

# Media access over PLC technology using Shapley-Shubik power index

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*Abstract:* Due to the growing demand of new services, IEEE 1901 has embedded a mechanism of Access to the hybrid means, supported in CSMA/CA and TDMA in order to offer accurate levels of quality of service (QoS). Nevertheless, such strategy has shown low efficiency, as the number of nodes increases. In the view of the above, this paper proposes a mechanism of media access over OFDMA (Orthogonal Frequency-Division Multiple Access), based on the weighted voting games, supported in the Shapley-Shubik's power index in order to optimize the allocation of resources in the time and frequency domain. This optimizing will, thus, improve the network performance and allow multiple nodes to be able to transmit simultaneously. According to the results, it was possible to evidence that the usage of weighted voting games supported in the Shapley-Shubik's power index can be considered as an alternative to establish a mechanism of access to the adequate media in PLC networks and its feasibility of being implemented in low-cost embedded systems due to the fact that such alternative does not require a high computational complexity for its development.

*Key-Words:* Resource management, Computer networks, Cooperative Game Theory, Optimization methods, Shapley-Shubik's power index, Power Line Communications.

## 1 Introduction

Power Line Communications (PLC), makes reference to a group of technologies that allow to establish communication processes by means of the usage of the electric network as a physical media of transmission, where HomePlug AV (IEEE 1901) is one of the most accepted standards on PLC technology [1]. IEEE 1901 uses OFDM as multiplexing technique and an access mechanism to the hybrid media supported in CSMA/CA and TDMA, in which CSMA/CA is meant to be used for data package transmission, while TDMA is used for voice and video package transmission, so that it can offer accurate levels of QoS [2].

Despite IEEE 1901 can reach transmission rates close to 500 Mbps, the Standard lacks Policies and a media access mechanism suitable for the right allocation of resources in the domain of frequency and time. Such fact highly affects the provision of ser-

vices on IP and network performance, due to the increase of users amount and channel dynamic conditions [1][3][4]. Considering the above, the next questions raises: ¿What can be done to optimize access to a PLC channel, taking into account the channel conditions and the established requirements for each node, in order to offer adequate levels of QoS without affecting the performance of a different service? In the face of the above, it is proposed the usage of Game theory of weighted voting, supported in the Shapley-Shubik's Power Index, as a strategy to optimize the PLC channel access.

A PLC network is a setting in which the nodes constantly present particular transmission needs. This occurrence origins conflicts to media access and inequality situations and resource waste. Nonetheless, it is possible to establish an order of PLC channel access by means of the usage of democratic systems sup-

ported in Game theory of weighted Voting. Moreover, when considering the network nodes as players that can work cooperatively, it increases the possibility of getting a superior profit compared to the one of an individual performance [5].

Although the concept of power is considered to be an illusion, it is a very useful criterion for decision making in several systems and the Game theory goes deep in the study of this particular subject[6]. Each player can have different power levels on the voting process, which depends on the value or amount of votes associated to him/her. However, there is no a lineal reliance with the vote number due to the fact that there is a chance that each player generates coalitions with other players.

In the view of the above, the present paper proposes a methodology to establish a Scheduler by Homeplug AV period (33,33ms), supported in the Shapley-Shubik's Power index, to optimize the spectrum allocation to each node and service class, in the domain of time and frequency, considering the bit-rate (BW) allocation, and QoS requirements. Moreover, the paper lays out the usage of OFDMA (Orthogonal Frequency-Division Multiple Access) instead of OFDM (Orthogonal Frequency Division Multiplexing), which allows more than one node to transmit simultaneously through PLC channel [7][8].

Generation of scheduler is a process managed and socialized by the node that serves the function of central coordinator (CCo) according to what is established by IEEE 1901 Standard. The Scheduler is considered to be a strategy aiming to maximize the throughput and minimize the collisions presented on a multiuser setting under a common collision domain, as in a PLC Network [9].

It is important to mention that the processes related to provision of voice, data and video service are actual research topics and they draw interest in the PLC context, due to the increasing of the network service demand, as well as the need for having adequate levels of QoS in order to meet the needs, despite the less favorable network existing conditions [10].

## 2 Methodology

### 2.1 Weighted voting games

**Definition 1** A weighted voting game can be defined as a triad  $(N, w, q)$  where  $N = \{1, 2, 3 \dots n\}$  is the set of players,  $w = \{w_1, w_2, \dots, w_n\}$  with  $w_i \geq 0$  for  $i = 1, 2, \dots, n$  is a weight distribution and  $q \in R^+$  is the fee or majority demanded to win a voting process.

Generally  $q > \frac{T}{2}$ , where  $T$  corresponds to the total sum of the weights  $T = w_1 + w_2 + \dots + w_n$ . A

weighted voting game is represented as follows [11]:

$$[q : w_1, w_2, \dots, w_n] \quad (1)$$

When in a voting game, no voters has enough votes to even the established fee, it is necessary to carry out a cooperation process among voters in order to join votes and work in alliances called coalitions. Such coalitions allow voters to exceed the established fee and thus win the voting process. Some concepts that are extremely important on the basis of Weighted Voting games are displayed above:

- A Coalition is a winner if the sum of votes of each member allows to even or exceed the established fee.
- A Coalition is a winner if the sum of votes of each member allows to even or exceed the established fee.
- A Coalition is a block when the sum corresponding to the vote number of each member is higher than the difference between the total sum of the rest of the voters and the game fee.
- A Coalition is a loser if it is no winner and at the same time if it is not a block.
- A Coalition is a minimum or minimal winner when all the voters are essentials.
- A essential or tilting voter is someone who abandons the Coalition at the moment of voting, generating such coalition stops from being a winner and thus becoming loser.
- A parade voter is someone that in case of abandoning a winner or block coalition does not alter the condition of such coalition.

### 2.2 Power Indexes

The Power Index is a number allocated to each player in order to indicate the power level such player has over the game, and can also be understood as the percentage of power that each group may have within the system [6]. The Power Indexes measure the cooperation possibilities each group has within winner coalitions. The bigger the weight, the bigger the power. However, there could be some situations in which despite the difference in group weights, indexes of equal powers occur [6]. In this paper, a Shapley-Shubik power index will be used as a strategy in order to generate a Scheduler for accessing a Network channel PLC.

*Shapley-Shubik power index*

Power index for a determined participant  $P$ , in a weighted voting game, consists in obtaining all the voters permutations in the game and in each one of the permutations in which  $P$  appears. Then, one should check if the voters preceding  $P$  in this permutation, form a loser coalition and thus if adding  $P$  value, such coalition would become winner. When this situation takes place, this permutation is called Pivot of  $P$ .

The power Index of  $P$  participant is defined as the quotient resulting from dividing the Pivot number  $P$  by the total number of permutations. If there is  $N$  members, and there is a number of permutations  $SS = n!$  and a Pivot total  $k(P)$ . The general expression to calculate the Shapley-Shubik power index is as follows:

$$I_{Sh}(P) = \frac{k(P)}{S} \tag{2}$$

*Algorithms for weighted voting games*

Calculating power indexes in weighted voting games is an issue that involves high levels of computational complexity when such game involves more than 20 players, due to the use of permutations within classical algorithms [16]. Fortunately, there is a way to carry out this process without inspecting all possible coalitions, by means of the use of generating functions. Those functions offer, in terms of combinatory analysis, an indirect method for calculating the number of configurations of determined types.

Be  $\{a_j\} \forall j > 0$ , a sequence of real numbers, which can be represented by means of the function:

$$f(x) = \sum_{j=0}^{\infty} a_j x^j \tag{3}$$

Such function  $f(x)$  is called Ordinary generating function of sequence  $\{a_j\} \forall j > 0$ .

Ordinary generating functions provide a method to count the number of elements  $c(k)$  from a finite set. It is possible to represent an ordinary generating function as follows:

$$f(x) = \sum_{k=0} c(k) x^k \tag{4}$$

The number of subsets of  $k$  elements from  $N = 1, 2, n$  can be determined by means of the expression that allows calculating Binomial coefficients:

$$\binom{n}{k} = \frac{n(n-1) \dots (n-k+1)}{k!} \tag{5}$$

Therefore, the ordinary generating function that estimates the binomial coefficients is given by

$$\sum_{k>0} \binom{n}{k} x^k = (1+x)^n \tag{6}$$

In Ribnikov [13] it is possible to find information that best describes aspects related to use and properties of generating functions. As follows, the use of generating functions as a strategy to calculate Shapley-Shubik power index in a weighted voting game  $(N, v)$ , defined by  $v = [q; w_1, w_2, \dots, w_n]$ .

**2.3 Estimating shapley-shubik power index through generatrice functions**

The expression to calculate the Shapley-Shubik power index, for each player  $i \in N$ , on a simple game is as follows (7):

$$\begin{aligned} \phi_i(v) &= \sum_{S \notin W: S \cup i \in W} \frac{s!(n-s-1)!}{n!} \\ &= \sum_{j=0}^{n-1} \frac{j!(n-j-1)!}{n!} d_j^i \end{aligned} \tag{7}$$

where  $d_j^i$  represents the pivots number from the player  $i$  in coalitions of a  $j$  size; that is, the number of coalitions of size  $j$  that become winners when player  $i$  is added to the coalition. In a weighted voting game  $(N, v)$ , defined by  $v = [q; w_1, w_2, \dots, w_n]$  it is possible to calculate  $d_j^i$  under the use of generating functions described above. Thus,

$$d_i^j = \sum_{k=q-w_i}^{q-1} a_{k,j}^i \tag{8}$$

where  $a_{k,j}^i$  the number of pivots from player  $i$  in coalitions formed with  $j$  number of players, different from  $i$ , and with a coalitional weight  $k$ .

**Definition 2** Cantor (1962) [14], established that: for any weighted voting game  $(N, v)$  defined as  $v = [q; w_1, w_2, \dots, w_n]$ , to any  $i \in N$ , the generating function to estimate  $a_{k,j}^i$  coalitions  $S \subseteq N$  formed by  $j$  number of players, with  $i \notin S$  and  $w(S) = k$ , is the following

$$\begin{aligned} S_i(x, z) &= \prod_{j=1, j \neq i}^n (1 + x^{w_j} z) \\ &= \sum_{k=0}^{w(N)} \sum_{j=0}^n a_{k,j} x^k z^j \end{aligned} \tag{9}$$

where  $j$  and  $k$  represent the size of the coalition  $S$  and its weight, respectively.

Knowing the coefficients  $a_{k,j}^i$  by means of the use of generating function  $S_i(x, z)$  allows determining the value of coefficients  $d_j^i$  that are needed in order to calculating Shapley-Shubik power index. Thanks to that,

values  $d_j^i$  can be identified as the coefficients from a polynomial that follows this expression

$$g_i(z) = \sum_{j=0}^{n-1} d_j^i z^j \tag{10}$$

And taking into account that

$$d_j^i = \sum_{k=q-w_i}^{q-1} a_{kj}^i \tag{11}$$

When one replaces, one can get

$$g_i(z) = \sum_{j=0}^{n-1} d_j^i z^j = \sum_{j=0}^{n-1} \left[ \sum_{k=q-w_i}^{q-1} a_{kj}^i \right] z^j \tag{12}$$

Evening  $g_i(z)$  with the expression  $S_i(x, z)$  it is possible to deduce that in order to determine the coefficients  $g_i(z)$  it is necessary to select the coefficient of the monomial  $x^k z^j$  in which the exponent  $q - w_i \leq k \leq q - 1$ .

**Example 3** Consider the weighted voting game  $(N, v)$  given as follows

$$v = [q : w_1, w_2, w_3, w_4] = [q : 14, 116, 2] \tag{13}$$

*Step 1:* Value of  $q$  is calculated

$$q = \frac{(14 + 11 + 6 + 2)}{2} = 16,5 \approx 17$$

*Step 2:* Calculating each function  $S_i(x, z)$  using the following expression

$$S_i(x, z) = \prod_{j=1, j \neq i}^n (1 + x^{w_j} z) \tag{14}$$

The result for each one of the functions is:

$$\begin{aligned} S_1(x, z) &= (1 + x^{11}z)(1 + x^6z)(1 + x^2z) \\ S_1(x, z) &= x^{19}z^3 + x^{17}z^2 + x^{13}z^2 + x^{11}z \\ &\quad + x^8z^2 + x^6z + x^2z + 1 \\ S_2(x, z) &= (1 + x^{14}z)(1 + x^6z)(1 + x^2z) \\ S_2(x, z) &= x^{22}z^3 + x^{20}z^2 + x^{16}z^2 + x^{14}z \\ &\quad + x^8z^2 + x^6z + x^2z + 1 \\ S_3(x, z) &= (1 + x^{14}z)(1 + x^{11}z)(1 + x^2z) \\ S_3(x, z) &= x^{27}z^3 + x^{25}z^2 + x^{16}z^2 + x^{14}z \\ &\quad + x^{13}z^2 + x^{11}z + x^2z + 1 \\ S_4(x, z) &= (1 + x^{14}z)(1 + x^{11}z)(1 + x^6z) \\ S_4(x, z) &= x^{31}z^3 + x^{25}z^2 + x^{20}z^2 + x^{17}z^2 \\ &\quad + x^{14}z + x^{11}z + x^6z + 1 \end{aligned}$$

By means of the generating function  $S_i(x, z)$  it is possible to determine the numbers  $\{d_j^i\}_{j \geq 0}$  that are needed to calculate the Shapley-Shubik power index.

*Step 3:* Calculating polynomials  $g_i(z)$  by means of the use of the expression

$$g_i(z) = \sum_{j=0}^{n-1} d_j^i z^j = \sum_{j=0}^{n-1} \left[ \sum_{k=q-w_i}^{q-1} a_{kj}^i \right] z^j \tag{15}$$

$$g_1(z) = \sum_{j=0}^3 \left[ \sum_{k=3}^{16} a_{kj}^1 \right] z^j$$

For the polynomial  $S_1(x, z)$  it is necessary to extract such monomial in which the exponent of  $x$  is between 3 and 16:

$$x^{13}z^2 + x^{11}z + x^8z^2 + x^6z$$

Evaluating  $x = 1$ , the result is:

$$g_1(z) = \sum_{j=0}^3 d_j^1 z^j = 2z^2 + 2z$$

Applying the same process to the polynomials  $S_2(x, z)$ ,  $S_3(x, z)$  y  $S_4(x, z)$ ; the values  $g_2(z)$ ,  $g_3(z)$  y  $g_4(z)$  are:

$$\begin{aligned} g_2(z) &= 2z^2 + 2z \\ g_3(z) &= 2z^2 + 2z \\ g_4(z) &= 0 \end{aligned}$$

*Step 4:* The value of Shapley-Shubik  $\phi_i(v)$  for each player  $i$  according to the values registered in the first polynomials  $g_i(z)$ . In the case of  $\phi_1(v)$ , the result is as follows:

$$\begin{aligned} \phi_1(v) &= \sum_{j=0}^{n-1} \frac{j!(n-j-1)!}{n!} d_j^1 \\ &= \frac{1}{4!} [1!(4-1-1)!d_1^1 + 2!(4-2-1)!d_2^1] \end{aligned}$$

Taking into account

$$g_1(z) = \sum_{j=0}^3 d_j^1 z^j = 2z^2 + 2z$$

It is possible to observe that  $d_1^1 = 2$  and  $d_2^1 = 2$ . Replacing those values in  $\phi_1(v)$ , the result one can obtain is

$$\begin{aligned} \phi_1(v) &= \frac{1}{4!} [1!(4-1-1)!2 + 2!(4-2-1)!2] \\ \phi_1(v) &= \frac{1}{4!} [1!2!2 + 2!1!2] \end{aligned}$$

$$\phi_1(v) = \frac{1}{24}[4 + 4] = \frac{1}{3}$$

Carrying the same process for the rest of the players, the value of Shapley-Shubik is the following:

$$\phi_2(v) = \frac{1}{3}, \quad \phi_3(v) = \frac{1}{3}, \quad \phi_4(v) = 0.$$

In order to facilitate future research processes related to power indexes, a Matlab routine was developed, which allows estimating the Shapley-Shubik power index. The routine goes as follows:

```
V=[14; 11; 6; 2];
% The vector of weight for each
% player is defined
q=ceil(sum(V)/2); % the value of q is estimated
Nj=4; % Nj contains the number of players

VSh=zeros(Nj,1);
syms x z y
for i=1:Nj
    P=zeros(Nj-1,1);
    s=1;
    u=0;
    V1=zeros(Nj,1);
    for l=1:Nj
        if l~=i
            u=u+1;
            V1(u)=V(l);
        end
    end
    s=y*(1+(x^V1(1)*z*y))*(1+(x^V1(2)*z*y^2));
    PotY=4;
    for l=3:Nj-1 % Routine to establish the
                % Generating functions Bi(x)
        s=s*(1+(x^V1(l)*z*y^PotY));
        PotY=2^l;
    end
    s1=expand(s);
    % perform the symbolic multiplication
    % of the polynomial Si(x,z)
    [cxz cy]=coeffs(s1,y);
    % extract the coefficient of a polynomial cxz
    % Take values of xz and cy the values of y
    % (auxiliary % variable)
    cx=cxz/(z^Nj);
    % extract the X factor of each monomial

    [num, den]=numden(cx);
    cx=num;

    PotX=subs(log10(cx),x,10);
    % evaluate the polynomial in the value of x=10

    PotX=simplify(PotX);
    % simplify values by extracting exponents of x

    a=x.^PotX;
    cz=cxz./(a);
    % extract factor z in each monomial

    [num, den]=numden(cz);
    cz=num;

    PotZ=subs(log10(cz),z,10);
    % evaluate the polynomial in the value of
    % t=10 and register the total weight of
```

```
% each coalition

PotZ=simplify(PotZ);
% simplify values extracting
% the exponents of t

n=length(PotX);
Dij=zeros(n,1);
Linf=q-V(i); % q-Wi
Lsup=q-1; % q-1
for k=1:n-1
    if (PotX(k)>=Linf) & (PotX(k)<=Lsup)
        % if the coefficients of the player
        % i are found between the upper and
        % lower limits

        Dij(PotZ(k))=Dij(PotZ(k))+1;
        % Calculate the parameter Dij
    end
end
Aux=0;
% subroutine to calculate Shapley-Shubik
% value for the player k
for j=1:Nj-1
    Aux=Aux+(factorial(j)*Dij(j)*factorial(Nj-j-1));
end
VSh(i)=(1/(factorial(Nj)))*Aux;

end
VSh
% Print the vector corresponding to Shapley-
% Shubik power index allocated to each player
```

### 3 Results and Discussion

As practical setting, a Home Area Network (HAN) has been proposed. Such network is composed by eight (8) PLC nodes. Each node has a PLC adapter and a traffic source. Each traffic source can generate more than one type of traffic  $r$  simultaneously (Voice, data, video and telemetry). The node 8 will be considered as main node or o Coordinator (CCo). In Table 1, it is displayed the types of traffic and established Codec (only applied to voice and video) for each node  $i$  and type  $r$  that compose the proposed setting.

Table 1: Type of traffic for each node  $i$  and type  $r$

Node $i$	Types of Traffic			
	Voice ( $r = 1$ )	Video ( $r = 2$ )	Control ( $r = 3$ )	Data ( $r = 4$ )
1	G.711		X	X
2	G.723.1		X	X
3		MPEG-4		X
4	G.729		X	X
5		MPEG-2	X	
6				X
7	G.711			X
8	X	X	X	X

Table 2 shows the Bandwidth (BW) required for each type of traffic for the proposed setting.

Table 2: BW required for each node  $i$  and type  $r$ 

Node $i$	Types of Traffic			
	Voice [kpbs]	Videoa [Mbps]	Control [kpbs]	Data [Mbps]
1	81.6		1.5	3.5
2	23.2		2.2	10.2
3		18.4		8.4
4	35.8		2.7	12.7
5		15.38	1.8	
6				16.5
7	102.6			11.6
8 (CCo)	243.2	33.78	8.2	62.9

To estimate the channel conditions, the tool known as "Channel generator PLC" (GC\_PLC stands for "Generador de Canal PLC"), written in MATLAB and developed by PhD Francisco Javier Cañete, who belongs to the PLC research group at University of Malaga. GC\_PLC allows estimating the behavior of a PLC channel, according to the parameters associated to the network topology on a typical residential environment. Furthermore, such tool carries out a channel evaluation process under 30MHz band (due to the fact that the PLC network adapters under the standard HPAV work in such frequency band). In [15] it is presented information about the use of GC\_PLC tool.

For the proposed setting (Table 2), the tool GC\_PLC generated the values of 159.72 Mbps, 120.65 Mbps and 83.59 Mbps for the PLC channel conditions of excellent, moderate and deficient. It is possible to observe that the total BW required is of 193,863 Mbps, which is superior to the total BW available in the PLC channel. This indicates that the HAN Network operation mode will be under a saturation state.

Table 3 shows the values corresponding to BW allocated for each node and type of traffic by means of the use of the Shapley value as a resource optimization strategy in PLC network. In the paper Shapley value [16] it is possible to find the methodology used to estimate the BW allocated for each player.

### 3.1 Weight calculation for each node

Considering that there is no evidence of a methodology to calculate the player's weight in a weighted voting game, the following criteria to establish each required weights was adopted: Due to the fact that each player can present more than one type of service, it is necessary to calculate a specific weight for each type  $r$  and player  $i$ , with  $r = 1, 2, 3, 4$  for services such as *Data*, *Control*, *Video*, *Voice* respectively. The total weight for the player  $i$  can be calculated by means of

Table 3: Estimated weights for each node and type of service

Player	CoS	BW
1	Voice	0,0449
	Control	0,0008
	Data	1,9253
2	Voice	0,0128
	Control	0,0012
	Data	5,6429
3	Video	9,8756
	Data	4,5084
4	Voice	0,0197
	Control	0,0015
	Data	6,992
5	Video	8,4300
	Control	0,0010
6	Data	9,0280
7	Voice	0,0566
	Data	6,3954
8	Voice	0,1699
	Video	23,5972
	Control	0,0057
	Data	43,9392

the use of the following expression:

$$W_i = \sum_{r=1}^4 W_{i,r}$$

On the other hand, the weight for each type  $r$  can be calculated by means of the use of the following expression (16):

$$W_{i,r} = FP_{i,r} \left[ \left( \frac{BW_{i,r}}{\min_{j \in N, k=r} BW_{j,k}} \right) \right] \quad (16)$$

where  $\min_{j \in N, k=r} BW_{j,k}$  corresponds to the minimal value of BW allocated between all players whose class of service is  $k = r$ ,  $FP_{i,r}$ . This is the priority factor type  $r$ , player  $i$  and  $[*]$  represents the operator that shows the highest whole number, closest to the registered operation.

**Example 4** You want to calculate total weight for the player  $i = 3$ , based on the data registered in Table 3.

**Answer:** Table 3 shows that player 3 will support data traffic ( $r = 1$ ) and video ( $r = 3$ ). The first step will be to calculate the weights for each of the types of services and the second step will be to calculate the total weight.

*Step 1:* Calculate the weights for each type of service ascribed to player  $i = 3$ , by means of the use of the proposed expression. The result is as follows:

$$\begin{aligned} W_{i,r} = W_{3,1} &= FP_{i,r} \left[ \left( \frac{BW_{i,r}}{\min_{j \in N, k=r} BW_{j,k}} \right) \right] \\ &= 1 \left[ \left( \frac{4, 5084}{1, 9253} \right) \right] = [2, 34] = 3 \end{aligned}$$

$$\begin{aligned} W_{i,r} = W_{3,3} &= FP_{i,r} \left[ \left( \frac{BW_{i,r}}{\min_{j \in N, k=r} BW_{j,k}} \right) \right] \\ &= 3 \left[ \left( \frac{9, 8756}{8, 4300} \right) \right] \\ &= 3[1, [17]] = 3(2) = 6 \end{aligned}$$

*Step 2:* Calculate the total weight of the player.

$$W_i = W_1 = W_{1,1} + W_{1,2} = 3 + 6 = 9$$

Finally, it is necessary to calculate the weights for each node according to the proposed criteria. The outcome of this process is registered in Table 4.

Table 4: Estimated weights for each node and type of service

$i$	CoS ( $r$ )	Factor of Priority ( $FP_{i,r}$ )	BW Allocated (Shapley) ( $BW_{i,r}$ )	Weight ( $W_{i,r}$ )	Total $W_i$
1	Voice	4	0,0449	16	19
	Control	2	0,0008	2	
	Data	1	1,9253	1	
2	Voice	4	0,0128	4	11
	Control	2	0,0012	4	
	Data	1	5,6429	3	
3	Video	3	9,8756	6	9
	Data	1	4,5084	3	
4	Voice	4	0,0197	8	16
	Control	2	0,0015	4	
	Data	1	6,9928	4	
5	Video	3	8,4300	3	7
	Control	2	0,0010	4	
6	Data	1	9,0280	5	5
7	Voice	4	0,0566	20	24
	Data	1	6,3954	4	
8 CCo	Voice	4	0,1699	56	30
	Video	3	23,5972	9	
	Control	2	0,0057	16	
	Data	1	43,9392	24	

Player (Node)  $i$ ;

In order to analyse if the PLC channel condition can have an effect in some way on the weight allocated to one player  $i$ , the calculation of each player's weight was carried out, under excellent, moderate and deficient channel conditions. The outcome of this estimating process is registered in Table 5 shows that the

total weight and the weight for each type of service are very similar for the three channel conditions. Thus, it can be inferred that the PLC channel status does not influence greatly on the weight estimated for a player status will be taken into account.

Table 5 shows that  $W_N$  presents a very high value compared to  $W_i$ . This could have as a result  $I_{dp_{i \notin N}} = 0$  and  $I_{dp_{i \in N}} = 1$ , and would not allow estimating the power level or influence of each node accurately, since node  $N$  would be considered as a dictator player. The dictator would be the only essential player in all coalitions and the rest of the nodes would be considered as null players. That is such players would not have any power in the game. Considering the above, the use of the following expression to calculate the adjust value  $W_N^*$  is proposed:

$$W_N^* = \left( \left[ \max_{i \in N, inoteqN} W_i/10 \right] + 1 \right) * 10$$

where  $W_N^*$  corresponds to the total weight adjusted for Node  $N$  (CCo) and [ $*$ ] is the whole part of the registered operation.

**Example 5** Considering that the maximum weight between the nodes different from CCo is 24, the value of  $W_N^*$  would be

$$W_N^* = \left( \left[ \max_{i \in N, inoteqN} W_i/10 \right] + 1 \right) * 10$$

$$W_N^* = ([24/10] + 1) * 10$$

$$W_N^* = ([2, 4] + 1) * 10 = (2 + 1) * 10 = 30$$

**Example 6** Considering that the maximum weight between the nodes different from CCo is 28, the value of  $W_N^*$  would be

$$W_N^* = \left( \left[ \max_{i \in N, inoteqN} W_i/10 \right] + 1 \right) * 10$$

$$W_N^* = ([28/10] + 1) * 10$$

$$W_N^* = ([2, 8] + 1) * 10 = (3 + 1) * 10 = 40$$

### 3.2 Scheduler generation

The last objective of this papers is to establish the chronogram of media access (scheduler), by means of the allocation of time domain and the frequency domain for each node  $i$ . This process is managed by CCo and  $y$  propagated to each node, reducing the collision possibility and optimizing the use of the resource.

Figure 1 presents the proposed map corresponding to time and frequency distribution of a PLC channel, supported in OFDMA. The frequency domain is

Table 5: Estimate of the total weight for player  $i$  vs PLC channel condition

Channel conditions			Excellent			Moderate			Deficient		
Player (Node) $i$	CoS	Factor of Priority	BW Allocated (Shapley)	Weight (W)	Total $W_i$	BW Allocated (Shapley)	WeWght (i)	Total $W_i$	BW Allocated (Shapley)	Weight (W)	Total $W_i$
1	Voice	4	0,0580	16	19	0,0449	16	19	0,0386	16	19
	Control	2	0,0011	2		0,0008	2		0,0007	2	
	Data	1	2,4870	1		1,9253	1		1,6557	1	
2	Voice	4	0,0167	4	11	0,0128	4	11	0,0110	4	11
	Control	2	0,0016	4		0,0012	4		0,0010	4	
	Data	1	7,3497	3		5,6429	3		4,8459	3	
3	Video	3	13,7348	6	9	9,8756	6	9	8,9741	6	9
	Data	1	6,2702	3		4,5084	3		4,0969	3	
4	Voice	4	0,0259	8	16	0,0197	8	16	0,0170	8	16
	Control	2	0,0020	4		0,0015	4		0,0013	4	
	Data	1	9,1822	4		6,9928	4		6,0367	4	
5	Video	3	11,1597	3	7	8,4300	3	7	7,3611	3	7
	Control	2	0,0013	4		0,0010	4		0,0009	4	
6	Data	1	11,9820	5	5	9,0280	5	5	7,9210	5	5
7	Voice	4	0,0741	20	24	0,0566	20	24	0,0488	20	24
	Datos	1	8,3769	4		6,3954	4		5,5132	4	
8 CCo	Voice	4	0,2233	56	105 (30**)	0,1699	56	102 (30**)	0,0930	36	67 (30**)
	Video	3	31,0153	9		23,5972	9		12,9131	6	
	Control	2	0,0075	16		0,0057	14		0,0031	10	
	Data	1	57,7519	24		43,9392	23		24,0448	15	

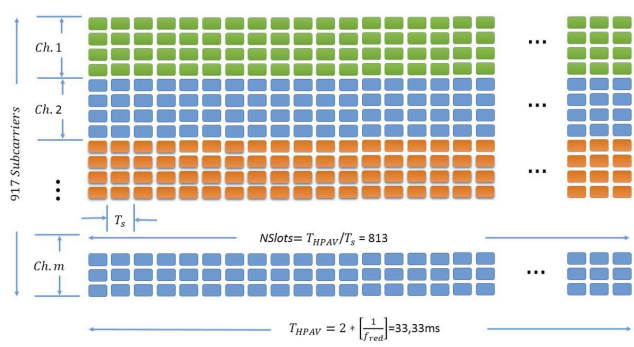


Figure 1: Proposed map for the time and frequency distribution of a PLC channel

formed by the 917 subcarriers, and the time domain by 813 time slots. On this Time-frequency map the allocation of resources for each node will be carried out, according to BW and the allocated power index.

The proposed methodology for the scheduler generation is the following:

*Step 1:* To establish the value of BW allocated for each node that is part of HAN network, through Shapley value [16].

*Step 2:* To determine the total weight for each node and based on this information, to calculate the Shapley-Shubik power index.

*Step 3:* Using Engset formula, to estimate the recommended channel number by which the entire PLC channel will be divided. During this optimization process, it is need to establish the number  $m$  of channels from which 917 subcarriers will be distributed. This will be done in order to carry out latter resource allocation process, in time and frequency, for each type of traffic generated from each node  $i$ . For this purpose, Engset formula (named after the Norwegian mathematician and engineer *Thorir Olaus Engset*) is used. This formula allows to estimate the number of chan-

nels needed according to the traffic intensity generated by the finite population of stations. Besides, by using this expression, it is possible to estimate the blocking and loss probability in each node  $i$  [17]. Engset formula is given by the following expression (17):

$$P_{bl}(m, \rho, N) = \frac{\rho^m \binom{N}{m}}{\sum_{i=1}^m \rho^i \binom{N}{i}} \quad (17)$$

where,

$P_{bl}$ : Blocking probability

$\rho$ : Traffic intensity generated by each node

$m$ : Estimate channel number

$N$ : Node number conforming PLC network

In order to make such calculations, the following function in Matlab is used:

```
% Engset formula to calculate the
% channels number according to the
% traffic intensity and the
% available bandwidth.
```

```
Pl=0.01;
% Blocking probability 1% suggested.
```

```
[server, realPl, realPb]=find_server
(NNodos, Carga_Erlangs, Pl);
NCanales=server;
% Suggested number of channels
% realPl:
% Estimated probability of block.
```

*Step 4:* To estimate the level of access priority to each of the  $m$  channels in each node, arranged by descending order, taking into account the PLC channel conditions and the tone mapping presented in each node. The result from this process is a matrix where it is possible to register which channels are intended



to be used the most by each node when requesting for media access.

*Step 5:* To conform a Results Matrix (RM) in which it can be possible to consolidate all parameters mentioned before for each node. Such RM must be organized by descending order depending on the turn allocated to media accessing. Starting from this results matrix, we shall now proceed to establish the time slots equivalent to allocated BW and the channel/channels suggested for the node, in an organized way and according to the number of available slots per channel. It is important to take into account that the highest number of time slots per channel is 813 and that the allocation of resources can be distributed in more than one channel. The process is repeated until all nodes are allocated.

Table 6 shows all the consolidated information in order to generate the scheduler where the Shapley-Shubik power index and media accessing turn for each node part of the proposed setting stand out.

Table 6 shows Steps 1 and 2 consolidated of the proposed methodology, where BW and the Power index for each node have been established. This has been established by using the Shapley Value and the Shapley-Shubik power index, respectively.

*Step 3:* For the proposed setting, the entrance parameters are:

$$NNodos = 8;$$

$$Pl = 0,01;$$

$$Ro = 1,606 \left( \frac{BW_{Requested}}{BW_{Allocated}} = \frac{193,86Mbps}{120,65Mbps} \right)$$

$$[server, realPl, realPb] = find_{server}(NNodes, Ro, Pl)$$

The use of the function corresponding to the Engset formula showed a result of 8 servers, which equals to dividing the PLC channel in 8 channels with an estimated blocking probability (realPb) of 0,0208.

*Step 4:* For practical effects, it was possible to generate random numbers from 1 to 8, which represent the order of priority of each node for channel accessing according to the possible channel answer and generated tone mapping. The outcome of this process is found in Table 7, which obeys to a matrix where is registered the channels intended to be used the most by each node when requesting media accessing. It is possible to observe that for the case of node 1, the channels with the highest and the lowest priority of being selected for the information transmitting are the channel 6 and 4 respectively.

*Step 5:* Table 7 shows the resulting matrix RM, which consolidates all information that is required for the scheduler construction. It is possible to observe that RM matrix data are organized in ascending order in turns of node i channel access.

Consequently, a routine elaborated in Matlab is shown in order to generate the scheduler based on the RM matrix guidelines.

```
% Routine to establish the PLC network schedule
% Organize the matrix RM in an ascending order
% according to the media access turn

Kmax=NCanales+3;
for i=1:Nj-1
    for j=i+1:Nj
        if MR(j,3)<MR(i,3)
            for k=1:Kmax
                Aux=MR(i,k);
                MR(i,k)=MR(j,k);
                MR(j,k)=Aux;
            end
        end
    end
end

Total_Slots=NSimbolos*NCanales;

for i=1:Nj

    %ACanal is the matrix that manages the channel
    %availability
    ACanal(i,1)=i;      % AChannel(i,1) --> Node
    ACanal(i,2)=0;     % AChannel(i,2) --> Initial Slot
end

k=0;                  % Register 1 of Schedule

for i=1:Nj
    k=k+1;
    % Estimation of the required slot number
    % proportionalto the BW allocated.

    Num_Slots=ceil(Total_Slots*MR(i,2)/sum(MR(:,2)));
    NPri=4;           % Position of channel priority
    CPri=MR(i,NPri); % Channel priority
    Ok=0;

    while Ok==0
        Schedule(k,1)=MR(i,1); % Number or node
        Schedule(k,2)=MR(i,2); % Allocated bandwidth
        Schedule(k,3)=MR(i,3); % Accessing media turn

        if (NSimbolos>=(ACanal(CPri,2)+Num_Slots))
            %If required Slots available in CPri channel

            Schedule(k,4)=CPri;
            % Allocated channel
            Schedule(k,5)=ACanal(CPri,2)+1;
            % Initial Slot
            Schedule(k,6)=ACanal(CPri,2)+Num_Slots;
            % Final Slot
            ACanal(CPri,2)=ACanal(CPri,2)+Num_Slots;
            % New value Initial Slot
            Ok=1;

        Else %Otherwise
            Schedule(k,4)=CPri;
            % Channel allocated
            Aux=NSimbolos-ACanal(CPri,2);
            Schedule(k,5)=ACanal(CPri,2)+1;
            % Initial slot part 1
            Schedule(k,6)=ACanal(CPri,2)+Aux;
            % Slot Final Parte 1
            ACanal(CPri,2)=ACanal(CPri,2)+Aux;
            %New value initial slot
            k=k+1;
            %New register of Schedule
            NPri=NPri+1; % next position of channel priority
            if NPri<(NCanales+4)
                CPri=MR(i,NPri); %New channel priority
                Num_Slots=Num_Slots-Aux;
            else
                Ok=1;
            end
        end
    end
end
end
Schedule %Print the schedule generated
```

For the particular case, the initial parameters required for the former routine, and for the generation of the scheduler are as follows:

r=[1	1.9710	3	6	2	1	7	5	8	3	4;
2	5.6570	5	5	6	7	1	3	2	4	8;
3	14.3840	6	4	7	8	2	6	1	5	3;
4	7.0140	4	7	5	8	3	2	4	1	6;
5	8.4310	7	3	1	2	8	6	5	4	7;
6	9.0280	8	5	2	3	7	1	8	4	6;

Table 6: Estimate of the total weight for player  $i$  vs PLC channel condition

Player (Node)	CoS	Factor of Priority	$BW_{i,r}$ Allocated (Shapley)	BW Total	Weight ( $W_{i,r}$ )	Total $W_i$	$I_{dpShapley-Shubik}$	Turn to media accessing
	Voice	4	0,0449	1,9710	16	19	0,1452	3
	Control	2	0,0008		2			
	Data	1	1,9253		1			
2	Voice	4	0,0128	5,6570	4	11	0,0833	5
	Control	2	0,0012		4			
	Data	1	5,6429		3			
3	Video	3	9,8756	14,3840	6	9	0,069	6
	Data	1	4,5084		3			
4	Voice	4	0,0197	7,0140	8	16	0,1167	4
	Control	2	0,0015		4			
	Data	1	6,9928		4			
5	Video	3	8,4300	8,4310	3	7	0,069	7
	Control	2	0,0010		4			
6	Data	1	9,0280	9,0280	5	5	0,0357	8
	Voice	4	0,0566		20			
7	Data	1	6,3954	6,4520	4	24	0,2024	2
	Voice	4	0,1699		56			
8 CCo	Video	3	23,5972	67,7120	9	102 (30**)	0,2786	1
	Control	2	0,0057		14			
	Data	1	43,9392		23			
	Voice	4	0,0008		2			

Table 7: Channel priority for node for the proposed setting

Node	Channel Priority							
	Opt. 1	Opt. 2	Opt. 3	Opt. 4	Opt. 5	Opt. 6	Opt. 7	Opt. 8
1	6	2	1	7	5	8	3	4
2	5	6	7	1	3	2	4	8
3	4	7	8	2	6	1	5	3
4	7	5	8	3	2	4	1	6
5	3	1	2	8	6	5	4	7
6	5	2	3	7	1	8	4	6
7	6	2	5	1	4	8	7	3
8	2	8	4	6	3	1	5	7

7 6.4520 2 6 2 5 1 4 8 7 3;  
 8 67.7120 1 2 8 4 6 3 1 5 7];

```

NNodos=8; % Number of nodes part of the HAN network
Nj=NNodos; % Number of players = number of nodes

% Estimation of parameters proper of HPAV for 60Hz
Tred=1/60; % Period of an electric network
% signal to 60Hz

Tsimbolo=40.96e-6; % Time of a OFDM symbol for HPAV
NSimbolos=floor(2*Tred/Tsimbolo);
% Number of symbols by duration of Beacon HPAV
(2 Network cycles) ? 813 symbols
% Use of Engset formula to calculate the channel
number according to the intensity of traffic
and available bandwidth.

Pl=0.01;
% Blocking Probability 1% suggested.

Carga_Erlangs=1.606; %Charge.

[server,realPl,realPb]=find_server(NNodos,Carga_Erlangs,Pl);

NCanales=server; %Number of estimated channels
    
```

**Generated Scheduler:** The result of the process corresponding to the scheduler generated for the proposed setting is as follows

Table 9 presents the established timetable for the proposed scenario (schedule), where it indicates the node, the BW allocated, the media accessing turn, the channel or channels allocated to each node in order to

carry out the transmitting process and media accessing time period by means of the definition of initial and final for each node  $i$ .

According to the results, it is possible to observe that channels 2, 4, 6 and 8 were fully assigned to node 8 or CCo. Channel 3 was assigned from the slot 1 to 399. Node 7 was assigned to channel 5 from slot 1 to 348. In this way, it is possible to verify the frequency and time assigned for the remaining nodes. Additionally, it can be stated that the resource allocation was carried out properly and without slot wastes, which are part of the frequency-time spectrum.

## 4 Conclusions

The use of power indexes when defining the Scheduler for a transmission period on a PLC network is considered to be a very useful strategy. The power index represents the degree of influence that the node exercises on the network and according to the allocated value, a channel accessing order is set. Using the obtained results, it was possible to observe that the value corresponding to the power index can be estimated in function to the reflected weight, which depends on several

Table 8: Resulting matrix for proposed setting

Node $i$	$BW_i$ [Mbps]	Turn $i$	Channel Priority							
			Opt.1	Opt.2	Opt.3	Opt.4	Opt.5	Opt.6	Opt.7	Opt.8
8	67.7120	1	2	8	4	6	3	1	5	7
7	6.4520	2	6	2	5	1	4	8	7	3
1	1.9710	3	6	2	1	7	5	8	3	4
4	7.0140	4	7	5	8	3	2	4	1	6
2	5.6570	5	5	6	7	1	3	2	4	8
3	14.3840	6	4	7	8	2	6	1	5	3
5	8.4310	7	3	1	2	8	6	5	4	7
6	9.0280	8	5	2	3	7	1	8	4	6

Table 9: Scheduler of the proposed setting

Node $i$	BW Allocated (Shapley) [Mbps]	Media accessing turn	Channel	Initial Slot	Final Slot
8	67,712	1	2	1	813
8	67,712	1	8	1	813
8	67,712	1	4	1	813
8	67,712	1	6	1	813
8	67,712	1	3	1	399
7	6,452	2	5	1	348
1	1,971	3	1	1	107
4	7,014	4	7	1	379
2	5,657	5	5	349	653
3	14,384	6	7	380	813
3	14,384	6	1	108	449
5	8,431	7	3	400	813
5	8,431	7	1	450	490
6	9,028	8	5	654	813
6	9,028	8	1	491	813

factors such as allocated BW, priority level and traffic type. At the end of the process, the CCo organizes in a descending way the power indexes in order to optimize the allocation of PLC channel resources, in the time domain and frequency domain for each node. In case that two nodes present equal power indexes, the weight will be considered a tie-breaker factor.

The process of calculating the power index for each node from a PLC network can be considered as a main issue since it increases the number of nodes. This happens due to the fact that the classical algorithms use permutations, increasing the required computational complexity levels. Nonetheless, a Matlab routine was designed. Such routine allows to estimate the Shapley-Shubik power index by means of the use of generating functions and offering a lower time and computational complexity in future low-cost embedded systems. The development of such routine in Matlab can be considered as a great contribution for future research due to the fact that it will be used in other simulation settings as required.

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