Measure of bullwhip effect considering stochastic disturbance based on price fluctuations in a supply chain with two retailers

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Abstract: This paper establishes a new price-sensitive demand model which considers stochastic disturbance similar to ARMA(1,1) model. We examine the impact of two forecasting methods on the bullwhip effect in a two-stage supply chain with two retailers. It is assumed that two retailers face the same demand model and an order-up-to inventory policy is employed. The lead-time demand is forecasted respectively by the moving average (MA) and exponential smoothing(ES) methods. The effect of various parameters is investigated by numerical simulation and the bullwhip effect under two forecasting methods is compared. The results show that the MA forecasting method is better than the ES method based on our demand process. Besides, conclusions indicate that both the extent of consumers concerning about the historical price volatility and the lead time play significant roles on reducing the bullwhip effect, and stochastic disturbance impacts the bullwhip effect differently based on the lead time. The larger the variance of stochastic disturbance of the retailer which has a longer lead time, the greater the bullwhip effect in the supply chain. The moving average coefficient of stochastic disturbance generally has a little different impact on the bullwhip effect under the different relationship of the lead time between two retailers. Moreover, some proposals are present to help managers to take appropriate measures and select the forecasting method that yields the lowest bullwhip effect.

Key–Words: Supply chain, Bullwhip effect, Demand forecasting, Price fluctuations, Stochastic disturbance

1 Introduction

For many companies, bullwhip effect is a high-risk phenomenon prevalent in their marketing activities, and has serious impacts on operational costs. With its influence on integrating business activities, bullwhip effect has become a focus in supply chain management research, attracting the attention of researchers and practitioners. It refers to the empirical and theoretical observation that as moving up from a downstream member to an upstream member, the demand variability placed by the downstream member to its (immediate) upstream member tends to be amplified in a supply chain.

The first evidence of this phenomenon can be traced back to Forrester (1958, 1961) who discovered its causes and possible remediation in the context of industrial dynamics. After that, many researchers also acknowledged the existence of the bullwhip effect in supply chains, such as Blinder (1982), Blanchard (1983), Burbidge (1984), Caplin (1985), Blinder (1986) and Kahn (1987). Then, a well-known classic beer game which has been used in business schools for decades is experimented by Sterman (1989) to illustrate the bullwhip effect. The bullwhip effect is first named by Lee et al. (1997a, b), whose works is the milestone for the research of bullwhip effect, who identified that demand signal processing, non-zero lead time, supply shortage, order batching, and price fluctuation are the five main causes of the bullwhip effect in supply chains.

Base on Lee et al. (1997a, b), the bullwhip effect may be mitigated by eliminating its main causes. With various causes of the bullwhip effect, the demand process and forecasting methods are considered most frequently because they directly affected the inventory system of supply chains. Different demand processes and different forecasting methods are employed in a lot of papers.

Lee et al. (2000) followed a first-order autoregressive process and a simple order-up-to inventory policy with a minimum mean square error (MMSE) forecasting technique measuring the benefit of information sharing between a retailer and a manufacturer in a two-stage supply chain. Using a first-order autoregressive (AR(1)) demand process similar to Lee et al. (1997a, b), Chen et al. (2000a,b) investigated the impact of the simple moving average(MA) and exponential smoothing(ES) forecasts on the bullwhip effect for a simple, two-stage supply chain with one supplier and one retailer. Likewise, Zhang (2004) also used an AR(1) demand process to investigate the impact of different forecasting methods on the bullwhip effect. Luong (2007) measured the bullwhip effect in a simple supply chain including one retailer and one supplier by performing the AR(1) demand forecasting process on the base stock policy for their inventory under the MMSE forecasting technique. As a sequel of Luong (2007), Luong and Phien(2007) investigated autoregressive models with higher order, first handling AR(2) demand process and considering the general AR(p) demand process with the MMSE method. By quantifying the bullwhip effect, all the above papers using an AR(1) process, investigated the behavior of autoregressive coefficients and order lead-time and showed effects of different forecasting methods on bullwhip effect measures. Moreover, Duc et al. (2010) continued using AR(1) process to examine the effect of a third-party warehouse on the bullwhip effect and inventory cost in a three-stage supply chain with a supplier ,a third-party warehouse and two retailers.

Note that several academics (Lee et al. (1997a,b), Chen et al. (2000 a,b), Xu et al. (2001), Zhang (2004) et al.) assumed a pure autoregressive process and Graves (1999) assumed a pure moving average process. However, the demand model seldom has characteristics of a pure autoregressive process or a pure moving average process. While the demand process usually has characteristics of both moving average and autoregressive process, Pindyck et al. (1998) proposed that a mixed autoregressive-moving average (ARMA) demand process is more suitable for the time series of the market demand than the AR model. Subsequently, the ARMA model is frequently used. Disney et al.(2006) quantified the bullwhip effect for the mixed first-order autoregressive-moving average(ARMA(1,1)) demand pattern under the ES forecasting method in a single supply chain echelon with the base stock policy for replenishment. Duc et al.(2008) investigated the impact of the autoregressive coefficient, the moving average parameter, and the lead time on the bullwhip effect via a ARMA(1,1) model when the demand forecast is performed by the MMSE method. Likewise, by using the dynamic simulation, Feng (2008) evaluated the different effects of three forecasting methods, i.e., MA, ES and MMSE methods, for the simple supply with an ARMA(1,1) model chain as is done in the research of Disney et al. (2006). As an extension of Feng (2008), Ma(2013) made a new supply chain with one supplier and two retailers who both employ the ARMA(1,1) demand process, and analyzed and compared the impact of parameters on the bullwhip effect under various forecasting methods. As a further development of ARMA model, Gilbert(2005) used a new Autoregressive Integrated Moving Average (ARIMA) time-series model to present the causes of the bullwhip effect and managerial insights about reducing the bullwhip effect in a multistage supply chain model. Based on the ARIMA demand pattern, Dhahri (2007) alleviated the bullwhip effect in two respects-namely increase of the stock level and reduction of the service given back to customers.Claudimar(2014) also used ARIMA model for demand forecasting in the food retail.

Performance of a supply chain is affected not only by demand forecasting but also by price fluctuation. The price has an important effect on the demand variability. However, while many researchers explore the demand process, inventory policy and forecasting technique, seldom papers consider price in the demand process. However, in other supply chain research, kinds of price-demand models are used, such as many recent papers: Junhai Ma(2014), Lei Xie(2014), Fang Wu(2014) and Lisha Wang(2014). Refer to these price models and classic cournot model, prices can be added into the demand process for studying the influence on the bullwhip effect. Hamister et al.(2008) improved the first-order autoregressive and considered the effect of current prices on the demand based on the correlation with the previous demand. Rong et al.(2009) put forward that the demand is not only affected by the current price, but also by the previous period of price, for analyzing the impact of price fluctuation and consumer response on the supply chain in the supply disruptions. Furthermore, a nonlinear demand process about retail price was established by Ma et al. (2008) and this paper studied retailer's demand decision and the complex behavior between the retailer and the whole supply chain. Recently, Ma et al.(2012a) established a price-sensitive linear demand model by considering the current price and the one period-ahead price in a two-stage supply chain with one supplier and one retailer, and analyzed the impact of price forecasting behavior by consumers on the BWE and inventory level. In a sequel, Ma et al.(2012b) extended the research to the case in which n period-ahead price is used in the demand process to investigate the impact of price forecast, furthermore, extended their results to multiple retailers and derived the analytical total order quantity.

This paper continues to study the price-sensitive demand process on the impact of bullwhip effect. In the current research, we will quantify the bullwhip effect in a two-stage supply chain with one supplier and two retailers on a price-sensitive demand process. Furthermore, a measure of the bullwhip effect will be developed under the simple ES and MA forecasting technique, based on a replenishment model which is similar to the one used by Chen et al. (2000a,b). Meanwhile the retailers both employ the base stock policy for replenishment. We also analyze the impact of parameters on the BWE under the MA and ES forecasting methods and compare the different effect of two methods on the bullwhip effect.

Our paper differs from the previous research in the following ways. First, we develop a new demand process in which we add the first-order moving average random disturbance into the price-sensitive demand model used in the work of Ma et al. (2012b), in order to evaluate the impact of price fluctuation and the random disturbance parameter on the bullwhip effect. Second, the current research aims at determining an exact measure of the bullwhip effect in a supply chain with one supplier and two retailers, which is closer to the actual situation, while Ma et al.(2012a, b) mainly developed a bullwhip effect measure for a simple two-stage supply chain with one supplier and one retailer only.

The remaining part of this paper is organized as follows. In section 2, we present the stationary property of the new price-sensitive demand process in a new supply chain with one supplier and two retailers which both employ the order-up-to inventory policy. In section 3, we quantify the bullwhip effect under MA and ES forecasting methods and derive the expression of important parameters such as lead-time demand forecast and variance of lead-time demand forecast error. We investigate its behavior and discuss the effects of parameters on the bullwhip effect under different forecasting methods, then, compare the impact of two forecasting methods on the bullwhip effect in section 4. Finally, a short summary is concluded for this paper in section 5. Proofs for some expressions in this paper are summarized in the Appendix.

2 Supply chain model

This research is conducted in two stages with one supplier and two retailers(see Figure.1). In the paper, we assume that our two retailers are in a duopolistic competitive industry. The two retailers face the customer demands which consider price in a ARMA(1,1) process, and place orders to the supplier respectively. The new supply chain model is presented in this section.

2.1 Demand process

The demand forecast is performed through AR(p)(Lee et al., 1997a,b; Chen et al., 2000a,b; Zhang, 2004; Duc, 2010),ARMA(1,1) (Duc, 2008; Feng, 2008; Ma, 2013) and ARIMA(Gilbert, 2005; Dhahri, 2007) in

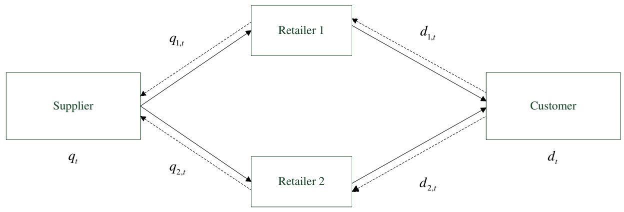

Figure 1: Two-stage supply chain model

most research which dont consider pricing .However, in our paper, demand is contingent on price and price volatility has great impacts on the demand.Only considering current price, we can get price-sensitive demand function model:$d_t = a - bp_t$. In fact, price volatility may cause consumers to purchase in advance. Similar to the customer behavior of MaY.G.et al.(2012),we also emphasize on the impact of current and previous prices of the demand based on the autoregressive-moving average model. We add stochastic disturbance into the demand model.Retailers order and replenish their stock from a supplier on a fixed time interval to supply customer demand. All shortages are backordered.

Consider retailer 1 facing demand of the form

$$d_{1,t} = a - bp_{1,t} + rb(p_{1,t} - \sum_{i=1}^{n} p_{1,t-i}/n) + \varepsilon_{1,t} - \theta_1 \varepsilon_{1,t-1} \quad (1)$$

Here,$d_{1,t}$ represents unit demand in the period t, which is linearly decreasing in price. $p_{1,t}$ is the market price during period t, which is independent and identically distributed random variables.a is the market demand scale and b is a price sensitive coefficient, where $a > 0, b > 0$. r is the extent of customer concern about historical price volatility. The assumption of $0 \le r < 1$ assures that the negative impact of price during period t still dominates on the demand in the period t. When $r = 0$, the market demand is only related with the price in period t, ignoring the impact of the price-prediction behavior of consumers on demand under price fluctuation. $\varepsilon_{1,t}$ is independent and identically distributed from a normal distribution with mean 0 and variance ${\sigma_1}^2$. θ_1 is the first-order moving average coefficient of retailer 1, where $-1 < \theta_1 < 1$. n is the span of previous prices for the demand model.

For the demand process to be stationary, we must have

$$E(d_{1,t}) = E(d_{1,t-1}) = E(d_1), \quad \forall t$$

Hence, a stationary condition can be given as

$$E(d_1) = a - bE(p_{1,t}) + rb(E(p_{1,t}) - \sum_{i=1}^{n} E(p_{1,t-i})/n).$$

In this paper, we assume

$$E(p_{1,t}) = E(p_{1,t-1}) = E(p_1), \forall t$$

and then, we can get

$$E(d_1) = a - bE(p_1) \tag{2}$$

In addition, the variance of demand can be

$$Var(d_{1,t}) = b^2(1-r)^2 Var(p_{1,t}) + \frac{r^2b^2}{n^2} Var(\sum_{i=1}^{n} p_{1,t-i}) + \sigma_1^2 + \theta_1^2\sigma_1^2$$

Because of

$$Var(p_{1,t}) = Var(p_{1,t-1}) = Var(p_1)$$

we have

$$Var(d_{1,t}) = [b^2(1-r)^2 + \frac{b^2r^2}{n}]Var(p_1) + (1+\theta_1^2)\sigma_1^2 \tag{3}$$

and it results in

$$Var(d_{1,t}) = Var(d_{1,t-1}) = Var(d_1) \tag{4}$$

Similarly, retailer 2 has the same demand model with retailer 1

$$d_{2,t} = a - bp_{2,t} + rb(p_{2,t} - \sum_{i=1}^{n} p_{2,t-i}/n) + \varepsilon_{2,t} - \theta_2\varepsilon_{2,t-1} \tag{5}$$

Here, θ_2 has the same property with θ_1 accordingly. n is the same with retailer 1. And, $\varepsilon_{2,t}$ has the same meaning with $\varepsilon_{1,t}$, with mean 0 and variance ${\sigma_2}^2$. So we can have

$$Var(d_{2,t}) = Var(d_{2,t-1}) = Var(d_2), \tag{6}$$

$$E(d_2) = a - bE(p_2), \tag{7}$$

$$Var(d_{2,t}) = [b^2(1-r)^2 + \frac{b^2r^2}{n}]Var(p_2) + (1+\theta_2^2)\sigma_2^2. \tag{8}$$

2.2 Inventory policy

To supply the demand, we adopt the order-up-to inventory policy in the system similar to Lee et al. (1997a). We assume that two retailers both employ the order-up-to inventory policy in which the order-up-to level is determined to achieve a desired service level. Retailer 1 place an ordered quantity $q_{1,t}$ to the supplier at the beginning of period t to be delivered at the beginning of period $t + L_1$, where L_1 is the fixed lead time for the supplier to fulfill an order of retailer 1. The order quantity $q_{1,t}$ can be given as

$$q_{1,t} = y_{1,t} - y_{1,t-1} + d_{1,t-1} \tag{9}$$

where $y_{1,t}$ is the order-up-to inventory position at the beginning of period t of retailer 1 after placing the order in period t. While the base stock policy (Duc, 2008) is employed, the order-up-to level $y_{1,t}$ can be determined by the sum of forecasted lead-time demand and the safety stock as

$$y_{1,t} = \hat{D}_{1,t}^{L_1} + z\hat{\sigma}_{1,t}^{L_1} \tag{10}$$

in which $\hat{D}_{1,t}^{L_1}$ is the forecast for the lead-time demand of retailer 1 which depends on the forecasting method and lead time L_1, $\hat{\sigma}_{1,t}^{L_1}$ is the standard deviation of lead-time demand forecast error, z is the normal z score determined based on a desired service level[1].

Similarly, for retailer 2, the order quantity $q_{2,t}$, which is placed to the supplier in period t to be delivered at the beginning of period $t+L_2$, where L_2 is the fixed lead time of retailer 1, can be given as

$$q_{2,t} = y_{2,t} - y_{2,t-1} + d_{2,t-1} \tag{11}$$

The order-up-to level of retailer 2 at period t is

$$y_{2,t} = \hat{D}_{2,t}^{L_2} + z\hat{\sigma}_{2,t}^{L_2} \tag{12}$$

Eqs.9-10 have the same meaning with Eqs.7-8.

2.3 Forecasting methods

In this paper, we assume that two retailers both use the same forecasting method to forecast the lead-time demand.Commonly, there are three forecasting technique: MA, ES and MMSE, for demand forecast. As mentioned abovethey have been used in most similar

[1]The optimal order-up-to level y_t can be implicitly determined from inventory holding cost and shortage cost for backorders (Heyman and Sobel, 1984; ZhangX., 2004). However, since it is usually not easy to estimate these costs accurately in practice, the approach of using the service levels often employed when the order-up-to level is to be determined.

research. Here, in our paper, according to the demand process, we choose the first two forecasting methods.

In section 3, the bullwhip effect will be measured respectively under the MA and ES forecasting methods. Those two forecasting methods will be introduced in this section firstly.

2.3.1 The MA forecasting method

Using the MA forecasting method, we first have the τ-period-ahead demand forecast given by

$$\hat{d}_{t+\tau} = \hat{d}_t = \frac{1}{k}\sum_{i=1}^{k} d_{t-i}, \tau \geq 1 \tag{13}$$

where k is the span (number of date points) for the MA forecasting method. Then the lead-time demand forecast is given as

$$\hat{D}_t^L = \frac{L}{k}\sum_{i=1}^{k} d_{t-i} \tag{14}$$

2.3.2 The ES forecasting method

The ES forecasting method is an adaptive algorithm in which one-period-ahead forecast is adjusted with a fraction of the forecasting error. The demand forecast with ES can be written as

$$\hat{d}_t = \alpha d_{t-1} + (1-\alpha)\hat{d}_{t-1} \tag{15}$$

where α denotes the fraction used in this process, also called the smoothing factor, and $0 < \alpha < 1$.

3 Bullwhip effect with various forecasting technique

In this section, we derive the measure of the bullwhip effect of a supply chain with one supplier and two retailers under the MA and ES forecasting methods mentioned above respectively.From an early start, the bullwhip effect is a phenomenon in which the variance of demand information is amplified when moving upstream in a supply chain. Thus, it is reasonable to measure the bullwhip effect by the ratio of the variance of order quantities experienced by the supplier to the actual variance of demand quantities. This means has been used in previous research such as those of Chen et al. (2000a, b), Duc (2010) and MaY.G. (2012), and it is adopted in our research as well.

Total demand which two retailers face is

$$d_t = d_{1,t} + d_{2,t} \tag{16}$$

Take the variance of d_t, we have

$$\begin{aligned} Var(d_t) &= Var(d_{1,t} + d_{2,t}) \\ &= Var(d_{1,t}) + Var(d_{2,t}) + 2Cov(d_{1,t}, d_{2,t}) \end{aligned} \tag{17}$$

Because $p_{1,t}$ and $p_{2,t}$ are both independent and identically distributed, we have

$$Cov(d_{1,t}, d_{2,t}) = 0^2 \tag{18}$$

Take Eq.(3), Eq.(8) and Eq.(18) into Eq.(17), the variance of customer demand can be written as

$$\begin{aligned} Var(d_t) &= [b^2(1-r)^2 + \frac{b^2r^2}{n}]Var(p_1) + [b^2(1-r)^2 \\ &+ \frac{b^2r^2}{n}]Var(p_2) + (1+\theta_1^2)\sigma_1^2 + (1+\theta_2^2)\sigma_2^2. \end{aligned} \tag{19}$$

3.1 Bullwhip effect with MA forecasting of lead-time demand

According to Eqs.(9)-(10) and Eq.(14), the order of retailer 1 can be determined as

$$\begin{aligned} q_{1,t} &= y_{1,t} - y_{1,t-1} + d_{1,t-1} \\ &= \frac{L_1}{k}(\sum_{i=1}^{k} d_{1,t-i} - \sum_{i=1}^{k} d_{1,t-1-i}) \\ &+ z(\hat{\sigma}_{1,t}^{L_1} - \hat{\sigma}_{1,t-1}^{L_1}) + d_{1,t-1} \\ &= (1+\frac{L_1}{k})d_{1,t-1} - \frac{L_1}{k}d_{1,t-k-1} \\ &+ z(\hat{\sigma}_{1,t}^{L_1} - \hat{\sigma}_{1,t-1}^{L_1}). \end{aligned} \tag{20}$$

By the definition, the variance of lead-time demand forecast error of retailer 1 at period t, $(\hat{\sigma}_{1,t}^{L_1})^2$ is given as

$$\begin{aligned} (\hat{\sigma}_{1,t}^{L_1})^2 &= Var(D_{1,t}^{L_1} - \hat{D}_{1,t}^{L_1}) \\ &= Var(D_{1,t}^{L_1}) + Var(\hat{D}_{1,t}^{L_1}) \\ &- 2Cov(D_{1,t}^{L_1}, \hat{D}_{1,t}^{L_1}). \end{aligned} \tag{21}$$

According to $Var(p_{1,t}) = Var(p_{1,t-1}) = Var(p_1)$, we can prove that the three terms in the fol-

[2] The demand between the two retailers doesn't have the linear correlation, but it does not mean that there is no contact. While the total market demand is certain under some circumstances, more demand for products of retailer 1 somehow inhibits the demand for retailer 2 (Ma Y.G., 2012).

lowing equation can be shown to reduce to:

$$\begin{aligned}
&Var(D^{L_1}_{1,t}) = L_1 Var(d_1) - 2\theta_1(L_1-1)\sigma_1^2 \\
&+ 2Var(p_1)[-\frac{L_1(L_1-1)}{2}\cdot\frac{rb^2}{n}(1-r) \\
&+ \frac{r^2b^2}{n^2}\sum_{i=1}^{L_1-1}(L_1-i)(n-i)], \\
&Var(\hat{D}^{L_1}_{1,t}) = \frac{L_1^2}{k}Var(d_1) - 2\theta_1(k-1)(\frac{L_1}{k})^2\sigma_1^2 \\
&+ 2(\frac{L_1}{k})^2 Var(p_1)[-\frac{k(k-1)}{2}\cdot\frac{rb^2}{n}(1-r) \\
&+ \frac{r^2b^2}{n^2}\sum_{i=1}^{k-1}(k-i)(n-i)], \\
&Cov(D^{L_1}_{1,t},\hat{D}^{L_1}_{1,t}) = -\frac{\theta_1 L_1}{k}\sigma_1^2 + [2(nrbL_1)^2 \\
&\cdot(4n-k-L_1) - 4n^3rb^2L_1^2]Var(p_1).
\end{aligned} \tag{22}$$

Proof: See the Appendix.

These three terms don't depend on t. Consequently, the variance of lead-time demand forecasting error of retailer 1 remains constant over time as well and has no influence on the bullwhip effect. So, the order quantity of retailer 1 is easily obtained as

$$q_{1,t} = (1+\frac{L_1}{k})d_{1,t-1} - \frac{L_1}{k}d_{1,t-k-1}. \tag{23}$$

Similarly, the retailer 2 has the same span k with the retailer 1, and also has $\hat{\sigma}^{L_2}_{2,t} = \hat{\sigma}^{L_2}_{2,t-1}$, so the order quantity placed by the retailer 2 is

$$q_{2,t} = (1+\frac{L_2}{k})d_{2,t-1} - \frac{L_2}{k}d_{2,t-k-1}. \tag{24}$$

Then, total order quantity of retailers in period t under the MA forecasting method is

$$\begin{aligned}
q_t &= q_{1,t} + q_{1,t} \\
&= (1+\frac{L_1}{k})d_{1,t-1} - \frac{L_1}{k}d_{1,t-k-1} \\
&+ (1+\frac{L_2}{k})d_{2,t-1} - \frac{L_2}{k}d_{2,t-k-1}.
\end{aligned} \tag{25}$$

Proposition 1 *The variance of the total order quantity at period t under the MA forecasting method can be given as*

$$\begin{aligned}
&Var(q_t)_{n\geq k} \\
&= \{[(1+\frac{L_1}{k})^2 + (\frac{L_1}{k})^2](1-r)^2 \\
&+[1-2\frac{L_1}{k} - 2(\frac{L_1}{k})^2]\frac{r^2}{n} + (2L_1 + 2\frac{{L_1}^2}{k})\frac{r^2}{n^2} \\
&+ 2\frac{L_1}{k}(1+\frac{L_1}{k})\frac{r}{n}\}b^2Var(p_1) \\
&+ \{[(1+\frac{L_2}{k})^2 + (\frac{L_2}{k})^2](1-r)^2 \\
&+ [1-2\frac{L_2}{k} - 2(\frac{L_2}{k})^2]\frac{r^2}{n} \\
&+ (2L_2 + 2\frac{{L_2}^2}{k})\frac{r^2}{n^2} \\
&+ 2\frac{L_2}{k}(1+\frac{L_2}{k})\frac{r}{n}\}b^2Var(p_2) \\
&+ [(1+\frac{L_1}{k})^2 + (\frac{L_1}{k})^2](1+\theta_1^2)\sigma_1^2 \\
&+ [(1+\frac{L_2}{k})^2 + (\frac{L_2}{k})^2](1+\theta_2^2)\sigma_2^2
\end{aligned} \tag{26}$$

with $n \geq k$, and

$$\begin{aligned}
&Var(q_t)_{n<k} \\
&= \{[(1+\frac{L_1}{k})^2 + (\frac{L_1}{k})^2][(1-r)^2 + \frac{r^2}{n}] \\
&+ 2\frac{L_1}{k}(1+\frac{L_1}{k})\frac{r}{n}(1-r)\}b^2Var(p_1) \\
&+ \{[(1+\frac{L_2}{k})^2 + (\frac{L_2}{k})^2][(1-r)^2 + \frac{r^2}{n}] \\
&+ 2\frac{L_2}{k}(1+\frac{L_2}{k})\frac{r}{n}(1-r)\}b^2Var(p_2) \\
&+ [(1+\frac{L_1}{k})^2 + (\frac{L_1}{k})^2](1+\theta_1^2)\sigma_1^2 \\
&+ [(1+\frac{L_2}{k})^2 + (\frac{L_2}{k})^2](1+\theta_2^2)\sigma_2^2
\end{aligned} \tag{27}$$

with $n < k$.

Proof: See the Appendix.

For simplicity, Eq.26 can be written as

$$Var(q_t)_{n\geq k} = b^2A_1Var(p_1) + b^2A_2Var(p_2) + A_3 \tag{28}$$

where b^2A_1 is the coefficient of $Var(p_1)$, b^2A_2 is the coefficient of $Var(p_2)$, A_3 is the constant term in the Eq.26.

Eq.27 can be written as

$$Var(q_t)_{n<k} = b^2B_1Var(p_1) + b^2B_2Var(p_2) + B_3 \tag{29}$$

where b^2B_1 is the coefficient of $Var(p_1)$, b^2B_2 is the coefficient of $Var(p_2)$, B_3 is the constant term in the Eq.27.

Then, from Eq.19 and Eqs.28-29, the bullwhip effect measure, denoted with BWE_{MA} in this case, can be derived as

$$BWE_{MA} = \frac{Var(q_t)}{Var(d_t)} = \begin{cases} \frac{b^2Var(p_1)(A_1+A_2\gamma^2)+A_3}{b^2Var(p_1)[(1-r)^2+\frac{r^2}{n}](1+\gamma^2)+(1+\theta_1^2)\sigma_1^2+(1+\theta_2^2)\sigma_2^2}, n \geq k \\ \frac{b^2Var(p_1)(B_1+B_2\gamma^2)+B_3}{b^2Var(p_1)[(1-r)^2+\frac{r^2}{n}](1+\gamma^2)+(1+\theta_1^2)\sigma_1^2+(1+\theta_2^2)\sigma_2^2}, n < k \end{cases} \quad (30)$$

where $\gamma=\sqrt{\frac{Var(p_2)}{Var(p_1)}}$ and means the consistency of price volatility between the two retailers.

3.2 Bullwhip effect with ES forecasting of lead-time demand

In this section, we use the exponential smoothing forecasting technique to perform demand forecast. According to Zhang (2004), we know the forecasting demand of retailer 1 at period t is

$$\hat{d}_{1,t} = \sum_{i=0}^{\infty} \alpha_1(1-\alpha_1)^i d_{1,t-i-1} \quad (31)$$

where α_1 is the smoothing exponent of retailer 1. Therefore, the $\hat{d}_{1,t}$ can be interpreted as the weighted average of all past demand of retailer 1 with exponentially declining weights. Then, the τ-period-ahead forecasting demand for retailer 1 with ES method simply extends the one-period-ahead forecast similar to the MA case where

$$\hat{d}_{1,t+\tau} = \hat{d}_{1,t}.\tau \geq 1 \quad (32)$$

Because of Eq.32, the forecast for the lead-time demand of retailer 1 can be expressed as

$$\hat{D}_{1,t}^{L_1} = L_1\hat{d}_{1,t}. \quad (33)$$

Using the Eq.15 and Eq.33, we also have

$$\hat{D}_{1,t}^{L_1} - \hat{D}_{1,t-1}^{L_1} = \alpha_1 L_1(d_{1.t-1} - \hat{d}_{1,t-1}). \quad (34)$$

The variance of lead-time demand forecasting error $(\hat{\sigma}_{1,t}^{L_1})^2$ under ES forecast method again is a constant term over time. Its derivation parallels the MA case where

$$\begin{aligned}(\hat{\sigma}_{1,t}^{L_1})^2 &= Var(D_{1,t}^{L_1} - \hat{D}_{1,t}^{L_1}) \\ &= Var(D_{1,t}^{L_1}) + Var(\hat{D}_{1,t}^{L_1}) - 2Cov(D_{1,t}^{L_1}, \hat{D}_{1,t}^{L_1}).\end{aligned}$$

The first term $Var(D_{1,t}^{L_1})$ is identical to the case of MA forecast. The second term in the variance formula can be obtained from the following expression:

$$\begin{aligned}Var(\hat{D}_{1,t}^{L_1}) =& \frac{\alpha_1 L_1^2}{(2-\alpha_1)}Var(d_1) + 2L_1^2 Var(p_1) \\ & \{\frac{r^2b^2}{n^2[1-(1-\alpha_1)^2]}[\alpha_1(1-\alpha_1)(n-1) \\ & - (1-\alpha_1)^2 + (1-\alpha_1)^{n+1}] \\ & - \frac{rb^2(1-r)n\alpha_1(1-\alpha_1)}{n^2[1-(1-\alpha_1)^2]}\}.\end{aligned} \quad (35)$$

Proof: See the Appendix.

The last covariance term between lead-time demand and forecasting lead-time demand can be derived similarly. So, these three terms all dont depend on t.

Then, according to Eqs.9-10 and Eq.34, the order quantity of retailer 1 at period t is

$$q_{1,t} = \alpha_1 L_1(d_{1.t-1} - \hat{d}_{1,t-1}) + d_{1.t-1}. \quad (36)$$

Likewise, the order quantity placed by retailer 2 at the beginning of period t can be identified respectively, as

$$q_{2,t} = \alpha_2 L_2(d_{2.t-1} - \hat{d}_{2,t-1}) + d_{2.t-1} \quad (37)$$

where α_2 is the smoothing exponent of retailer 2, and $\hat{d}_{2,t-1}$ is the forecast of the demand at period t for retailer 2.

From the above, total order quantity received by the supplier at the beginning of period t under ES forecasting method is

$$\begin{aligned}q_t &= q_{1,t} + q_{2,t} \\ &= \alpha_1 L_1(d_{1.t-1} - \hat{d}_{1,t-1}) + d_{1.t-1} \\ &+ \alpha_2 L_2(d_{2.t-1} - \hat{d}_{2,t-1}) + d_{2.t-1}.\end{aligned} \quad (38)$$

Proposition 2 *The variance of the total order quantity at period t under the ES forecasting method can be given as*

$$\begin{aligned}Var(q_t) \quad &= [(1+\alpha_1 L_1)^2 + \tfrac{\alpha_1^3 L_1^2}{2-\alpha_1}](1+\theta_1^2)\sigma_1^2 \\ &+[(1+\alpha_2 L_2)^2 + \tfrac{\alpha_2^3 L_2^2}{2-\alpha_2}](1+\theta_2^2)\sigma_2^2 \\ &+\{[(1+\alpha_1 L_1)^2 + \tfrac{\alpha_1^3 L_1^2}{2-\alpha_1}][(1-r)^2 + \tfrac{r^2}{n}]\end{aligned}$$

$$
\begin{aligned}
&+\frac{(2r\alpha_1L_1)^2(1-\alpha_1)}{n(2-\alpha_1)}+\frac{2r^2\alpha_1L_1^2(1-\alpha_1)^{n+1}}{n^2(2-\alpha_1)}\\
&-\frac{2r\alpha_1L_1^2(1-\alpha_1)(r+n\alpha_1)}{n^2(2-\alpha_1)}\\
&-2(1+\alpha_1L_1)\alpha_1L_1\\
&[\frac{2r^2-r}{n}-\frac{r^2[1-(1-\alpha_1)^n]}{n^2\alpha_1}]\}b^2Var(p_1)\\
&+\{[(1+\alpha_2L_2)^2+\frac{\alpha_2^3L_2^2}{2-\alpha_2}][(1-r)^2+\frac{r^2}{n}]\\
&+\frac{(2r\alpha_2L_2)^2(1-\alpha_2)}{n(2-\alpha_2)}+\frac{2r^2\alpha_2L_2^2(1-\alpha_2)^{n+1}}{n^2(2-\alpha_2)}\\
&-\frac{2r\alpha_2L_2^2(1-\alpha_2)(r+n\alpha_2)}{n^2(2-\alpha_2)}\\
&-2(1+\alpha_2L_2)\alpha_2L_2\\
&[\frac{2r^2-r}{n}-\frac{r^2[1-(1-\alpha_2)^n]}{n^2\alpha_2}]\}b^2Var(p_2).
\end{aligned}
\tag{39}
$$

Proof: See the Appendix.

Because the variance of total order quantity is too complex, we set

$$
\begin{aligned}
Var(q_t) &= b^2C_1Var(p_1)+b^2C_2Var(p_2)\\
&\quad +C_{31}(1+\theta_1^2)\sigma_1^2+C_{32}(1+\theta_2^2)\sigma_2^2
\end{aligned}
\tag{40}
$$

where $C_{31}(1+\theta_1^2)\sigma_1^2+C_{32}(1+\theta_2^2)\sigma_2^2$ is the constant term, b^2C_1 is the coefficient of $Var(p_1)$, b^2C_2 is the coefficient of $Var(p_2)$ in Eq.39.

Then, the bullwhip effect measure, denoted with BWE_{ES} in this case, can be derived from (19) and (39) as

$$
BWE_{ES}=\frac{b^2Var(p_1)(C_1+C_2\gamma^2)+C_{31}(1+\theta_1^2)\sigma_1^2+C_{32}(1+\theta_2^2)\sigma_2^2}{b^2Var(p_1)[(1-r)^2+\frac{r^2}{n}](1+\gamma^2)+(1+v\theta_1^2)\sigma_1^2+(1+\theta_2^2)\sigma_2^2}
\tag{41}
$$

similarly, where $\gamma=\sqrt{\frac{Var(p_2)}{Var(p_1)}}$

4 Behavior and comparison of bullwhip effect for MA and ES forecasting methods

In the last section, we have the exact measure of the bullwhip effects for different forecasting technique. We will explore and illustrate the impact of various parameters on the bullwhip effect, such as price-sensitive coefficient, lead time, the span of MA method and so on, by using the numerical experiments in this section.Then, according to the analysis results, some economic and managerial proposal can be achieved for the members of supply chain on reducing the bullwhip effect.

4.1 Behavior of the bullwhip effect under the MA forecasting technique

As shown in Eq.30, the expression of the bullwhip effect under the MA forecasting technique has two cases where they are $n \geq k$ and $n < k$.

Case 1:$n \geq k$

First, we would like to consider the case 1: the bullwhip effect in the condition of $n \geq k$. Figs. 2-12 simulate the bullwhip effect in case 1 and illustrate the impact of parameters on the BWE. Other than the most previous research, we mainly explore how the extent of customer concerning about historical price volatility effects the BWE under the MA forecasting method.

Fig. 2 depicts the impact of r on the bullwhip effect, for the case which $n = 5, k = 2, b = 2.5, L_2 = 4, \theta_1 = \theta_2 = 0.3$,$\sigma_1 = \sigma_2 = 1, Var_1 = 4, \gamma = 1$ for simplification. This figure shows that with the increase of r from 0 to 1, the BWE_MA first increases slowly, and then decreases rapidly after it reaches the maximum value at a specific r value. We also observe that, in this situation, by shifting the lead-time of retailer 1, the bullwhip effect becomes to be greater with the increase of L_1. When the r tends to 1, the bullwhip effect is smaller than that which r tends to 0, while the bullwhip effects are both greater than 2. These results can be presented clearly in the Table 1. So, if r is smaller than a certain value, we can lessen r to decrease the bullwhip effect. In a similar way, when r is larger than the certain value, the larger the better. From Table 1, it can be seen that as the value of L_1 increases, the r_{max} is fixed in spite of the bullwhip effect increasing for any value of $r > 0$. In a word, if customers pay attention to the fluctuations of historical price enough, the bullwhip effect will be smaller.

Figs.3-4 show the impact of b on the bullwhip effect under the different values of r which we vary the price-sensitive coefficient b from 0 to 10 and we set $n = 5, k = 2, L_2 = 4, \theta_1 = \theta_2 = 0.3, \sigma_1 = \sigma_2 = 1, Var_1 = 4, \gamma = 1$. Fig.3 shows that when $r = 0.3$, the bullwhip effect increases a little slowly. And then, when b increases to a small certain value, the bullwhip effect tends to be stable in spite of the continue increase of b.However, Fig.4 shows the complete contrary condition. When $r = 0.8$, the bullwhip effect decreases slowly, and the same, the bullwhip effect tends to be stable with the continuous increase of b, after b increases to a small certain value. In addition, from the figs.3-4, we can observe the certain value of b in the case of $r = 0.3$ is smaller than that in the case of $r = 0.8$. As seen, in the market, when we face that customers pay close attention to the historical price volatility of product, that is r is bigger than a certain

Table 1: Partial values of the bullwhip effect measure for different r under the MA

r	L_1					
	2	3	4	5	6	7
0	9.000	10.750	13.000	15.750	19.000	22.750
0.05	9.078	10.845	13.117	15.893	19.175	22.961
0.1	9.157	10.941	13.236	16.040	19.353	23.177
0.15	9.237	11.039	13.355	16.186	19.533	23.394
0.2	9.315	11.134	13.472	16.330	19.708	23.605
0.25	9.388	11.223	13.582	16.465	19.873	23.805
0.3	9.453	11.302	13.679	16.584	20.018	23.980
0.35	9.502	11.362	13.753	16.676	20.130	24.116
0.4	9.529	11.395	13.793	16.725	20.190	24.188
0.45	9.522	11.386	13.782	16.712	20.173	24.168
0.5	9.466	11.318	13.699	16.609	20.048	24.016
0.55	9.344	11.170	13.517	16.385	19.775	23.686
0.6	9.139	10.920	13.209	16.007	19.314	23.129
0.65	8.834	10.548	12.751	15.444	18.627	22.299
0.7	8.420	10.043	12.130	14.680	17.694	21.172
0.75	7.902	9.412	11.353	13.726	16.530	19.765
0.8	7.306	8.686	10.459	12.627	15.189	18.145
0.85	6.675	7.917	9.513	11.464	13.769	16.430
0.9	6.061	7.168	8.592	10.332	12.388	14.760
0.95	5.512	6.499	7.768	9.319	11.152	13.267
r_{max}	0.42	0.42	0.42	0.42	0.42	0.42

value, we could have a larger value of b for the smaller bullwhip effect. On the contrary, if the production is insensitive of the customer concern about the historical price volatility, we can decrease the bullwhip effect by decreasing the value of b.

Figs.5-8 mainly show the effect of the first-order moving average coefficient of retailer 2 θ_2 on the bullwhip effect. From Eq.30, we can know that the bullwhip effect is a function of ${\theta_2}^2$ and the range of θ_2 is from -1 to 1. So the bullwhip effect is a symmetric function of θ_2. It can be seen that the bullwhip effect decreases very slowly to the minimum and becomes to increase slowly with the increase of θ_2 when $L_1 < L_2$ in the Fig.5. By contrast, from Fig.6, the bullwhip effect increases very slowly when $-1 < \theta_2 < 0$ and then the bullwhip effect decreases very slowly when it reaches the maximum at $\theta_2 = 0$ under the condition of $L_1 > L_2$. We can know that from Figs. 5-6, the range of growth of the bullwhip effect become larger when shifting the gap of the lead time between retailer 1 and retailer 2. From our research, we also discover that these results are proper for any value of $0 < r < 1$ (In order to make the article concise, this effect is not shown in the paper). In a word, the square of the moving average coefficient of the retailer with a longer lead time is larger, the bullwhip effect is greater.

However, there are some differences in the case of $L_1 = L_2$ for θ_2. Fig.7 shows a surface of the BWE corresponding to the different values of r and θ_2 when we set $L_1 = L_2 = 4, b = 2.5, n = 5, k = 2, \theta_1 = 0.3, \sigma_1 = \sigma_2 = 1, Var_1 = 4, \gamma = 1$. The

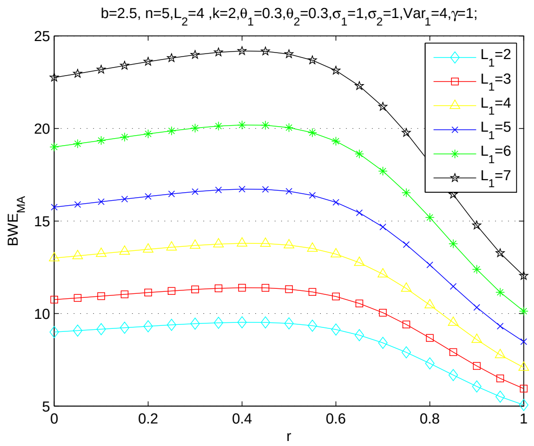

Figure 2: Effect of r on the BWE under the MA method

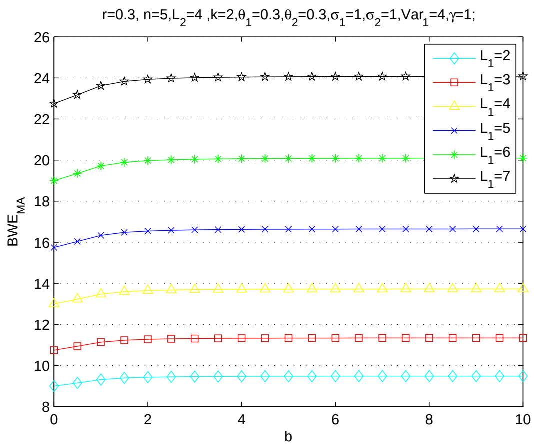

Figure 3: Effect of b on the BWE under the MA method in the case of $r = 0.3$

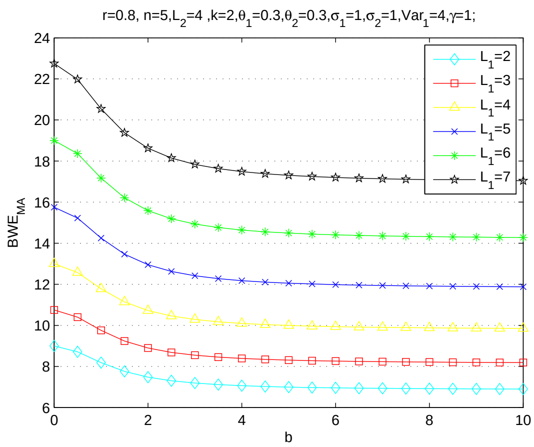

Figure 4: Effect of b on the BWE under the MA method in the case of $r = 0.8$

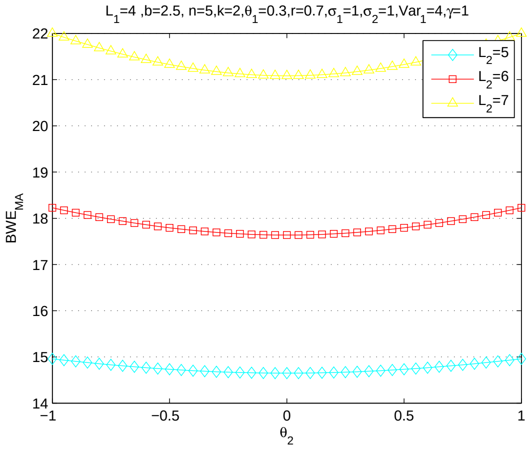

Figure 5: Effect of θ_2 on the bullwhip effect under the MA in the case of $L_1 < L_2$

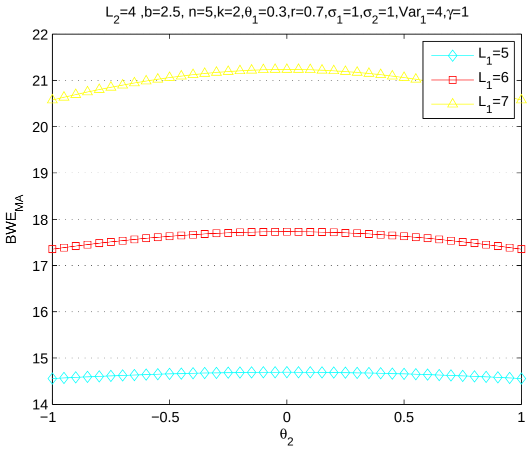

Figure 6: Effect of θ_2 on the bullwhip effect under the MA in the case of $L_1 > L_2$

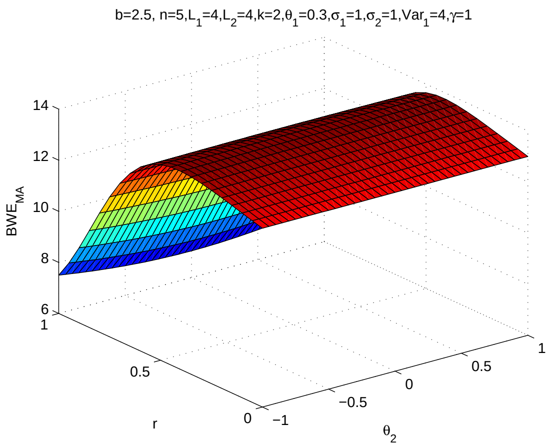

Figure 7: BWE corresponding to r and θ_2 values under the MA in the case of $L_1 = L_2$

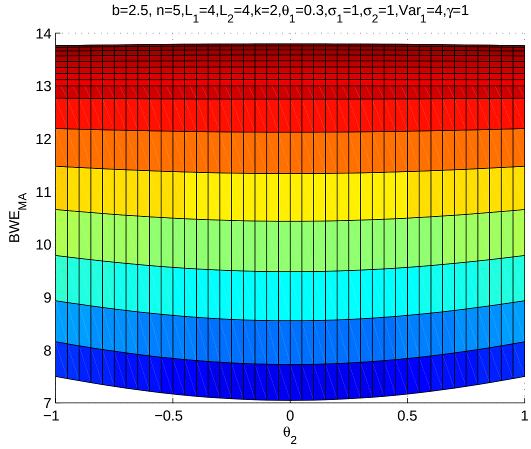

Figure 8: Effect of θ_2 on the bullwhip effect in the case of $L_1 = L_2$(The projection of Fig.7)

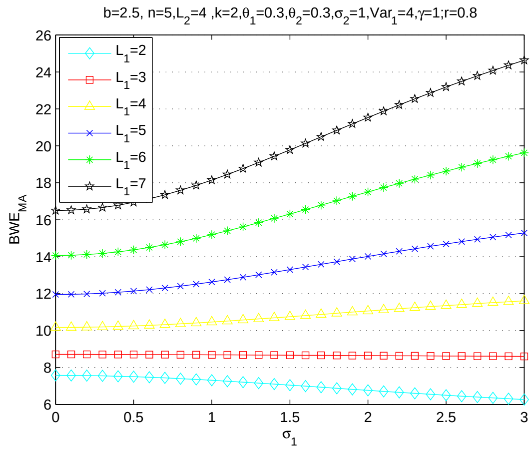

Figure 9: Effect of σ_1 on the bullwhip effect under different L_1

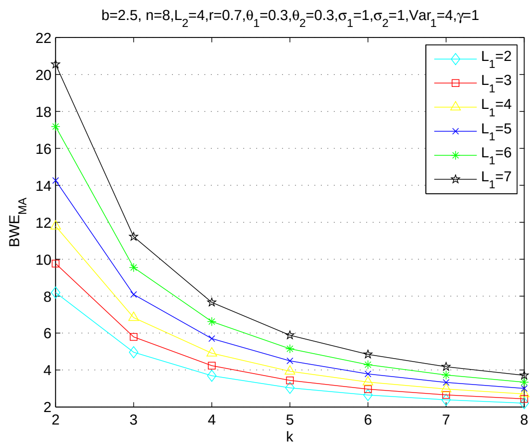

Figure 10: Effect of k on bullwhip effect for different L_1 under the MA method

curve in Fig.8 is the X-Z projection for the surface in Fig.7. From Figs.7-8, first, we can observe that the variation tendency between the bullwhip effect and θ_2 is changeful with the increase of r. The Fig.8 clearly investigates how parameter θ_2 affects the bullwhip effect for different r when we set $L_1 = L_2$. It can be seen that when r is very small, the bullwhip effect increases a little slowly firstly, then decreases slowly as well until r increases to a certain value. Then, the trend is in the opposite direction. With the continue increase of r, the bullwhip effect firstly decreases a little slowly, and then increases slowly, and moreover, the range of change become to be bigger and bigger. In addition, the bullwhip effect reaches the minimum or the maximum when $\theta_2 = 0$.

From Figs.5-8 and the above results, we can conclude that the change of θ_2 has a little impact on the bullwhip effect, and the greater relatively the square of moving average parameter of the retailer which has a shorter lead time is, the smaller the bullwhip effect is. So the impact of θ on the bullwhip effect is affected by other parameters. Only when the customers concern about the historical price volatility enough, a member of the supply chain can change θ_2 to reduce the bullwhip effect.

σ_1 is the standard deviation of stochastic disturbance of retailer 1. From Eq.30 and Eq.41, we discover that the structure of the function of σ_1 on the bullwhip effect is the same under the different forecasting methods. Thus, σ_1 affects the bullwhip effect under different methods similarly. So we analyze the influence of σ_1 only in the case of $n > k$ under the MA method.

Fig.9 shows that how σ_1 affects the bullwhip effect for different L_1 by varying σ_1 from 0 to 3 and fixing other parameters. We can observe that, as σ_1 increases, the bullwhip effect increases when $L_1 > L_2$, and the bullwhip effect decreases as σ_1 increases when $L_1 < L_2$. This phenomenon implies that, the larger the variance of stochastic disturbance of the retailer which has a longer lead time, the greater the bullwhip effect in the supply chain.And we also know that when $L_1 = L_2$, the increase of σ_1 also can lead to the increase of the bullwhip effect.

Fig.10 depicts the relationship between the span for the MA forecasting method k and the bullwhip effect for different L_1. From Fig.10, we can observe that k is an important factor to the bullwhip effect. The bullwhip effect is a decreasing function of k and we can weaken the bullwhip effect by increasing k. But k is not the bigger the better, since that when k reaches a certain value, the bullwhip effect almost will not decrease with the increase of k again. Moreover, the great k may increase costs associated with data collection.

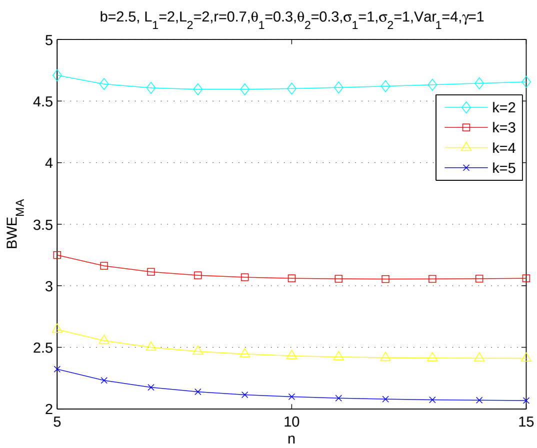

Figure 11: Effect of n on bullwhip effect for different k under the MA

We shift the span of previous prices to examine how the parameter n affects the bullwhip effect for different k in Fig.11. We set $n \geq 5$ in order to assure the model is correct. Fig.11 implies that the great increase of n results in a little change of the bullwhip effect for a given value of k. So, in real life, the member of a supply chain should consider many aspects and select an appropriate value rather than a big value for n.

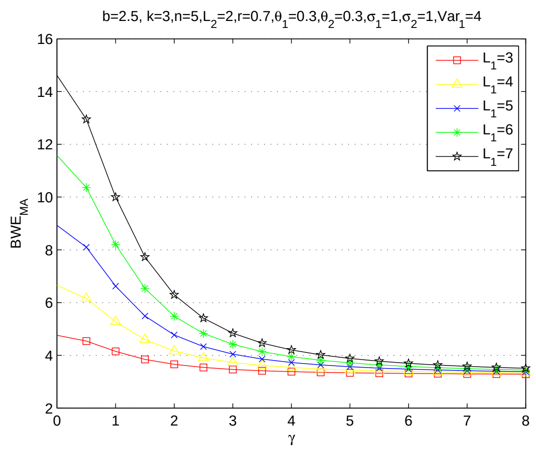

Figure 12: Effect of γ on bullwhip effect for different $L_1 > L_2$ under the MA

Figs.12-13 show that γ affects the bullwhip effect differently as the relationship of lead-time between retailer 1 and retailer 2 is different. Fig.12 depicts the bullwhip effect is a decreasing function of γ when we vary L_1 from 3 to 7 guaranteeing $L_1 > L_2 = 2$. Fig.13 depicts the bullwhip effect is an increasing function of γ when we set $L_1 = 2$ guaranteeing $L_1 < L_2$. From the definition, γ represents the consistency of price volatility between two retailers, and the

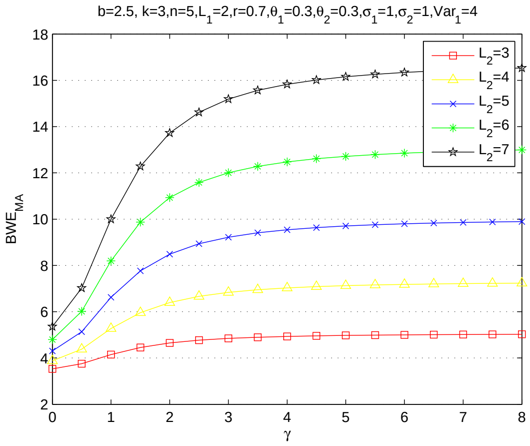

Figure 13: Effect of γ on bullwhip effect in the case of $L_1 > L_2$ under the MA

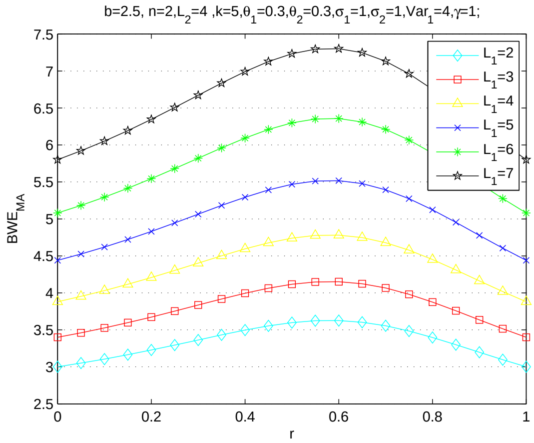

Figure 14: Effect of r on the BWE under the MA in the case of $n < k$

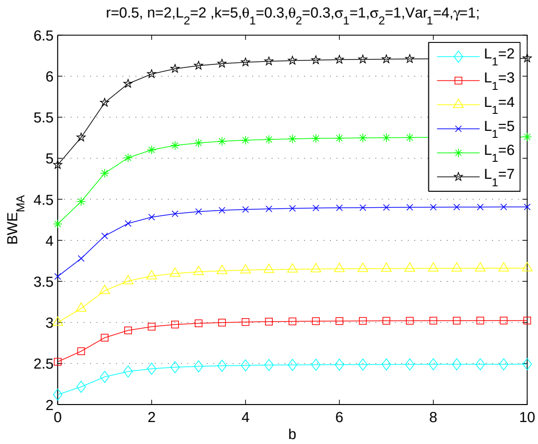

Figure 15: Effect of b on the BWE under the MA in the case of $n < k$

greater γ means the larger $Var(p_2)$ at a given value of $Var(p_1)$. From these two figures, it is clear that the bullwhip effect measure increases as price volatility of the retailer with a longer lead time increases, and the bullwhip effect measure decreases as price volatility of the retailer with a shorter lead time increases.

Case 1:$n < k$

According to the above analysis of parameters, we have known some properties about the bullwhip effect. In the following, we discuss the change of the bullwhip effect in the case of $n < k$ in Figs.14-16 briefly. Similar with the case $n \geq k$, we can observe that the greater r is the greater the bullwhip effect is when r is less than a certain number, and the greater r is the less the bullwhip effect is when r is greater than that certain number. It reaches the maximum value where r is in the vicinity of 0.6 in Fig.14. For fixed values of L_1, the bullwhip effect reaches the minimum value when both $r = 0$ and $r = 1$. Compared to Fig.2, we can discover that the bullwhip effect when $n = 2, k = 5$ is a lot less than that when $n = 5, k = 2$ at a given L_1 value for any value of r. This phenomenon indicates that k plays a more important role than n at reducing the bullwhip effect. Moreover, the shift of lead-time also can lead to a big decline of the bullwhip effect.

According to the case $n \geq k$, we have already grasped the main property of b on the bullwhip effect and based on our research, we discover that the impact of b in the case of $n < k$ is similar to the case 1. So we select r value arbitrarily and make the relationship between price sensitive coefficient and the bullwhip effect. Fig.15 shows the impact of the price sensitive coefficient on the bullwhip effect when we shift L_1 and fix $r = 0.5, n = 2, k = 5, L_2 = 2, \theta_1 = \theta_2 = 0.3, \sigma_1 = \sigma_2 = 1, Var_1 = 4, \gamma = 1$. It can be seen that, when the extent of concerning about historical prices for customers is neutral, the bullwhip effect increases slowly and then keeps stable as b increases. In this case, it implies that when the price-sensitive coefficient for product becomes higher and higher, demand fluctuation will be more serious leading to a larger bullwhip effect.

In our model, r is a key factor on affecting the bullwhip effect. Fig.16 mainly investigates the impact of n and k on the bullwhip effect based on the change of r. This figure depicts that the increase of n can lead to the decrease of the bullwhip effect with data ages $k = 5$ and 6,respectively. When r is near one, accurately speaking, from approximately 0.85, the impact of n on the bullwhip effect is hardly existing any more. From the figure, we also clearly observe that the greater decrease of the bullwhip effect results from the increase of k.

Fig.17 depicts the impact of γ on the bullwhip ef-

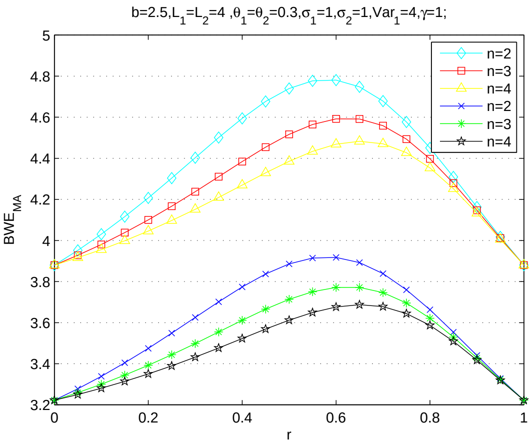

Figure 16: Effect of n and k on the bullwhip effect under the MA in the case of $n < k$

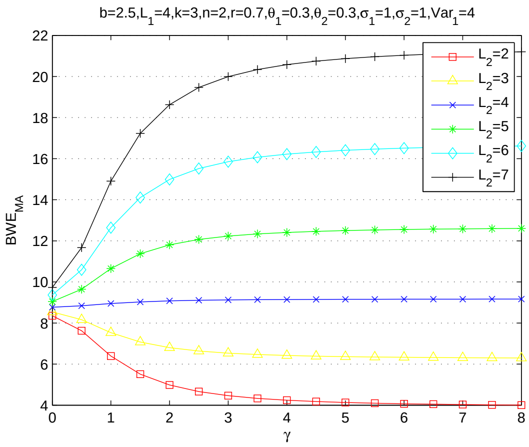

Figure 17: Effect of γ on the BWE under the MA in the case of $n < k$

fect similar to Figs.12-13. From the figure, we can conclude that the bullwhip effect measure decreases as price volatility $Var(p)$ of the retailer with a shorter lead time increases and the greater the price variance $Var(p)$ of the retailer who has a longer lead time the greater the bullwhip effect is.

From the two cases, in general, we can summarize that the influence of the consistency γ on the bullwhip effect has something important with the relationship of lead time between two retailers regardless of the values of n and k under the MA forecasting method.

4.2 Behavior of the bullwhip effect under the ES forecasting technique

In this section, we will discuss the impact of parameters on the bullwhip effect under the using of the ES forecasting method for lead-time demand forecast. Figs.18-24 simulate the expression of the bullwhip effect and illustrate the parameters impact on the bullwhip effect under the ES forecasting technique. The smoothing parameter α is a significant factor for the ES forecasting. In the following, we will add the analysis of α.

Fig.18 depicts the impact of r on the bullwhip effect when we shift the lead time L_1 and fix all other parameters values $\alpha_1 = \alpha_2 = 0.5, b = 2.5, n = 2, L_2 = 4, \theta_1 = \theta_2 = 0.3, \sigma_1 = \sigma_2 = 1, Var_1 = 4, \gamma = 1$. It is shown that the trend is consistent for different L_1 similar with the case 1 under the MA forecasting and the bullwhip effect increases slowly, and then decreases as r increases. Specially, the bullwhip effect attains the maximum when r is all about 0.5 for different L_1. And we can observe that the lead time has serious influence on the bullwhip effect. The bullwhip effect becomes to be great with the increase of L_1.

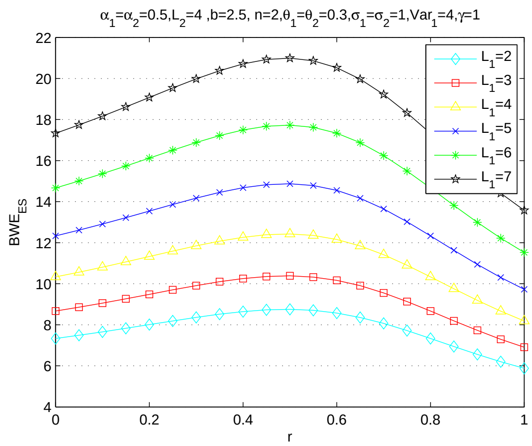

Figure 18: Effect of r on the bullwhip effect for different L_1 under the ES

Figs.19-20 indicates the relationship between the bullwhip effect and with the increase of r on the condition of $\alpha_1 = \alpha_2 = 0.5, n = 2, L_1 = L_2 = 4, \theta_1 = \theta_2 = 0.3, \sigma_1 = \sigma_2 = 1, Var_1 = 4, \gamma = 1$. Fig.19 shows the surface of the bullwhip effect corresponding to b and r values. Fig.20 is the X-Z projection of the surface in order to see the relationship between the bullwhip effect and the price-sensitive coefficient clearly. By contrast, firstly, with the increase of b, the bullwhip effect increases slowly in the beginning , and then is insensitive to changes after b is larger than about 4 as reflected by the flatness of the curve when r is less than a certain value from Figs.19-20. And for this stage of r, we can observe from the surface that the initial magnitude of the increase of the bullwhip effect first gradually becomes larger and then becomes less until no change. When r is greater than that certain value, the bullwhip effect decreases slowly at first, then is almost unchanged as b increases.

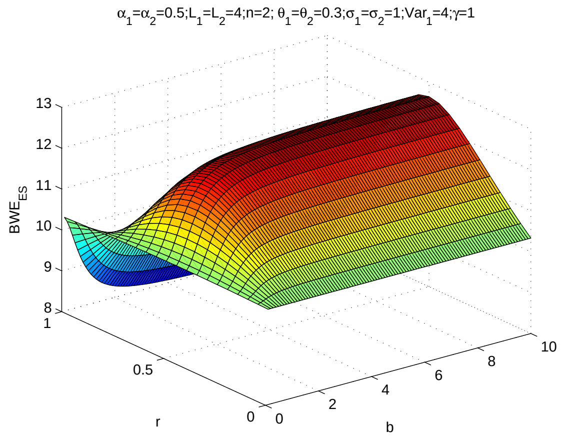

Figure 19: The bullwhip effect corresponding to r and b values under the ES

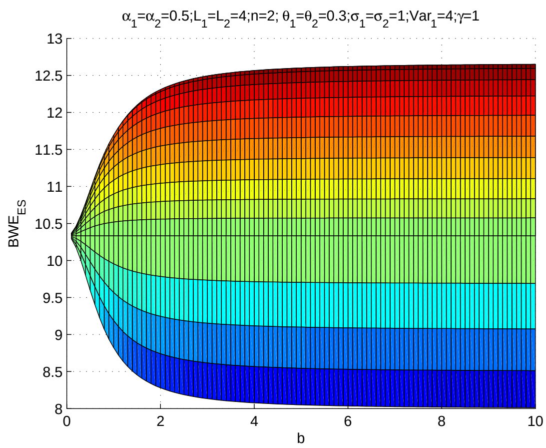

Figure 20: Effect of b on the bullwhip effect for different r(The projection of Fig.19)

We can observe that the bullwhip effect increases significantly with increase of the smoothing exponent α_1 and it decreases with the increase of n in Fig.21. On one hand, the bullwhip effect converges to the same value for fixed α_1 and different n as r approaches one. On the other hand, only when consumers pay more attention to the impact of historical price volatility on the demand in period t, the increase of n, that is more terms of historical price, reduces the bullwhip effect more effectively.

θ_2 is the first-order moving average coefficient of retailer 2 of the random error for the demand model. From Eq.41, we can know that the bullwhip effect is a function of θ_2^2 and the range of θ_2 is from -1 to 1. Thus, it is the same with cases under MA forecasting and the bullwhip effect is also a symmetric function of θ_2. It can be seen that the bullwhip

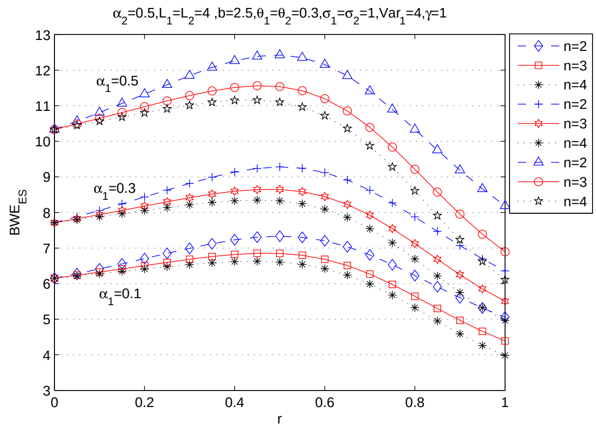

Figure 21: Effect of α_1 and n on the bullwhip effect under the ES

effect decreases very slowly to the minimum and becomes to increase slowly with the increase of θ_2 when $L_1 < L_2$ in the Fig.22. From Fig.23, the bullwhip effect increases very slowly when $-1 < \theta_2 < 0$ and then decreases very slowly when $0 < \theta_2 < 1$ under the condition of $L_1 > L_2$. From these figures, we can know, in general, that the properties of θ_2 under the ES are basically the same with the relationship between the bullwhip effect and θ_2 under the MA. Thus, the square of the moving average coefficient of the retailer with a longer lead time is larger, the bullwhip effect is greater.

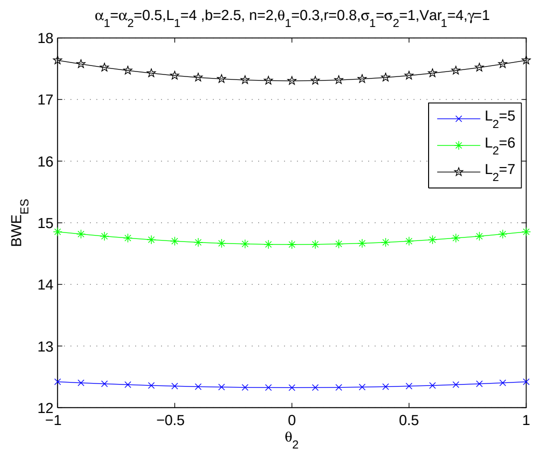

Figure 22: Effect of θ_2 on the bullwhip effect under the ES in the case of $L_1 < L_2$

Fig.24 depicts how the consistency of the two retailers' price γ affects the bullwhip effect when we fix $\alpha_1 = \alpha_2 = 0.5, n = 2, b = 2.5, L_1 = 4, \theta_1 = \theta_2 = 0.3, \sigma_1 = \sigma_2 = 1, Var_1 = 4$. This figure clearly shows it has different variation trends for different L_2. Compared with Figs.12-13, we can discover that

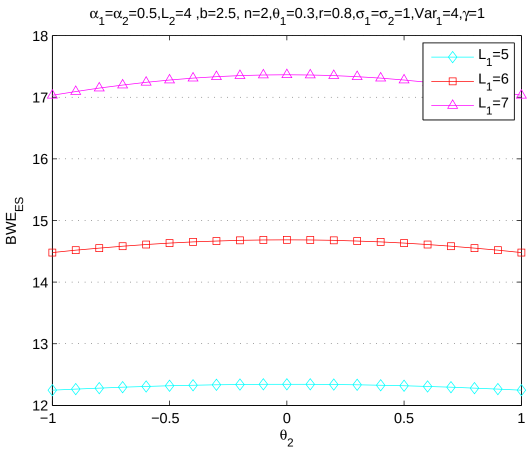

Figure 23: Effect of θ_2 on the bullwhip effect under the ES in the case of $L_1 > L_2$

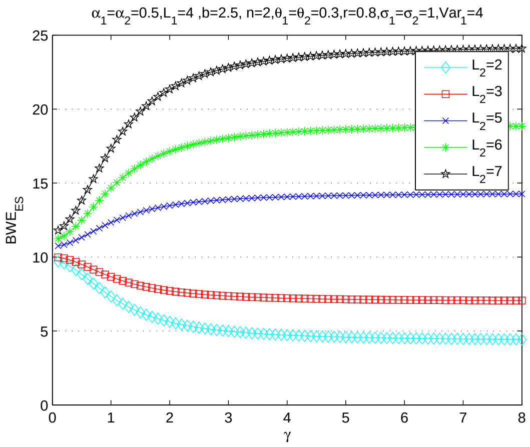

Figure 24: Effect of γ on the bullwhip effect for different L_2 under the ES

it is also similar to MA forecasting. When $L_1 > L_2$, the bullwhip effect is a decreasing function of γ and on the contrary, when we set $L_1 < L_2$ the bullwhip effect is an increasing function of γ. So we cant enlarge γ to reduce the bullwhip effect blindly.

4.3 The comparison of MA and ES forecasting methods

In order to compare the bullwhip effects for two methods, we should put a constraint on the span k and the smoothing factors α_1 and α_2. According to Zhang (2004), the average data ages which are defined as the weighted average of the age for data points should be the same for the MA and ES forecasting methods. When we set the $\frac{k+1}{2}$ equal to $\frac{1}{\alpha}$, we obtain that the smoothing exponents are selected as $\alpha_1 = \alpha_2 = \frac{2}{k+1}$.

Figs.25-27 depict the comparison between the MA and ES by varying r from 0 to 1 and varying

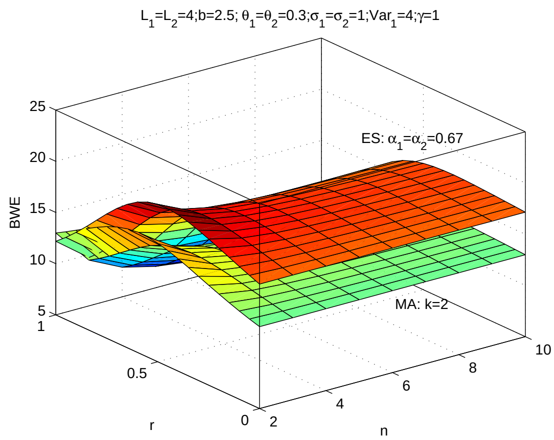

Figure 25: Comparison for the MA and ES forecasting methods by varying r and n(k=2)

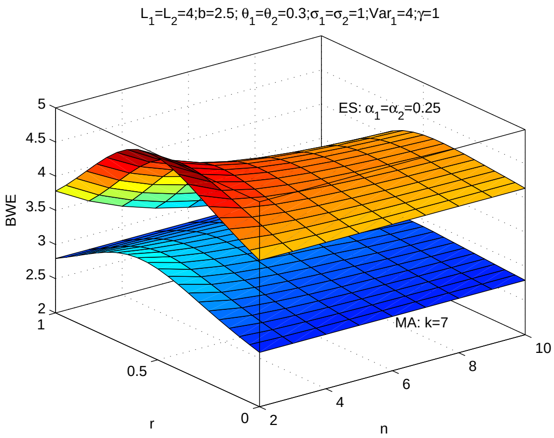

Figure 26: Comparison for the MA and ES forecasting methods by varying r and n(k=7)

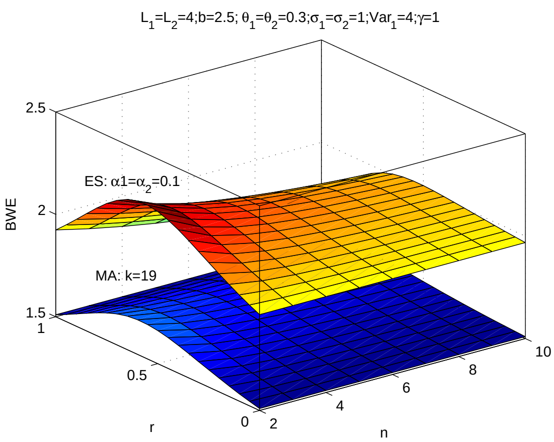

Figure 27: Comparison for the MA and ES forecasting methods by varying r and n(k=19)

n from 2 to 10, respectively. From these figures, we can conclude that BWE_{ES} always is higher than BWE_{MA} whatever r and n are as long as $\alpha_1 = \alpha_2 = \frac{2}{k+1}$. In Fig.25, we set $k = 2, \alpha_1 = \alpha_2 = 0.67$ and fix other parameters. Because of $n \geq k$, we just need to select the expression of the bullwhip effect in the case 1 to measure the bullwhip effect under the MA forecasting method. This figure shows that, only when r is near one and $n = 2$, BWE_{ES} is lower than BWE_{MA}. In Fig.26, let $k = 7, \alpha_1 = \alpha_2 = 0.25$ and we vary the value of n, so when $2 \leq n < 7$, the case 1 applies to simulate the bullwhip effect under the MA forecasting technique, and when $7 \leq n \leq 10$, the case 2 applies. The result indicates that BWE_{ES} is higher than BWE_{MA} no matter what r and n is as long as $\alpha_1 = \alpha_2 = \frac{2}{k+1}$.In Fig.27, we set $k = 19, \alpha_1 = \alpha_2 = 0.1$ and fix other parameters, so we can get $n < k$ no matter how n changes. Applying the case 2, we get the Fig.27 which still shows that BWE_{ES} is higher than BWE_{MA}. From these three figures, we can observe additionally that BWE_{ES} and BWE_{MA} get lower gradually with the decrease of α_1 and α_2 or with the increase of k. In conclusion, these phenomena reveal that when we consider the price fluctuation in the demand process, the MA forecasting method is better than the ES method almost whatever the extent of consumers concerning about the historical price volatility is.

5 Conclusions

The bullwhip effect is one of the most common problems in the supply chain management and price fluctuation is an important cause of the bullwhip effect. In our paper, we first establish a new linear demand model which considers the impact of market price of product and stochastic disturbance on the t-period demand. Second, other than previous papers which also study the price fluctuations, we build a two-stage supply chain containing one supplier and two retailers who both face the same demand process. Third, this paper contrasts the bullwhip effect under two different forecasting methods by simulating the impact of parameters on the bullwhip effect comprehensively.

In general, we first deduce the bullwhip effect expression statistically based on the established model in our paper.Due to the difficulty of applying the algebraic analysis, we use the numerical simulation. We study the effect of lead-times on the bullwhip effect similar to the results of Lee et al. (1997a,b), Chen et al. (2000a,b) and Zhang (2004). Increasing the lead times of two retailers will enhance the bullwhip effect regardless of the forecasting methods applied. The results also show that the extent of consumers concerning about the historical price volatility r plays a significant role on reducing the bullwhip effects for both the MA method and the ES method. The bullwhip effect both firstly increases then decreases with the increase of r for these two forecasting methods and the size of the impact depends on the forecasting methods and has some differences. Moreover, the impact of the price-sensitive coefficient on the bullwhip effect is affected by the size of r. When r is smaller than a certain value, the more sensitive demand is to price changing the greater the bullwhip effect is whatever the forecasting methods are. Another parameter n which is the span of previous market prices is important based on different r. Only when r is large to a certain extent, the increase of n can reduce the bullwhip effect obviously.

In our paper, we focus on the stochastic disturbance and investigate the impact of θ and σ^2 on the bullwhip effect. We discover that the impact of the moving average coefficient θ of stochastic disturbance on the bullwhip effect is not very significant. The variance σ^2 affects the bullwhip effect more significantly than θ. It implies that the impact of these two parameters both has something with lead time. The larger any one of the two parameters of the retailer which has a longer lead time, the greater the bullwhip effect in the supply chain.

From our model, we also study the consistency of price volatility between two retailers. It is clear that the bullwhip effect increases as price volatility of the retailer with a longer lead time increases, and the bullwhip effect decreases as price volatility of the retailer with a shorter lead time increases.

Under the MA and ES methods, the average data age has an important impact on the bullwhip effect, and the larger it is, the weaker the bullwhip effect is. By comparison, we also know that the MA forecasting method is clearly the winner between the two methods when we submit to our demand model in this paper.

Our findings not only are theoretical but also can give the member of a supply chain some managerial proposals. First, the findings suggest that managers should keep a close eye on the price-predicting behavior of consumers because it affects the bullwhip effect directly as well as indirectly by affecting many other parameters. The larger concern extent of historical price volatility can effectively reduce the bullwhip effect within limits, and if consumers pay enough attention to the impact of historical price volatility on the t-period demand, they observing the historical prices in the longer term can further reduce the bullwhip effect. Hence, it has a guiding significance for reducing the bullwhip effect that managers guide consumers concerning the impact of the historical prices rationally. Second, in order to reduce the bullwhip effect, managers should control the price volatility of the retailer which has a longer lead time as low as possible. Third, in order to reduce the bullwhip effect, the retailer which has a longer lead time should keep the variance of stochastic disturbance as small as possible.At last, the MA method may be optimal for the price-sensitive demand model, and managers should firstly select the MA method for the forecasting of the lead-time demand.

As a main cause of the bullwhip effect, the price fluctuations can be further studied in many aspects. First, this research did not consider the one-period-ahead demand in the demand model. The demand process in our paper is only related to the price, however, in practice, the previous demand data may affect the -period demand somehow. Further researches can be recommended to incorporate this factor into the developed demand model. Next, some general inventory policies should be studied. The simple order-up-to inventory policy could be misleading under the condition of existing of an obvious fixed ordering cost. The general (s,S) policy may have more practical significance. Third, managers may comprehensively pay attention to the inventory cost and the size of the bullwhip effect when they evaluate the forecasting methods and take measures to reduce the bullwhip effect. Lastly, our model did not consider the case when there has correlation between the prices of two retailers. This may be added in developed mathematical models to expand their applicability.

Appendix. Proofs

Proof of Eq.(22).

$$\begin{aligned}
&Var(D_{1,t}^{L_1}) = Var(d_{1,t} + d_{1,t+1} + \cdots + d_{1,t+L-1})\\
&= Var(d_{1,t}) + Var(d_{1,t+1}) + \cdots + Var(d_{1,t+L-1})\\
&+2[Cov(d_{1,t}, d_{1,t+1}) + Cov(d_{1,t}, d_{1,t+2}) + \cdots\\
&+Cov(d_{1,t}, d_{1,t+L-1}) + Cov(d_{1,t+1}, d_{1,t+2})\\
&+Cov(d_{1,t+1}, d_{1,t+3}) + \cdots + Cov(d_{1,t+1}, d_{1,t+L-1})\\
&+Cov(d_{1,t+2}, d_{1,t+3}) + Cov(d_{1,t+2}, d_{1,t+4}) + \cdots\\
&+Cov(d_{1,t+2}, d_{1,t+L-1}) +\\
&\cdots + Cov(d_{1,t+L-2}, d_{1,t+L-1})]
\end{aligned} \tag{A.1}$$

According to the basic statistical knowledge, we can have

$$\begin{aligned}
&Cov(d_{1,t+j}, d_{1,t+j+1}) = Cov(a - bp_{1,t+j} + rb(p_{1,t+j}\\
&-\sum_{i=1}^{n} p_{1,t+j-i}/n) + \varepsilon_{1,t+j} - \theta_1\varepsilon_{1,t+j-1}, d_{1,t+j+1})\\
&= (1-r)bCov(p_{1,t+j}, d_{1,t+j+1})\\
&-\frac{rb}{n}Cov(\sum_{i=1}^{n} p_{1,t+j-i}, d_{1,t+j+1})\\
&+Cov(\varepsilon_{1,t+j}, d_{1,t+j+1}) - \theta_1 Cov(\varepsilon_{1,t+j-1}, d_{1,t+j+1})\\
&= (1-r)bCov(p_{1,t+j}, a - bp_{1,t+j+1} + rb(p_{1,t+j+1}\\
&-\sum_{i=1}^{n} p_{1,t+j+1-i}/n) + \varepsilon_{1,t+j+1} - \theta_1\varepsilon_{1,t+j})\\
&-\frac{rb}{n}Cov(\sum_{i=1}^{n} p_{1,t+j-i}, a - bp_{1,t+j+1} + rb(p_{1,t+j+1}\\
&-\sum_{i=1}^{n} p_{1,t+j+1-i}/n) + \varepsilon_{1,t+j+1} - \theta_1\varepsilon_{1,t+j})\\
&+Cov(\varepsilon_{1,t+j}, a - bp_{1,t+j+1} + rb(p_{1,t+j+1}\\
&-\sum_{i=1}^{n} p_{1,t+j+1-i}/n) + \varepsilon_{1,t+j+1} - \theta_1\varepsilon_{1,t+j})\\
&-\theta_1 Cov(\varepsilon_{1,t+j-1}, a - bp_{1,t+j+1} + rb(p_{1,t+j+1}\\
&-\sum_{i=1}^{n} p_{1,t+j+1-i}/n) + \varepsilon_{1,t+j+1} - \theta_1\varepsilon_{1,t+j})\\
&= (1-r)bCov(p_{1,t+j}, -\frac{rb}{n}p_{1,t+j})\\
&-\frac{rb}{n}Cov(\sum_{i=1}^{n} p_{1,t+j-i} - \frac{rb}{n}\sum_{i=1}^{n} p_{1,t+j+1-i})\\
&+Cov(\varepsilon_{1,t+j}, -\theta_1\varepsilon_{1,t+j}) + 0\\
&= -\frac{rb^2}{n}(1-r)Var(p_{1,t+j}) + \frac{r^2b^2}{n^2}[Var(p_{1,t+j-1})\\
&+Var(p_{1,t+j-2}) + \cdots + Var(p_{1,t+j-n+1})]\\
&-\theta_1 Var(\varepsilon_{1,t+j})\\
&= [\frac{r^2b^2}{n^2}(n-1) - \frac{rb^2}{n}(1-r)]Var(p_1) - \theta_1\sigma_1^2
\end{aligned} \tag{A.2}$$

and, based on this, we also have

$$\begin{aligned}
&Cov(d_{1,t+j}, d_{1,t+j+2}) = Cov(a - bp_{1,t+j}\\
&+rb(p_{1,t+j} - \sum_{i=1}^{n} p_{1,t+j-i}/n) + \varepsilon_{1,t+j}\\
&-\theta_1\varepsilon_{1,t+j-1}, d_{1,t+j+2})\\
&= (1-r)bCov(p_{1,t+j}, a - bp_{1,t+j+2} + rb(p_{1,t+j+2}\\
&-\sum_{i=1}^{n} p_{1,t+j+2-i}/n) + \varepsilon_{1,t+j+2} - \theta_1\varepsilon_{1,t+j+1})\\
&-\frac{rb}{n}Cov(\sum_{i=1}^{n} p_{1,t+j-i}, a - bp_{1,t+j+2} + rb(p_{1,t+j+2}\\
&-\sum_{i=1}^{n} p_{1,t+j+2-i}/n) + \varepsilon_{1,t+j+2} - \theta_1\varepsilon_{1,t+j+1})
\end{aligned}$$

$$
\begin{aligned}
&+Cov(\varepsilon_{1,t+j}, a-bp_{1,t+j+2}+rb(p_{1,t+j+2}\\
&-\sum_{i=1}^{n}p_{1,t+j+2-i}/n)+\varepsilon_{1,t+j+2}-\theta_1\varepsilon_{1,t+j+1})\\
&-\theta_1Cov(\varepsilon_{1,t+j-1}, a-bp_{1,t+j+2}+rb(p_{1,t+j+2}\\
&-\sum_{i=1}^{n}p_{1,t+j+2-i}/n)+\varepsilon_{1,t+j+2}-\theta_1\varepsilon_{1,t+j+1})\\
&=-\frac{rb^2}{n}(1-r)Var(p_{1,t+j})+\frac{r^2b^2}{n^2}[Var(p_{1,t+j-1})\\
&+Var(p_{1,t+j-2})+\cdots+Var(p_{1,t+j-n+2})]\\
&=[\frac{r^2b^2}{n^2}(n-2)-\frac{rb^2}{n}(1-r)]Var(p_1)
\end{aligned} \tag{A.3}
$$

By applying this procedure, we can get

$$
Cov(d_{1,t+j}, d_{1,t+j+m})=
\begin{cases}
[\frac{r^2b^2}{n^2}(n-m)-\frac{rb^2}{n}(1-r)]Var(p_1), & m\geq 2, n\geq m\\
-\frac{rb^2}{n}(1-r)Var(p_1), & m\geq 2, n<m
\end{cases} \tag{A.4}
$$

Using the Eq. (A.2)- (A.4) and Eq.(4), the $Var(D_{1,t}^{L_1})$ can be identified as

$$
\begin{aligned}
&Var(D_{1,t}^{L_1})=L_1Var(d_1)+2\{(L_1-1)[(\frac{r^2b^2}{n^2}(n-1)\\
&-\frac{rb^2}{n}(1-r))Var(p_1)-\theta_1\sigma_1^2]\\
&+(L_1-2)[\frac{r^2b^2}{n^2}(n-2)-\frac{rb^2}{n}(1-r)]Var(p_1)\\
&+(L_1-3)[\frac{r^2b^2}{n^2}(n-3)-\frac{rb^2}{n}(1-r)]Var(p_1)\\
&+\cdots+(L_1-L_1+1)[\frac{r^2b^2}{n^2}(n-L_1+1)\\
&-\frac{rb^2}{n}(1-r)]Var(p_1)\}\\
&=L_1Var(d_1)-2\theta_1(L_1-1)\sigma_1^2+2Var(p_1)\\
&[-\frac{L_1(L_1-1)}{2}\cdot\frac{rb^2}{n}(1-r)\\
&+\frac{r^2b^2}{n^2}\sum_{i=1}^{L_1-1}(L_1-i)(n-i)], n\geq L_1-1
\end{aligned} \tag{A.5}
$$

For the MA forecasting method, we have

$$
\begin{aligned}
&Var(\hat{D}_{1,t}^{L_1})=Var(\frac{L_1}{k}\sum_{i=1}^{k}d_{1,t-i})\\
&=(\frac{L_1}{k})^2Var(d_{1,t-k}+d_{1,t-k+1}+\cdots+d_{1,t-2}+d_{1,t-1})
\end{aligned} \tag{A.6}
$$

and, similar to the $Var(D_{1,t}^{L_1})$, from (A.2-A.4), $Var(D_{1,t}^{L_1})$ can be written as

$$
\begin{aligned}
&Var(\hat{D}_{1,t}^{L_1})=Var(\frac{L_1}{k}\sum_{i=1}^{k}d_{1,t-i})\\
&=(\frac{L_1}{k})^2Var(d_{1,t-1}+d_{1,t-2}+\cdots+d_{1,t-k})\\
&=(\frac{L_1}{k})^2\{kVar(d_1)+2\{(k-1)[(\frac{r^2b^2}{n^2}(n-1)\\
&-\frac{rb^2}{n}(1-r))Var(p_1)-\theta_1\sigma_1^2]\\
&+(k-2)[\frac{r^2b^2}{n^2}(n-2)-\frac{rb^2}{n}(1-r)]Var(p_1)\\
&+(k-3)[\frac{r^2b^2}{n^2}(n-3)-\frac{rb^2}{n}(1-r)]Var(p_1)\\
&+\cdots+(k-k+1)[\frac{r^2b^2}{n^2}(n-k+1)\\
&-\frac{rb^2}{n}(1-r)]Var(p_1)\}\}\\
&=\frac{L_1^2}{k}Var(d_1)-2\theta_1(k-1)(\frac{L_1}{k})^2\sigma_1^2\\
&+2(\frac{L_1}{k})^2Var(p_1)[-\frac{k(k-1)}{2}\cdot\frac{rb^2}{n}(1-r)\\
&+\frac{r^2b^2}{n^2}\sum_{i=1}^{k-1}(k-i)(n-i)]
\end{aligned} \tag{A.7}
$$

where we set $n\geq k-1$. In addition, we have

$$
\begin{aligned}
&Cov(D_{1,t}^{L_1}, \hat{D}_{1,t}^{L_1})=Cov(d_{1,t}+d_{1,t+1}+\cdots\\
&+d_{1,t+L_1-1}, \frac{L_1}{k}\sum_{i=1}^{k}d_{1,t-i})\\
&=\frac{L_1}{k}[Cov(d_{1,t}, d_{1,t-1})+\cdots+Cov(d_{1,t}, d_{1,t-k})\\
&+Cov(d_{1,t+1}, d_{1,t-1})+\cdots+Cov(d_{1,t+1}, d_{1,t-k})\\
&+\cdots+\\
&Cov(d_{1,t+L_1-1}, d_{1,t-1})+\cdots+Cov(d_{1,t+L_1-1}, d_{1,t-k})]
\end{aligned} \tag{A.8}
$$

and from (A.2) and (A.4), it can result in

$$
\begin{aligned}
&Cov(D_{1,t}^{L_1}, \hat{D}_{1,t}^{L_1})=\frac{L_1}{k}\{[\frac{r^2b^2}{n^2}(n-1)\\
&-\frac{rb^2}{n}(1-r)]Var(p_1)-\theta_1\sigma_1^2\\
&+[\frac{r^2b^2}{n^2}(n-2)-\frac{rb^2}{n}(1-r)]Var(p_1)+\cdots\\
&+[\frac{r^2b^2}{n^2}(n-k)-\frac{rb^2}{n}(1-r)]Var(p_1)
\end{aligned}
$$

$$
\begin{aligned}
&+[\frac{r^2b^2}{n^2}(n-2)-\frac{rb^2}{n}(1-r)]Var(p_1)+\cdots\\
&+[\frac{r^2b^2}{n^2}(n-k-1)-\frac{rb^2}{n}(1-r)]Var(p_1)\\
&+\cdots\\
&+[\frac{r^2b^2}{n^2}(n-L_1)-\frac{rb^2}{n}(1-r)]Var(p_1)+\cdots\\
&+[\frac{r^2b^2}{n^2}(n-L_1-k+1)-\frac{rb^2}{n}(1-r)]Var(p_1)\}\\
&=-\frac{\theta_1 L_1}{k}\sigma_1^2+\frac{L_1}{k}Var(p_1)\{\frac{r^2b^2}{n^2}[(n-1+n-2\\
&+\cdots n-k)+(n-2+n-3+\cdots n-k-1)+\cdots\\
&+(n-L_1+n-L_1-1+\cdots n-L_1-k+1)]\\
&-kL_1\frac{rb^2}{n}(1-r)\}\\
&=-\frac{\theta_1 L_1}{k}\sigma_1^2+\frac{L_1}{k}Var(p_1)\{\frac{r^2b^2}{n^2}[knL_1-\frac{k}{2}(k+1\\
&+k+3+\cdots+k+2L_1-1)]-kL_1\frac{rb^2}{n}(1-r)\}\\
&=-\frac{\theta_1 L_1}{k}\sigma_1^2+[2(nrbL_1)^2(4n-k-L_1)\\
&-4n^3rb^2L_1^2]Var(p_1)
\end{aligned}
\tag{A.9}
$$

This completes the proof for the Eq.(22).

Proof of Proposition 1.

The total quantity of products received by the supplier at the beginning of period t under the MA forecasting technique is

$$
\begin{aligned}
q_t &= q_{1,t}+q_{1,t}\\
&=(1+\frac{L_1}{k})d_{1,t-1}-\frac{L_1}{k}d_{1,t-k-1}\\
&+(1+\frac{L_2}{k})d_{2,t-1}-\frac{L_2}{k}d_{2,t-k-1}
\end{aligned}
\tag{A.10}
$$

Taking the variance for q_t, we can get

$$
\begin{aligned}
&Var(q_t)=(1+\frac{L_1}{k})^2Var(d_{1,t-1})\\
&+(\frac{L_1}{k})^2Var(d_{1,t-k-1})+(1+\frac{L_2}{k})^2Var(d_{2,t-1})\\
&+(\frac{L_2}{k})^2Var(d_{2,t-k-1})\\
&-2\frac{L_1}{k}(1+\frac{L_1}{k})Cov(d_{1,t-1},d_{1,t-k-1})\\
&+2(1+\frac{L_1}{k})(1+\frac{L_2}{k})Cov(d_{1,t-1},d_{2,t-1})\\
&-2\frac{L_2}{k}(1+\frac{L_1}{k})Cov(d_{1,t-1},d_{2,t-k-1})\\
&-2\frac{L_1}{k}(1+\frac{L_2}{k})Cov(d_{1,t-k-1},d_{2,t-1})\\
&+2\frac{L_1}{k}\frac{L_2}{k}Cov(d_{1,t-k-1},d_{2,t-k-1})\\
&-2\frac{L_2}{k}(1+\frac{L_2}{k})Cov(d_{2,t-1},d_{2,t-k-1})
\end{aligned}
\tag{A.11}
$$

Because $p_{1,t}$ and $p_{2,t}$ are both independent and identically distributed, we have

$$
\begin{aligned}
&Cov(d_{1,t-1},d_{2,t-k-1})=0,\\
&Cov(d_{1,t-k-1},d_{2,t-1})=0.
\end{aligned}
\tag{A.12}
$$

Substituting Eq.(3), Eq.(6), Eq.(18) and Eq.(A.12) into Eq. (A.11), we can get

$$
\begin{aligned}
&Var(q_t)=[(1+\frac{L_1}{k})^2+(\frac{L_1}{k})^2]Var(d_1)\\
&+[(1+\frac{L_2}{k})^2+(\frac{L_2}{k})^2]Var(d_2)\\
&-2\frac{L_1}{k}(1+\frac{L_1}{k})Cov(d_{1,t-1},d_{1,t-k-1})\\
&-2\frac{L_2}{k}(1+\frac{L_2}{k})Cov(d_{2,t-1},d_{2,t-k-1})
\end{aligned}
\tag{A.13}
$$

According to (A.4), the covariance term can be derived as

$$
Cov(d_{1,t-1},d_{1,t-k-1})=
\begin{cases}
[\frac{r^2b^2}{n^2}(n-k)-\frac{rb^2}{n}(1-r)]Var(p_1), & n\ge k\\
-\frac{rb^2}{n}(1-r)Var(p_1), & n<k
\end{cases}
\tag{A.14}
$$

The same, we also have

$$
Cov(d_{2,t-1},d_{2,t-k-1})=
\begin{cases}
[\frac{r^2b^2}{n^2}(n-k)-\frac{rb^2}{n}(1-r)]Var(p_2), & n\ge k\\
-\frac{rb^2}{n}(1-r)Var(p_2), & n<k
\end{cases}
\tag{A.15}
$$

Substituting Eq.(3), Eq.(8), Eq. (A.14) and Eq. (A.15) into Eq. (A.13), then take the simplification, we can get two cases:

Case 1 : $n \ge k$.

The variance of q_t can be given as

$$
\begin{aligned}
&Var(q_t)_{n\ge k}=\{[(1+\frac{L_1}{k})^2+(\frac{L_1}{k})^2](1-r)^2\\
&+[1-2\frac{L_1}{k}-2(\frac{L_1}{k})^2]\frac{r^2}{n}+(2L_1+2\frac{{L_1}^2}{k})\frac{r^2}{n^2}\\
&+2\frac{L_1}{k}(1+\frac{L_1}{k})\frac{r}{n}\}b^2Var(p_1)
\end{aligned}
$$

$$
\begin{aligned}
&+\{[(1+\frac{L_2}{k})^2+(\frac{L_2}{k})^2](1-r)^2\\
&+[1-2\frac{L_2}{k}-2(\frac{L_2}{k})^2]\frac{r^2}{n}\\
&+(2L_2+2\frac{{L_2}^2}{k})\frac{r^2}{n^2}\\
&+2\frac{L_2}{k}(1+\frac{L_2}{k})\frac{r}{n}\}b^2Var(p_2)\\
&+[(1+\frac{L_1}{k})^2+(\frac{L_1}{k})^2](1+\theta_1^2)\sigma_1^2\\
&+[(1+\frac{L_2}{k})^2+(\frac{L_2}{k})^2](1+\theta_2^2)\sigma_2^2.
\end{aligned}
$$

Case 2 : $n < k$**.**

The variance of q_t can be given as

$$
\begin{aligned}
&Var(q_t)_{n<k}\\
&=\{[(1+\frac{L_1}{k})^2+(\frac{L_1}{k})^2][(1-r)^2+\frac{r^2}{n}]\\
&+2\frac{L_1}{k}(1+\frac{L_1}{k})\frac{r}{n}(1-r)\}b^2Var(p_1)\\
&+\{[(1+\frac{L_2}{k})^2+(\frac{L_2}{k})^2][(1-r)^2+\frac{r^2}{n}]\\
&+2\frac{L_2}{k}(1+\frac{L_2}{k})\frac{r}{n}(1-r)\}b^2Var(p_2)\\
&+[(1+\frac{L_1}{k})^2+(\frac{L_1}{k})^2](1+\theta_1^2)\sigma_1^2\\
&+[(1+\frac{L_2}{k})^2+(\frac{L_2}{k})^2](1+\theta_2^2)\sigma_2^2.
\end{aligned}
$$

This completes the proof for the proposition 1.

Proof of Eq.(35).

The variance of forecast error for lead-time demand under ES forecast is

$$
\begin{aligned}
&(\hat{\sigma}_{1,t}^{L_1})^2=Var(D_{1,t}^{L_1}-\hat{D}_{1,t}^{L_1})\\
&=Var(D_{1,t}^{L_1})+Var(\hat{D}_{1,t}^{L_1})-2Cov(D_{1,t}^{L_1},\hat{D}_{1,t}^{L_1}).
\end{aligned}
\tag{A.16}
$$

The second term in the variance formula can be obtained from the following expression:

$$
Var(\hat{D}_{1,t}^{L_1})=Var(L_1\hat{d}_{1,t})=L_1^2Var(\hat{d}_{1,t}).\tag{A.17}
$$

According to Eq.(31), we know the variance of $\hat{d}_{1,t}$ is

$$
\begin{aligned}
&Var(\hat{d}_{1,t})=Var(\sum_{i=0}^{\infty}\alpha_1(1-\alpha_1)^i d_{1,t-i-1})\\
&=\alpha_1^2[\sum_{i=0}^{\infty}(1-\alpha_1)^{2i}Var(d_{1,t-i-1})\\
&+2\sum_{i=0}^{\infty}\sum_{j>i}^{\infty}(1-\alpha_1)^i(1-\alpha_1)^jCov(d_{1,t-i-1},d_{1,t-j-1})]
\end{aligned}
$$

From Eqs.(4) and (A.4), the variance of $\hat{d}_{1,t}$ can be written as

$$
\begin{aligned}
&Var(\hat{d}_{1,t})=\alpha_1^2[Var(d_1)\sum_{i=0}^{\infty}(1-\alpha_1)^{2i}+2\sum_{i=0}^{\infty}\\
&[\sum_{j-i=1}^{n}(1-\alpha_1)^{i+j}[\frac{r^2b^2}{n^2}(n-(j-i))-\frac{rb^2}{n}(1-r)]Var(p_1)\\
&+\sum_{j-i=n+1}^{\infty}(1-\alpha_1)^{i+j}[-\frac{rb^2}{n}(1-r)]Var(p_1)]]
\end{aligned}
\tag{A.18}
$$

For simplicity, Eq.(A.18) can be derived as

$$
\begin{aligned}
&Var(\hat{d}_{1,t})\\
&=\alpha_1^2\{Var(d_1)\frac{1}{\alpha_1(2-\alpha_1)}+2Var(p_1)\sum_{i=0}^{\infty}\\
&[\sum_{j-i=1}^{n}(1-\alpha_1)^{i+j}[\frac{r^2b^2}{n^2}(n-(j-i))-\frac{rb^2}{n}(1-r)]\\
&+\sum_{j-i=n+1}^{\infty}(1-\alpha_1)^{i+j}[-\frac{rb^2}{n}(1-r)]]\}\\
&=\alpha_1^2\{Var(d_1)\frac{1}{\alpha_1(2-\alpha_1)}+2Var(p_1)\\
&\cdot[\frac{r^2b^2}{n^2}\sum_{i=0}^{\infty}\sum_{j-i=1}^{n}(1-\alpha_1)^{i+j}(n-j+i)\\
&-\frac{rb^2}{n}(1-r)\sum_{i=0}^{\infty}\sum_{j=i+1}^{\infty}(1-\alpha_1)^{i+j}]\}\\
&=\alpha_1^2Var(d_1)\frac{1}{\alpha_1(2-\alpha_1)}+2\alpha_1^2Var(p_1)\\
&\{\frac{r^2b^2}{n^2}\frac{1}{\alpha_1}[\sum_{i=0}^{\infty}(n-1)(1-\alpha_1)^{2i+1}\\
&-\frac{1-(1-\alpha_1)^{n-1}}{\alpha_1}\sum_{i=0}^{\infty}(1-\alpha_1)^{2i+2}]\\
&-\frac{rb^2}{n}(1-r)\sum_{i=0}^{\infty}\frac{(1-\alpha_1)^{2i+1}}{\alpha_1}\}\\
&=\alpha_1Var(d_1)\frac{1}{(2-\alpha_1)}+\frac{2}{n^2[1-(1-\alpha_1)^2]}Var(p_1)\\
&\{r^2b^2[\alpha_1(1-\alpha_1)(n-1)-(1-\alpha_1)^2\\
&+(1-\alpha_1)^{n+1}]-rb^2(1-r)n\alpha_1(1-\alpha_1)\}
\end{aligned}
\tag{A.19}
$$

Substituting (A.19) into (A.17), we can get

$$Var(\hat{D}^{L_1}_{1,t}) = \frac{\alpha_1 L_1^2}{(2-\alpha_1)} Var(d_1) + 2L_1^2 Var(p_1) \{\frac{r^2b^2}{n^2[1-(1-\alpha_1)^2]}[\alpha_1(1-\alpha_1)(n-1) - (1-\alpha_1)^2 + (1-\alpha_1)^{n+1}] - \frac{rb^2(1-r)n\alpha_1(1-\alpha_1)}{n^2[1-(1-\alpha_1)^2]}\}. \tag{A.20}$$

This completes the proof for Eq.(35).

Proof of Proposition 2.

The total order quantity at period t of the ES forecasting method is

$$q_t = (1+\alpha_1 L_1)d_{1.t-1} - \alpha_1 L_1 \hat{d}_{1,t-1} + (1+\alpha_2 L_2)d_{2.t-1} - \alpha_2 L_2 \hat{d}_{2,t-1} \tag{A.21}$$

Therefore, the variance is

$$\begin{aligned} Var(q_t) &= Var(\alpha_1 L_1(d_{1.t-1} - \hat{d}_{1,t-1}) + d_{1.t-1} + \alpha_2 L_2(d_{2.t-1} - \hat{d}_{2,t-1}) + d_{2.t-1}) \\ &= (1+\alpha_1 L_1)^2 Var(d_{1.t-1}) + (\alpha_1 L_1)^2 Var(\hat{d}_{1.t-1}) \\ &+ (1+\alpha_2 L_2)^2 Var(d_{2.t-1}) + (\alpha_2 L_2)^2 Var(\hat{d}_{2.t-1}) \\ &- 2(1+\alpha_1 L_1)\alpha_1 L_1 Cov(d_{1.t-1}, \hat{d}_{1,t-1}) \\ &+ 2(1+\alpha_1 L_1)(1+\alpha_2 L_2) Cov(d_{1.t-1}, d_{2.t-1}) \\ &- 2(1+\alpha_1 L_1)\alpha_2 L_2 Cov(d_{1.t-1}, \hat{d}_{2,t-1}) \\ &- 2\alpha_1 L_1(1+\alpha_2 L_2) Cov(\hat{d}_{1.t-1}, d_{2,t-1}) \\ &+ 2\alpha_1 L_1 \alpha_2 L_2 Cov(\hat{d}_{1.t-1}, \hat{d}_{2,t-1}) \\ &- 2(1+\alpha_2 L_2)\alpha_2 L_2 Cov(d_{2.t-1}, \hat{d}_{2,t-1}) \end{aligned} \tag{A.22}$$

From (18), we know $Cov(d_{1,t-1}, d_{2,t-1}) = 0$, $Cov(d_{1,t-1}, \hat{d}_{2,t-1}) = 0$, $Cov(\hat{d}_{1,t-1}, d_{2,t-1}) = 0$ and $Cov(\hat{d}_{1,t-1}, \hat{d}_{2,t-1}) = 0$. By substituting these relationships into (A.22), we have

$$\begin{aligned} Var(q_t) &= (1+\alpha_1 L_1)^2 Var(d_{1,t-1}) + (\alpha_1 L_1)^2 Var(\hat{d}_{1,t-1}) \\ &+ (1+\alpha_2 L_2)^2 Var(d_{2,t-1}) + (\alpha_2 L_2)^2 Var(\hat{d}_{2,t-1}) \\ &- 2(1+\alpha_1 L_1)\alpha_1 L_1 Cov(d_{1,t-1}, \hat{d}_{1,t-1}) \\ &- 2(1+\alpha_2 L_2)\alpha_2 L_2 Cov(d_{2,t-1}, \hat{d}_{2,t-1}) \end{aligned} \tag{A.23}$$

Using Eq.(31), we can derive

$$\begin{aligned} & Cov(d_{1.t-1}, \hat{d}_{1,t-1}) \\ &= Cov(d_{1.t-1}, \sum_{i=0}^{\infty} \alpha_1(1-\alpha_1)^i d_{1,t-1-i-1}) \\ &= \alpha_1 \sum_{i=0}^{\infty} (1-\alpha_1)^i Cov(d_{1.t-1}, d_{1,t-2-i}) \end{aligned} \tag{A.24}$$

According to (A.4), and take the simplification, the Eq.(A.24) can be written as

$$\begin{aligned} Cov(d_{1.t-1}, \hat{d}_{1,t-1}) &= \alpha_1\{[\frac{r^2b^2}{n^2}\sum_{i=0}^{n-1}(1-\alpha_1)^i(n-i-1) \\ &- \frac{rb^2}{n}(1-r)\sum_{i=0}^{n-1}(1-\alpha_1)^i]Var(p_1) \\ &+ \sum_{i=n}^{\infty}(1-\alpha_1)^i[-\frac{rb^2}{n}(1-r)]Var(p_1)\} \\ &= \alpha_1 Var(p_1)[\frac{r^2b^2}{n^2}\sum_{i=0}^{n-1}(1-\alpha_1)^i(n-i-1) \\ &- \frac{rb^2}{n}(1-r)\sum_{i=0}^{n-1}(1-\alpha_1)^i - \frac{rb^2}{n}(1-r)\sum_{i=n}^{\infty}(1-\alpha_1)^i] \\ &= \alpha_1 Var(p_1)[\frac{r^2b^2}{n^2}\sum_{i=0}^{n-1}(1-\alpha_1)^i(n-i-1) \\ &- \frac{rb^2}{n}(1-r)\sum_{i=0}^{\infty}(1-\alpha_1)^i] \\ &= \alpha_1 Var(p_1)[\frac{r^2b^2}{n^2}[(n-1)\frac{1-(1-\alpha_1)^n}{\alpha_1} \\ &- \frac{(1-\alpha_1)[1-(1-\alpha_1)^n]}{\alpha_1^2} + \frac{n(1-\alpha_1)^n}{\alpha_1}] \\ &- \frac{rb^2}{n}(1-r)\frac{1}{\alpha_1}] \\ &= Var(p_1)[\frac{r^2b^2}{n^2}[1-(1-\alpha_1)^n](n-\frac{1}{\alpha_1}) \\ &+ \frac{r^2b^2}{n}(1-\alpha_1)^n - \frac{rb^2}{n}(1-r)] \end{aligned} \tag{A.25}$$

The same, we also have

$$Cov(d_{2.t-1}, \hat{d}_{2,t-1}) = Var(p_2)\{\frac{r^2b^2}{n^2}[1-(1-\alpha_2)^n] \cdot (n-\frac{1}{\alpha_2}) + \frac{r^2b^2}{n}(1-\alpha_2)^n - \frac{rb^2}{n}(1-r)\} \tag{A.26}$$

From (A.19), we can get

$$Var(\hat{d}_{2.t-1}) = \frac{\alpha_2}{(2-\alpha_2)} Var(d_2) + \frac{2}{n^2[1-(1-\alpha_2)^2]} \cdot \{r^2 b^2[\alpha_2(1-\alpha_2)(n-1) - (1-\alpha_2)^2 + (1-\alpha_2)^{n+1}] - rb^2(1-r)n\alpha_2(1-\alpha_2)\} Var(p_2) \tag{A.27}$$

and

$$Var(\hat{d}_{2.t-1}) = \frac{\alpha_2}{(2-\alpha_2)} Var(d_2) + \frac{2}{n^2[1-(1-\alpha_2)^2]} \cdot \{r^2 b^2[\alpha_2(1-\alpha_2)(n-1) - (1-\alpha_2)^2 + (1-\alpha_2)^{n+1}] - rb^2(1-r)n\alpha_2(1-\alpha_2)\} Var(p_2) \tag{A.28}$$

Substituting Eq.(A.25)-(A.28), Eq.(3) and Eq.(8) into Eq.(A.23), then take the simplification, we have

$$\begin{aligned} Var(q_t) = {} & [(1+\alpha_1 L_1)^2 + \frac{\alpha_1^3 L_1^2}{2-\alpha_1}](1+\theta_1^2)\sigma_1^2 \\ & + [(1+\alpha_2 L_2)^2 + \frac{\alpha_2^3 L_2^2}{2-\alpha_2}](1+\theta_2^2)\sigma_2^2 \\ & + \{[(1+\alpha_1 L_1)^2 + \frac{\alpha_1^3 L_1^2}{2-\alpha_1}][(1-r)^2 + \frac{r^2}{n}] \\ & + \frac{(2r\alpha_1 L_1)^2(1-\alpha_1)}{n(2-\alpha_1)} + \frac{2r^2\alpha_1 L_1^2(1-\alpha_1)^{n+1}}{n^2(2-\alpha_1)} \\ & - \frac{2r\alpha_1 L_1^2(1-\alpha_1)(r+n\alpha_1)}{n^2(2-\alpha_1)} \\ & - 2(1+\alpha_1 L_1)\alpha_1 L_1 \\ & [\frac{2r^2-r}{n} - \frac{r^2[1-(1-\alpha_1)^n]}{n^2\alpha_1}]\} b^2 Var(p_1) \\ & + \{[(1+\alpha_2 L_2)^2 + \frac{\alpha_2^3 L_2^2}{2-\alpha_2}][(1-r)^2 + \frac{r^2}{n}] \\ & + \frac{(2r\alpha_2 L_2)^2(1-\alpha_2)}{n(2-\alpha_2)} + \frac{2r^2\alpha_2 L_2^2(1-\alpha_2)^{n+1}}{n^2(2-\alpha_2)} \\ & - \frac{2r\alpha_2 L_2^2(1-\alpha_2)(r+n\alpha_2)}{n^2(2-\alpha_2)} \\ & - 2(1+\alpha_2 L_2)\alpha_2 L_2 \\ & [\frac{2r^2-r}{n} - \frac{r^2[1-(1-\alpha_2)^n]}{n^2\alpha_2}]\} b^2 Var(p_2). \end{aligned} \tag{A.29}$$

This completes the proof for proposition 2.

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