE = \sqrt{\frac{1}{N_{sat}} \sum_{i=1}^{N_{sat}} S(i)}$$

where

$$S(i) = \sqrt{R_{ERR}^2(i) + 0.0192 \times [T_{ERR}^2(i) + N_{ERR}^2(i)]} \quad (20)$$

The symbol N_{sat} denotes the total number of satellites in constellation, $S(i)$ is the URE of i-th satellite, $R_{ERR}(i), T_{ERR}(i)$ and $N_{ERR}(i)$ are the radial error, along track error and normal error of i-th satellite, respectively. Equation (20) employs a factor 0.0192 for weighting the contribution of cross-track and along-track errors in the URE, and a common factor is used for all simulated satellites in this paper. All the satellites of MEO, GEO and IGSO are used to calculate the URE of this constellation.

Respectively, the calculation methods of RMS of radial error, along track error and normal error are

$$RMS_R = \sqrt{\frac{1}{N_{sat}} \sum_{i=1}^{N_{sat}} R_{ERR}^2(i)}$$

$$RMS_T = \sqrt{\frac{1}{N_{sat}} \sum_{i=1}^{N_{sat}} T_{ERR}^2(i)}$$

and

$$RMS_N = \sqrt{\frac{1}{N_{sat}} \sum_{i=1}^{N_{sat}} N_{ERR}^2(i)}$$

Comparisons between Kalman filter and TRKF are given. The results of the condition number, the UER and the RMS are shown as Fig.4-6 and Fig.10-12. The radial error, along track error and normal error of MEO A1, GEO1 and IGSO1 are given in Fig.7-9 and Fig.13-15.

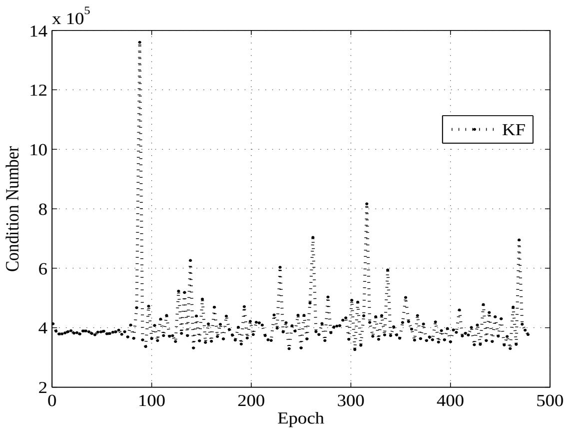

Fig. 4: Condition Number of KF

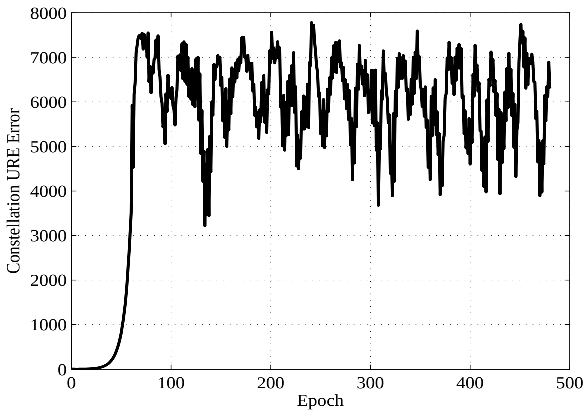

Fig. 5: URE of KF

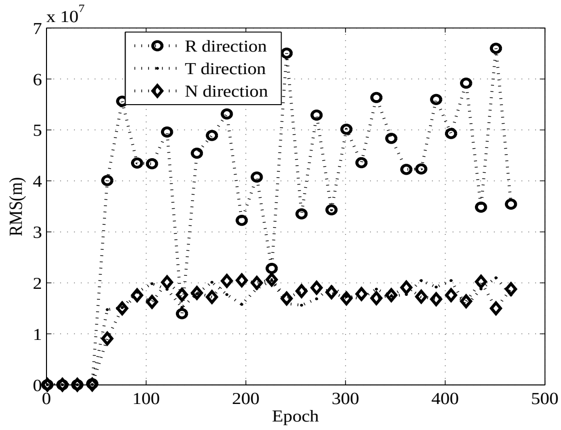

Fig. 6: RMS of KF

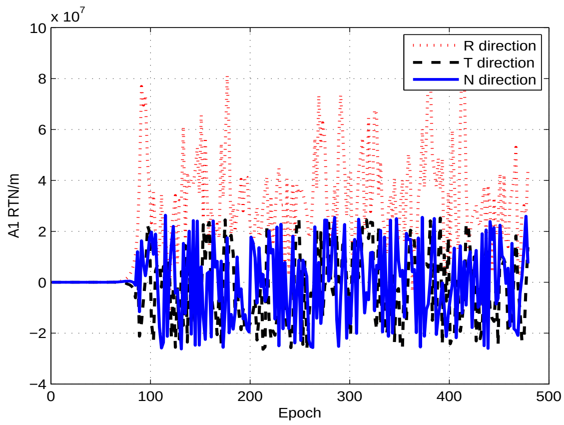

Fig. 7: RTN of MEO A1 when the KF is used

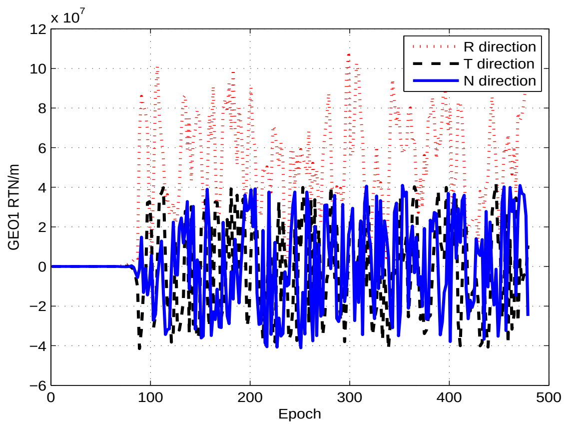

Fig. 8: RTN of GEO1 when the KF is used

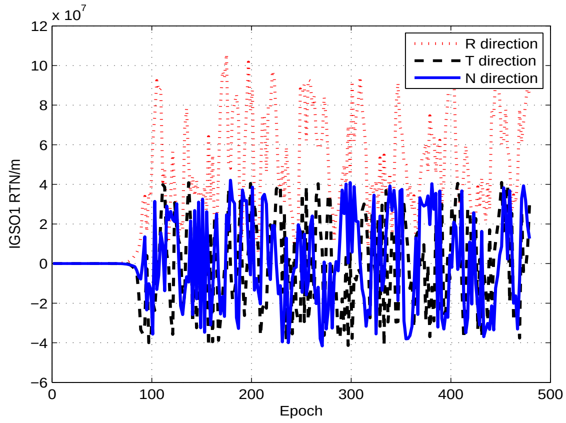

Fig. 9: RTN of IGSO1 when the KF is used

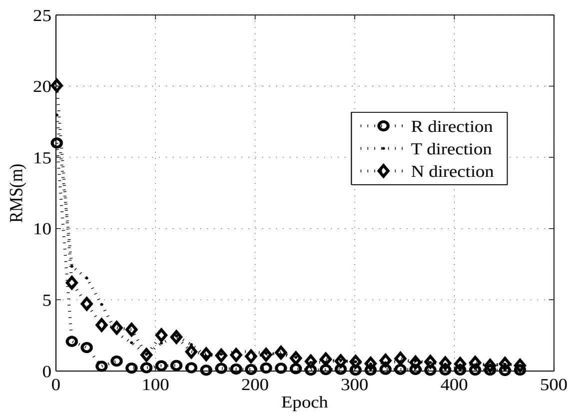

Fig. 12: RMS of TRKF

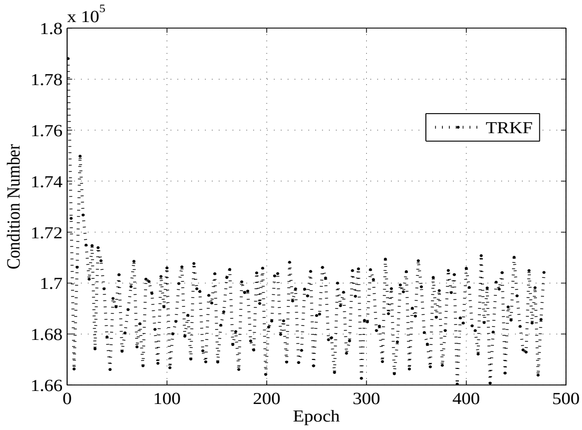

Fig. 10: Condition Number of TRKF

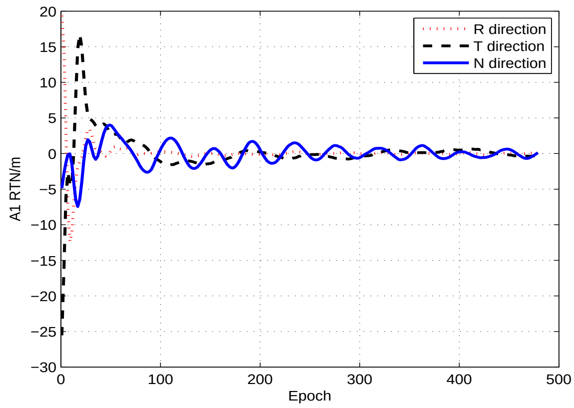

Fig. 13: RTN of MEO A1 when the TRKF is used

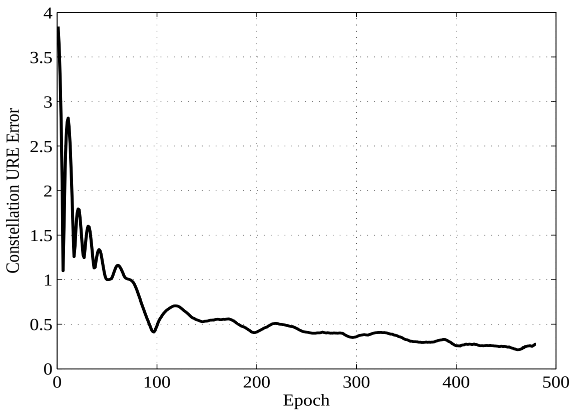

Fig. 11: URE of TRKF

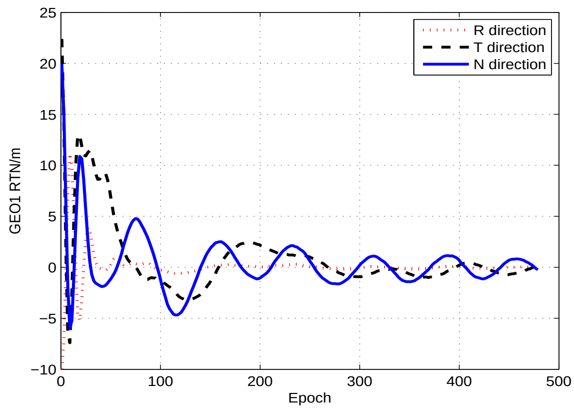

Fig. 14: RTN of GEO1 when the TRKF is used

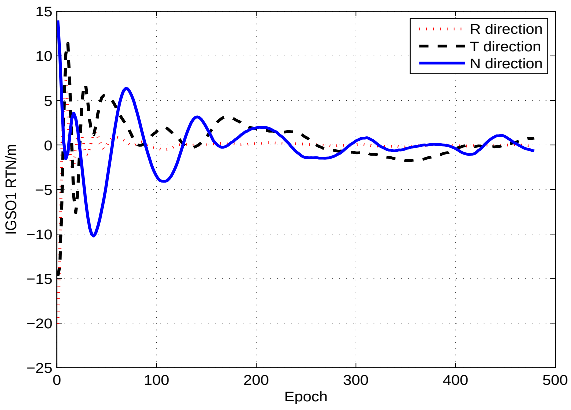

Fig. 15: RTN of IGSO1 when the TRKF is used

It can be concluded from Fig.4-15.

1) The observation matrix is seriously ill-conditioned for the observing geometry is rather weak in the autonomous orbit determination of BDS. Therefore, accuracy of the Kalman filter is poor.

2) By using the prior information about the state parameters to restrict the optimal estimator, TRKF can overcome the harmful effect of the ill-conditioned observation matrix on Kalman filter. Therefore, TRKF can prevent divergence character of traditional filter and can also improve the accuracy of it effectively.

7 Conclusion

1) The Kalman filter was seriously influenced by the ill-conditioning of observation matrix in discrete dynamic systems. Meanwhile, although the state equation and historical information can control the ill-conditioning to some extent, but the effect is not ideal.

2) Because the predicted estimator is used to restrict the optimal estimator, regularization parameter and regularization matrix are used to balance data filting part and constraint conditions in TRKF. Therefore, the harmful effect caused by ill-conditioned observation matrix on Kalman filter is weakened and TRKF is more accurate and more stability than Kalman filter.

3) The convergence speed of filter is faster and the precision is higher within TRKF than that of ordinary ridge-type Kalman filter.

4) The observation matrix is seriously ill-conditioned for the observing geometry is rather weak in the autonomous orbit determination of BDS. Therefore, accuracy of the Kalman filter is poor. However, TRKF can overcome the adverse effects of ill-conditioning effectively and improve the accuracy of autonomous orbit determination of BDS.

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