Contract design for 4PL service supply chain and its Pareto improvement

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Abstract: Considering the probability of the logistics business successfully completed is influenced by the effort level of 4PL and 3PL, it will cause bilateral moral risk in the case that both sides of the factors of production cannot be checked. Using the principal-agent theory, the paper discusses both sides contracts in the centralized decision and distributed decision, founding that short-term contract in distributed decision can’t make both sides realize system revenue maximization; In this paper, we develop a long-term development contract programming model to improve the system benefits by introducing earnings discount factor and designing contract on reservation revenue of 3PL which is called secondary revenue-sharing contract, where improving benefits is important. Numerical simulations are presented to show that long-term development contract programming model have good effects and verify the related conclusions.

Key–Words: 4PL, principal-agent, 3PL, Contract design, Pareto improvement

1 Introduction

In the global market competition, it is well known that integrating supply chain resources can reduce the supply chain cost greatly and enhance the competitiveness of the supply chain. But, 3PL can not meet the needs of the development of supply chain because of lacking the supply chain operation control ability as well as the supply chain coordination ability when we have gradually increasing demand for logistics service. Therefore, as a supply chain information aggregator, 4PL arises in the environment, 4PL is the central and the link which connect upstream logistics demand enterprises with downstream logistics service providers (3PL), realizing the supply chain integration and network.

As a supply chain integrator, 4PL has the comprehensive management and coordination ability. Through organically integrates resources with scientific and reasonable scheduling arrangement, logistics efficiency and cost continually draw near optimal level [1]. The 4PL system design goal is to realize industry resources interaction and sharing, and give full play to the advantage of the whole logistics industry[2].

Recently, many scholars have studied the 4PL operation model, Literature [3] consideres transportation and inventory outsourcing decisions of the 4PL under the 3PL competition, establishes three phase joint optimization model based on price, distribution frequency and the demand allocation proportion to analyze 4PL influencing on 3PL’s competition and to analyze the initiative for the influence of the individual and the system performance. By introducing inertia weight factor and contraction factor, literature [4] improves simulated annealing algorithm, optimizes the 4PL enterprise scheduling management. The literatures above study 4PL operation method, through the examples explaining the realistic significance of the study. Literature [5] sets up 4PL and 3PL long-term cooperation of the static model and dynamic model, it is concluded that under the 4PL in dynamic model, it can have better incentive effect when effort level adjustment coefficient is greater than zero.

The domestic and foreign scholars have interested in outsourced logistics management. Logan [6] analyzes the failure of the transportation outsourcing relationship; points out that we can establish a suitable outsourced contract using principal-agent theory. Lim [7] studies outsourced logistics contract design in the case of a third-party private logistics service quality and cost, establishes the incentive mechanism which guides the third-party of logistics to tell the truth, but this paper does not consider the effort of the third-party logistics for the improvement of the service quality. Cai [8] studies the decision problem of distributor on optimal preservation efforts in
the long-distance transport for fresh produce, but the paper does not refer to the quality of the outsourced logistics incentive. Liu Chang xian [9] studies optimal contract setting of the supply chain outsourcing with asymmetric information, but the paper does not consider the risk attitude of the decision maker. Wang Yong [11] studies the integration of resources and operations in the fourth party logistics mode. Although the literatures above have studied the outsourced logistics management, they do not fully consider the 4PL quality risk, 3PL risk averse and asymmetric information which affect contract design.

However, 4PL and 3PL is partnership in strategic perspective, the 4PL and 3PL are cooperative service relation tactically. The results performed by logistics business are influenced by joint efforts of 4PL and 3PL. A large number of empirical datum proved the existence and effectiveness of the contract between 4PL and 3PL, but less to make further study on how to design the contract to realize 4PL and 3PL system revenue maximization.

All the tasks, behavior, special knowledge technology investment cannot be verified in the cooperation service process can not be clearly defined in the formal contract, thus maintaining the contract has strong uncertainty. In a large number of bilateral moral hazardstudies of formal contract designing show that there is not a kind of allocation mechanism which can encourage both sides to achieve the optimal level of system to meet the budget constraint balance[12], thus the carefully designed short-term contract are usually not able to effectively motivate 3PL and 4PL to work hard together. No matter from the transaction cost or from the aspects of resource utilization, it should emphasize on the long-term cooperation of supply chain, it should form a kind of long-term cooperation based on the short term contract, in other words, the long term development alliance. Many scholars study the long-term development contract, study on the service outsourcing: in relational contract of the double-side moral hazard. The literature [13] studies the relational contract with the bilateral moral risk in the service outsourced, it shows that the relational contract encourage the both sides to work together to achieve the system optimal. The literature [14] using the theory of transaction cost has carried real diagnosis analysis for outsourcing contract of a number of companies, research shows that complex outsourced contract has two parts, namely formal contract and long-term development contract, and the treatment of the two sides are complementary[15]. Long term development contract is proportional to the service level when mutual trust degree is high [16].

Based on the perspective of relational contract, literature [17] establishes a model of principal-agent and then designs a set of optimal payment incentives by a dynamic game model which analyzes the participants in the construction projects.

Literature [18] analyzes the features of relational contract, constructs the relational motivation contracts, discusses the reward mechanism and its influencing factors between government and construction agent, and explores the management on relational motivation contracts. The results show that the optimal reward is dependent on several factors, such as how government values the future relation with construction agent, and market situation of construction agency.

Focusing on how to balance the incentive between the two kinds of tasks, the formal contract and relational contract are established based on the principal-agent theory, and then the impacts of discount rate and cost alternative degree to the contract are analyzed in literature [19].

The above literature analysis the relationship between the both sides in relational contract in different environment, but there is little study on the 3PL and 4PL in the relational contract with the bilateral risk.

The structures of this paper are as follows: Section 2 explains the model hypothesis and variable definition. Section 3 discusses 4PL and 3PL model in centralized decision-making and decentralized decision-making. Section 4 constructs improved model. Section 5 presents some numeric examples and offers some qualitative discussion of the solutions. Finally, Section 6 summarizes the work presented in this paper and offers some areas for potential development.

2 Model assumptions

This model is based on the following assumptions.

(1) Suppose that logistics business output function is

\[ \pi(a, e) = kf(a, e) + \theta \]  

where \( f(a, e) = \sqrt{ae} \) is probability of logistics business ideal output, \( a \) is effort level of 3PL, \( e \) is effort level of 4PL, \( \theta \) is a random perturbation item of external uncertain factors on the output and \( k \) is logistics business ideal revenue. When both sides effort make \( f(a, e) \) realize maximum, \( \pi \) is output function of system. If \( k \) is bigger, the logistics business output is bigger. Therefore, function(1)satisfies:

\[ \frac{\partial \pi(a, e)}{\partial a} > 0, \quad \frac{\partial \pi(a, e)}{\partial e} > 0 \]
\[ \frac{\partial^2 \pi(a, e)}{\partial^2 a} < 0, \quad \frac{\partial^2 \pi(a, e)}{\partial^2 e} < 0 \]  

(2)
Based on the formulas above, we can find out, with the increase of \( a, e \), the logistics business output increases, but the amplitude of output increasing decrease gradually.

2) Reference literature [13], Suppose that 3PL effort cost is \( C(a) = \frac{1}{2}ba^2 \), 4PL effort cost is \( C(e) = \frac{1}{2}be^2 \), with the increase of the effort level, effort cost increases and has a marginal increasing law, satisfying the following formulas:

\[
\frac{\partial C(a, e)}{\partial a} > 0, \quad \frac{\partial C(a, e)}{\partial e} > 0,
\]

\[
\frac{\partial^2 C(a, e)}{\partial^2 a} > 0, \quad \frac{\partial^2 C(a, e)}{\partial^2 e} > 0
\]

(3) 3PL and 4PL are all risk neutral.

4) 4PL motivates 3PL to work hard by the revenue-sharing contract, the payment that 4PL pays for 3PL is the function of output, assume 3PL revenue payment function is \( S(\pi) = F + \beta \pi \), where \( F \) shows no matter how much the successful probability of logistics business, 3PL will get reservation revenue, \( k f(a, e) \) is 3PL sharing output revenue, \( \beta \) is output sharing coefficient, 3PL's reservation revenue is \( \omega_0 \).

5) Logistics business expectation outputs are \( E\pi(a, e) = k\sqrt{ae} \).

3 Model analysis

3.1 Centralized decision-making

Under the centralized decision-making, 3PL and 4PL form an integrated company, which means 3PL and 4PL seek unify of decision. Therefore, the integrated company is the risk neutral, integrated company's expected revenue is

\[
S_I = kf(a, e) - c(e) - c(a)
\]

\[
= k\sqrt{ae} - \frac{1}{2}ba^2 - \frac{1}{2}be^2
\]

(4)

Because \( S_I \) is a concave function, optimal solution exists, and

\[
\frac{\partial S_I}{\partial a} = \frac{1}{2}k\sqrt{\frac{e}{a}} - ba = 0,
\]

\[
\frac{\partial S_I}{\partial e} = \frac{1}{2}k\sqrt{\frac{a}{e}} - ba = 0
\]

(5)

Let \( a_I \) and \( e_I \) be the optimal solution: \( a_I = \frac{k}{2b} \), \( e_I = \frac{k}{2b} \). Let \( S^*_I \) be the system optimal output:

\[
S^*_I = \frac{k^2}{4b^2}
\]

(6) It can be seen that when \( k < 2b \) an ideal output prompts integrated company set high effort level, accordingly, a high effort cost coefficient reduces effort level of integrated company; When \( k < 2b \), a high ideal output will increase integrated company's expected revenue, a high cost reduces integrated company’s expected revenue; When \( k > 2b \), integrated company will select 1 as effort level, gets the most system revenue \( k - b \). so, when a powerful 4PL integrates a large amount of logistics business and its ideal output is very big, the 3PL will pay best effort level.

3.2 Distributed decision-making under bilateral moral risk

When both partners have some common output, maximize their own profits as their target, they determine their effort level in the cooperation. So both parties are likely to take some behaviors to reduce the effort level or resource input, which produce bilateral moral risk. In bilateral moral risk, the action that both partners take is non-cooperative, 4PL and 3PL make decision by himself, select the effort level considering to maximize their revenue, both efforts are restricted by incentive compatibility.

In the revenue-sharing contract, \( F + \beta k f(a, e) - c(e) \) is 3PL's revenue, where \( F \) shows no matter how the successful probability of logistics business is, 3PL will get reservation revenue, \( \beta k f(a, e) \) is 3PL sharing output revenue, \( \beta \) is output sharing coefficient, \( c(e) \) is 3PL service cost, 3PL target is to determine the optimal level of service to make expected revenue maximum, the optimal marginal profit of the 3PL can be calculated by the first-order conditions of 3PL’s revenue, and let the formula equals zero

\[
\frac{1}{2}\beta k \sqrt{\frac{e}{a}} - ba = 0.
\]

(7)

That is 3PL incentive compatibility constraint, express 3PL optimal reflect function to 4PL effort degree.

If expected utility of 3PL is less than \( \omega_0 \), 3PL will exist and

\[
F + \beta k f(a, e) - C(a) \geq \omega_0
\]

(8)

the expected utility of 4PL is \( E(4PL) = (1 - \beta)k f(a, e) - F - C(e) \).

Use programming problem \( P_1 \) to describe the problem above

\[
P_1 : \max_{F, \beta, a, e} (1 - \beta)k f(a, e) - F - C(e),
\]

(9)

\[
\frac{1}{2}\beta k \sqrt{\frac{e}{a}} - ba = 0,
\]

(10)

\[
F + \beta k f(a, e) - C(a) \geq \omega_0,
\]

(11)

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where (9) is earning objective function of 4PL, (10-11) is constraint conditions.

In the programming problem $P_1$, usually, 4PL can obtain positive expected revenue. Of course, 4PL’s expected revenue can be negative, when expected revenue is negative, shows that 4PL is still in its initial stage, there is no better integration on supply chain resources, expected revenue cannot make up for the cost loss of 4PL. If we want to make the supply chain resources integrate better, need the government to support in the initial, so as to promote the development of 4PL.

Programming problem $P_1$ meets Kuhn-Tucker conditions, so we can structure the Generalized Lagrange function:

$$L_1(F, a, c, \lambda_1, \lambda_2) = (1 - \beta)kF(a, e) - F - C(e) - \lambda_1(\beta kF(a, e) - ba) - \lambda_2(\omega_0 - F - \beta kF(a, e) + C(a))$$  (12)

where $\lambda_1, \lambda_2 \geq 0$ are Lagrange multipliers.

Make the first-order conditions on $L_1$ for $F$ and make it equals zero, we can obtain $\frac{\partial L_1}{\partial F} = -1 + \lambda_2 = 0$, thus $\lambda_2 = 1$, and

$$F = \omega_0 + C(a) - \beta kF(a, e).$$  (13)

Will (13) into 4PL objective function (9) to eliminate $F$, obtain 4PL objective function

$$kf(a, e) - C(a) - C(e) - \omega_0$$  (14)

System expected revenue $S$

$$S = kf(a, e) - C(a) - C(e) - \omega_0$$  (15)

Programming problem $P_1$ can be expressed by programming problem $P_2$

$$P_2 : \max_{a, e} (S - \omega_0)$$  (16)

$$\frac{1}{2} \beta k \sqrt{\frac{e}{a}} - ba = 0$$  (17)

In the programming problem $P_2$, $S - \omega_0$ equals system expected revenue minus $\omega_0$, $\omega_0$ is a constant. If system expected revenue is bigger, the 4PL revenue is bigger, programming problem $P_2$ can be interpreted as 4PL select the optimal effort level and revenue sharing coefficient, to guide both sides behavior to achieve higher system revenue. 4PL is contract designer which has the contract design right, 3PL will only get reservation revenue.

Programming problem $P_2$ meets the Kuhn-Tucker conditions, we can obtain the second optimal solution $a^*, e^*, \pi^*$.

$$a^* = \frac{k}{2b}((1 - \beta)\beta^3)^{\frac{1}{4}}$$
$$e^* = \frac{k}{2b}((1 - \beta)\beta^3)^{\frac{1}{4}}$$

$$S^* = \frac{3k^2}{8b} \sqrt{(1 - \beta)\beta - \omega_0}$$

$$S^* - S_1 = \frac{3k^2}{8b} \sqrt{(1 - \beta)\beta - \omega_0} - \frac{k^2}{4b}$$

We can see from the results that, in bilateral moral risk, output of 3PL will be shared, in other words, $\beta \neq 0$, 1 effort level of 3PL and 4PL is not equal to zero. Under bilateral moral risk, effort level in distributed decision-making is less than the one in centralized decision-making, $a_1 > a^*, e_1 > e^*$.

In bilateral moral risk, 4PL-dominated contract can’t guide both sides to work hard together, produce the system loss of profits, this is because under the distributed decisions, self-care decision of the two parties produce double marginal effect, optimal goal based on personal-oriented is often not optimal as a whole. This contract is a kind of short-term cooperative behavior, in this contract, fixed pay does not produce incentives, incentives for 3PL produce by revenue sharing coefficient is limited, in the moral risk, effort level of two parties level and system revenue won’t achieve optimal in distributed decision.

4 Pareto improvement of the distributed decision under bilateral moral risk

Short-term cooperation contract does not produce good incentive effect, produces system loss of profit, if they want to increase their revenues, they must optimize the contract, establish a long-term development league, production expectation for future earnings can have some incentive effect for the two parties. Under the long-term development league, $\beta, a, e$ mean respectively revenue sharing coefficient effort level of 3PL and effort level of 4PL so as to distinguish them in short-term cooperation contract. This paper quotes ”discount factor” of expected revenue to describe long-term cooperation contract, because of bilateral moral risk, there will be a possibility of breaching contract for 3PL and 4PL.

In the long-term cooperation contract, the expected revenue of 3PL are $\tilde{F} + \beta kF(\tilde{a}, \tilde{e}) - C(\tilde{a})$, If 3PL does not establish long-effect mechanism, 3PL
can only obtain the maximum revenue of short-term cooperation contract in the later cooperation, namely, 
\[ F + \beta^* k f(a^*, e^*) - C(a^*) \].

Assume \( \delta \) is discount factor, and in this paper 3PL and 4PL have the same \( \delta \); in the case of the cooperation, 3PL will get net revenue present value as follows
\[
\hat{\beta} k f(\hat{a} \hat{e}) - C(\hat{a}) + \hat{F} \\
+ \frac{\delta}{1 - \delta} [\hat{\beta} k f(\hat{a} \hat{e}) - C(\hat{a}) + \hat{F}]
\]

In the case of the non-cooperation, 3PL will get net revenue present value as follows
\[
\hat{\beta} k f(a \hat{e}) - C(a) + \hat{F} \\
+ \frac{\delta}{1 - \delta} [\hat{\beta} k f(a^* e^*) - C(a^*) - F]
\]
Constraint condition that 3PL participates in long-term cooperation contract is as follows
\[
\hat{\beta} k f(a \hat{e}) - C(a) + \hat{F} \\
+ \frac{\delta}{1 - \delta} [\hat{\beta} k f(a^* e^*) - C(a^*) - F]
\]
where \( a = \arg \max \hat{F} + \hat{\beta} k f(a \hat{e}) - C(a) \) is 3PL’s effort level of breaching contract which can maximize their revenue.

Similarly, constraint condition that 4PL participates in long-term cooperation contract is as following
\[
(1 - \hat{\beta}) k f(\hat{a}, e) - \hat{F} - C(e) \\
+ \frac{\delta}{1 - \delta} [(1 - \hat{\beta}) k f(a^* e^*) - F - C(e^*)]
\]
\[
\leq (1 - \hat{\beta}) k f(\hat{a}, \hat{e}) - \hat{F} - C(\hat{e}) \\
+ \frac{\delta}{1 - \beta} [(1 - \hat{\beta}) k f(\hat{a} \hat{e}) - \hat{F} - C(\hat{e})]
\]
where \( e = \arg \max (1 - \hat{\beta}) k f(\hat{a} \hat{e}) - C(e) - \hat{F} \) is 4PL’s effort level of breaching contract which can maximize their revenue.

Literature [13] designs a long-term development contract which can lead participator to improve effort level, but in which reservation revenue of service provider is equal to the one of formal contract numerical analysis shows that the revenue of service provider is always reservation revenue this contract did not reflect the interrelation between revenue and the effort level. This paper assumes that human nature is greedy, under the long-term cooperation contract, 3PL’s reservation revenue is \( \omega_0 + \gamma[S(\hat{a} \hat{e}) - s(a^* e^*)] \), so
\[
\hat{\beta} k f(\hat{a} \hat{e}) \hat{F} - C(\hat{a}) \\
\geq \omega_0 + \gamma[S(\hat{a} \hat{e}) - s(a^* e^*)]
\]
Use the programming problem \( P_3 \) to describe the problem above
\[
P_3 : \max_{F, \beta \hat{ae}} (1 - \hat{\beta}) k f(\hat{a} \hat{e}) - \hat{F} - C(\hat{e}).
\]
with
\[
\hat{\beta} k f(\hat{a} \hat{e}) - C(\hat{a}) + \hat{F} \\
+ \frac{\delta}{1 - \delta} [\hat{\beta} k f(\hat{a} \hat{e}) - C(\hat{a}) + \hat{F}]
\]
\[
\leq (1 - \hat{\beta}) k f(\hat{a} \hat{e}) - C(\hat{a}) + \hat{F} \\
+ \frac{\delta}{1 - \beta} [(1 - \hat{\beta}) k f(\hat{a} \hat{e}) - \hat{F} - C(\hat{e})]
\]
where \( a = \arg \max (1 - \hat{\beta}) k f(\hat{a} \hat{e}) - C(e) - \hat{F} \)
\[
\omega_0 + \gamma[S(\hat{a} \hat{e}) - S(a^* e^*)];
\]
\[
\omega_0 + \gamma[S(\hat{a} \hat{e}) - s(a^* e^*)].
\]
3PL’s participation constraint is binding constraint, namely,
\[
\hat{F} = -\hat{\beta} k f(\hat{a} \hat{e}) + C(\hat{a}) + \omega_0 + \gamma[S(\hat{a} \hat{e}) - S(a^* e^*)]
\]
(26)
Substitute (26) into the programming problem \( P_3 \), we can obtain the programming problem \( P_4 \) as follows
\[
P_4 : \max_{\beta \hat{ae}} S(\hat{a} \hat{e}) - \omega_0 - \gamma[S(\hat{a} \hat{e}) - s(a^* e^*)]
\]
with
\[
(1 - \hat{\beta}) k f(\hat{a} \hat{e}) - f(\hat{a} \hat{a}) - C(e) - C(\hat{e}) \\
\leq \frac{\delta}{1 - \delta} [(1 - \gamma) S(\hat{a} \hat{e}) - S(a^* e^*)];
\]
\[
\hat{\beta} k f(a \hat{e}) - f(\hat{a} \hat{e}) - C(a) - C(\hat{a}) \\
\leq \frac{\delta}{1 - \delta} \gamma[S(\hat{a} \hat{e}) - S(a^* e^*)];
\]
\[
\frac{1}{2} k \sqrt{\frac{\hat{e}}{\hat{a}} - b \hat{a}} = 0;
\]


\[ e = \arg \max (1 - \beta)k f(\hat{a}e) - C(e) - \hat{F}; \]
\[ a = \arg \max \hat{F} + \beta k f(\hat{a}e) - C(a). \quad (28) \]

**Proposition 1** When \( \delta = 0 \) the long-term cooperation contract of 3PL and 4PL is equivalent to short-term contract, 4PL’s effort level is \( e^* \) which is the effort level in short-term contract, in other words, both partners do not focus on the long-term benefits.

**Proof.** For formula

\[ (1 - \beta)k [f(\hat{a}e) - f(\hat{a}\hat{e})] - C(e) - C(\hat{e}) \leq \frac{\delta}{1 - \delta} (1 - \gamma) [S(\hat{a}e) - S(a^* e^*)] \quad (29) \]

The left express current revenue increase when 4PL tears up the contract, the right is later revenue discount when 4PL keeps contract, when \( \delta = 0 \) will be substituted (29), it becomes

\[ (1 - \beta)k [f(\hat{a}e) - f(\hat{a}\hat{e})] - C(e) - C(\hat{e}) \leq 0 \quad (30) \]

From (30), we can obtain \( e = \hat{e} \). \( \square \)

Similarly, form formula \( \beta k [f(\hat{a}e) - f(\hat{a}\hat{e})] - C(a) - C(\hat{a}) \leq \frac{\delta}{1 - \delta} (1 - \gamma) [S(\hat{a}e) - S(a^* e^*)] \) we can obtain \( a = \hat{a} \). Thus, \( e = \hat{e} = \arg \max (1 - \beta)k [f(\hat{a}e) - C(e) - \hat{F} \]
\( a = \hat{a} = \arg \max \hat{F} + \beta k f(\hat{a}e) - C(a) \). So, when \( \delta = 0 \), 4PL’s long-term cooperation contract can be expressed as programming problem \( P_3 \).

\[ \begin{align*}
P_3: & \max S(\hat{a}e) - \omega_0 - \gamma [S(\hat{a}e) - S(a^* e^*)] \\
& \text{with} \\
& \frac{1}{2} (1 - \beta)k \sqrt{\frac{\hat{a}}{e} - \hat{e} \hat{e}} = 0 \quad (31) \\
& \frac{1}{2} \beta k \sqrt{\frac{\hat{e}}{\hat{a}} - \hat{b} \hat{a}} = 0 \quad (32)
\end{align*} \]

**Proposition 2** Under long-term cooperation contract, the both parties’s revenue and system revenue are non-reduce function of discount factor \( (\delta) \).[13]

**Proof.** For programming problem \( P_4 \) structure the generalized Lagrange function.

\[ L_2 = \max \{S(\hat{a}e) - \omega_0 - \gamma [S(\hat{a}e) - s(a^* e^*)] - \lambda_1 (1 - \beta)k [f(\hat{a}e) - f(\hat{a}\hat{e})] - C(e) \]
\[ + C(\hat{e}) - \frac{\delta}{1 - \delta} (1 - \gamma) [S(\hat{a}e) - S(a^* e^*)] - \lambda_3 \beta k [f(\hat{a}e) - f(\hat{a}\hat{e})] - C(a) + C(\hat{a}) \]
\[ - \frac{1 - \gamma}{1 - \delta} [S(\hat{a}e) - S(a^* e^*)] \]

\[ - \lambda_3 [S(a^* e^*) - S(\hat{a}e)] - \lambda_4 \frac{1}{2} \beta k \sqrt{\frac{\hat{e}}{\hat{a}} - \hat{b} \hat{a}} \} \quad (33) \]

Take the first-order conditions on \( L_2 \) for \( \delta \), we can obtain

\[ \frac{\partial L_2}{\partial \delta} = \frac{\lambda_1}{1 - \delta} (1 - \gamma) [S(\hat{a}e) - S(a^* e^*)] \]
\[ + \frac{\lambda_2 \gamma}{(1 - \delta)^2} [S(\hat{a}e) - S(a^* e^*)] \quad (34) \]

According to (28), \( S(\hat{a}e) \geq S(a^* e^*) \), it holds that \( \frac{\partial L_2}{\partial \delta} > 0 \). So under long-term cooperation contract, both parties revenue and system revenue are non-reduce function of discount factor \( (\delta) \). \( \square \)

5 Numerical simulation

Now, assume that \( k = 12, b = 6, \omega_0, \gamma = 0.3, \delta \) is a exogenous variable, then we analysis different discount factor will cause the influence on the long-term cooperation contract.

As a research reference, we calculate the results are as follows.

\[ S_{t} = 5.143, a_{t} = 0.857, e_{t} = 0.857 \]

\[ S(a^* e^*) = 3.857, a^* = 0.429, a^* = 0.429 \]

In domain of definition, \( \delta \in [0, 1] \), select step length \( \delta = 0.1 \), the calculation results can see in Table 1.

**Table 1**: parameter value of long-term contract under different discount factor

<table>
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<th>( A_2 )</th>
<th>( A_3 )</th>
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<td>0.694</td>
<td>0.727</td>
<td>3.347</td>
<td>1.7897</td>
<td>4.988</td>
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<td>0.800</td>
<td>3.349</td>
<td>1.794</td>
<td>5.094</td>
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<tr>
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<td>0.945</td>
<td>0.824</td>
<td>0.853</td>
<td>3.349</td>
<td>1.794</td>
<td>5.137</td>
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<tr>
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<td>0.857</td>
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<td>1.794</td>
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<td>1.794</td>
<td>5.143</td>
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</tbody>
</table>

In the table, \( A_1 \) is discount factor, \( A_2 \) is revenue sharing coefficient, \( A_3 \) is 3PL effort level, \( A_4 \) is 4PL effort level, \( A_5 \) is 4PL revenue, \( A_6 \) is 3PL revenue, \( A_7 \) is Total revenue.
When $\gamma = 0.6$, in domain of definition, $\delta \in [0, 1]$, select step length $\delta = 0.1$, the calculation results can see in Table 2.

From table 2, we can see that with the increasing of $\gamma$, the revenue of 3PL is increasing which make the 3PL and 4PL to construct the longer corporation.

The bigger discount factor is, the more both sides concern about future revenue, it can be seen from table1, when discount factor $\delta = 0$, long-term cooperation contract is equivalent to short-term contract; When $\delta \in [0, 0.74]$, the revenues of both sides and system revenues are gradually increasing under the long-term contract; When $\delta = 0.74$, the revenues of both sides and system revenues reach maximum value, after that, discount factor no longer have affect on the system revenues, numerical simulation results above verify the correctness of the proposition 2. Under the long-term contract, 3PL reservation revenues embody the secondary distribution of the revenue, namely, secondary distribution revenue contract. In secondary distribution revenue contract, along with the effort level increase 3PL revenue add, which really reflect the incentive effect.

Fig.1 shows that effort level of 4PL has been more bigger than it of 3PL, this paper studies the supply chain which is dominated by 4PL, when 3PL see 4PL pay more efforts, 3PL also improve its effort level. With the discount factor rising continuously, the revenue sharing coefficient is gradually bigger, 4PL puts more output for 3PL and shows that 4PL pays more attention to the long-term development which is showed in the Fig.2.

Fig.3 shows that with the increasing of the discount factor, the 3PL’s revenue and 4PL’s revenue increase gradually. When discount factor equals to 0.1, 4PL’s revenue increase faster, after that, 4PL’s revenue increases slowly.

Table 2: parameter value of long-term contract under different discount factor

<table>
<thead>
<tr>
<th>$A1$</th>
<th>$A2$</th>
<th>$A3$</th>
<th>$A4$</th>
<th>$A5$</th>
<th>$A6$</th>
<th>$A7$</th>
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<tbody>
<tr>
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<td>0.5</td>
<td>0.429</td>
<td>0.429</td>
<td>2.125</td>
<td>1</td>
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<tr>
<td>0.1</td>
<td>0.547</td>
<td>0.476</td>
<td>0.491</td>
<td>2.991</td>
<td>1.173</td>
<td>4.164</td>
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<tr>
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<td>0.552</td>
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</tr>
<tr>
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<td>0.577</td>
<td>0.598</td>
<td>3.319</td>
<td>1.314</td>
<td>4.633</td>
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<tr>
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<td>0.633</td>
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</tr>
<tr>
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<td>0.727</td>
<td>3.568</td>
<td>1.420</td>
<td>4.988</td>
</tr>
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<td>0.6</td>
<td>0.861</td>
<td>0.758</td>
<td>0.800</td>
<td>3.642</td>
<td>1.452</td>
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<td>0.7</td>
<td>0.945</td>
<td>0.824</td>
<td>0.853</td>
<td>3.672</td>
<td>1.456</td>
<td>5.137</td>
</tr>
<tr>
<td>0.74</td>
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<td>0.857</td>
<td>3.676</td>
<td>1.467</td>
<td>5.143</td>
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<tr>
<td>0.8</td>
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</table>

Next, we analysis the situation when $\gamma = 0.6$ which shows in Fig.4 and Fig.5. We can see that with the increasing of $\gamma$, the revenue of 3PL increases and the revenue of 4PL reduce in the different of the sharing coefficient. When the discount factor equal to 0.1, the revenue of 4PL increases slower than it at $\gamma = 0.3$. We also obtain that effort level of 4PL has been more bigger than it of 3PL, when 3PL see that 4PL pays more efforts, 3PL also improves its effort level, but when the effort level equals to 0.857, revenues of the 3PL and 4PL are all not increasing, so, we should choose the quite value of discount factor.

With the discount factor rising continuously, the revenue sharing coefficient is gradually bigger, 4PL puts more output for 3PL which shows that 4PL pays...
more attention to the long-term development which is the same to Fig.2.

Fig.6 shows that with the increasing of the discount factor the revenue-added of the total revenue decrease with the different value of $\gamma = 0.6$, that is to say, we should choose the appropriate value of discount factor to make the both side obtain the biggest revenue.

![Figure 3](image3.png)

Figure 3: The relationship diagram between discount factor and both sides’s revenue and system revenue when $\gamma = 0.3$

![Figure 4](image4.png)

Figure 4: The relationship diagram between discount factor and sharing coefficient and both sides’s effort level when $\gamma = 0.6$

![Figure 5](image5.png)

Figure 5: The relationship diagram between discount factor and both sides’s revenue and system revenue when $\gamma = 0.6$

![Figure 6](image6.png)

Figure 6: The relationship diagram between discount factor and the increased revenue of the system

6 Conclusions

It is a kind of cooperation service relationship between 3PL and 4PL, the two sides arise bilateral moral risk in the process of cooperation under incomplete information. Short-term cooperation contract, which two sides sign, cannot produce effective incentive, and can’t effectively solve bilateral moral risk. This paper establishes the long-term cooperation contract, and we construct the long-term cooperation contract mode on the basis of considering the 3PL which reservation revenue should be change along with the change of revenue. Simulation results show that the model increases incentive effect significantly with the increase of the discount factor. When the discount factor equals 0.71, the system reaches optimal state. Discount factor reflects the attention degree for future revenue, represents the cooperation prestige, when a party’s prestige is good, the other party will think op-
ponent’s discount factor is bigger and will pay relatively high effort level. So, enterprisers should pay attention to credit construction, and gradually establish a long-term cooperation contract.

Because different side has different attitude, the results of service contract is difficult to estimate. In this paper we only consider the contract designing for 3PL and 4PL in risk neutral conditions, we will consider the contract designing for 3PL and 4PL in risk aversion conditions in further research.

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References: