Complexity Analysis in Evolutionary Game System in the Real Estate Market

YUJING YANG
Tianjin University
College of Management Economic
Tianjin 300072
CHINA
cghnju@aliyun.com

JUNHAI MA
Tianjin University
College of Management Economic
Tianjin 300072
CHINA
mjhtju@aliyun.com

Abstract: In this paper, an evolutionary game model is constructed by introducing reciprocity theory which based on behavioral economics. The complexity and dynamic characteristics are analyzed by mapping out the bifurcation diagrams, Lyapunov index, the strange attractors, power spectrum and sensitivity depending on the initial conditions which mark the chaotic phenomena. The conclusion can be drawn as disorderly chaotic phenomena is as a result of specific parameter conditions in evolutionary game model. The conditions push the evolutionary game system into a specific area in the form of exponential convergence, and inhibit the evolutionary game from evolving towards the desired direction. Finally a linear feedback controlled method is adopted to achieve the goal of driving the system to a steady state, and the goal of evolving to the desired direction is realized.

Key–Words: Behavioral economics, Evolutionary game,Chaos, Linear feedback control.

1 Introduction

In recent years, chaotic economists have used the theory of chaos to improve the existing models of economic phenomena. Chaos is applied in the fields of social economy, information and biology. Globally, researchers pay much attention on the generation and application of chaos and solve a series of economies question by combing chaos and duopoly game. Chaos is applied in analyzing Cournot model, Bertrand model, StanKerrberg model, and the mixed game model of Cournot -Bertrand to make analysis of electricity market, telecommunication market, steel market, insurance market and the closed-loop supply chain.

Gory et al.[1-4] have focused on repeated game based on Bertrand model and Cournot model, and found other chaotic phenomena in the repeated game. Ma et al.[5-9] have considered the repeated game based on Bertrand model which is applied in repeated game, the complexity of trio-poly price game with different rationality, four oligarch different decisions rules and its chaos control, delayed complexity based on nonlinear price game of insurance market, and output gaming analysis and control among enter-prises of rational difference in a two-level supply chain. Chen et al.[10-12]have made analysis on hopf bifurcation in a four-dimensional hyper-chaotic system. C.Veller and V.Rajpaul[13] have focused on the system stability in purely competitive evolutionary games. D. G. Hernndez and D. H.Zanette [14] have made analysis on evolutionary dynamics of resource allocation in the Colonel Blotto Game. Li et al.[15] have made analysis on dynamic behaviors of evolutionary game based control for biological systems with applications in drug delivery. A.Szolnoki and M.Perc [16] have focused on effectiveness of conditional punishment for the evolution of public cooperation. Xia and Chen [17] have applied the fuzzy chaos method in the evaluation of nonlinearly evolutionary game of rolling bearing performance, and obtained useful conclusions. J.YuichiroWakano and L.Lehmann [18] have studied the convergence stability for continuous phenotypes in finite populations derived from two-allele models, and obtained the conclusion as that convergence stability is enough to characterize long-term evolution under the trait substitution sequence assumption in two-allela systems in finite populations. C. Diks, C. Hommes and P. Zeppini [19] have found that more memory under evolutionary leaning may lead to chaos. Based on this, chaos has become a hot topic in the evolutionary game recently.

Chaos has sensitivity to the initial values of parameters. Slight changes of the initial values will make an enormous influence on the results, and make a tremendous negative impact on the system. It restrains the system from evolving towards the expected directions for both behaviors. when the system converges to the unexpected particular points, it will get into a state of unrestricted circulation. Thus, we should join disturbance or control to delay the happening time of chaos. Motivated by this, if we want to avoid chaos appearing, proper measures should be taken in the system.

In this paper, we describe the behaviors by analyzing a phase diagram in the real estate market. We outline two innovations and one of which is the introduction of the 'psychological game', and construct evolutionary path applying psychological theory. Then, we analyze analytic parameters of phase diagrams in the system. The conclusions are that chaos appears when the parameters are in a certain range. We give a detailed explanation through numerical simulations, which has not been done in the previous work.

This paper is organized as follows: in chapter 2, we establish the dynamics of the evolutionary game and describe the evolution and conditions. In chapter 3, we analyze the stability, bifurcation diagrams, the equilibrium point and Lyapunov exponent through numerical simulations. We find out that, there are obvious chaotic phenomena and the system goes into chaos when the parameters are beyond certain ranges. Finally, the complex characteristics are displayed via strange attractors, power spectrum and butterfly effect. In chapter 4, the linear controller is taken to realize the goal of chaos control. Chapter 5 is our conclusions and recommendations.

2 A Model Based on the Harmonious Evolution Game

Basic conditions to establish the model:

Conditions 1: the capital must be intensive, and is affected evidently by the region in the real estate market. There is a huge redistribution of interests for the developers who seek profit maximization and buyers who pursue the good quality and reasonable price of the property; developers and buyers are the two major interest groups to determine whether it is harmonious in the real estate market; thus, the model of evolutionary game is constructed based on mutual harmony between developers and buyers.

Condition 2: because the type of profit pursuit and reciprocal type individual will have different mental preference for the payment in the same competition environment, both sides will take different strategies. In order to facilitate the analysis of model, we set the expected target dose on the unit.

Condition3: in the process of the evolutionary game, the developers and buyers must take the corresponding strategies under the situation of considering the decision of the other; and the premise condition of the establishment of the strategy space is that it is free to choose reciprocity model and profit model for both the developers and buyers.

Condition 4: on the basis of the information asymmetry, the strategy space for developers is reciprocity model, profit model; strategy space for buyers is reciprocity model, profit model.

Condition 5: the probability choosing the reciprocity model for developers is p, and the probability to pursue the profit is $(1-p)$, according to the psychological expectations of the buyers; the probability of choosing the reciprocity model for buyers is r, and the probability to pursue the profit is $(1-r)$ according to the psychological expectations of the developers.

Conditions 6: for quantized payment of an expected return, if developers take the strategy of reciprocal type, buyers choose to buy the houses when they achieve certain expected targets. The utility for each other is(a, b); developers take the strategy of reciprocal type and the buyers choose the profit model because they fall to achieve the anticipated target, then the utility of each other is(c, d); similarly, then utility is(e, f) for developers take the strategy to pursue profit style and the buyers decide to buy the house; the utility is $(g, h)$when developers take the strategy to pursue profit pursuit and the buyers take the same strategy.

The game matrix payment formed by the conditions above can be shown in Table 1.

Table 1: The utility matrix of developers and home buyers in the evolutionary game

Probability		Buyers	
		R-T(r)	P-T$(1-r)$
Develpers	R-T(p)	a, b	c, d
	P-T$(1-r)$	e, f	g, h

2.1 The Built of Dynamic Equation

According to Table 1 and basic conditions for establishment of the model, when the buyers have psychological preferences of taking the strategy of reciprocal type, the utility for developers taking the strategy of reciprocal type is as follows:

$$E_H(k) = r \times a + (1-r) \times c \tag{1}$$

When developers take the strategy of profitable type, the utility is as follows:

$$E_L(k) = r \times e + (1-r) \times g \tag{2}$$

The average expected utility for developers is as follows:

$$\overline{E(k)} = p \times E_H(k) + (1 - p) \times E_L(k) \quad (3)$$

according to formula (3) , the dynamic copy equation for developers taking the strategy of reciprocal type is as follows:

$$\begin{aligned} dp/dt &= p[E_H(k) - \overline{E(k)}] \\ &= p \times (1 - p)[E_H(k) - E_L(k)] \quad (4) \\ &= p \times (1 - p)[r(a - e - c + g) + (c - g)] \end{aligned}$$

according to formula (4) the probability of equilibrium is as follows:

$$r^* = (g - c)/[(a - e) + (g - c)] \quad (5)$$

Based on the developers taking the strategy of reciprocal type, the expected utility for buyers taking the strategy of reciprocal type is as follows:

$$E_B(G) = p \times b + (1 - p) \times f \quad (6)$$

the utility of profitable pursuit for buyers is as follows:

$$E_N(G) = p \times d + (1 - p) \times h \quad (7)$$

The average expected utility for buyers is as follows:

$$\overline{E(G)} = r \times E_B(G) + (1 - r) \times E_N(G) \quad (8)$$

according to formula (8), the dynamic copy equation for buyers taking the strategy of reciprocal type is as follows:

$$\begin{aligned} dr/dt &= r[E_B(G) - \overline{E(G)}] \\ &= r \times (1 - r)[p(b - d - f + h) + (f - h)] \end{aligned} \quad (9)$$

the probability of equilibrium is as follows:

$$p^* = (h - f)/[(b - d) + (h - f)] \quad (10)$$

In summary, the probability of evolutionary equilibrium is as follows

$$\frac{(h - f)}{[(b - d) + (h - f)]}, \frac{(g - c)}{[(a - e) + (g - c)]}.$$

2.2 Dynamic Phase Diagram Analysis for an Evolutionary Game in the Real Estate Market

First, analyze copy dynamic equation on the developers taking the strategy of reciprocal type. When $r = r^*$, dp/dt always equals to 0, that is to say, all the values p are in a steady state; when $r\rangle r^*$, two states of $p^* = 0$ and $p^* = 1$ exist, in which $p^* = 1$ is steady strategy. Similarly, when $r\langle r^*$, there are also two states of $p^* = 0$ and $p^* = 1$, in which $p^* = 0$ is the steady strategy. Circumstances phase and steady state are about three states with the changes of p as shown in Figure 1.

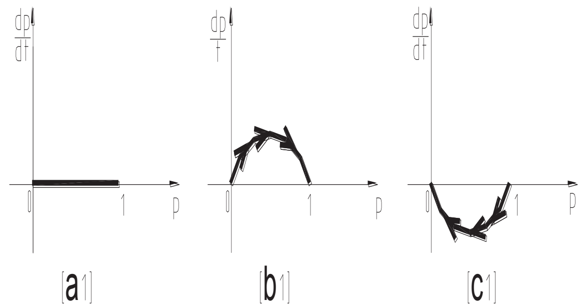

Figure 1: The dynamic phase diagram of developers

Similarly, we analyze the copy dynamic equation for the buyers taking the strategy of reciprocity when the developers have psychological preferences of taking the strategy of reciprocal type; when $p = p^*$, dr/dt always equals to 0, all the values of r are in a steady state; when $p\rangle p^*$, two states of $r^* = 0$ and $r^* = 1$ exist, in which $r^* = 1$ is evolutionary steady strategy; when $p\langle p^*$, both $r^* = 0$ and $r^* = 1$ are in a steady state, in which $r^* = 0$ is the evolutionary steady strategy; thus, circumstances phase and steady state are about three states with the change of r as shown in Figure 2.

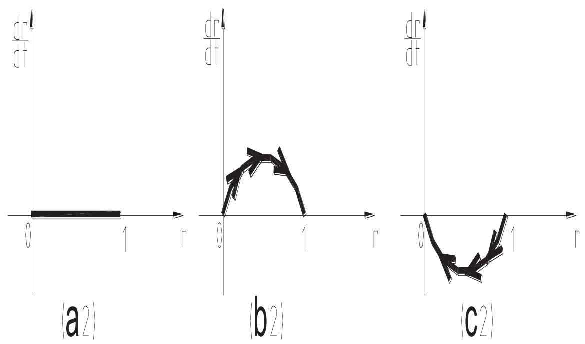

Figure 2: The dynamic phase diagram of buyers

We use the same coordinate relations system to describe the dynamic changing proportion of copy for both developers and buyers taking the strategy of reciprocal type, as shown in Figure 3. In Figure 3, $p = 0, r = 0$ and $p = 1, r = 1$ are steady strategies.

In the duplicate dynamic evolutionary game, when the initial condition is located in area II, it will converge to the stability of strategies $p = 1, r = 1$. That is to say, when the initial condition is located in area IV , it will converge to the stability of strategies $p = 0, r = 0$, on the basis of mutual benefit both the developers and buyers taking the strategy ofreciprocal type, reciprocal type. Both of the developers and buyers take the strategies ofprofitable type, profitable type. When the initial condition is located in areas I and III, the system usually can converge to the evolutionary steady strategy $p = 0, r = 0$, according to the evolution of psychology in the mutual preference.

Therefore, with the point p^*,r^* reducing, the areas I, II and III will increase, and the system will converge to the steady state of strategies $p = 1, r = 1$. The area of the probability of evolutionary steady strategy of reciprocal type, reciprocal type increases for both sides. It can also be said that when p^* and r^* are relatively small in the circumstances, both sides of the game take the reciprocal type strategies finally . In order to make p^* and r^* reducing, we should make the value of $(g-c)$ decrease, the values of $(a-e)$ and $(b-d)$ increasing, and $(h-f)$ decrease.

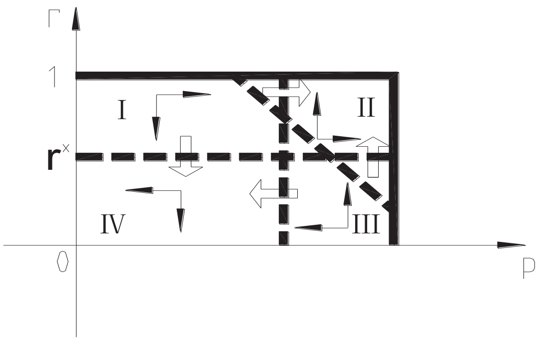

Figure 3: The dynamic phase diagram for both developers and buyers

3 Complex Dynamics Analysis of Equation

For a deeper step analysis, we make $p = x$, $r = y$, $f - h = q$, $c - g = n$, $(b - d) + (h - f) = p$, $(a - e) + (g - c) = m$. The original equation is transformed into differential equation:

$$\begin{cases} \dot{x} = mxy + nx - mx^2y - nx^2 \\ \dot{y} = pxy + qy - pxy^2 - qy^2 \end{cases} \tag{11}$$

3.1 The Equilibrium Point and Stability Analysis

The equilibrium points of the system is as follows: $E1(0,0)$, $E2(1,0)$, $E3(0,1)$, $E4(-q/p, -n/m)$ $E5(1,1)$.

The Jacobian matrix is:

$$J = \begin{bmatrix} my+n-2mxy-2nx & mx - mx^2 \\ py - py^2 & px+q-2pxy-2qy \end{bmatrix}$$

According to Routh-Hurwitz Criteria, the characteristic polynomial is

$$f(\lambda) = \lambda^2 + a_1\lambda + a_2$$

the conditions for the local stability of equilibrium are $a_1 > 0$ and $a_2 > 0$.

The characteristic polynomial of equilibrium$E1(0,0)$ is

$$f(\lambda) = (\lambda - n)(\lambda - q) = \lambda^2 - (n + q)\lambda + nq,$$

the conditions for the local stability of equilibrium$E1(0,0)$ are $-(n+q) > 0$ and $nq > 0$;

Similarly, the characteristic polynomial of equilibrium E2(0,1) is

$$\begin{aligned} f(\lambda) &= [\lambda - (m+n)](\lambda + q) \\ &= \lambda^2 + [q - (m+n)]\lambda - q(m+n) \end{aligned}$$

the conditions for the local stability of equilibrium E2(0,1) are $[q - (m+n)] > 0$ and $-q(m+n) > 0$;

The characteristic polynomial of equilibrium $E3(1,0)$ is

$$\begin{aligned} & f(\lambda) = [\lambda - (m+n)][\lambda - (p+q)] \\ & = \lambda^2 - [(m+n) + (p+q)]\lambda + (m+n)(p+q), \end{aligned}$$

the conditions for the local stability of equilibrium $E3(1,0)$ are $-[(m+n) + (p+q)] > 0$ and $(m+n)(p+q) > 0$;

The characteristic polynomial of equilibrium $E4(-q/p, -n/m)$ is

$$f(\lambda) = \lambda^2 - (\frac{mq^2 - mq}{p})(-\frac{pn^2 - pmn}{m^2}).$$

Obviously, $E4(-q/p, -n/m)$ is unstable equilibrium.

The polynomial characteristic of the equilibrium $E5(1,1)$ is

$$\begin{aligned} & f(\lambda) = [\lambda + (m+n)][\lambda + (p+q)] \\ & = \lambda^2 + (m+n+p+q)\lambda + (m+n)(p+q) \end{aligned}$$

the conditions for the local stability of equilibrium $E5(1,1)$ are $(m+n+p+q) > 0$,$(m+n)(p+q) > 0$.

Explained from the economics of evolutionary game model, both developers and buyers pursue maximum profit in equilibrium point $E1(0, 0)$ in the real estate market. Persons who take the strategy of reciprocal type withdraw from the market completely. If they don't quit, the reciprocal game subjects are conducting the business under the condition of the absolute loss, unless there is a national financial incentives or rewards given to the subjects who choose the mutual decision, or impose punitive measures on the actors to pursue maximum profit, otherwise, they will exit from the real estate market as rational decision makers. In the equilibrium points $E2(0, 1)$ and $E3(1, 0)$, a party as behavior subject take the reciprocal type strategy, and the other party take simple maximize profit strategy. In equilibrium point $E4(-q/p, -n/m)$, profit margin is zero for both behaviors, and they reach to balance temporarily. In equilibrium point $E5(1, 1)$, both behaviors take mutual type strategy. Evolutionary game of reciprocal type is formed, but this is only a temporary balance. The parameters change because of the change of the external factors, which is equivalent to add disturbance in the system. Because equilibrium point $E5(1, 1)$ is the most promising result for the built of reciprocal evolutionary game model. Therefore, only the stability of equilibrium point $E5(1, 1)$ is studied here.

3.2 Complex Dynamic Characteristics Analysis of the Model

We study the effects of parameters on the stability of the equilibrium points. In order to identify the chaotic behavior, we present examples of bifurcation diagram, Lyapunov exponent, and corresponding chaotic strange attractors with the change of parameters p, q, r, n. The initial value of x is selected as 0.88, and the initial value of y is selected as 0.98; that is to say, the initial probability to take reciprocal strategy for developers is 0.88, and the probability for buyers is 0.98.

(1) We analyze the influence of parameters on the stability of the system.

(a) Fix parameters $m = 2, n = 0.5, q = -3$. The initial value of x is 0.88, and the initial value of y is 0.98; Figure 4 is bifurcation diagram with the change of p for developers; Figure 5 is bifurcation diagram with the change of p for buyers; Figure 6 is the corresponding Lyapunov exponent with the change of p.

From Figure 4, in the range of $6 < p < 8$, the developers are steady. It means that all the developers take the reciprocity strategy.

Accordingly from Figure 5, in the range of $p < 6.32$, the probability to take reciprocity strategy for buyers has been stable at the equilibrium point $(1, 1)$; in the range of $p > 6.32$, equilibrium point becomes unstable, and the period - doubling bifurcation phenomenon appears, namely 2 cycles; in the range of $7.08 < p < 7.23$, 4 cycles appear; when the value of p is larger than 7.23, the system goes into chaos, and the buyers go to a chaotic state in the real estate market.

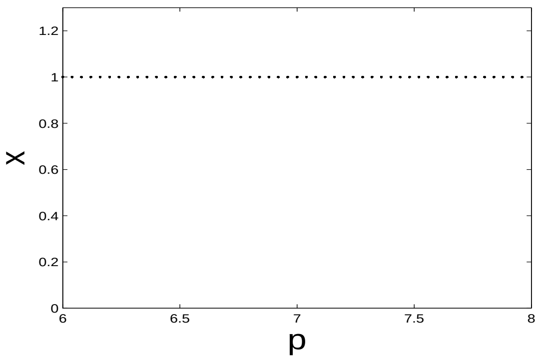

Figure 4: Bifurcation diagram with the change of p for developers with $m = 2, n = 0.5, q = -3$

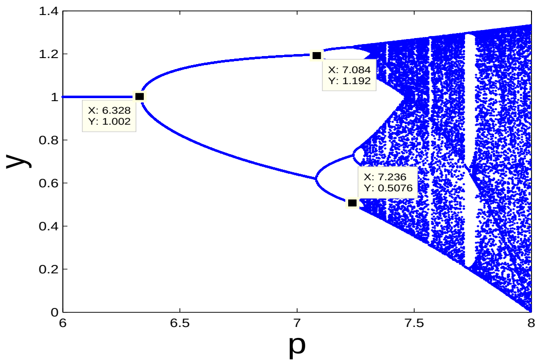

Figure 5: Bifurcation diagram with the change of p in the system for buyers with $m = 2, n = 0.5, q = -3$

From Figure 6, in the range of $p < 6.32$, Lyapunov index of the system is less than zero. The periodic bifurcation appears when p is equal to 6.32, and Lyapunov index equals to zero. When p is larger than 7.23, positive Lyapunov index exists, and the system goes into chaos. Obviously, Lyapunov index and bifurcation diagrams are consistent.

Compare Figure 5 with Figure 4, it shows that it is earlier for developers to enter a state of chaos than buyers with the increase of value of p. From analysis on phase diagram, because $(b - d) + (h - f) = p$ in the system, the buyers show greater

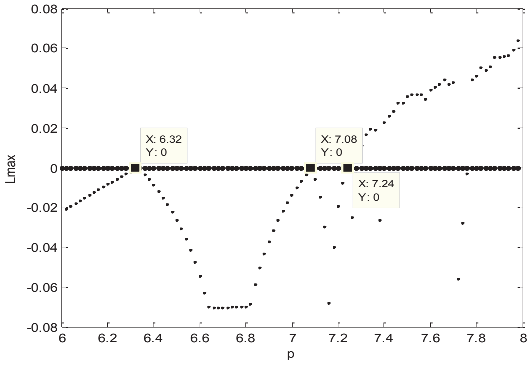

Figure 6: The Lyapunov index with the change of p in the system with $m = 2, n = 0.5, q = -3$

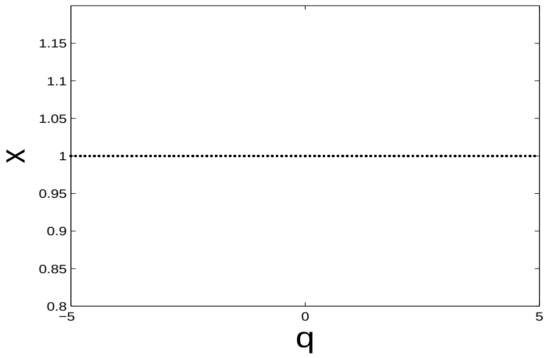

Figure 7: Bifurcation diagram with the change of q in the system for developers with $m = 2, n = 0.5, p = 0.005$

sensitivity than buyers under certain preconditions of$q = -(h - f)$. The significance of the system is that when the value of $q = -(h - f)$ is certain, buyers are sensitive to the value of p very muchand they are more interested in the increase of $(b - d)$. For$(b - d)$which means the income difference for the buyers to choose the mutual beneficial strategy and strategy of profit maximization, when the developers take the mutual beneficial strategy. According to the previous phase diagram, it is required to increase gradually for the value of p, thus the system will move towards the expected direction for both behaviors. That is to say, when the value of $q = -(h - f)$ is certain, the value of $(b - d)$ must increase gradually. It means that when the developers take the mutual beneficial strategythe income becomes more and more to choose the mutual beneficial strategy than the profit maximization strategy for the buyers. However, from Figure 5 and Figure 6, this theory is effective when the value of p increases in a certain range. The system will enter a chaotic state when the value of p exceeds 7.24 and $(b - d)$ exceeds 4.24. It has not exceed 4.24 of the income difference for the buyers to choose the mutual beneficial strategy and the profit maximization strategy when the system moves to the expected direction for both behaviors as mutual strategic, under the precondition of the developers taking the mutual beneficial strategy. As can be seen from Figure 4, developers have been in a state of chaos when the value is between six and eight.

(b) Fix parameters $m = 2, n = 0.5, p = 0.005$. The initial value of x is 0.88, and the initial value of y is 0.98; Figure 7 is bifurcation diagram with the change of q for developers, Figure 8 is bifurcation diagram with the change of q for buyers, Figure 9 is the corresponding Lyapunov index of the system.

From Figure 7, it has been stable at $x = 1$ for the probability of developers taking the mutually beneficial strategy in the range of $[-5, 5]$. This means that developers appear not so sensitive to the change of q.

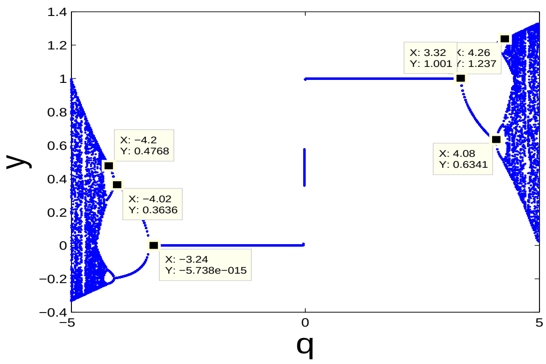

Figure 8: Bifurcation diagram with the change of q in the system for buyers with$m = 2, n = 0.5, p = 0.005$

As it can be seen from Figure 8, it has been stable at equilibrium $y = 1$ in the system in the range of [0,3.32] for the probability of buyers taking the mutually beneficial strategy. When q is larger than 3.32, the equilibrium point gets into an unstable state. Thus, 2 cycles appear when q=3.32, and 4 cycles appear when $q = 4.08$, 8 cycle...; when q is larger than 4.26, the buyers get into chaos in the system. What is deserved to be mentioned is that when q is in the range of [-5,0], the behavior appears so similar in the range of [0,5]. Actually, it is stable at $x = 0$ for the buyers taking the mutually beneficial strategy when q is in the range of [-3.24,0]; 2 cycles appear when $q = 3.24$, and 4 cycles appear when $q = -4.2$. The buyers rushes into chaos from the point $x = 0$.

We can draw the conclusion that buyers taking the mutually beneficial strategy are in chaotic state for q in the range of [-5,0] in the whole process, which a special chaotic phenomenon in the system.

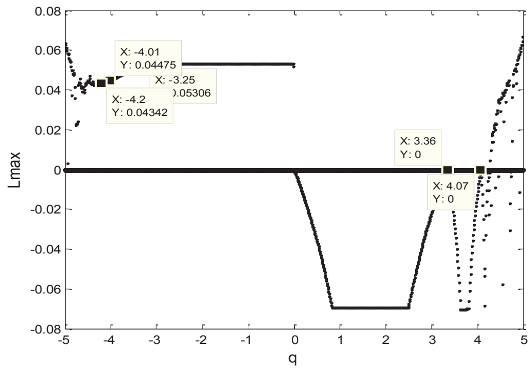

Figure 9: The Lyapunov index with the change of q in the system with $m = 2, n = 0.5, p = 0.005$

Accordingly, from Figure 9 it can be seen that, when q is in the range of [-5,0], the Lyapunov index is positive, and the system is chaotic. Lyapunov index of the system is always less than zero within the range of $[0, 3.36]$, and the system is in a steady state. When $q = 3.36$ and $q = 4.07$, Lyapunov index equals to zero, and 2 cycles and 4 cycles appear. When q is larger than 4.33, positive Lyapunov index appear once again, and the system gets into chaos a second time. Obviously, Figure 7, Figure 8 and Figure 9 are consistent.

From the payoff matrix built in the system, the more of the differences between buyers taking mutual beneficial strategies and making profit maximization of income as their targets, the more beneficial to the evolution for the system towards the expected direction, on the premise of developers taking the strategy of pursuing maximization of profit in the condition of the other parameters fixed. However, if the value of q is beyond 4.33, that is to say, if the difference is beyond a certain range for the buyers taking the two strategies, the system gets into chaos and markets are out of control, hindering the formation of the reciprocity model.

(c) Fix parameters $n = 0, q = 4.6, p = 7$. Make the initial value of x being 0.88, the initial value of y being 0.98; Figure 10 is the bifurcation diagram with the change of m for developers ; Figure 11 is the bifurcation diagram with the change of m for buyers ; while Figure 12 is corresponding :yapunov index.

Figure 10 shows that when m is smaller than 2.81, the probability of mutually beneficial strategy has been in a steady state in $x = 1$ for developers

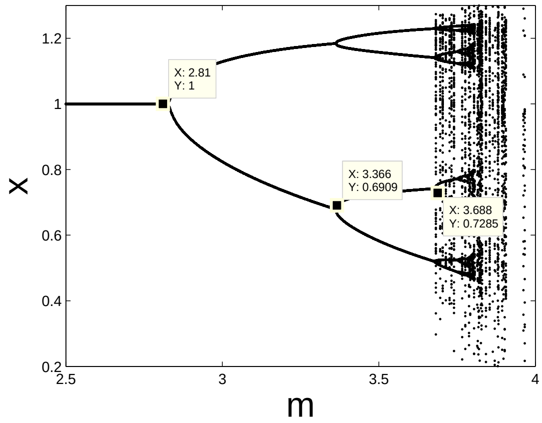

Figure 10: Bifurcation diagram with the change of m in the system for developers with $n = 0, q = 4.6, p = 7$

to choose happiness; when m is larger than 2.81, the probability has become unstable for developers to choose happiness, and the periodic doubling bifurcation phenomenon appears, such as 2 cycles at $m = 2.81$, 4 cycles at $m = 3.3.37$, 8 cycle...; when m is greater than 3.688, the developers will get into chaos.

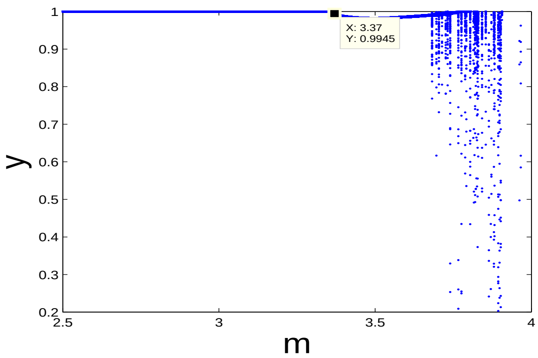

Figure 11: Bifurcation diagram with the change of m in the system for buyers with $n = 0; q = 4.6, p = 7$

As it can be seen from Figure 11, when m is less than 3.37, the probability of mutually beneficial strategy has been stable in $y = 1$ for buyers to choose happiness. When m is larger than 3.37, the equilibrium become unstable and the periodic doubling bifurcation phenomenon appears, such as 2 cycles, 4 cycles, 8 cycle.... When m is larger than 3.688, the system gets into chaos.

In front of analysis on phase of evolutionary game diagram, the greater of the $(a - e) + (g - c) =$

m, the more that the evolution towards the expected direction. If $-(g-c) = n$ is fixed, the larger of $(a-e)$, the more may lead developers and buyers towards the expected direction. The Figure 10 and Figure 12 show when $(a-e)$ is larger than 3.688, the buyers will enter a state of chaos.

Figure 10 shows that when $(a - e)$ is larger than 2.81, developers will get into chaos. Compared with Figure 11, it's said that developers are more sensitive to parameter m in the early stage and will get into doubling bifurcation phenomenon such as two periods, four cycles earlier. That is because the developers have a better professional knowledge and rich experience and been more capable to adopt to economic method when emergency occurs, and the developers will get into the bifurcation earlier and can delay the chaotic state through the control compared with the buyers.

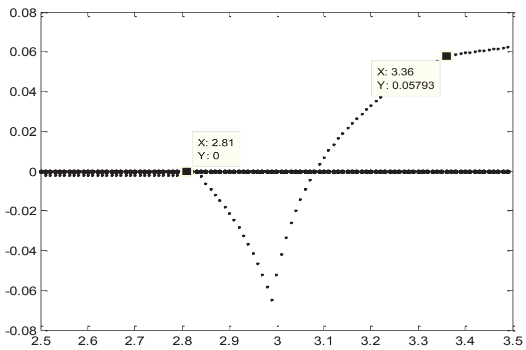

Figure 12: The corresponding Lyapunov index with$n = 0, q = 4.6, p = 7$

Accordingly, Figure 12 shows that when m is less than 2.81 , the largest Lyapunov index of system is always less than zero. The period of doubling bifurcation occurs when m is equal to 2.81, and the Lyapunov index equals to zero in the system. When m is larger than 3.08, the Lyapunov index is positive and the system gets into chaos. Obviously, the results of Figure 12, Figure 10 and Figure 11 are consistent to each other.

(d) Fix the parameters $m = 3.8, q = 4.6, p = 7$. Make initial value of x being 0.88, and y being 0.98. Figure 13 is the bifurcation diagram with the change of n for developers with $m = 3.8; q = -4.6; p = 7.9$; Figure 14 is the bifurcation diagram with the change of n for buyers with $m = 3.8; q = -4.6; p = 7.9$; Figure 15 is the corresponding Lyapunov index with $m = 3.8; q = -4.6; p = 7.9$.

As it can be seen from Figure 13, when $n$is larger than zero, developers have entered the periodic

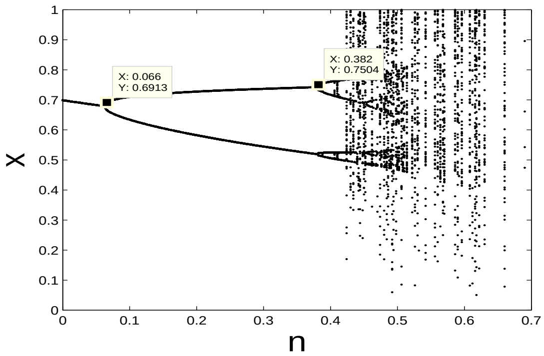

Figure 13: Bifurcation diagram with the change of n in the system for developers with $m = 3.8; q = -4.6; p = 7.9$

doubling bifurcation, namely, 2 cycles , 4 cycles at $n = 0.066$, 8 cycle... ; when n is larger than 0.382, developers get into the chaos.

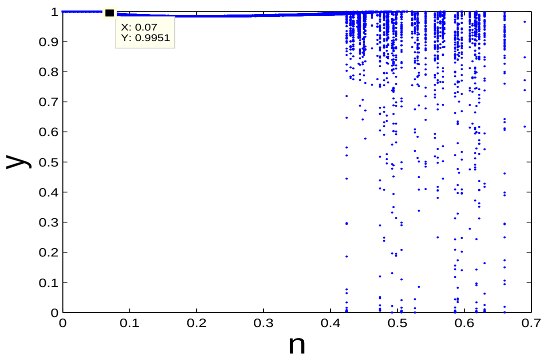

Figure 14: Bifurcation diagram with the change of n in the system for buyers with $m = 3.8; q = -4.6; p = 7.9$

From Figure 14 it can be seen that buyers and developers get into the chaos phase almost at the same time.However, it is the second bifurcation for developers, and the first bifurcation for buyers at the point $n = 0.066$. In other words, developers go into an unstable state earlier than buyers. Developers become more sensitive to the value of n, rushing into an unstable state, out of control more easily than buyers. Compared with payoff matrix built in the model, the smaller of n , the more evolution towards the expected direction for both behaviors in the system in the conditions of other parameters fixed. Namely to meet conditions of $n = c - g$ growing up all the time, the more of return value for developers taking the mutual beneficial strategies compared with the strategies of the simple pursuit of profit maximization, the closer to the target of making evolution towards the expected directions, under the premise of the maximization of profit for buyers. From Figure 13 and Figure 14, it can be seen that $n = 0.382$ is a cut-off, when n is greater than 0.382, system gets into chaos, and behaviors are out of control. Similar analysis like other parameters of p, q, m, Lyapunov index of the system for n is also negative within the range of certain parameters. When it is larger than certain parameters, system rushes into a chaotic state.

As it can be seen from the bifurcation diagram and the Lyapunov index, buyers are much easier to appear an unsteady state, and produce chaos compared with developers. The decision for developers can stay in a steady state all the time because of rich experience. When parameters m, n change, the developers go into an unsteady state or chaos earlier than buyers, and buyers remain in a steady state for a longer time relatively. From the payoff matrix Table above, we know that $p = (b - d) + (h - f), q = f - h$, and b, d, h, f represent the payoff of buyers under the premise of the different decision-making for developers. At the same time, $m = (a - e) + (g - c), n = c - g$ are the reduction to the payoff matrix, and a, e, g, c represent the payoff of buyers under the premise of different decision-making for developers. The values of the payoff b, d, h and f, are the main factors which affect the buyers, and they decide whether the state is steady, periodic , or chaotic. a, e, g, c are the main factors which influence the stability of developers, and whether the developers will rush into chaos. Therefore, the decisions of developers are mainly affected by earnings of developers. Their own profit is the main factor which affects developers rushing into chaotic state. while the decision of the buyers is mainly affected by the payoff of the buyers, whose payoff is the main factor deciding the buyers whether go into a chaotic state.

(2) Fix four parameters of p, q, r, n. It can be seen from Figure 15 that t not only stands for evolution rate in the system, but also said that the sensitive degree for the developers and buyers to the change of market.

That is to say, both developers and buyers are in a steady state in the range of [0,0.66] under the condition of constant fixed parameters with the increase of t. The system goes into 2 cycles bifurcation when $t = 0.66$, and 4 cycles bifurcation appear when $t = 0.818$. The system rushes into chaos when $t = 0.85$ at last. Namely the faster of the evolution rate, the more complicated of the real estate market. Many factors are filled with uncontrollability. It becomes more and more difficult to make right decisions for buyers. It becomes more easier to fall into a state of chaos for the market. It also can be seen that developers are always in a steady state from 0.6 to 1. This means that developers have ability to adopt the change of market because of knowledge and experience under the condition of fixed parameters. Compared with developers, buyers tend to lose control and rush into chaos because of lack of experience.

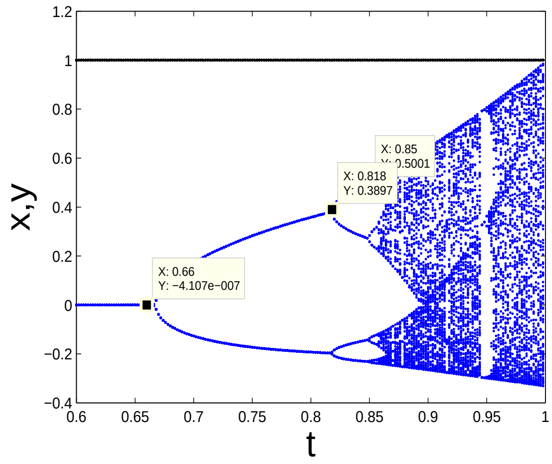

Figure 15: Bifurcation diagram with the change of t with $m = 2; n = 0.5; q = -3; p = 0.005;$

3.3 The Strange Attractor of the System

Figures below show strange attractors with varies of parameters (the decision maker's income level) under the condition of initial values $x = 0.88, y = 0.98$; Figure 16 shows the strange attractors when the values of p are $7.2; 7.5; 7.8$ respectively, under the conditions of $q = 5, m = 3.7, n = 0.4$; Figure 17 shows the strange attractors when the values of q are $-4.8; -4.6; -4.4$ respectively, under the conditions of $m = 3.7, n = 0.4, p = 7.2$; Figure 18 shows the strange attractors when the values of m are $3.7; 3.85, 3.95$ respectively, under the conditions of $n = 0.42, q = 4.8, p = 7.2$; Figure 19 shows the strange attractors when the values of n is $0.45, 0.5, 0.6$respectively, under the conditions of $m = 3.7, q = 5, p = 7.2$.

3.4 The Power Spectrum of Variables of the System

According to numerical simulation of the system, we take the cycle method to estimate the power spectrum of variables of the system, as shown from Figure 20 to Figure 24 with different parameters.

From the numerical simulation, the result shows that no matter how the initial payment matrix

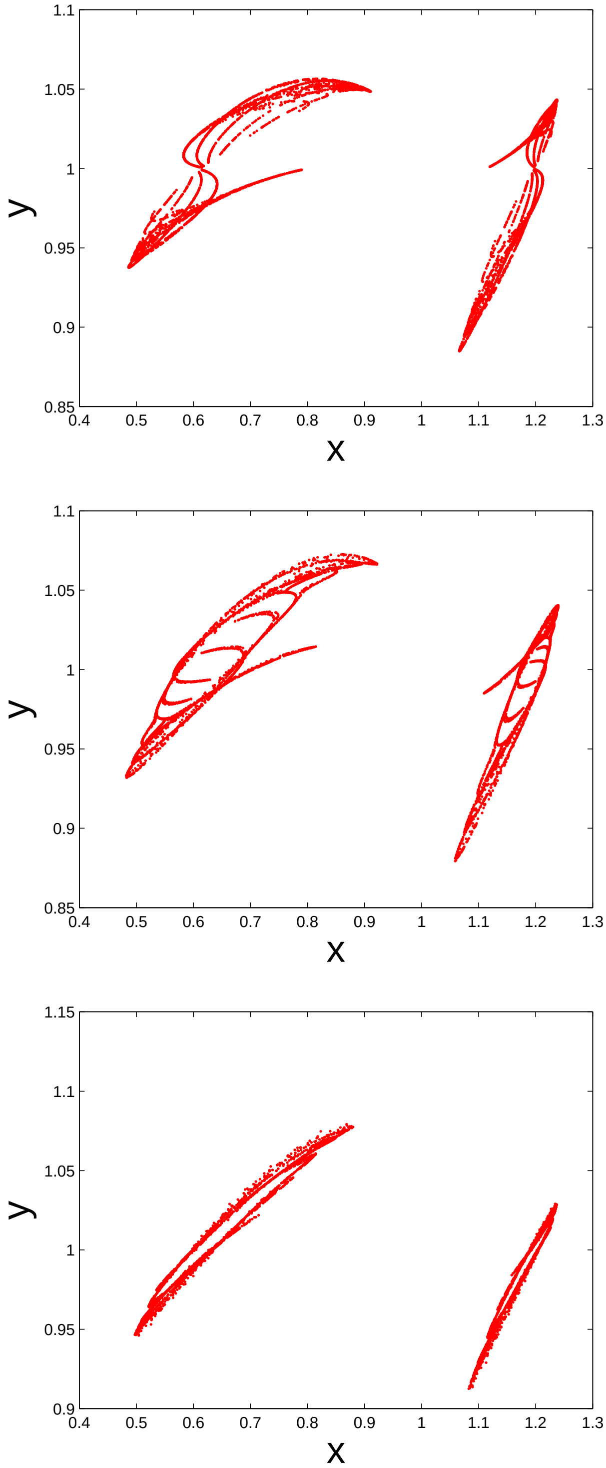

Figure 16: Strange attractor with $(p = 7.2, p = 7.5, p = 7.8)$

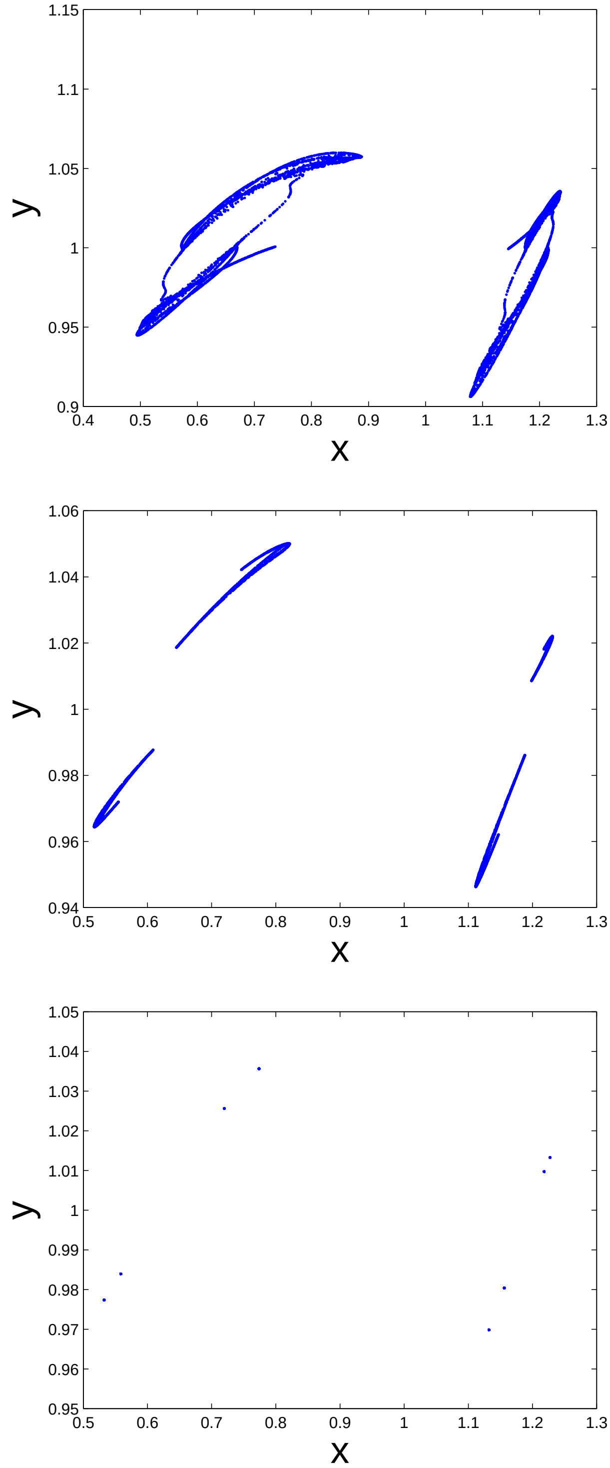

Figure 17: Strange attractor with $(q = -4.8, q = -4.6, q = -4.4)$

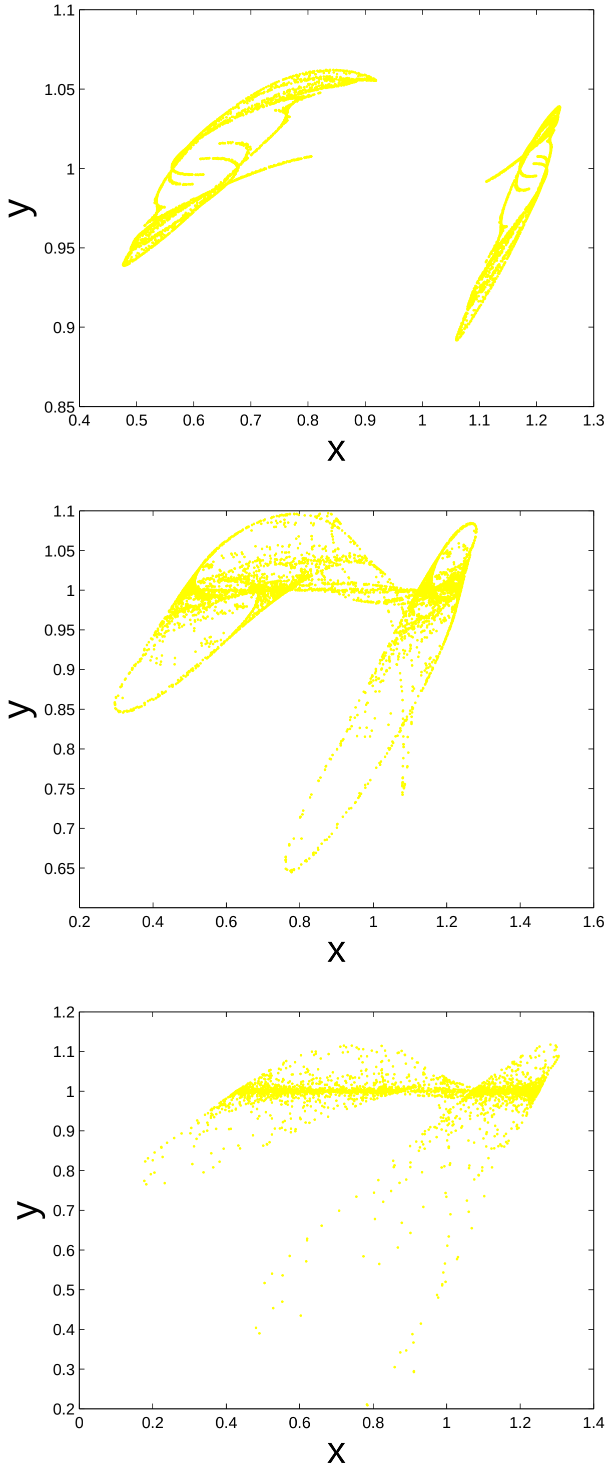

Figure 18: Strange attractor with$(m = 3.7, m = 3.8, m = 3.95)$

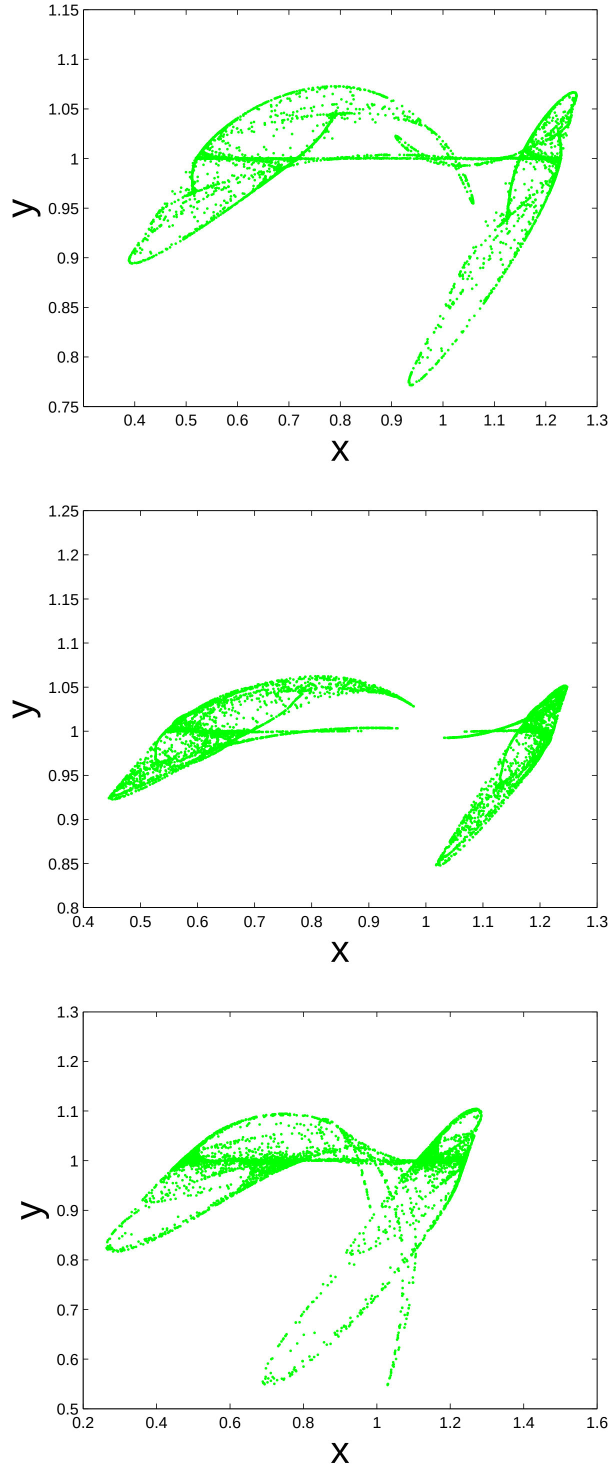

Figure 19: Strange attractor with $(n = 0.45, n = 0.5, n = 0.6)$

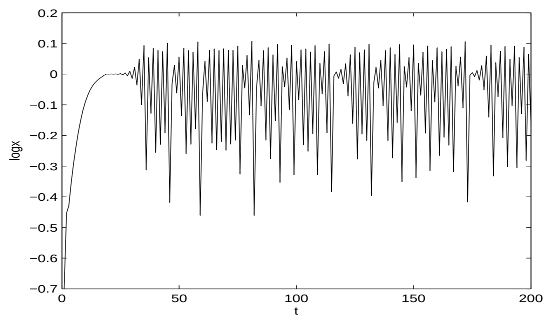

Figure 20: The power spectrum of developers where $m = 3.8; n = 0.45; q = -4.6; p = 7.9$

parameters change, namely the values of the initial parameter m, n, p and q can traverse the entire area with the passage of time, the probability will always been restricted in a certain range for behaviors in the system.

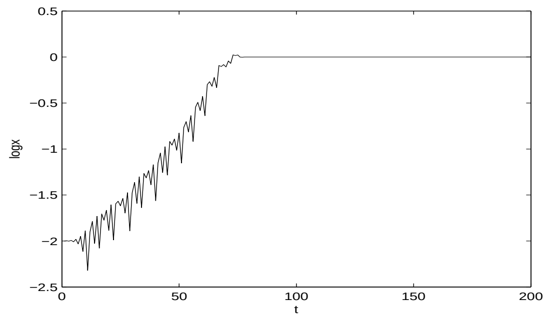

Figure 21: The power spectrum of developers where $m = 3.8; n = 0; q = -4.6; p = 0.005; x = 0.01; y = 0.003$

The diagram above has proved the evidence of chaos in the system: bounded-ness and ergodicity. From the external point of view, the chaotic movement is of a certain value space, and that is the chaotic attractor and the bounded-ness of chaos.

That is to say, the chaotic movement is no obvious regularity from the inside the system, and of certain value space from the view of the external point, namely the chaotic attractor and the bounded-ness of chaos. Especially, Figure 22 to Figure 24 show that power spectral density will be increased with the increase of the initial values under the conditions of the initial parameters $m = 2; n = 0.5; q = 3; p = 7$, namely the initial values vary from $x = 0.5; y = 0.63$ to $x = 0.88; y = 0.98$. Therefore, there are some ways to control chaos.

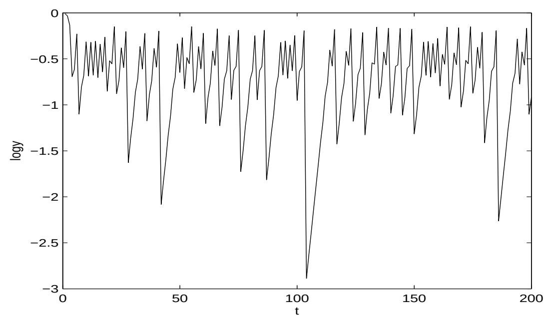

Figure 22: The power spectrum of developers where $m = 3.8; n = 0.5; q = -4.6; p = 0.005; x = 0.88; y = 0.98$

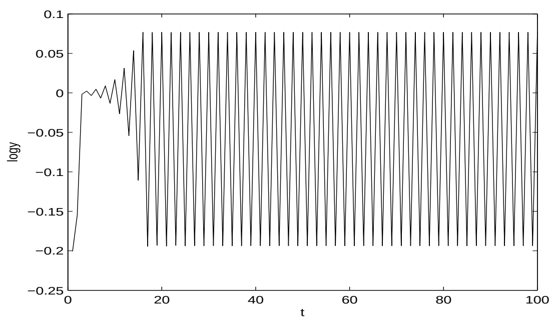

Figure 23: The power spectrum of developers where $m = 2; n = 0.5; q = -3; p = 7; x = 0.5; y = 0.63$

3.5 The Butterfly Effect of the Chaotic Movement

The characteristics of butterfly effect of chaotic movement differ from other steady movement. Slight changes of the initial values can make the adjacent orbital to separate in the exponential form after the evolution of multiple cycles. Through the analysis above, we know that the adjustment changes of the initial payment of the benefit of matrix can make the system go to a chaotic state. The sensitivity of initial values is the most obvious characteristic of chaos. We investigate how the slight changes of initial probability values effect on the system for both developers and buyers taking the strategy of reciprocity .

3.6 The Effect of Change of the Initial Probability for Buyers

Firstly, we examine how the system changes when the initial probability increases from $y = 0.3$to $y = 0.3001$ for buyers taking the mutually beneficial

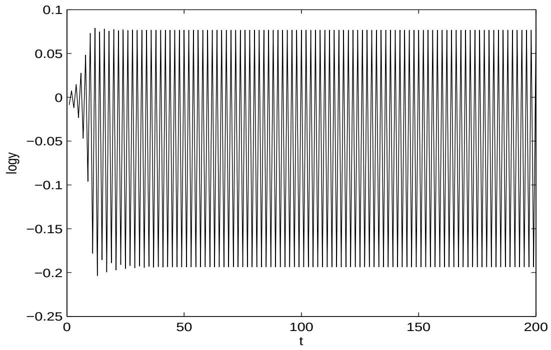

Figure 24: The power spectrum of developers where $m = 2; n = 0.5; q = -3; p = 7; x = 0.88; y = 0.98$

strategy.

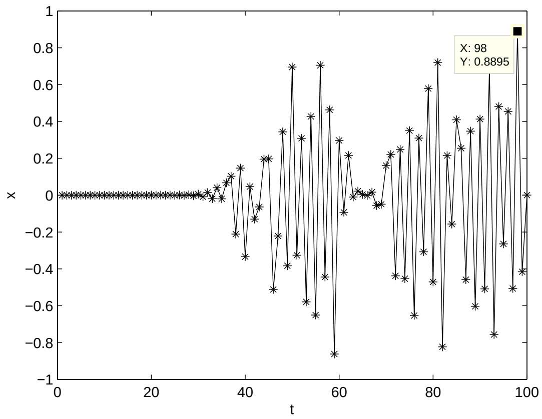

Figure 25: Sensitivity of system to slight changes of developers taking the strategy of reciprocity with $m = 3.8; n = 0.45; q = -4.6; p = 7.9$

In Figure 25 we can see that the varies are similar in the initial period of time, and difference increases with the increase of iteration times, which directly expresses that the system has sensitive dependence on initial values. When tiny change of the initial value is 0.0001, the change will be amplified after several iterations for buyers taking the strategy of reciprocity. At last, the scope of change is (0.25, 0.2) for the probability of buyers taking the mutually beneficial strategy. As can be seen from Figure 26, the slight varies of probability for buyers taking the strategy of reciprocity will have dramatic impact on the probability of developers taking mutually beneficial strategy, and tiny variations are also amplified at the early stage. That makes the probability of developers taking mutually beneficial strategy fluctuate within

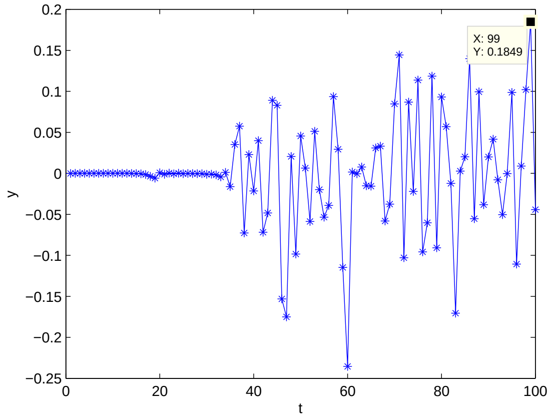

Figure 26: Sensitivity of system to slight changes of buyers taking the strategy of reciprocity with $m = 3.8; n = 0.45; q = -4.6; p = 7.9$

the scope of $(-1, 1)$ in the process of evolutionary game.

3.7 The Effect of Change of Initial Probability of Developers

Now we examine how the system changes when the initial probability increases from $x = 0.2$ to $x = 0.2001$ for developers taking the mutually beneficial strategy as shown below.

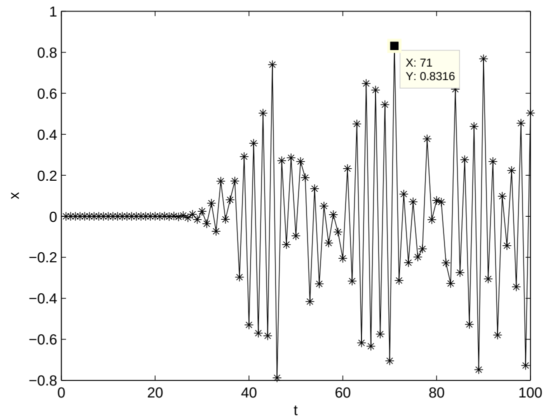

Figure 27: Sensitivity of system to slight changes of buyers taking the Strategy of reciprocity with $m = 3.8; n = 0.45; q = -4.6; p = 7.9$

In Figure 27, the variables are similar in the initial period of time. Obviously the differences increase gradually with the increase of iteration times. Slight change will be amplified, when tiny changes of the initial value is 0.0001 for developers taking the strategy of reciprocity after several iterations. This makes the probability fluctuate within the scope of $(-1, 1)$ for buyers taking mutually beneficial strategy. As can be seen from Figure 28, the slight variations of probability for developers taking the strategy of reciprocity will have dramatic impact on the probability of buyers taking mutually beneficial strategy, and tiny variations are also amplified at the early stage. This makes the probability fluctuate within the scope of$(-0.4, 0.3)$ for developers taking mutually beneficial strategy.

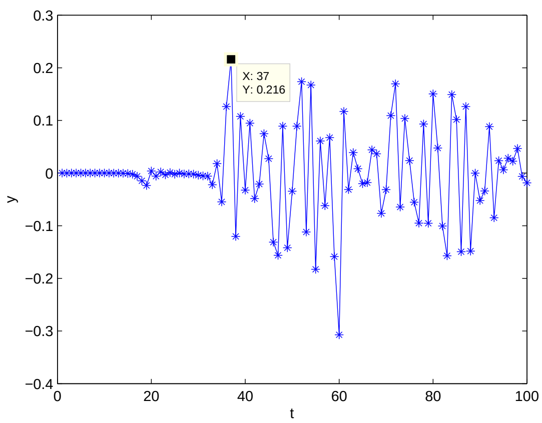

Figure 28: Sensitivity of system to slight changes of developers taking the strategy of reciprocity with $m = 3.8; n = 0.45; q = -4.6; p = 7.9$

From Figure 25 to Figure 28, it is shown that when it is in a state of chaos. Tiny changes of probability for taking the mutually beneficial strategy for behaviors will have an important effect on both of the parties, and the future evolutionary direction has a strong sensitivity to the changes in the current period. When the main body of purchasing behaviors is in this state, the subject will have difficulty in making accurate judgment for the future.

At the same time from Figure 25 to Figure 28 it can be seen that there are more effects on the developers than the buyers at the initial values, and the reason is asymmetric information. Compared with buyers, developers have more professional knowledge to analyze the market. Therefore, developers can make earlier decisions, and buyers take relative rational actions after observing the behaviors of developers. Strategies of the buyers are better than that of developers. Obviously, chaos appears and system is out of control because different parameters are taken in the system. It also draws a conclusion that not all the initial game matrix can evolve to points of stability for mutual or profit maximization. In evolutionary game, we should avoid the scope which produces chaos when initial parameters are taken. Therefore, the initial revenue is a key factor which not only decides the direction of the evolution, but also make system go into chaos. As a result, guidance is needed to introduce punitive measures in some of evolutionary game.

The sensitivity analysis shows that when the tiny changes of initial values for buyers are from 0.3 to 0.3001, the wave scope is [0,0.8895]for developers, and [0,0.1849] for buyers; when the tiny changes of initial values of developers is from 0.2 to 0.2001, the wave scope is [0,0.8316] for developers and [0,0.0216] for buyers. Therefore, when the system is in a chaotic state, whether developers or buyers having slight changes can have influence on each other. As it can be seen from the numerical simulation above, whether the tiny changes are for developers or for buyers, the effects on the developers is much greater than buyers. What indicates that when chaos occurs, it has much more serious effects on developers than buyers, which is mainly because there are the characteristics of capital intensive in the real estate market, and developers are in a dominant position, holding much more money. Every decision will have an important influence on the result. Compared with developers, the buyers are relatively scattered, holding small amount of money, and the effect of chaos is relatively weaker on them.

4 Chaos control

What should be noted is that chaotic state makes many negative impacts of the system that we need to apply some measures to delay its occurrence. In this paper, the linear feedback control is taken to delay or control the chaos, and there are three characteristics for the linear feedback control: firstly, both unstable periodic orbits and the fixed point can be controlled by this method, in other words, it can achieve control for any solution in the original system; secondly, it has the characteristics of anti-interference, so the linear feedback control can not be affected by the impact of the slight changes of parameters; finally, there are certain difficulties for some systems because of the exist of interaction of many system variables.

Firstly, we have a transformation at the equilibrium point E4 as the following rules:

$$\begin{cases} X = x + \frac{q}{p} \\ Y = y + \frac{n}{m} \end{cases} \tag{12}$$

Then the system can be rewritten as follows:

$$\begin{cases} \dot{X} = m(X-\frac{q}{p})(Y-\frac{n}{m}) - m(X-\frac{q}{p})^2(Y-\frac{n}{m}) \\ \quad +n(X-\frac{q}{p}) - n(X-\frac{q}{p})^2 \\ \dot{Y} = p(X-\frac{q}{p})(Y-\frac{n}{m}) - p(X-\frac{q}{p})(Y-\frac{n}{m})^2 \\ \quad +q(Y-\frac{n}{m}) - q(Y-\frac{n}{m})^2 \end{cases} \tag{13}$$

We assume that the controlled system is written as follows:

$$\begin{cases} \dot{X} = m(X-\frac{q}{p})(Y-\frac{n}{m}) - m(X-\frac{q}{p})^2(Y-\frac{n}{m}) \\ \quad +n(X-\frac{q}{p}) - n(X-\frac{q}{p})^2 - kX \\ \dot{Y} = p(X-\frac{q}{p})(Y-\frac{n}{m}) - p(X-\frac{q}{p})(Y-\frac{n}{m})^2 \\ \quad +q(Y-\frac{n}{m}) - q(Y-\frac{n}{m})^2 - kY \end{cases} \tag{14}$$

The Jacobian matrix of the controlled system is as follows:

$$J = \begin{bmatrix} -k & -\frac{qm}{p} - \frac{mq^2}{p^2} \\ -\frac{np}{m} - \frac{n^2 p}{m^2} & -k \end{bmatrix}$$

when the characteristic polynomial is

$$f(\lambda) = \lambda^2 + a_1\lambda + a_2$$

the conditions for the local stability of equilibrium are $a_1 > 0$ and $a_2 > 0$.

$$\begin{cases} a_1 = 2k \\ a_2 = k^2 - \frac{n^2q^2p+n^2q}{m} + npq^2 + nq \end{cases} \tag{15}$$

the local stability of equilibrium point E4 can be gained as follows:

$$\begin{cases} 2k > 0 \\ k^2 - \frac{n^2q^2p+n^2q}{m} + npq^2 + nq > 0. \end{cases} \tag{16}$$

When $q = -3, p = 5, m = 2, n = 0.5$, the control gain $k = -1$, the characteristic values are $\lambda_1 = 1.0000 + 0.8660i, \lambda_2 = 1.0000 - 0.8660i$. At this time, the system is in a chaotic state. When we choose the control gain $k = 0.6$, the characteristic values are: $\lambda_1 = -0.6000+0.8660i, \lambda_2 = -0.6000-0.8660i$. Obviously, the system turns into a steady state. Thus, the chaos can be controlled by linear feedback control, and achieve the goal of driving the system to a steady state, inhibiting the system from going towards the desired direction.

Then we select $k = 0.2$ and check the effect of the control.

Figure 30 is bifurcation diagram with the change of p for buyers with $m = 2; n = 0.5; p = -3$ with the linear controller. Compare Figure 30 with Figure 4, it is easy to see that the first bifurcation point is delayed from 6.328 to 6.91, and the point which rushes into chaos is delayed from 7.326 to 7.91. The effect of the control is obvious.

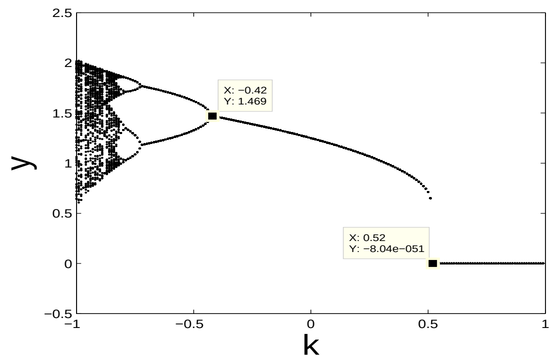

Figure 29: Bifurcation diagram with the change of control gain k in the system for system with $q = -3, p = 5, m = 2, n = 0.5$

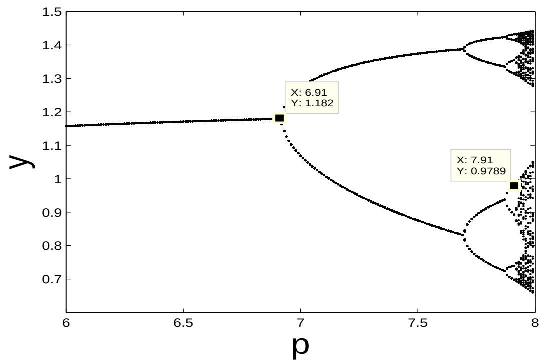

Figure 30: Bifurcation diagram with the change of p in the system for buyers with $m = 2; n = 0.5; p = -3$ under the condition of control

5 Conclusion

This research constructs an evolutionary game based on the theory of mutual harmony in the behavioral economics. It draws the conclusion that when the conditions are $r > r^*$ and $p > p^*$, the system evolves towards the expected directions, and both the developers and buyers achieve the highest degree of happiness.

Besides this, there are several useful conclusions expressed as follows:

(1) The payoff of the developers is the main factor which urges the developer groups to rush into a chaotic state; and the revenue of the buyers is the main factor which makes the buyer groups produce chaos.

(2) Chaos has more tremendous negative impact on developers than buyers, because each party can make much more sensitivity dependence on

developers than buyers in chaotic state. Therefore, it is worth to be mentioned that the developers should pay much more attention on chaos control than buyers.

(3) The linear feedback control method is adopted to achieve the goal of driving the chaotic state to a steady state.

Acknowledgements: My conscience will not serve me well if I failed to extend my sincerest thanks and appreciations to the institutions and people who have worked collaboratively to see that the aims and objectives to complete this research is attained. I must seize this opportunity to thank the following institutions: Tianjin University and The National Nature Science Foundation of China (grant No. 61273231)

References:

[1] L. Gori, M. Sodini, Nonlinear Dynamics in an OLG Growth Model with Young and Old Age Labour Supply: The Role of Public Health Expenditure, *Computational Economics*, 38(3), 2011, pp. 261–275

[2] Y. Saiki, A. C. L. Chian, H. Yoshida, Economic Intermittency in a Two-country Model of Business Cycles Coupled by Investment, *Chaos, solutions and Fractals*, 44(6), 2011, pp. 418–428

[3] J.G. Du, T. W. Huang, Z. H. Sheng, Analysis of Decision-making in Economic Chaos Control,*Nonlinear Analysis: Real World Applications*, 10(4), 2009, pp. 2493–2501.

[4] C. H. Tremblay, V. J. Tremblay, The Cournot-Bertrand Model and the Degree of Product Differentiation, *Economic Letters*, 111, 2011, pp: 233–235.

[5] J.H. Ma, Q. Gao, Stability and Hopf Bifurcations in a Business Cycle Model with Delay, *Applied Mathematics and Computation*, 215, 2009, pp: 829–834.

[6] J. H. Ma, L. L. Mu, Improved Piece-wise Linear and Nonlinear Synchronization of a Class of Discrete Chaotic Systems, *International Journal of Computer Mathematics*, 87, 2010, pp: 619–628.

[7] J. L. Zhang, J. H. Ma, Research on Delayed Complexity Based on Nonlinear Price Game of Insurance Market, *WSEAS Transactions on Mathematics*, 10, 2011, pp: 368–376.

[8] G. H. Wang, J. H. Ma, Output Gaming Analysis and Chaos Control among Enterprises of Rational Difference in a Two-level Supply Chain, *WSEAS Transactions on Mathematics*, 6, 2012, pp: 209–219.

[9] H. W. Wang, J. H. Ma, Chaos Control and Synchronization of a Fractional-order Autonomous System, *WSEAS Transactions on Mathematics.*, 11, 2012, pp: 716–727.

[10] Z. Q. Chen, W. H. Ip, C. Y. Chan, K. L. Yung, Two-level Chaos-based Video Cryptosystem on H.263 Codec, *Nonlinear Dynamics.*, 62, 2010, pp: 647–664.

[11] Z. Q. Chen, Y. Yang, Z. Z. Yuan, A Single Three-wing or Four-wing Chaotic Attractor Generated from a Three-dimensional Smooth Quadratic Autonomous System, *Chaos, Solitons and Fractals.*, 38, 2008, pp: 1187–1196.

[12] J. W. Wen, Z. Q. Chen, Hopf Bifurcation and Intermittent Transition to Hyperchaos in a Novel Strong Four-dimensional Hyperchaotic System,*Nonlinear Dyn*, 60, 2012, pp: 615–630.

[13] C. Veller, V. Rajpaul, Purely Competitive Evolutionary Dynamics for Games, *Physical Review E.*, 041907, 2012, pp: 1–11.

[14] D. G. Hernndez, D. H. Zanette, Evolutionary Dynamics of Resource Allocation in the Colonel Blotto Game, *Stat Phys*, 151, 2013, pp: 623–636.

[15] X. B. Li, ScottC. Lenaghan, Evolutionary Game Based Control for Biological Systems with Applicationsin Drug Delivery, *Journal of Theoretical Biology*, 326, 2013, pp: 58–69.

[16] A. Szolnoki, M. Perc, Effectiveness of Conditional Punishment for the Evolution of Public Cooperation, *Journal of Theoretical Biology*, 326, 2013, pp: 58–69.

[17] X. T. Xia, L. Chen, Fuzzy Chaos Method for Evaluation of Nonlinearly Evolutionary Process of Rolling Bearing Performance, *Measurement*, 46, 2013, pp: 1349–69.

[18] J. Yuichiro Wakano, L. Lehmann, Evolutionary and Convergence Stability for Continuous Phenotypes Infinite Populations Derivedfromtwo-allelemodels, *Journal of Theoretical Biology*, 310, 2012, pp: 206–215.

[19] C. Diks, C. Hommes, P. Zeppini, More Memory under Evolutionary Learning May Lead to Chaos, *Physica A: Statistical Mechanics and its Applications*, 392, 2012, pp: 808–812.