Differential pricing and recycling decisions based on market segmentation

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Abstract: This paper studies differential pricing and recycling decisions in a closed supply chain with one manufacturer and one retailer. The manufacturer makes the new product directly from raw material and remanufactured products with recycling used one. Considering the market segmentation and the consumer preference, we establish four decision models, which contain a centralized decision model and three Stankelberg game models, to explore the chain members' optimal strategies on price and profit. By using game theory, the firms optimal strategies and some managerial insights are obtained.

Key–Words: Differential pricing, Game theory, Recycling decisions, Market segmentation

1 Introduction

With the industrialization and the population growth, the environment is wanton destruction. More and more companies and governments pay growing attention to producing green and eco-friendly products for the environmentally sustainable development. With the increased environmental consciousness, the consumer awareness and stringent environmental laws, the management of closed-loop supply chains has gained more attention from both business and academic research[1]. A closed-loop supply chain includes the forward supply chain and the reverse supply chain, and the reverse supply chain can be defined as the logistics activities all the way from used products no longer required by the customer to products again usable in the market[2].

Even if many producers specialize in remanufacturing products, some original equipment manufacturers (OEM) may choose to combine manufacturing and remanufacturing activities together[3]. It is said that the costs derived from reverse-logistics activities in the USA exceed \$35 billion per year; remanufacturing is a \$53 billion industry in the USA[1]. In the last few years, the importance of remanufacturing has been widely recognized in the literature, quite a large number of researches have been done about it (Debo et al.[4], Liang et al.[5], Wee et al.[6], Hsueh[7], Vadde et al.[8], Wei et al.[9]). Previous literature assume that new products and remanufactured ones are not distinguishable, or both new and remanufactured products have the same wholesale prices. Huang et al.[10] study on the efficiency of the closed-loop supply chains with remanufacture based on third-party collecting. Guide et al.[11] consider managing product returns for remanufacturing. Wei and Zhao[12] report the results of a study that explores the decisions of reverse channel choice in a fuzzy closed-loop supply chain.

More recently, studies take into account that remanufactured products are differ from the new ones. Debo et al.[4] discuss the market and technology drivers of product remanufacturability and identify some phenomena of managerial importance that are typical of a remanufacturing environment. Konstantaras et al.[13] study a periodic review inventory model with finite horizon and remanufacturing, manufacturing options. Wu[14] studies the price and service decisions between new and remanufactured products, which respectively produced by two manufacturers.

Si and Ma[15] discussed the relationship between cooperation level and prices through a theoretical model in a closed-loop supply chain, which consisted of one reclaimer, one dealer and one manufacturer. But they do not consider market segmentation and the preference of the consumer with the new and remanufactured products.

Our paper differs from the afore mentioned studies in two aspects. Firstly, we consider the reverse supply chain under market segmentation and study the consumer preference coefficient of exchanging for the remanufactured products. Secondly, we establish four different decision models, which contain a centralized decision model and three Stackelberg game models (similar as Esmaeili et al.[16], Zhao et al.[17], etc.), and compare the corresponding equilibrium solutions. Moreover, we carry out the sensitivity analysis through numerical studies of some key parameters for examining their influences on the chain members' optimal decisions and maximal profits. We also study that how the industry or a chain will be better off if the supply chain in the industry is centralized or decentralized. Thus, an analysis of our paper would provide useful insights to supply chain members.

The rest of this paper is organized as follows. In Section 2, the problem description and assumptions are presented. Four decision models are discussed in Section 3. In Section 4, we give numerical examples to compare the results obtained in four models and to study the channel members' behaviors facing changing parameters. Finally, the conclusion including summary of the main results and some directions for future research is given in Section 5.

2 Problem description

2.1 Problem description and assumptions

This paper considers a closed-loop supply chain of reverse channel choice with one manufacturer and one retailer. The manufacturer can make new products from raw material (incurs a manufacturing cost per unit c_n) and remanufactured products with recycling used one (incurs a remanufacturing cost per unit c_r) in this situation. The manufacturer wholesales the new and remanufactured products to the retailer with unit wholesale price w_n, w_r, respectively, satisfied $c_n < w_n, c_r < w_r$, then the retailer sells them to the consumer with unit retail price p_n, p_r, respectively, satisfied $w_n < p_n, w_r < p_r$. We assume the new product and remanufactured one are substitutable, and all activities occur in a single period. Consumers can decide to exchange used product for new one, remanufactured one or no participation with the retailer. We denote consumer preference coefficient of exchanging for the remanufactured products as δ, $0 < \delta < 1$, and the availability of used products as k, $0 < k < 1$ (which means that the value of used products is km when the price is m). In the reverse supply chain, the expense of the retailer, which pays for one unit trading (i.e. transport, store) from the end consumer, is denoted $s(\gamma) = \frac{\theta}{2}\gamma^2$, which is a function of the used-product exchanging rats γ (θ is a scaling parameter) and the manufacturer would pay a subsidy b for the retailer to stimulating the participation of the retailer. The consumer demands are non-negative in the real world, thus $\delta(k\gamma + 2)^2 > k^2\gamma^2$. In our models, the manufacturer determines the wholesale prices of new product and remanufactured product, and the retailer determines the retail prices of two products. The structure of the supply chain is depicted in Fig.1.

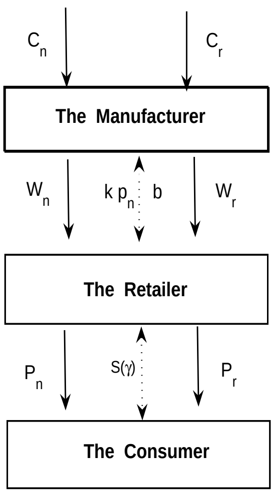

Fig. 1: The structure of the supply chain.

We do not assume any collusion or cooperation among firms. All channel members try to maximize their own profits and behave as if they have perfect information of the demand and the cost structures of other channel members. From the above descriptions, the manufacturer's profit function can be described as follows:

$$\begin{aligned}\Pi_m(w_n, w_r) &= (w_n - c_n)q_n + (w_r - c_r)q_r \\ &\quad + (kp_n - b)\gamma(q_n + q_r) \qquad (1)\end{aligned}$$

where q_n is the consumer demand of new products and q_r the remanufactured one. Both of the demands are influenced by the retailer prices p_n and p_r, and we will describe detailedly in the next section.

The profit function of the retailer can be expressed as follows:

$$\begin{aligned}\Pi_r(p_n, p_r) &= (p_n - w_n)q_n + (p_r - w_r)q_r \\ &\quad + b\gamma(q_n + q_r) - \frac{\theta}{2}\gamma^2 \qquad (2)\end{aligned}$$

The profit function of the whole supply chain can be expressed as follows:

$$\begin{aligned}\Pi_c(p_n,p_r) &= (p_n-c_n)q_n+(p_r-c_r)q_r \\ &\quad +kp_n\gamma(q_n+q_r)-\frac{\theta}{2}\gamma^2 \qquad (3)\end{aligned}$$

2.2 An important description

In this section, we introduce a useful lemma that describes the self-selection quantities associated with the prices of the new and remanufactured product. In previous studies, similar demand function has been used widely in some economic and marketing research literatures (Ferrer and Swaminathan[18][19], Wang et al.[20]).

Lemma: Assume that A characterize the market base of all products, and Δ consumers' valuation of the new product. Let δ, satisfying $\delta\in(0,1)$, indicate the consumer preference of exchanging for the remanufactured product. Large values of δ indicate that consumers accept exchanging for the remanufactured product better than if δ is small. The utility that a consumer of type Δ enjoys when exchanging for a product is $u_n(\Delta)=\Delta-p_n$ and $u_r(\Delta)=\delta\Delta-p_r$, respectively. When$\delta\in[\frac{p_r}{p_n},1+\frac{p_r-p_n}{A}]$, the consumer demand for new product q_n and consumer demand for remanufactured one q_r are:

$$q_n = A+\frac{p_r}{1-\delta}-\frac{p_n}{1-\delta} \qquad (4)$$

$$q_r = \frac{1}{1-\delta}(p_n-\frac{p_r}{\delta}) \qquad (5)$$

which means the demand is decreasing in its own retail price and increasing in the substitutable one's retail price. The exact coefficients for the demand function can be provided in Lemma (the proof of Lemma appears in Wang et al.[20]).

Remark: From the Lemma, we can easily obtain that, if $\delta<\frac{p_r}{p_n}$, q_n equal to 0, while, if $\delta>1+\frac{p_r-p_n}{A}$, q_r equal to 0. Each firm would only consider the non-negative demand, so we just discuss the pricing decisions satisfying $\delta\in[\frac{p_r}{p_n},1+\frac{p_r-p_n}{A}]$.

Together using Eqs. (1)-(5), the respective profit functions of the manufacturer, the retailer and the whole supply chain can be denoted as:

$$\begin{aligned}\Pi_m(w_n,w_r) &= (w_n-c_n)(A+\frac{p_r}{1-\delta}-\frac{p_n}{1-\delta}) \\ &\quad +(w_r-c_r)\frac{1}{1-\delta}(p_n-\frac{p_r}{\delta}) \\ &\quad +(kp_n-b)\gamma(A-\frac{p_r}{\delta}) \qquad (6)\end{aligned}$$

$$\begin{aligned}\Pi_r(p_n,p_r) &= (p_n-w_n)(A+\frac{p_r}{1-\delta}-\frac{p_n}{1-\delta}) \\ &\quad +(p_r-w_r)\frac{1}{1-\delta}(p_n-\frac{p_r}{\delta}) \\ &\quad +b\gamma(A-\frac{p_r}{\delta})-\frac{\theta}{2}\gamma^2 \qquad (7)\end{aligned}$$

$$\begin{aligned}\Pi_c(p_n,p_r) &= (p_n-c_n)(A+\frac{p_r}{1-\delta}-\frac{p_n}{1-\delta}) \\ &\quad +(p_r-c_r)\frac{1}{1-\delta}(p_n-\frac{p_r}{\delta}) \\ &\quad +kp_n\gamma(A-\frac{p_r}{\delta})-\frac{\theta}{2}\gamma^2 \qquad (8)\end{aligned}$$

3 Main analytical results

To analyze our model, we first consider the decisions of the centralized supply chain as a benchmark and establish the centralized decision model (CD model). Then we consider the situations where the manufacturer and the retailer implement the corresponding decentralized decision models using Stackelberg game. (1) Manufacturer-leader Stackelberg (MS) game model, where the manufacturer is leader and the retailer is follower in the supply chain; (2)Retailer-leader Stackelberg (RS) game model, where the manufacturer first announces the wholesale price, then the retailer decides the retail price; (3) Vertical Nash game (NG) model, where every firm has equal bargaining power, thus, they make their decisions simultaneously. Such modeling enables us to capture the chain members' competitive dynamics under different power structures. The leader in each scenario makes his decision to maximize his own profit, conditioned on the follower's response.

3.1 Centralized decision (CD) model

In this scenario, we consider a supply chain operated by an integrated-firm which can also be regarded as the manufacturer and the retailer making cooperation. We ignore the subsidy b which the manufacturer paid for the retailer to stimulating the participation of the retailer in the reverse supply chain. Furthermore, the wholesale price w_n and w_r are regarded as inner transfer price, which do not affect the profit of the whole supply chain system. The integrated firm tries to maximize his profit, denoted as $\Pi_c(p_n,p_r)$. Thus, we establish the CD model for the centralized scenario as follows.

$$\max_{p_n,p_r}\Pi_c(p_n,p_r) \qquad (9)$$

Proposition 1 *The profit $\Pi_c(p_n,p_r)$ defined in equation (9) is jointly concave in (p_n,p_r). The optimal retail prices of the new product and the remanufactured one, denoted as p_n^{c*} and p_r^{c*} respectively, are given as* *follows.*

$$p_n^{c*} = \frac{\frac{1}{\delta(1-\delta)}(\delta c_n - c_r)(\frac{2}{1-\delta} - \frac{k\gamma}{\delta})}{(\frac{2}{1-\delta} - \frac{k\gamma}{\delta})^2 - \frac{4}{\delta(1-\delta)^2}} - \frac{\frac{1}{\delta(1-\delta)}((1+k\gamma)A - \frac{c_r - c_n}{1-\delta})}{(\frac{2}{1-\delta} - \frac{k\gamma}{\delta})^2 - \frac{4}{\delta(1-\delta)^2}} \quad (10)$$

$$p_r^{c*} = \frac{(\frac{2}{1-\delta} - \frac{k\gamma}{\delta})(\frac{c_r - c_n}{1-\delta} - (1+k\gamma)A)}{(\frac{2}{1-\delta} - \frac{k\gamma}{\delta})^2 - \frac{4}{\delta(1-\delta)^2}} + \frac{\frac{2\delta c_n - 2c_r}{\delta(1-\delta)^2}}{(\frac{2}{1-\delta} - \frac{k\gamma}{\delta})^2 - \frac{4}{\delta(1-\delta)^2}} \quad (11)$$

Proof. From Eq. (8), the first-order and second-order partial derivatives of $\Pi_c(p_n, p_r)]$ with respect to p_n and p_r can be shown as

$$\frac{\partial \Pi_c(p_n, p_r)}{\partial\, p_n} = -\frac{2}{1-\delta}p_n + (\frac{2}{1-\delta} - \frac{k\gamma}{\delta})p_r + \frac{c_n - c_r}{1-\delta} + (1+k\gamma)A \quad (12)$$

$$\frac{\partial \Pi_c(p_n, p_r)}{\partial\, p_r} = (\frac{2}{1-\delta} - \frac{k\gamma}{\delta})p_n - \frac{2}{\delta(1-\delta)}p_r + \frac{c_r - \delta c_n}{\delta(1-\delta)} \quad (13)$$

$$\frac{\partial^2 \Pi_c(p_n, p_r)}{\partial^2 p_n} = -\frac{2}{1-\delta} < 0 \quad (14)$$

$$\frac{\partial^2 \Pi_c(p_n, p_r)}{\partial^2 p_r} = -\frac{2}{\delta(1-\delta)} \quad (15)$$

$$\frac{\partial^2 \Pi_c(p_n, p_r)}{\partial p_n \partial p_r} = \frac{\partial^2 \Pi_c(p_n, p_r)}{\partial p_r \partial p_n} = \frac{2}{1-\delta} - \frac{k\gamma}{\delta} \quad (16)$$

By Eqs. (14)-(16), the Hessian matrix

$$H_1 = \begin{bmatrix} \frac{\partial^2 \Pi_c(p_n,p_r)}{\partial p_n^2} & \frac{\partial^2 \Pi_c(p_n,p_r)}{\partial p_n \partial p_r} \\ \frac{\partial^2 \Pi_c(p_n,p_r)}{\partial p_r \partial p_n} & \frac{\partial^2 \Pi_c(p_n,p_r)}{\partial p_r^2} \end{bmatrix} = \begin{bmatrix} -\frac{2}{1-\delta} & \frac{2}{1-\delta} - \frac{k\gamma}{\delta} \\ \frac{2}{1-\delta} - \frac{k\gamma}{\delta} & -\frac{2}{\delta(1-\delta)} \end{bmatrix}.$$

which is negative definite since $0 < \delta < 1$ and the assumption $\delta(k\gamma + 2)^2 > k^2\gamma^2$. So $\Pi_c(p_n, p_r)$ is jointly concave in (p_n, p_r). By setting Eqs. (12)-(13) to zero and solving them simultaneously, the optimal retail prices can be obtained as in Eqs. (10) and (11).

Substituting p_n^{c*} and p_r^{c*} into Eq. (8), the whole supply chain system's maximal profit, denoted as Π_c^{c*}, can be given as follows

$$\Pi_c^{c*} = (p_n^{c*} - c_n)(A + \frac{p_r^{c*}}{1-\delta} - \frac{p_n^{c*}}{1-\delta}) + (p_r^{c*} - c_r)\frac{1}{1-\delta}(p_n^{c*} - \frac{p_r^{c*}}{\delta}) + kp_n^{c*}\gamma(A - \frac{p_r^{c*}}{\delta}) - \frac{\theta}{2}\gamma^2 \quad (17)$$

3.2 Manufacturer-leader Stackelberg (MS) game model

The sizes of the manufacturer is larger compared to the retailer's, so the manufacturer is the Stackelberg leaders, and the retailer is the Stackelberg follower. The manufacturer first announces his wholesale prices of new products and remanufactured one, respectively. Then, the retailer decides the retail prices charging for two products. The objective of each participant is to maximize his own profits. The MS game model can be formulated as follows:

$$\begin{cases} \max\limits_{w_n, w_r} \Pi_m(w_n, w_r, p_n^*(w_n, w_r), p_r^*(w_n, w_r)) \\ \text{subject to} \\ \quad p_n^*(w_n, w_r), p_r^*(w_n, w_r) \text{ are derived from} \\ \qquad \text{solving the problem} \\ \qquad \max\limits_{p_n, p_r} \Pi_r(p_n, p_r) \end{cases} \quad (18)$$

The problem can be solved backwards, and we obtain the following proposition.

Proposition 2 *In the MS game model, given the wholesale prices w_n and w_r made earlier by the manufacturer, the retailer's optimal response functions are*

$$p_n^*(w_n, w_r) = \frac{1}{2}(w_n + A - b\gamma) \quad (19)$$

$$p_r^*(w_n, w_r) = \frac{1}{2}(w_r + \delta A - b\gamma) \quad (20)$$

Proof. The first-order and second-order partial derivatives of Eq. (7) with respect to p_n and p_r can be shown as

$$\frac{\partial \Pi_r(p_n, p_r)}{\partial p_n} = -\frac{2}{1-\delta}p_n + \frac{2}{1-\delta}p_r + \frac{1}{1-\delta}w_n - \frac{1}{1-\delta}w_r + A \quad (21)$$

$$\frac{\partial \Pi_r(p_n, p_r)}{\partial p_r} = \frac{2}{1-\delta}p_n - \frac{2}{\delta(1-\delta)}p_r - \frac{1}{1-\delta}w_n + \frac{1}{\delta(1-\delta)}w_r - \frac{b\gamma}{\delta} \quad (22)$$

$$\frac{\partial^2 \Pi_r(p_n, p_r)}{\partial^2 p_n} = -\frac{2}{1-\delta} < 0 \quad (23)$$

$$\frac{\partial^2\Pi_r(p_n,p_r)}{\partial^2 p_r} = -\frac{2}{\delta(1-\delta)} \tag{24}$$

$$\frac{\partial^2\Pi_r(p_n,p_r)}{\partial p_n\partial p_r} = \frac{\partial^2\Pi_r(p_n,p_r)}{\partial p_r\partial p_n} = \frac{2}{1-\delta} \tag{25}$$

It follows from $0<\delta<1$, the Hessian matrix

$$H_2 = \begin{bmatrix} -\frac{2}{1-\delta} & \frac{2}{1-\delta} \\ \frac{2}{1-\delta} & -\frac{2}{\delta(1-\delta)}] \end{bmatrix} = \frac{4}{\delta(1-\delta)}$$

is negative definite, so $\Pi_r(p_n,p_r)$ is concave in (w_n,w_r). By setting Eqs. (21) and (22) to zero and solving them simultaneously, Eqs. (19) and (20) can be obtained.

Then, after knowing the retailer's reaction functions, the manufacturer sets the optimal wholesale prices of two products to maximize his profit $\Pi_m(w_n,w_r,p_n^*(w_n,w_r),p_r^*(w_n,w_r))$. The following proposition gives the closed form solution of manufacturer's optimal wholesale prices.

Proposition 3 *In the MS game model, the manufacturer's equilibrium wholesale prices, denoted as w_n^{M*} and w_r^{M*}, are given as*

$$w_n^{M*} = \frac{B_1B_3+\delta B_1B_4-\frac{B_2B_4}{4}}{(\delta B_1-\frac{B_2}{4})^2-\delta B_1^2} \tag{26}$$

$$w_r^{M*} = \frac{\delta B_1B_3+\delta B_1B_4-\frac{B_2B_3}{4}}{(\delta B_1-\frac{B_2}{4})^2-\delta B_1^2} \tag{27}$$

where

$$B_1=\frac{1}{\delta(1-\delta)}, B_3=\frac{c_r-c_n}{2(1-\delta)}-\frac{(2+k\gamma)A}{4}-\frac{kb\gamma^2}{4\delta}$$

$$B_2=\frac{k\gamma}{\delta}, \qquad B_4=\frac{\delta c_n-c_r}{2\delta(1-\delta)}-\frac{4b\gamma+k\gamma^2-k\gamma A}{4\delta}$$

Proof. By substituting Eqs. (19) and (20) into Eq. (6), the profits $\Pi_m(w_n,w_r)$ can be expressed as

$$\begin{aligned}\Pi_m \quad (w_n,w_r) &= (w_n-c_n)(A+\frac{p_r^*(w_n,w_r)}{1-\delta}\\ &-\frac{p_n^*(w_n,w_r)}{1-\delta})+(w_r-c_r)\frac{1}{1-\delta}\\ &\times(p_n^*(w_n,w_r)-\frac{p_r^*(w_n,w_r)}{\delta})\\ &+(kp_n^*(w_n,w_r)-b)\gamma(A-\frac{p_r^*(w_n,w_r)}{\delta})\end{aligned} \tag{28}$$

The first-order and second-order partial derivative of Eq. (28) with respect to (p_n,p_r) can be shown as

$$\begin{aligned}\frac{\partial\Pi_m(w_n,w_r)}{\partial w_n} &= -\frac{1}{1-\delta}w_n+(\frac{1}{1-\delta}-\frac{k\gamma}{4\delta})w_r\\ &+\frac{c_n-c_r}{2(1-\delta)}+\frac{(2+k\gamma)A}{4}+\frac{kb\gamma^2}{4\delta}\end{aligned} \tag{29}$$

$$\begin{aligned}\frac{\partial\Pi_m(w_n,w_r)}{\partial w_r} &= (\frac{1}{1-\delta}-\frac{k\gamma}{4\delta})w_n-\frac{1}{\delta(1-\delta)}w_r\\ &+\frac{c_r-\delta c_n}{2\delta(1-\delta)}+\frac{4b\gamma+k\gamma^2-k\gamma A}{4\delta}\end{aligned} \tag{30}$$

$$\frac{\partial^2\Pi_m(w_n,w_r)}{\partial^2 w_n} = -\frac{2}{1-\delta}<0 \tag{31}$$

$$\frac{\partial^2\Pi_m(w_n,w_r)}{\partial^2 w_r} = -\frac{2}{\delta(1-\delta)} \tag{32}$$

$$\frac{\partial^2\Pi_m(w_n,w_r)}{\partial w_n\partial w_r} = \frac{\partial^2\Pi_m(w_n,w_r)}{\partial w_r\partial w_n} = \frac{2}{1-\delta} \tag{33}$$

For $0<\delta<1$ and the assumption $\delta(k\gamma+2)^2>k^2\gamma^2$, we have a negative definite Hessian matrix

$$H_3 = \begin{bmatrix} -\frac{1}{1-\delta} & \frac{1}{1-\delta}-\frac{k\gamma}{4\delta} \\ \frac{1}{1-\delta}-\frac{k\gamma}{4\delta} & -\frac{1}{\delta(1-\delta)}] \end{bmatrix}$$

so $\Pi_m(w_n,w_r)$ is concave in (w_n,w_r). Let Eqs. (29) and (30) be equal to zero and solve them, w_n^{M*} and w_r^{M*} are the optimal wholesale prices for the manufacturer.

By substituting Eqs. (26) and (27) into Eqs. (19) and (20), the following proposition can be easily obtained.

Proposition 4 *In the MS game model, the retailer's optimal retail prices p_n^{M*} and p_r^{M*} can be obtained, respectively, as*

$$p_n^{M*} = \frac{1}{2}(w_n^{M*}+A-b\gamma) \tag{34}$$

$$p_r^{M*} = \frac{1}{2}(w_r^{M*}+\delta A-b\gamma) \tag{35}$$

The manufacturer and retailer attain their maximal profits value as follows

$$\begin{aligned}\Pi_m^{M*} &= (w_n^{M*}-c_n)(A+\frac{p_r^{M*}}{1-\delta}-\frac{p_n^{M*}}{1-\delta})\\ &+(w_r^{M*}-c_r)\frac{1}{1-\delta}(p_n^{M*}-\frac{p_r^{M*}}{\delta})\\ &+(kp_n^{M*}-b)\gamma(A-\frac{p_r^{M*}}{\delta})\end{aligned} \tag{36}$$

$$\begin{aligned}\Pi_r^{M*} &= (p_n^{M*}-w_n^{M*})(A+\frac{p_r^{M*}}{1-\delta}-\frac{p_n^{M*}}{1-\delta})\\ &+(p_r^{M*}-w_r^{M*})\frac{1}{1-\delta}(p_n^{M*}-\frac{p_r^{M*}}{\delta})\\ &+b\gamma(A-\frac{p_r^{M*}}{\delta})-\frac{\theta}{2}\gamma^2\end{aligned} \tag{37}$$

3.3 Retailer-leader Stackelberg (RS) game model

The RS game scenario represents a market wherein there are one larger retailer and one relatively smaller manufacturer. In addition, the market is controlled by retailer, the relationship between the manufacturer and the retailer is modeled as a sequential noncooperative game, where the manufacturer is the follower. The retailer selects retail prices in the first step. The manufacturer observes the decisions made by the retailer and simultaneously makes his responses to those decisions in the second step (by choosing the wholesale prices w_n and w_r). The following RS game model is formulated as

$$\begin{cases} \underset{p_n,p_r}{max}\,\Pi_r(p_n,p_r,w_n^*(p_n,p_r),w_r^*(p_n,p_r)) \\ \text{subject to} \\ \quad w_n^*(p_n,p_r),w_r^*(p_n,p_r) \text{ are derived from} \\ \qquad \text{solving the problem} \\ \qquad \underset{w_n,w_r}{max}\,\Pi_m(w_n,w_r) \end{cases} \tag{38}$$

Similarly, the problem is solved backwards. We first derive the manufacturer's response function in the following proposition.

Proposition 5 *In the RS game model, for given retail prices of the new products and the remanufactured one p_n and p_r, the manufacturer's optimal response functions are*

$$\begin{aligned} w_n^*(p_n,p_r) &= -(1+k\gamma)p_n-\frac{k\gamma}{\delta}p_r \\ &\quad +c_n+b\gamma+(1+k\gamma)A \qquad (39) \\ w_r^*(p_n,p_r) &= -k\gamma p_n-(1+k\gamma)p_r \\ &\quad +c_r+b\gamma+\delta(1+k\gamma)A \qquad (40) \end{aligned}$$

Proof. By letting $p_n = w_n + t_n, p_r = w_r + t_r$ into Eqs. (6), where t_n is the new products' profit margins, t_r is the remanufactured products' profit margins, the first-order and second-order partial derivatives of $\Pi_m(w_n,w_r)$ to (w_n,w_r) can be derived as follows

$$\begin{aligned} \frac{\partial\Pi_m(w_n,w_r)}{\partial w_n} &= -\frac{1}{1-\delta}w_n+\frac{1}{1-\delta}w_r \\ &\quad -\frac{1}{1-\delta}p_n+(\frac{1}{1-\delta}-\frac{k\gamma}{\delta})p_r \\ &\quad +\frac{c_n-c_r}{1-\delta}+(1+k\gamma)A \qquad (41) \\ \frac{\partial\Pi_m(w_n,w_r)}{\partial w_r} &= \frac{1}{1-\delta}w_n-\frac{1}{\delta(1-\delta)}w_r \\ &\quad +(\frac{1}{1-\delta}-\frac{k\gamma}{\delta})p_n-\frac{1}{\delta(1-\delta)}p_r \\ &\quad +\frac{c_r-\delta c_n}{\delta(1-\delta)}+\frac{b\gamma}{\delta} \qquad (42) \end{aligned}$$

$$\frac{\partial^2\Pi_m(w_n,w_r)}{\partial^2 w_n} = -\frac{1}{1-\delta} < 0 \tag{43}$$

$$\frac{\partial^2\Pi_m(w_n,w_r)}{\partial^2 w_r} = -\frac{1}{\delta(1-\delta)} \tag{44}$$

$$\frac{\partial^2\Pi_m(w_n,w_r)}{\partial w_n\partial w_r} = \frac{\partial^2\Pi_m(w_n,w_r)}{\partial w_r\partial w_n} = \frac{1}{1-\delta} \tag{45}$$

we can see the Hessian matrix is negative definite. By setting Eqs. (41)-(42) to zero and solving them simultaneously, Eqs. (39)-(40) can be obtained.

Having the information about the reaction functions of the manufacturer, the retailer would use them to maximize his expected profit, denoted as $\Pi_r(p_n,p_r,w_n^*(p_n,p_r),w_r^*(p_n,p_r))$. The optimal retail prices can be derived as follows.

Proposition 6 *In the RS game model, the optimal decision p_n^{R*} and p_r^{R*} of the retailer can be derived as*

$$p_n^{R*} = \frac{2B_1C_1+2\delta B_1C_2-B_2C_2}{2(2\delta B_1-B_2)^2-8\delta B_1^2} \tag{46}$$

$$p_r^{R*} = \frac{2\delta B_1C_1+2\delta B_1C_2-B_2C_1}{2(2\delta B_1-B_2)^2-8\delta B_1^2} \tag{47}$$

where

$$C_1 = \frac{c_r-c_n}{1-\delta}-(3+2k\gamma)A, C_2 = \frac{\delta c_n-c_r}{\delta(1-\delta)}-\frac{k\gamma A}{\delta}$$

Setting Eqs. (39) and (40) into Eq. (7), the profit can be expressed as

$$\begin{aligned} \Pi_r(p_n,p_r) &= (p_n-w_n^*(p_n,p_r))(A+\frac{p_r}{1-\delta}-\frac{p_n}{1-\delta}) \\ &\quad +(p_r-w_r^*(p_n,p_r))\frac{1}{1-\delta}(p_n-\frac{p_r}{\delta}) \\ &\quad +b\gamma(A-\frac{p_r}{\delta})-\frac{\theta}{2}\gamma^2 \end{aligned}$$

We can obtain that the Hessian matrix is negative definite, so $\Pi_r(p_n,p_r)$ is jointly concave in (p_n,p_r). Thus, the equilibrium retail prices can be derived and be easily obtained as Eqs. (34) and (35).

By substituting Eqs. (46) and (47) into Eq. (39) and (40), the optimal wholesale prices of the new products and the remanufactured one can be obtained in the following proposition.

Proposition 7 *In the RS game model, the manufacturer's optimal decisions are*

$$\begin{aligned} w_n^{R*} &= -(1+k\gamma)p_n^{R*}-\frac{k\gamma}{\delta}p_r^{R*} \\ &\quad +c_n+b\gamma+(1+k\gamma)A \qquad (48) \\ w_r^{R*} &= -k\gamma p_n^{R*}-(1+k\gamma)p_r^{R*} \\ &\quad +c_r+b\gamma+\delta(1+k\gamma)A \qquad (49) \end{aligned}$$

Then, by substituting $w_n^{R*}, w_r^{R*}, p_n^{R*}$ and p_r^{R*} into Eqs. (6) and (7), the maximal profits value of the manufacturer and the retailer can be obtained

$$\begin{aligned}\Pi_m^{R*} &= (w_n^{R*} - c_n)(A + \frac{p_r^{R*}}{1-\delta} - \frac{p_n^{R*}}{1-\delta}) \\ &+ (w_r^{R*} - c_r)\frac{1}{1-\delta}(p_n^{R*} - \frac{p_r^{R*}}{\delta}) \\ &+ (kp_n^{R*} - b)\gamma(A - \frac{p_r^{R*}}{\delta}) \qquad (50)\end{aligned}$$

$$\begin{aligned}\Pi_r^{R*} &= (p_n^{R*} - w_n^{R*})(A + \frac{p_r^{R*}}{1-\delta} - \frac{p_n^{R*}}{1-\delta}) \\ &+ (p_r^{R*} - w_r^{R*})\frac{1}{1-\delta}(p_n^{R*} - \frac{p_r^{R*}}{\delta}) \\ &+ b\gamma(A - \frac{p_r^{R*}}{\delta}) - \frac{\theta}{2}\gamma^2 \qquad (51)\end{aligned}$$

3.4 Vertical Nash game (NG) model

This scenario arises in a market in which there are relatively small to medium-sized manufacturer and retailer. Every firm has equal bargaining power and makes his decision simultaneously. So, the NG model can be formulated as

$$\begin{cases} \max\limits_{w_n, w_r} \Pi_m(w_n, w_r) \\ \max\limits_{p_n, p_r} \Pi_r(p_n, p_r) \end{cases} \qquad (52)$$

Solving the NG model, we can obtain the following proposition.

Proposition 8 *In the NG model, the optimal strategies of the manufacturer and the retailer can be expressed as follows:*

$$p_n^{N*} = \frac{3B_1F_1 - 3\delta B_1F_2 + B_2F_2}{9\delta B_1^2 - (3\delta B_1 - B_2)^2} \qquad (53)$$

$$p_r^{N*} = \frac{3\delta B_1F_2 - 3\delta B_1F_1 + B_2F_1}{9\delta B_1^2 - (3\delta B_1 - B_2)^2} \qquad (54)$$

$$w_n^{N*} = \frac{6B_1F_1 - 6\delta B_1F_2 + 2B_2F_2}{9\delta B_1^2 - (3\delta B_1 - B_2)^2} - A + b\gamma \qquad (55)$$

$$w_r^{N*} = \frac{6\delta B_1F_2 - 6\delta B_1F_1 + 2B_2F_1}{9\delta B_1^2 - (3\delta B_1 - B_2)^2} - \delta A + b\gamma \qquad (56)$$

where

$$F_1 = \frac{c_r - c_n}{1-\delta} - (2 + k\gamma)A, \quad F_2 = \frac{\delta c_n - c_r}{\delta(1-\delta)}$$

Proof. From the MS game, the retailer response function for the given wholesale prices are obtained in Eqs. (19) and (20). From the RS game model, for given retail prices, the manufacturer's response functions are obtained in Eqs. (39) and (40). Solving these equations yields the Nash equilibrium solutions, i.e., Eqs. (53)-(56). So Proposition 8 holds.

By Proposition 8, we can easily obtain the manufacturer and retailer's maximal profits value as follows

$$\begin{aligned}\Pi_m^{N*} &= (w_n^{N*} - c_n)(A + \frac{p_r^{N*}}{1-\delta} - \frac{p_n^{N*}}{1-\delta}) \\ &+ (w_r^{N*} - c_r)\frac{1}{1-\delta}(p_n^{N*} - \frac{p_r^{N*}}{\delta}) \\ &+ (kp_n^{N*} - b)\gamma(A - \frac{p_r^{N*}}{\delta}) \qquad (57)\end{aligned}$$

$$\begin{aligned}\Pi_r^{N*} &= (p_n^{N*} - w_n^{N*})(A + \frac{p_r^{N*}}{1-\delta} - \frac{p_n^{N*}}{1-\delta}) \\ &+ (p_r^{N*} - w_r^{N*})\frac{1}{1-\delta}(p_n^{N*} - \frac{p_r^{N*}}{\delta}) \\ &+ b\gamma(A - \frac{p_r^{N*}}{\delta}) - \frac{\theta}{2}\gamma^2 \qquad (58)\end{aligned}$$

4 Numerical examples

The optimal strategies obtained in this paper are in a very complicated form, so we have to use numerical examples to compare the results obtained from the above different decision models and to explore the behavior of each firm facing changing environments. In this section, we can easily compare the expressions of the optimal wholesale prices, retail prices, and optimal profits of the manufacturer and retailer under four different decision scenarios.

We adopt the similar data in Zhao et al.[1] except the values of parameters b, k and δ, which have been appropriately manipulated before being employed to comply with certain assumptions of this research. We think these data can represent the real-world condition as closely as possible due to the difficulty of accessing the actual industry data.

Discussion 1 *Comparison of the optimal decisions under four different scenarios.*

The following values of parameters are assumed: the market base of all products A=400, the cost of new products c_n=23, the cost of remanufactured products c_r=11, scaling parameter θ=1000, consumer preference coefficient of exchanging for the remanufactured products δ=0.3, the availability of used products k=0.3, the subsidy which the manufacturer paid for the retailer b=3, the used-product exchanging rate γ=0.18. The corresponding results are shown as in Tables 1-2.

From Tables 1 and 2, the following results are obtained:

(1-1) $w_n^{M*} > w_n^{N*} > w_n^{R*}$, $w_r^{M*} > w_r^{N*} > w_r^{R*}$, $p_n^{R*} > p_n^{M*} > p_n^{N*} > p_n^{C*}$, $p_r^{R*} > p_r^{M*} > p_r^{N*} > p_r^{C*}$, The optimal retail prices of both products are the lowest under centralized decision scenario. This indicates that consumers are better off when no channel member in a dominant position. In Manufacturer-leader Stackelberg game model, the optimal wholesale price is larger than other decision models, while, in Retailer-leader Stackelberg game model, the retail price is larger than other models.

Table 1: The optimal profits of the whole supply chain system or every firm

$scenario$	Π_m	Π_r	Π_c
CD	–	–	37687
MS	19370	9399.1	28769
RS	9425.8	18835	28261
NG	17372	16421	33793

Table 2: The optimal decisions of retail prices and wholesale prices

$scenario$	w_n^*	w_r^*	p_n^*	p_r^*
CD	–	–	211.076	61.384
MS	206.641	58.693	303.050	89.076
RS	106.778	25.932	305.538	90.692
NG	144.279	39.544	271.870	79.502

(1-2) Obviously, the optimal retail price is always larger than the optimal wholesale price, i.e. $p_i^{j*} > w_i^{j*}$ $(i = n, r; j = M, R, N)$. The price of the new product is larger than that of the remanufactured one, i.e. $p_n^{j*} > p_r^{j*}$ $(j = M, R, N)$, and $w_n^{j*} > w_r^{j*}$ $(j = M, R, N)$. This conclusion is completely coincident with the fact.

(1-3) $\Pi_m^{M*} > \Pi_m^{N*} > \Pi_m^{R*}, \Pi_r^{R*} > \Pi_r^{N*} > \Pi_r^{M*}$, $\Pi_c^{C*} > \Pi_c^{N*} > \Pi_c^{M*} > \Pi_c^{R*}$. The maximal profit of the whole supply chain system under centralized decision case is higher than that under decentralized decision case. In the MS game model, the manufacturer's maximal profit is always larger than the retailer's. That's because the manufacturer is leader of the market. While, in the RS game model, the retailer obtained the highest maximal profit. That's because the market is controlled by the retailer.

Discussion 2 *Sensitivity analysis of the parameter* δ.

We investigate the change of the optimal decisions and profits with the consumer preference coefficient of exchanging for the remanufactured products δ. Consider the values of the parameters as before: A=400, c_n=23, c_r=11, θ=1000, k=0.3, b=3, γ=0.18 except $\delta \in (0.1, 0.9)$. Figs. 2-5 show the results.

Through the analysis, we gain the following intuitive insights.

(2-1) From Fig. 2, we can obtained: in the MS model, as parameter δ increases, the new products' optimal retail price and the optimal wholesale price

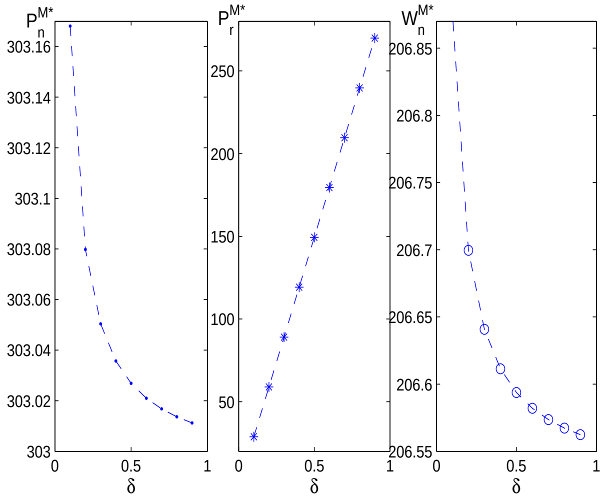

Fig. 2: Changes of prices with δ in MS model.

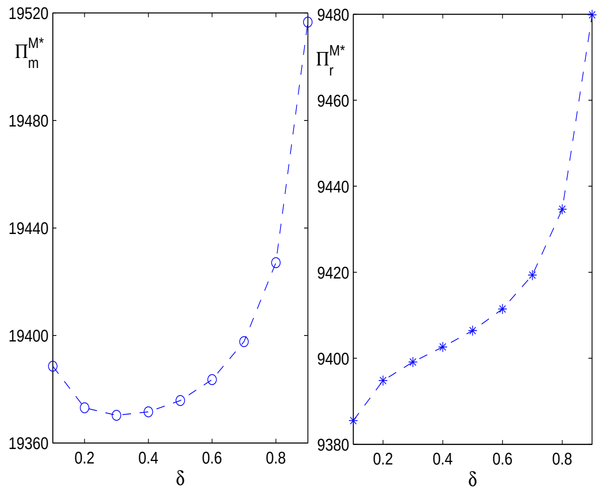

Fig. 3: Changes of two products' optimal profits with δ in MS model.

will decrease slightly. On the contrary, the remanufactured products' optimal retail price is increasing obviously. This is because more and more consumers choose exchanging for the remanufactured products, and the retailer should bring the price of new products down.

(2-2) From Fig. 3, we know: the optimal profits of remanufactured products increase as parameter δ increase. This is because of the increase in demands of remanufactured products resulting from the increase of parameter δ. When the parameter $0.1 < \delta < 0.3$, the optimal profits of new products slightly decrease, while increase obviously when $0.3 < \delta < 0.9$.

(2-3) As parameter δ increases, the optimal wholesale prices of new products will slowly decrease, and the optimal wholesale price of remanufactured products will sharply increase both in MS and RS game models from Fig. 4.

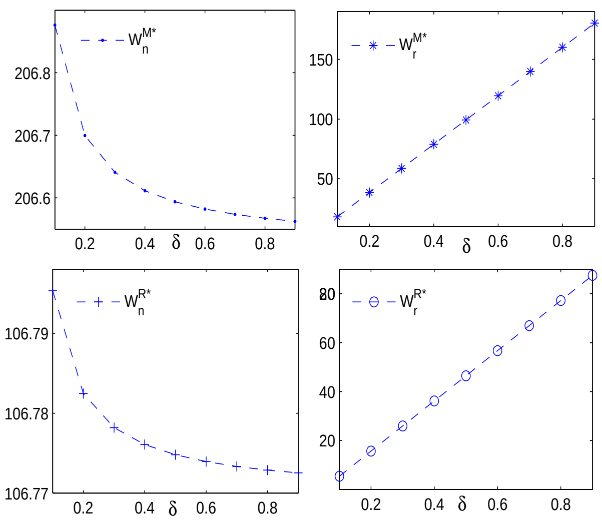

Fig. 4: Changes of wholesale prices with δ.

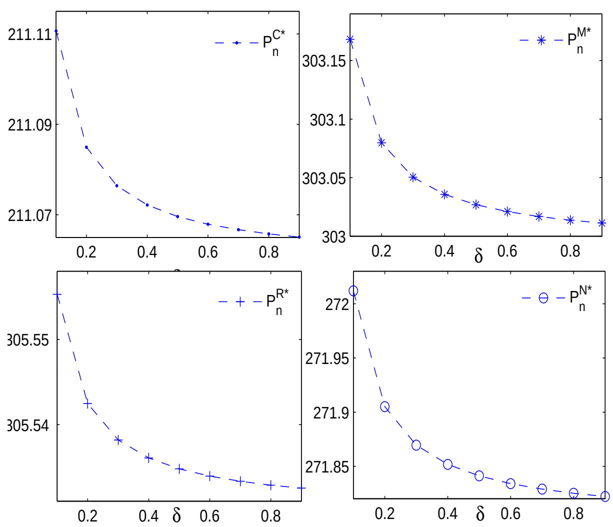

Fig. 5: Changes of retail prices with δ.

(2-4) The new products' retial prices will decrease slowly when the parameter δ increases in four different decision models, which is a good news for the consumers.

Discussion 3 *Sensitivity analysis of the parameter k.*

In this section, we explore the effects of the availability of used products k. The values of parameters are assumed as before: A=400, c_n=23, c_r=11, θ=1000, δ=0.3, b=3, γ=0.18 except $k \in (0.1, 0.9)$. The results can be illustrated in Figs. 6-8.

Through the analysis, we gain the following intuitive insights.

(3-1) From Figs. 6 and 7, we can obtain that the wholesale prices of the remanufactured products will

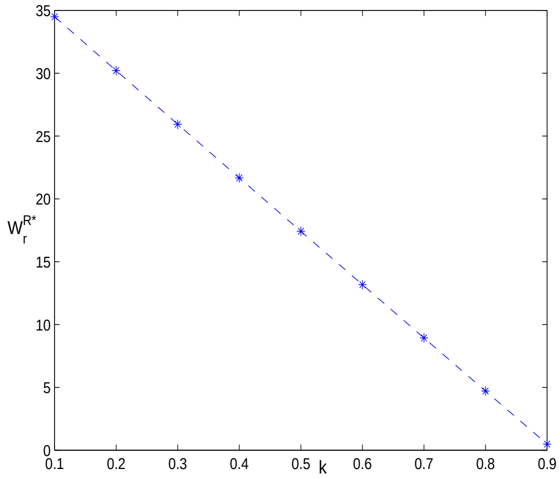

Fig. 6: The change of remanufactured products' wholesale price with k in RS model.

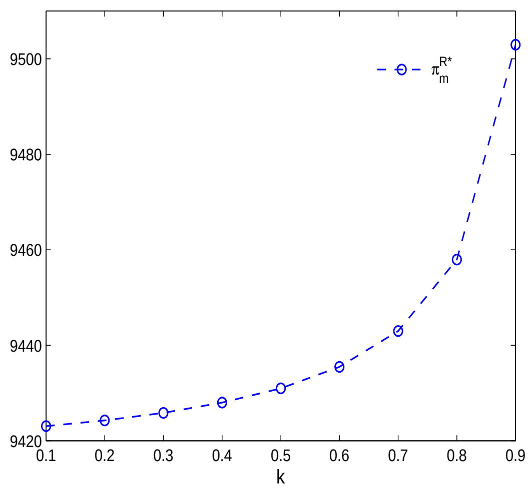

Fig. 7: The change of manufacturer's optimal profit with k in RS model

decrease while the manufacturer's maximal optimal profit increase when the parameter k increases in the RS model. The phenomenon implies consumers that, keeping the value of used products, they will benefit from the lower retail prices and each firm of the whole system benefits from the higher profits. That is because the increased demand resulting from lower wholesale prices more than offsets the loss of revenue per unit due to the lower wholesale prices. So, the manufacturer and the whole system are better off as k

increases.

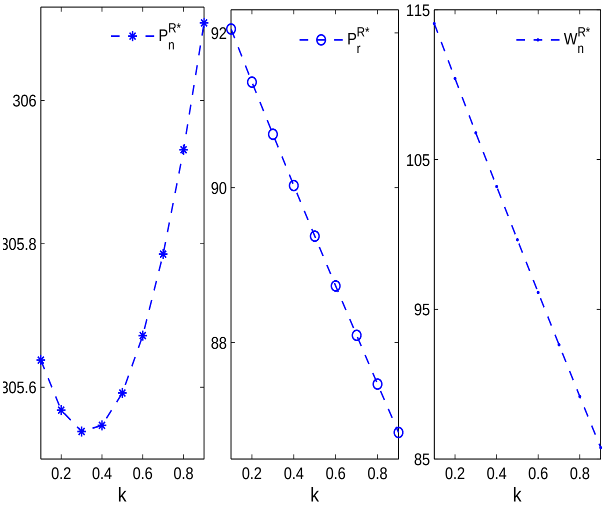

Fig. 8: Changes of product prices with k in RS model

(3-2) As parameter δ increases, the new products' optimal retail price will increase slightly in the RS game model. On the contrary, the remanufactured products' optimal retail price and the new products' optimal wholesale price is decreasing obviously.

Discussion 4 *Sensitivity analysis of the parameter* γ.

The values of parameters are assumed as before: A= 400, c_n=23, c_r=11, θ=1000, δ=0.3, b=3, k=0.3 except $\gamma \in (0.1, 0.9)$. We can gain the changes of each firm's maximal optimal profits with the parameter γ in Fig. 9.

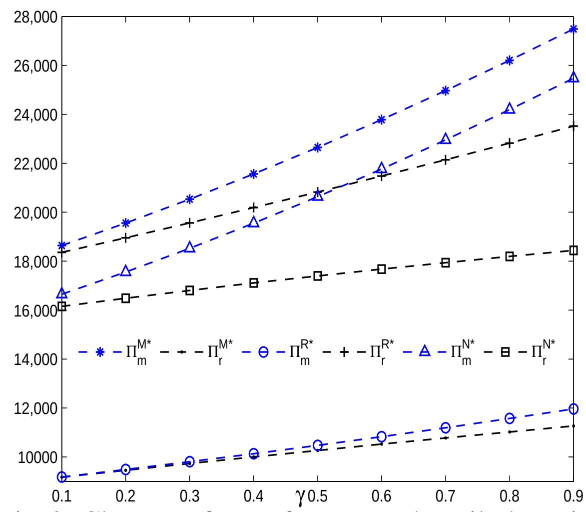

Fig. 9: Changes of manufacturer and retailer's optimal profit with γ.

(4-1) Under three different Stacklebeg game models, each firm's maximal optimal profit is always increasing as γ increases. That's to say, the whole system and his members are better off when the retailer tries his best to reverse the used products, which is also helping the environment.

(4-2) The maximal optimal profits of each firm in supply chain satisfy: $\Pi_m^{R*} < \Pi_m^{N*} < \Pi_m^{M*}$, $\Pi_r^{M*} < \Pi_r^{N*} < \Pi_r^{R*}$. The firm who is the leader in supply chain has the advantage to get the higher profits. For example, the manufacturer's profit under MS game scenario is the highest, while the retailer has his own maximal profit under RS. This is consistent with result (1-3).

5 Conclusions

This paper considers differential pricing and reverse channel decisions with one manufacturer and one retailer. By considering the market segmentation and using game-theoretic approach, the closed form solutions for four decision models are obtained, i.e. the Centralized decision model, the Manufacturer-leader Stackelberg game model, the Retailer-leader Stackelberg game model and the Vertical Nash game model. Through numerical analysis, we compare the results obtained from the above different decision scenarios and give some managerial analysis.

The optimal retail prices of both products are the lowest under centralized decision scenario. This indicates that consumers are better off when no channel member in a dominant position. In Manufacturer-leader Stackelberg game model, the optimal wholesale price is larger than other decision models, while, in Retailer-leader Stackelberg game model, the retail price is larger than other models. If more and more consumers choose exchanging for the remanufactured products, the retailer will bring the price of the new products down. Keeping the value of used products, the consumers will benefit from the lower retail prices and each firm of the whole system benefits from the higher profits. That is because the increased demand resulting from lower wholesale prices more than offsets the loss of revenue per unit due to the lower wholesale prices. In addition, the whole system and his members are better off when the retailer tries his best to reverse the used products, which is helping the environment.

However, several extensions to the analysis in this paper are possible. First, in our study we have assumed the supply chain with information symmetry, then, a plausible research direction is to consider the supply chain with information asymmetry. Second, we just consider the supply chain with one manufacturer and one retailer, however, the supply chain with many manufacturers and many retailers, and the model over multiple periods can also be considered in the future. Finally, this paper consider the case with linear price-sensitive demand functions. Further research can extend the model to include different or more general forms of demand functions.

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