Dual-Channel Supply Chain Coordination Strategy with Dynamic Cooperative Advertising

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Abstract: With the rapid development of electronic commerce, many manufacturers choose to establish dual-channel supply chains. To promote sales, the manufacturer in a dual-channel supply chain usually invests in national advertising and provides support to the retailer who promotes the goodwill through local advertisement. In our research, we use a Stackelberg differential game theoretic model to show that the manufacturer who establishes an e-channel can adopt the cooperative advertising strategy with the retailer, so that the profits of both the manufacturer and the retailer will increase. We analyze the advantages brought by the e-channel which is established by the manufacturer. Further, a revenue sharing contract is given to coordinate the dual-channel supply chain.

Key–Words: Electronic Commerce; Dual-channel Supply Chain; Cooperative Advertising; Stackelberg Differential Game; Nerlove-Arrow Model; Revenue Sharing Contract

1 Introduction

Cooperative advertising (co-op advertising) is often used to boost sales (Ahmadi et al. [1]). Some scholars showed that advertising could increase sales to avoid overstocking (Xie et al. [2]). Recently, with electronic commerce growing, more and more manufacturers who traditionally distribute their products through retailers, have opened a e-channel as their new channels (Tsay et al. [3], Wang et al. [4]) which is denoted as dual-channel supply chains, such as IBM, Apple, Dell, Nike, and Estee Lauder etc. Because the manufacturers in e-channels have much stronger advertising ability than the retailers in her channels, the retailers may complain that those orders placed through manufacturers' online e-channels are orders which should belong to them (Chiang et al. [5]). To mitigate the channel conflict, the manufacturers provide supports for the retailers in cooperative advertising. That is, cooperative advertising between the manufacturer and the retailer is an agreement where the local advertising cost which should be paid by the retailer may be partly supported by the manufacturer. Therefore, in a dual-channel supply chain, the cooperation and competition coexist between the manufacturer and the retailer.

The first paper about cooperative advertising was proposed by Berger et al. [6]. Then many scholars have done some researches about cooperative advertising. The models related to cooperative advertising are divided into static models and dynamic models. In this paper, we focus on the continuous-time models which are recent hot topics. Previous studies related to dynamic cooperative advertising have mainly focused on a single channel. For example, Chintagunta et al. [7] explored equilibrium dynamic cooperative advertising in a single supply chain for a manufacturer and a retailer. Jørgensen et al. [8] examined long term and short term advertising efforts of channel members in supply chain to enhance consumer sales. Jørgensen et al. [9] found that the goodwill stock depended on the advertising effort of the manufacturer rather than that of the retailer. Bass et al. [10] defined generic advertising and brand advertising, and examined how the brand advertising should be done. Karray et al. [11] considered the carryover effects of brand advertising over time. Dynamic models have been extensively used to analyze various advertising problems in supply chain systems (Huang et al. [12]). Zhang et al. [13] introduced the reference price effect into cooperative advertising in a vertical supply chain. A comprehensive review about cooperative advertising is also given out recently (Aust et al. [14]).

There are few references on cooperative advertising in dual-channel supply chains. Tsay et al. [3] investigated the effects of dynamic brand and advertising on market share in a dual-channel supply chain.

The competitors' advertising strategies in both the Bertrand game and Stackelberg game was considered in a dual-channel supply chain by Yan et al. [15]. Referring to the above results, in the paper we construct a dual-channel supply chain consisting of a manufacturer and a retailer where the manufacturer who acts as a leader in a Stackelberg game opens an e-channel. To promote sales, the manufacturer invests in national advertising and the retailer focuses on local advertising. To enhance the cooperation, the manufacturer pays a portion of the retailer's local advertising costs. Different from the previous research, we consider the cooperative advertising problem in a dynamic dual-channel supply chain. Although dynamic cooperative advertising has been studied in a single channel extensively, the dynamic scenario in dual-channel supply chains has not been considered. In addition, previous papers mainly pay attention to comparing advertising strategies in different game modes, while we focus on the coordination mechanism in dual channel supply chain.

In this paper, we apply a Stackelberg differential game model to analyze the cooperative advertising strategy between the retailer and the manufacturer who opens an e-channel. There are state variables and control variables of game players in the Stackelberg differential game. The differential equations are used to describe the evolution of the state variables. The control variables are determined to maximize the corresponding objective functions. The equilibrium solutions are obtained respectively under decentralized situation and centralized situation. We compare them with the equilibrium solutions in a single supply chain and find the profits of both the manufacturer and the retailer will increase in dual channel supply chain with the cooperative advertising. Finally, a revenue sharing contract is proposed to coordinate the dual-channel supply chain.

The remainder of this paper is organized as follows. Section 2 introduces the problem, notations and the demand functions. Section 3 respectively derives the equilibrium solutions for decentralized and centralized supply chain under cooperative advertising. Section 4 gives a revenue sharing contract to coordinate the dual-channel supply chain. Numerical analysis on key parameters is performed to examine their impacts on the equilibrium results and profits. The last section summarizes the major results and points out the directions for future research.

2 Model Description

In a dynamic dual-channel supply chain system which consists of a manufacturer and an independent traditional retailer. The manufacturer creates and distributes his product through a dual-channel supply chain, i.e., the traditional retail channel and its own e-channel. To promote sales, the manufacturer invests in national advertising and the retailer makes a decision on local advertising. National advertising mainly focuses on the manufacturer's long-term brand awareness and image. Competition between the two members is formulated by a Stackelberg differential game, in which the manufacturer acts as a leader and has enough power to influence the decisions about advertising (Zhang et al. [13], Si et al. [16]) and the retailer is a follower. The manufacturer's and the retailer's market shares in a dual-channel supply chain change over time which depend on current and hold-over effect of past advertising efforts.

We use $a_m(t)$ and $a_r(t)$ to express the manufacturer's national advertising level and the retailer's local advertising level over time t. Let π_m denote the margin profit of the manufacturer who sells the product to the retailer through the traditional retail channel, let π_d be the margin profit of the manufacturer who sells the product to consumers by his e-channel, and π_r is the margin profit of the retailer. Our main focus lies in the impacts on the profits of the dual-channel supply chain brought by optimal advertising levels. π_m, π_d and π_r are kept as the fixed constants to avoid the price decision making in order to concentrate on the study of cooperating advertising (Zhang et al. [13]).

We assume that the changing of the goodwill of the product in the e-channel and the retail channel follow the Nerlove-Arrow framework as follows,

$$\begin{aligned} &\dot{G}(t) = u_1 a_m(t) + u_2 a_r(t) - \delta G(t), \\ &G(0) = G_0. \end{aligned} \tag{1}$$

where $G(t)$ is the accumulated goodwill over time t, $G_0 \geq 0$ is the initial goodwill, $u_1 > 0$ reflects the national advertising which has positive effect on the brand image, $u_2 > 0$ reflects the local advertising which has positive impact on the accumulated goodwill, and $\delta > 0$ is the diminishing rate of goodwill, which means that consumers may forget the brand to some extent.

In practice, the firms seldom increase their advertising levels infinitely. According to Sethi et al. [17], we assume that the upper bound exists for the control variables, that is,

$$0 \leq a_m(t), a_r(t) \leq M, \tag{2}$$

where M is a large enough constant. Hereafter, we need use the above conditions in the process of solving related profit maximizing problems.

Let $D_m(t)$ denote the market demand of the manufacturer at time $t \geq 0$ which depends on its own and the retailer's advertising levels, and the goodwill of the product. Similarly, the retailer's sales response volume $D_r(t)$ at time t is assumed to be affected mainly by the manufacturer's national advertising level, the retailer's local advertising level, and the goodwill of the product. The manufacturer and the retailer make their own beneficial decisions by utilizing a cooperative advertising strategy. According to Jørgensen et al. [9], the advertising cost functions are supposed to be quadratic with respect to the different advertising levels, That is

$$c_m(t) = \frac{1}{2}{a_m}^2(t), c_r(t) = \frac{1}{2}{a_r}^2(t). \quad (3)$$

This is an extensively-accepted assumption in advertising literature (Nair et al. [18], Sigu et al. [19]). The demand functions of the retail channel and the e-channel are denoted by:

$$D_r(t) = (1-\theta)G(t) + a_r(t) + ba_m(t), \quad (4)$$

$$D_d(t) = \theta G(t) + a_m(t) + ba_r(t), \quad (5)$$

where $D_r(t), D_m(t) > 0$, b is the cross elastic coefficient between the two kinds of advertising levels, $0 < b < 1$. The value of the parameter θ denotes the market share of the e-channel, which can be seen as the customer loyalty to the e-channel.

Assume that the two members have the same discount rate ρ over time. To compensate for the retailer's lost and stimulate the retailer to invest more in local advertising, the manufacturer adopts the cooperative advertising strategy in which the manufacturer will share a part of the retailer's advertising expenditure.

Assuming that ϕ is the participation rate that the manufacturer will compensate the retailer's advertising cost. Furthermore, we assume the condition that $\delta \geq \frac{u_2(1-\theta)\pi_r}{b\pi_d+\pi_m-\pi_r} - \rho$ to avoid trivial cases. (If $\delta < \frac{u_2(1-\theta)\pi_r}{b\pi_d+\pi_m-\pi_r} - \rho$ then it is not profitable for the retailer to adopt co-op advertising, which is not in the paper's consideration.)

In the following sections, we will calculate the optimal national and local advertising levels as well as the optimal participation rate paid by the manufacturer based on the Stackelberg differential game.

3 Cooperative advertising based on Stackelberg differential game

3.1 The decentralized situation

When the two members make decisions independently, the sequence of decision making is assumed as follows. As the leader, the manufacturer acts as the first mover by choosing its participation rate ϕ and national advertising level $a_m(t)$ over time t. Sometimes, in order to stimulate the retailer to invest more, ϕ may be the subsidy given to the retailer by the manufacturer. After the participation rate and the national advertising level are given, the retailer will decide local advertising level $a_r(t)$ over time t according to the manufacturer's decision making. Since the advertising levels may change all the time, according to the Stackelberg model of Zhang et al. [13], we assume that the two members simultaneously make their decisions.

The objective functions are the discounted profit streams over an infinite planning horizon. Given the cost and demand functions in Eqs. (1), (3)-(5), the manufacturer's problem is specified as

$$\begin{aligned} \max_{a_m} J_m &= \max_{a_m} \int_0^{+\infty} e^{-\rho t} \textstyle\prod_m dt \\ &= \max_{a_m} \int_0^{+\infty} e^{-\rho t} Y\, dt, \end{aligned} \quad (6)$$

where $Y = \pi_d D_d(t) + \pi_m D_r(t) - c_m(t) - \phi c_r(t)$.

$$\begin{aligned} &\max_{a_r} J_r = \max_{a_r} \int_0^{+\infty} e^{-\rho t} \textstyle\prod_r dt \\ &= \max_{a_r} \int_0^{+\infty} e^{-\rho t} [\pi_r D_r(t) - (1-\phi)c_r(t)]\, dt. \end{aligned} \quad (7)$$

When the participation rate ϕ is fixed, we calculate the optimal equilibrium solutions for the manufacturer and the retailer. The manufacturer knows that the retailer will make its own decision based on the given participation rate and the national advertising level is not dependent on the participation rate. Then, the manufacturer's and the retailer's current value Hamiltonian functions are

$$\begin{aligned} H_m = &\pi_d D_d(t) + \pi_m D_r(t) \\ &-c_m(t) - \phi c_r(t) + \lambda_m \dot{G}(t), \end{aligned} \quad (8)$$

$$H_r = \pi_r D_r(t) - (1-\phi)c_r(t) + \lambda_r \dot{G}(t). \quad (9)$$

Substitute Eqs. (1), (3)-(5) into the Eqs. (8) and (9), we get

$$\begin{aligned} &H_m = \\ &\pi_d(\theta G(t) + a_m(t) + ba_r(t)) + \pi_m((1-\theta)G(t) \\ &+a_r(t) + ba_m(t)) - \tfrac{1}{2}{a_m}^2(t) - \tfrac{1}{2}\phi {a_r}^2(t) \\ &+\lambda_m(u_1 a_m(t) + u_2 a_r(t) - \delta G(t)). \end{aligned}$$

$$\begin{aligned} &H_r = \pi_r((1-\theta)G(t) + a_r(t) + ba_m(t)) \\ &-\tfrac{1}{2}(1-\phi){a_r}^2(t) + \lambda_r(u_1 a_m(t) + u_2 a_r(t) - \delta G(t)). \end{aligned}$$

Proposition 1 *When the manufacturer offers the support for the retailer's local advertising expenditure, the manufacturer's equilibrium solution of the Stackelberg differential game is* $\bar{a}_m = \frac{\rho+\delta-1}{\rho+\delta}(\pi_d + b\pi_m)+$

$\frac{\theta\pi_d+(1-\theta)\pi_m}{\rho+\delta}u_1$, *and that of the retailer is* $\bar{a}_r = b\pi_d + \pi_m$.

In the decentralized channel, the manufacturer's optimal participate rate is

$$\overline{\phi} = \begin{cases} \phi_1, & if\ \delta \geq \frac{u_2(1-\theta)\pi_r}{b\pi_d+\pi_m-\pi_r} - \rho \\ 0 & \text{otherwise} \end{cases}$$

where $\phi_1 = 1 - \frac{\pi_r}{b\pi_d+\pi_m}\left(1 + \frac{u_2(1-\theta)}{\rho+\delta}\right)$.

The manufacturer's and the retailer's optimal profits respectively are

$$\begin{aligned}\Pi_m^d &= \pi_d(1+\theta u_1)\times \\ &\left(\tfrac{\rho+\delta-1}{\rho+\delta}(\pi_d+b\pi_m)+\tfrac{\theta\pi_d+(1-\theta)\pi_m}{\rho+\delta}u_1\right) \\ &+\pi_d(b+\theta u_2)(b\pi_d+\pi_m) \\ &+\pi_m(1+(1-\theta)u_2)(b\pi_d+\pi_m) \\ &+\pi_m(b+(1-\theta)u_1)\times \\ &\left(\tfrac{\rho+\delta-1}{\rho+\delta}(\pi_d+b\pi_m)+\tfrac{\theta\pi_d+(1-\theta)\pi_m}{\rho+\delta}u_1\right) \\ &-\tfrac{1}{2}\left(\tfrac{\rho+\delta-1}{\rho+\delta}(\pi_d+b\pi_m)+\tfrac{\theta\pi_d+(1-\theta)\pi_m}{\rho+\delta}u_1\right)^2 \\ &-\tfrac{1}{2}\left(1-\tfrac{\pi_r}{b\pi_d+\pi_m}\left(1+\tfrac{u_2(1-\theta)}{\rho+\delta}\right)\right)(b\pi_d+\pi_m)^2,\end{aligned}$$

The retailer's optimal profit is

$$\begin{aligned}\Pi_r^d &= \pi_r[(1-\theta)u_1+b]\times \\ &\left[(1-\tfrac{1}{\rho+\delta})(\pi_d+b\pi_m)+\tfrac{\theta\pi_d+(1-\theta)\pi_m}{\rho+\delta}u_1\right] \\ &+\pi_r(b\pi_d+\pi_m)\left[u_2(1-\theta)-\tfrac{u_2(1-\theta)}{2(\rho+\delta)}+\tfrac{1}{2}\right].\end{aligned}$$

The Proof of Proposition 1 is shown in appendix A.

By differentiating the equilibrium advertising levels with their marginal profits, we get $\frac{\partial\bar{a}_m}{\partial\pi_d} = 1 - \frac{1}{\rho+\delta} + \frac{\theta}{\rho+\delta}u_1 > 0$, $\frac{\partial\bar{a}_m}{\partial\pi_m} = b\left(1-\frac{1}{\rho+\delta}\right) + \frac{(1-\theta)}{\rho+\delta}u_1 > 0$, $\frac{\partial\bar{a}_r}{\partial\pi_d} = b$, $\frac{\partial\bar{a}_r}{\partial\pi_m} = 1$.

This implies that a higher marginal profit can stimulate the manufacturer and the retailer to invest more in their advertising levels. Note that the advertising levels have no impact on the retailer's marginal profit, then $\frac{\partial\bar{a}_m}{\partial\pi_r} = 0$, $\frac{\partial\bar{a}_r}{\partial\pi_r} = 0$,

$$\frac{\partial\overline{\phi}}{\partial\pi_r} = -\frac{1}{b\pi_d+\pi_m}\left(1+\frac{u_2(1-\theta)}{\rho+\delta}\right) < 0,$$

$$\frac{\partial\overline{\phi}}{\partial\pi_d} = \frac{b\pi_r}{(b\pi_d+\pi_m)^2}\left(1+\frac{u_2(1-\theta)}{\rho+\delta}\right) > 0,$$

$$\frac{\partial\overline{\phi}}{\partial\pi_m} = \frac{\pi_r}{(b\pi_d+\pi_m)^2}\left(1+\frac{u_2(1-\theta)}{\rho+\delta}\right) > 0.$$

This is to say that a higher π_d and π_m can stimulate the manufacturer's participate degree, but a higher π_r is bad for the manufacturer.

Proposition 2 *When the optimal national and local advertising levels are $\bar{a}_m$ and $\bar{a}_r$ respectively, the accumulated goodwill on the product over time t is*

$$G(t) = (G_0 - G_s)e^{-\delta t} + G_s, \tag{10}$$

where $G_s = u_1\bar{a}_m + u_2\bar{a}_r$.

Note that the goodwill in Eq. (10) will finally achieve the ready state G_s when $t \to \infty$, which is mainly influenced by the optimal national and local advertising levels.

Appendix B gives the proof of Proposition 2.

Through the expression of G_s, we know that the goodwill of the product is positively correlated to the two advertising levels. That is to say, advertising can accumulate a higher goodwill.

3.2 The benchmark under the centralized situation

When the manufacturer and the retailer are centralized controlled, the supply chain has the best performance. Then, we set the profit of the entire supply chain in the centralized situation as the benchmark. In this section, we consider the manufacturer and the retailer are vertically integrated, and calculate the system optimal decisions, i.e., the national and local advertising levels. Hence, the objective function is to maximize the present value of the total discounted profit.

When the two members coordinate as a vertical integrated system, their objective is

$$\begin{aligned}&\max_{a_m,a_r} J_c = \max_{a_m,a_r}\textstyle\int_0^{+\infty} e^{-\rho t}\prod_c dt \\ &= \max_{a_m,a_r}\textstyle\int_0^{+\infty} e^{-\rho t}M dt, \\ &M = \pi_d D_m(t) + (\pi_m+\pi_r)D_r(t) - c_m(t) - c_r(t).\end{aligned}$$

Given the cost function, goodwill function and demand functions in Equation (1)-(4) and utilizing the standard optimal control theory to solve the above problem, we define the current value profit function as

$$\max_{a_m,a_r}\Pi_c = \max_{a_m,a_r}\int_0^{+\infty} e^{-\rho t}N dt, \tag{11}$$

$$\begin{aligned}N &= \pi_d[\theta G + a_m + ba_r] - \tfrac{1}{2}{a_m}^2 - \tfrac{1}{2}{a_r}^2 \\ &+(\pi_m+\pi_r)[(1-\theta)G + a_r + ba_m].\end{aligned}$$

The HJB equation of the system is

$$\rho V_c = \max_{a_m,a_r}\int_0^{+\infty}(E+F)\,dt,$$

$$\begin{aligned}E &= \pi_d[\theta G + a_m + ba_r] \\ &+(\pi_m+\pi_r)[(1-\theta)G + a_r + ba_m], \\ F &= -\tfrac{1}{2}{a_m}^2 - \tfrac{1}{2}{a_r}^2 + \tfrac{\partial V_c}{\partial G}[u_1a_m + u_2a_r - \delta G],\end{aligned}$$

where $\frac{\partial V_c}{\partial G}$ denotes the co-state variable in the problem of the whole supply chain associated with the changing of consumers' goodwill level, which is the shadow price associated with the state variable G. The shadow price is the change in the optimal profit of an optimization problem, obtained by relaxing the constraint by one unit. It can be interpreted as the impact on the future profit by creating and selling one more unit of the product. If the shadow price is positive, the current price should be decreased for future benefits, and vice visa (Kalish et al. [20]).

Proposition 3 *When the manufacturer and the retailer coordinate as an integrated system, the optimal national and local advertising levels over time are both constants, the equilibrium solutions are as follows:*

$$a_m^* = (1 + \frac{\theta u_1}{\rho+\delta})\pi_d + (b + \frac{(1-\theta)u_1}{\rho+\delta})(\pi_m + \pi_r),$$

$$a_r^* = (b + \frac{\theta u_2}{\rho+\delta})\pi_d + (1 + \frac{(1-\theta)u_2}{\rho+\delta})(\pi_m + \pi_r).$$

The accumulated goodwill on the product over time t is $G(t) = (G_0 - G_{ss})e^{-\delta t} + G_{ss}$, where $G_{ss} = u_1 a_m^* + u_2 a_r^*$. Note that the goodwill in the above equation will finally achieve the ready state G_{ss}, when $t \to \infty$, which is mainly affected by the optimal national and local advertising levels.

The proof of Proposition 3 is seen as appendix C.

Substitute a_m^*, a_r^*, G_{ss} into Π_c, then we get

$$\begin{array}{l}
\Pi_c^* \\
= \frac{\theta\pi_d+(\pi_m+\pi_r)(1-\theta)}{\rho+\delta}(u_1{}^2\theta + u_2{}^2\theta + u_1 + bu_2)\pi_d \\
+(1 + b^2 + \theta u_1 + \theta u_2 b)\pi_d{}^2 \\
+\pi_d(4b + \theta u_2 + b\theta u_1)(\pi_m + \pi_r) \\
+(1-\theta)(\pi_m + \pi_r)(u_1 + bu_2)\pi_d \\
+\frac{\theta\pi_d+(\pi_m+\pi_r)(1-\theta)}{\rho+\delta}(1-\theta)(\pi_m + \pi_r)(u_1{}^2 + u_2{}^2) \\
+[1 + b^2 + (1-\theta)(bu_1 + u_2)](\pi_m + \pi_r)^2 \\
+\frac{\theta\pi_d+(\pi_m+\pi_r)(1-\theta)}{\rho+\delta}(u_2 + bu_1)(\pi_m + \pi_r) \\
-\frac{1}{2}\left[\pi_d + b(\pi_m + \pi_r) + \frac{\theta\pi_d+(\pi_m+\pi_r)(1-\theta)}{\rho+\delta}u_1\right]^2 \\
-\frac{1}{2}\left[b\pi_d + (\pi_m + \pi_r) + \frac{\theta\pi_d+(\pi_m+\pi_r)(1-\theta)}{\rho+\delta}u_2\right]^2
\end{array} \tag{12}$$

By comparing the equilibrium results under the centralized scenario and the decentralized situation, we easily get $\Pi_c^* > \Pi_m^d + \Pi_r^d, a_m^* > \overline{a}_m, a_r^* > \overline{a}_r$. Based on the above analysis, we know that the supply chain cannot be coordinated under the Stackelberg game. In what follows, we use a revenue sharing contract to coordinate the whole supply chain.

4 A revenue sharing contract

Before analyzing the supply chain's coordination, we firstly give the definition of coordination in this paper. On the basis of the definition of coordination proposed by Gan et al. [21] and Ma et al. [22], we give the following definition in this paper.

Definition of coordination. A contact can coordinate the supply chain if and only if the following conditions are satisfied:

(1) Each member's profit is not less than the profit in decentralized situation.

(2) Total profit of the whole supply chain is equal to that in centralized scenario.

Assume that the manufacturer's profit ratio is denoted by η, then $1-\eta$ is that of the retailer, where $0 < \eta < 1$. To implement the revenue sharing contract, the ratios of the two members must satisfy the following conditions:

$$\begin{cases} \eta\Pi_c^* \geq \Pi_m^d, \\ (1-\eta)\Pi_c^* \geq \Pi_r^d. \end{cases}$$

We obtain that $\eta \in \left[\frac{\Pi_m^d}{\Pi_c^*}, \frac{\Pi_c^* - \Pi_r^d}{\Pi_c^*}\right]$. In order to express the condition conveniently, let $\underline{\eta} = \min\left\{\frac{\Pi_m^d}{\Pi_c^*}, \frac{\Pi_c^* - \Pi_r^d}{\Pi_c^*}\right\}$, and $\overline{\eta} = \max\left\{\frac{\Pi_m^d}{\Pi_c^*}, \frac{\Pi_c^* - \Pi_r^d}{\Pi_c^*}\right\}$, then $\eta \in [\underline{\eta}, \overline{\eta}]$. This is a Pareto region in which the dual-channel supply chain can be coordinated.

Obviously, in the interval $[\underline{\eta}, \overline{\eta}]$, both the manufacturer and the retailer expect sharing more profit from the value created by their cooperation. If $\eta \to \bar{\eta}$, then the manufacturer will gain more profit. If $\eta \to \underline{\eta}$, then the retailer will make more profit. We will discuss the profit ratios of the two members in the following.

Note that the discount rates of the two members are same in the paper. Using the method proposed by Binmore et al. [23], we get that the manufacturer's profit ratio η is $\eta = \frac{\rho}{1+\rho}(\overline{\eta} - \underline{\eta}) + \underline{\eta}$.

At the same time, the value functions of the manufacturer and the retailer are

$$V_m{}' = \left[\frac{\rho}{1+\rho}(\overline{\eta} - \underline{\eta}) + \underline{\eta}\right]\Pi_c^*,$$

$$\begin{array}{l} V_r{}' = [1 - \frac{\rho}{1+\rho}(\overline{\eta} - \underline{\eta}) - \underline{\eta}]\Pi_c^* \\ = [1 - \frac{\rho}{1+\rho}\overline{\eta} - \frac{1}{1+\rho}\underline{\eta}]\Pi_c^* \end{array}.$$

Differentiating the above two equations, we get

$$\frac{\partial V_m{}'}{\partial \rho} = \frac{1}{(1+\rho)^2}(\overline{\eta} - \underline{\eta})\Pi_c^* > 0,$$

$$\frac{\partial V_r{}'}{\partial \rho} = -\frac{1}{(1+\rho)^2}(\overline{\eta} - \underline{\eta})\Pi_c^* < 0.$$

We can see that when ρ increases, the manufacturer will gain much more profit, while the retailer will lose more.

5 Simulation Experiments

In this section, we perform a series of simulation experiments to give more management implications. Here we suppose that the effect brought by the global advertising is larger than that of the local advertising, that is to say, The data are defined as follows: $u_1 = 0,5, u_2 = 0.2, \pi_m = 10, \pi_r = 25, \pi_d = 26, \delta = 0.6, \rho = 0.15, b = 0.9$. These parameters must satisfy the condition $\delta \geq \frac{u_2(1-\theta)\pi_r}{b\pi_d+\pi_m-\pi_r} - \rho$ to avoid trivial cases. (If $\delta < \frac{u_2(1-\theta)\pi_r}{b\pi_d+\pi_m-\pi_r} - \rho$ then the retailer will has no profit which is brought by cooperative advertising.)

To compare the results between a single traditional supply chain and the dual channel supply chain, we apply the demand function $D(t) = G(t) + a_r(t) + a_m(t)$ in order to specify the profit functions of the manufacturer the retailer as follows:

$$\max_{a_m} J_m = \max_{a_m} \int_0^{+\infty} e^{-\rho t} \prod_m dt \\ = \max_{a_m} \int_0^{+\infty} e^{-\rho t} [\pi_m D(t) - c_m(t) - \phi c_r(t)]\, dt,$$

$$\max_{a_r} J_r = \max_{a_r} \int_0^{+\infty} e^{-\rho t} \Pi_r dt \\ = \max_{a_r} \int_0^{+\infty} e^{-\rho t} [\pi_r D(t) - (1-\phi) c_r(t)]\, dt.$$

Here ρ is the same discount rate. Fig. 1 gives the comparison results of equilibrium advertising levels under a single channel and a dual-channel supply chain. We can see that with the customer loyalty to the e-channel θ increasing, the gap on the equilibrium advertising levels of the manufacturer under a single channel and a dual-channel will decrease, while the gap on the equilibrium advertising levels of the retailer will almost keep stable. It means that if more and more customers transfer to the e-channel, two members in supply chain will decrease their advertising investment levels, especially the manufacturer will get more benefit from his e-channel, because the customers may search easily the information from the manufacturer's e-channel and his e-channel has a great advertising effect instead of his national advertising. However, the retailer almost maintains her advertising level and she may reduce the level when she suffers from the great transfer of customers.

Fig. 2 shows the equilibrium participation rates. Obviously, the manufacturer's participation rate under a single channel is larger than that under a dual channel, because the retailer is a unique buyer of the manufacturer's product in this case. But, under a dual

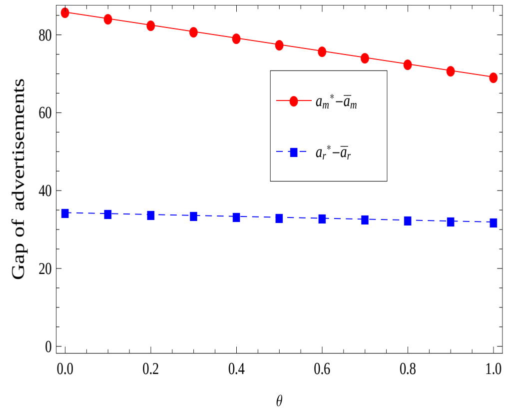

Fig. 1: Impacts of θ on advertising levels.

channel, the manufacturer can sell his product by his e-channel, so the participation rate of the manufacturer is much lower. We can know that with θ increasing, this rate will increase. This may be that when much more customers transfer to the manufacturer's e-channel, the manufacturer will make up for the retailer's sale by increasing the participation rate.

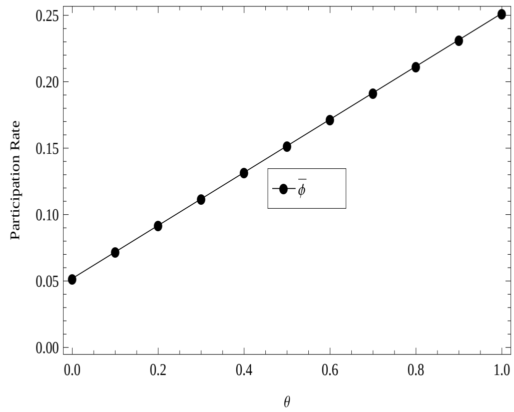

Fig. 2: Impacts of θ on participation rate of the manufacturer.

In Fig. 3, we can see that with the introduction of the direct channel, the profits of the manufacturer and the retailer both increase. The profit of the manufacturer will increase greatly with the increase of θ. On the contrary, the profit of the retailer will increase slowly as the increase of θ. That is to say, because the manufacturer will increase the participation rate, the retailer may benefit from it.

Further, let $\eta = \frac{\Pi_m^d}{\Pi_c^*}$, Fig. 4 gives the effects of the customer loyalty to the e-channel θ on two members' profits when considering the supply chain coordination. We note that the profits of the manufacturer and the retailer will increase after coordination. The manufacturer's profit increases much larger than the

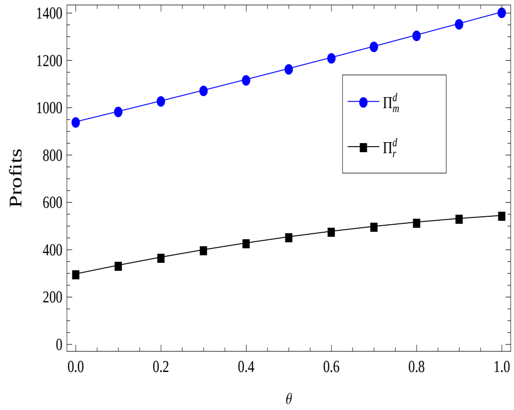

Fig. 3: Impacts of θ on the profits of supply chain members.

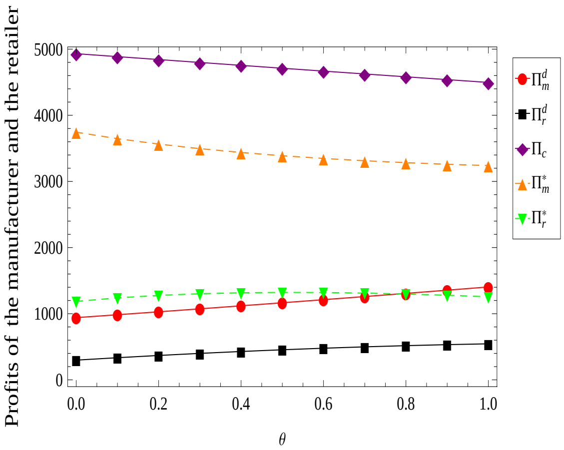

Fig. 4: A demonstration for supply chain coordination.

retailer's profit. In addition, the profit of the manufacturer will decrease slowly with the increase of θ, the reason may be that the advertisements invested by the manufacturer and the retailer will be slowly decrease as θ increases.

6 Concluding remarks

In this paper, we consider a dual-channel supply chain constituted by a manufacturer and a retailer, in which the manufacturer acts as a leader of the supply chain. The manufacturer distributes his product through the traditional retail channel and its own e-channel. The manufacturer invests in national advertising and the retailer makes a decision to local advertising in order to improve sales. The manufacturer's and the retailer's demands change over time which depend on current and hold-over effect of past advertising efforts. In this paper, we analyze the advantage when the manufacturer introduces the e-channel. We draw the conclusion that the supply chain can be coordinated under a revenue sharing contract. The papers considering competition in the demand functions are very few. If the demand is influenced by prices and the wholesale price, the problem becomes very complexity. But it is an extremely significant problem which deserves us to give more attention in the future.

Appendix A: Proof of Proposition 1.

$$\frac{\partial H_m}{\partial a_m} = 0, \qquad (\mathbf{A}1)$$

$$\frac{\partial H_m}{\partial \lambda_m(t)} = u_1 a_m(t) + u_2 a_r(t) - \delta G(t), \qquad (\mathbf{A}2)$$

$$\dot{\lambda_m}(t) = \rho\lambda_m(t) - \frac{\partial H_m}{\partial G(t)}, \qquad (\mathbf{A}3)$$

Eq. (**A**1) implies

$$\begin{aligned} a_m(t) &= \pi_d + b\pi_m + \lambda_m(t)u_1, \\ a_m(t) - \pi_d - b\pi_m &= \lambda_m(t)u_1. \end{aligned} \qquad (\mathbf{A}4)$$

Substituting $\frac{\partial H_m}{\partial G} = \theta\pi_d + (1-\theta)\pi_m - \delta\lambda_m(t)$ into Eq. (**A**3), we get

$$\dot{\lambda}_m(t) = (\rho+\delta)\lambda_m(t) - \theta\pi_d - (1-\theta)\pi_m. \qquad (\mathbf{A}5)$$

Differentiating Eq. (**A**4) with respect to time and substituting the time derivative of in Eq. (**A**5), we get

$$\begin{aligned} \dot{a}_m(t) = \pi_d + b\pi_m + (\rho+\delta)\lambda_m(t)u_1 \\ -\theta\pi_d u_1 - (1-\theta)\pi_m u_1. \end{aligned} \qquad (\mathbf{A}6)$$

Substituting for in Eq. (A6) by Eq. (A4), we obtain

$$\dot{a}_r(t) = (\rho+\delta)a_m(t) - (\rho+\delta-1)(\pi_d + b\pi_m)$$

$$-\theta\pi_d u_1 - (1-\theta)\pi_m u_1. \qquad (\mathbf{A}7)$$

Similarly, considering the retailer's decision making problem, we get

$$\dot{a}_r(t) = (\rho+\delta)a_r(t) - \tfrac{1}{1-\phi}[(\rho+\delta) + u_2(1-\theta)]\pi_r. \qquad (\mathbf{A}8)$$

Solving Eqs. (**A**7) and (**A**8) to obtain the time paths of $a_m(t)$ and $a_r(t)$, we get

$$a_m(t) = c_1(\rho+\delta)e^{(\rho+\delta)t} + \bar{a}_m, \qquad (\mathbf{A}9)$$

$$a_r(t) = c_2(\rho+\delta)e^{(\rho+\delta)t} + \bar{a}_r, \qquad (\mathbf{A}10)$$

$$\begin{aligned} \bar{a}_m = \tfrac{1}{\rho+\delta} \times \Big[(\rho+\delta-1)(\pi_d + b\pi_m) \\ + \theta\pi_d u_1 + (1-\theta)\pi_m u_1\Big], \end{aligned} \qquad (\mathbf{A}11)$$

$$\begin{aligned} \bar{a}_r &= \tfrac{1}{(1-\phi)(\rho+\delta)}[(\rho+\delta) + u_2(1-\theta)]\pi_r \\ &= \tfrac{1}{1-\phi}[1 + \tfrac{u_2(1-\theta)}{\rho+\delta}]\pi_r. \end{aligned} \qquad (\mathbf{A}12)$$

Note that c_1, c_2 are parameters to be determined, $\rho+\delta>0$. Once $c_1, c_2 \neq 0$, the equations given by Eqs. (**A**11) and (**A**12) will be infinite when $t \to +\infty$. This is impossible because Eqs. (**A**11) and (**A**12) do not satisfy the conditions given by Eq. (2). Thus, we must have, the equilibrium solutions are constants.

Substitute Eqs. (**A**11) and (**A**12) into

$$\begin{aligned}\Pi_m &= \pi_d(\theta G(t)+a_m(t)+ba_r(t)) \\ &\quad +\pi_m((1-\theta)G(t)+a_r(t)+ba_m(t)) \\ &\quad -\frac{1}{2}{a_m}^2(t)-\frac{1}{2}\phi {a_r}^2(t),\end{aligned}$$

we get

$$\begin{aligned}\prod_m &= (\pi_d\theta+\pi_m(1-\theta))(u_1\bar{a}_m+u_2\bar{a}_r) \\ &+ (b\pi_d+\pi_m)\left[\tfrac{1}{1-\phi}[1+\tfrac{u_2(1-\theta)}{\rho+\delta}]\pi_r\right] \\ &+\Big[\tfrac{1}{\rho+\delta}[(\rho+\delta-1)(\pi_d+b\pi_m)+\theta\pi_d u_1 \\ &+(1-\theta)\pi_m u_1]\Big](\pi_d+b\pi_m) \\ &-\tfrac{1}{2}\Big[\tfrac{1}{\rho+\delta}[(\rho+\delta-1)(\pi_d+b\pi_m) \\ &+\theta\pi_d u_1+(1-\theta)\pi_m u_1]\Big]^2 \\ &-\tfrac{1}{2}\phi\Big[\tfrac{1}{1-\phi}\left(1+\tfrac{u_2(1-\theta)}{\rho+\delta}\right)\pi_r\Big]^2.\end{aligned} \qquad (\mathbf{A}13)$$

Differentiating Eq. (**A**13) with respect to ϕ,

$$\begin{aligned}\tfrac{\partial\Pi_m}{\partial\phi} &= (b\pi_d+\pi_m)[1+\tfrac{u_2(1-\theta)}{\rho+\delta}]\pi_r\tfrac{1}{(1-\phi)^2} \\ &-\tfrac{1}{2}\Big[\tfrac{1}{1-\phi}[1+\tfrac{u_2(1-\theta)}{\rho+\delta}]\pi_r\Big]^2 \\ &-\tfrac{1}{2}[1+\tfrac{u_2(1-\theta)}{\rho+\delta}]^2{\pi_r}^2\tfrac{1+\phi}{(1-\phi)^3},\end{aligned} \qquad (\mathbf{A}14)$$

By solving (**A**14), we get

$$\overline{\phi} = 1-\frac{\pi_r}{b\pi_d+\pi_m}\left[1+\frac{u_2(1-\theta)}{\rho+\delta}\right]. \qquad (\mathbf{A}15)$$

Substitute Eqs. (**A**15) and (**A**13) into

$$\bar{a}_r = \frac{1}{1-\phi}\left[1+\frac{u_2(1-\theta)}{\rho+\delta}\right]\pi_r = b\pi_d+\pi_m. \qquad (\mathbf{A}16)$$

Similarly, we have

$$\frac{\partial H_r}{\partial a_r}=0, \qquad (\mathbf{A}17)$$

$$\frac{\partial H_r}{\partial \lambda_r}=u_1a_m+u_2a_r-\delta G, \qquad (\mathbf{A}18)$$

$$\dot{\lambda}_r=\rho\lambda_r-\frac{\partial H_r}{\partial G}, \qquad (\mathbf{A}19)$$

Eq. (**A**17) implies

$$a_r=\frac{1}{1-\phi}(\pi_r+u_2\lambda_r). \qquad (\mathbf{A}20)$$

Substituting $\frac{\partial H_r}{\partial G}=\pi_r(1-\theta)-\delta\lambda_r$ into Eq. (**A**19), we get

$$\dot{\lambda}_r=(\rho+\delta)\lambda_r-\pi_r(1-\theta). \qquad (\mathbf{A}21)$$

Differentiating Eq. (**A**20) with respect to time and substituting the time derivative of in Eq. (**A**21), we get

$$\dot{a}_r=\frac{1}{1-\phi}\left[(\rho+\delta)u_2\lambda_r-u_2\pi_r(1-\theta)\right]. \qquad (\mathbf{A}22)$$

Substituting for in Eq. (**A**22) by Eq. (**A**20), we obtain

$$\begin{aligned}\dot{a}_r &= \tfrac{1}{1-\phi}[(\rho+\delta)((1-\phi)a_r-\pi_r)-u_2\pi_r(1-\theta)] \\ &=(\rho+\delta)a_r-\tfrac{1}{1-\phi}\pi_r[(\rho+\delta)-u_2(1-\theta)].\end{aligned}$$

Appendix B: Proof of Proposition 2.

Substitute $a_m(t)=\bar{a}_m$ and $a_r(t)=\bar{a}_r$ into Eq. (1), we have

$$\frac{dG(t)}{dt}=u_1\bar{a}_m+u_2\bar{a}_r-\delta G(t). \qquad (\mathbf{B}1)$$

The general solution of Eq. (**B**1) is

$$G(t)=Ke^{-\delta t}+G_s. \qquad (\mathbf{B}2)$$

where $G_s=u_1\bar{a}_m+u_2\bar{a}_r$. K is an arbitrary constant. Let $t=0$, using $G(0)=G_0$, we get $K=G_0-G_s$.

The goodwill function is

$$G(t)=(G_0-G_s)e^{-\delta t}+G_s, \qquad (\mathbf{B}3)$$

where $G_s=u_1\bar{a}_m+u_2\bar{a}_r$.

Appendix C: Proof of Proposition 3.

$$\begin{aligned}\rho V_c &= \max_{a_m,a_r}\textstyle\int_0^{+\infty}\Big\{\pi_d[\theta G+a_m+ba_r] \\ &\quad+(\pi_m+\pi_r)[(1-\theta)G+a_r+ba_m]\Big\}dt \\ &\quad+\max_{a_m,a_r}\textstyle\int_0^{+\infty}\Big\{-\tfrac{1}{2}{a_m}^2-\tfrac{1}{2}{a_r}^2 \\ &\quad+\tfrac{\partial V_c}{\partial G}[u_1a_m+u_2a_r-\delta G]\Big\}dt.\end{aligned} \qquad (\mathbf{C}1)$$

The first-order conditions are

$$\pi_d+b(\pi_m+\pi_r)-a_m+\frac{\partial V_c}{\partial G}u_1=0, \qquad (\mathbf{C}2)$$

$$b\pi_d+\pi_m+\pi_r-a_r+\frac{\partial V_c}{\partial G}u_2=0. \qquad (\mathbf{C}3)$$

Then we obtain the optimal national and local advertising levels are

$$a_m^*=\pi_d+b(\pi_m+\pi_r)+\frac{\partial V_c}{\partial G}u_1, \qquad (\mathbf{C}4)$$

$$a_r^* = b\pi_d + \pi_m + \pi_r + \frac{\partial V_c}{\partial G} u_2. \qquad (\mathbf{C}5)$$

Substitute Eqs. (**C**4) and (**C**5) into Eq. (**C**1),

$$\begin{aligned}\rho V_c = & \max_{a_m,a_r} \int_0^{+\infty} \Big\{ [\theta\pi_d - \tfrac{\partial V_c}{\partial G}\delta + (\pi_m+\pi_r)(1-\theta)]G \\ & + \tfrac{1}{2}\Big(b\pi_d + \pi_m + \pi_r + \tfrac{\partial V_c}{\partial G}u_2\Big)^2 \Big\} dt \\ & + \max_{a_m,a_r} \int_0^{+\infty} \Big\{ \tfrac{1}{2}\Big[b(\pi_m+\pi_r) + \tfrac{\partial V_c}{\partial G}u_1 - \pi_d\Big]^2 \\ & + 2\pi_d \Big[b(\pi_m+\pi_r) + \tfrac{\partial V_c}{\partial G}u_1\Big] \Big\} dt. \end{aligned} \qquad (\mathbf{C}6)$$

Let

$$V_c{}^*(G) = h_1 G + h_2, \qquad (\mathbf{C}7)$$

$$\begin{aligned}\rho V_c = & \max_{a_m,a_r} \int_0^{+\infty} \Big\{ [\theta\pi_d - h_1\delta + (\pi_m+\pi_r)(1-\theta)]G \\ & + \tfrac{1}{2}(b\pi_d + \pi_m + \pi_r + h_1 u_2)^2 \Big\} dt \\ & + \max_{a_m,a_r} \int_0^{+\infty} \Big\{ \tfrac{1}{2}[b(\pi_m+\pi_r) + h_1 u_1 - \pi_d]^2 \\ & + 2\pi_d [b(\pi_m+\pi_r) + h_1 u_1] \Big\} dt. \end{aligned} \qquad (\mathbf{C}8)$$

By comparing the coefficients of (**C**7) and (**C**8), we get

$$h_1 = \frac{\theta\pi_d + (\pi_m+\pi_r)(1-\theta)}{\rho+\delta}, \qquad (\mathbf{C}9)$$

$$\begin{aligned} h_2 = & \tfrac{1}{2\rho}\Big(b\pi_d + \pi_m + \pi_r + \tfrac{\theta\pi_d + (\pi_m+\pi_r)(1-\theta)}{\rho+\delta}u_2\Big)^2 \\ & + \tfrac{1}{2\rho}\Big[b(\pi_m+\pi_r) + \tfrac{\theta\pi_d + (\pi_m+\pi_r)(1-\theta)}{\rho+\delta}u_1 - \pi_d\Big]^2 \\ & + \tfrac{2}{\rho}\pi_d\Big[b(\pi_m+\pi_r) + \tfrac{\theta\pi_d + (\pi_m+\pi_r)(1-\theta)}{\rho+\delta}u_1\Big]. \end{aligned} \qquad (\mathbf{C}10)$$

Then we obtain the optimal national and local advertising levels are

$$a_m^* = \pi_d + b(\pi_m+\pi_r) + \frac{\theta\pi_d + (\pi_m+\pi_r)(1-\theta)}{\rho+\delta}u_1, \qquad (\mathbf{C}11)$$

$$a_r^* = b\pi_d + \pi_m + \pi_r + \frac{\theta u_2\pi_d + (\pi_m+\pi_r)(1-\theta)u_2}{\rho+\delta}. \qquad (\mathbf{C}12)$$

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