Global asymptotic stability for second-order neutral type Cohen-Grossberg neural networks with time delays

CHUNFANG MIAO
Department of Mathematics
Shaoxing University
Shaoxing, Zhejiang,312000
CHINA
miaochf@usx.edu.cn

YUNQUAN KE*
Department of Mathematics
Shaoxing University
Shaoxing, Zhejiang,312000
CHINA
keyunquan@usx.edu.cn

Abstract: In this paper, the global asymptotic stability of second-order neutral type Cohen-Grossberg neural networks with time-varying delays are investigated. First, by choosing properly variable substitution the system is transformed to first-order differential equation. Second, some sufficient conditions which can ensure the existence and global asymptotic stability of equilibrium point for the system are obtained through using homeomorphism and the differential mean value theorem, constructing suitable Lyapunov functional and applying the positive matrix. Finally, two examples are given to illustrate the effectiveness of the results.

Key–Words: second-order neutral type; Cohen-Grossberg neural networks; Lyapunov functional; Homeomorphism theory; differential mean value theorem; asymptotic stability

1 Introduction

We note that second-order neutral type differential equations have applications in problems dealing with vibrating masses attached to an elastic bar and in some variational problems of physics, biology, and economics. There is a constant interest in obtaining new sufficient conditions for the oscillation or non-oscillation of the solutions of varietal types of the second-order equations, see, [1-7]. For example, In [1], the authors discussed the positive periodic solutions for second-order neutral differential equations with functional delays described by

$$\ddot{x}(t)+p(t)\dot{x}(t)+q(t)x(t)=C\dot{x}(t-\tau(t)) \\ +f(t,h(x(t)),g(x(t)-x(t-\tau(t)))).$$

In [2], the authors studied the periodic solutions for a second-order nonlinear neutral differential equations with variable delays:

$$\ddot{x}(t)+p(t)\dot{x}(t)+q(t)x(t) \\ =\dot{g}(t,x(t-\tau(t)))+f(t,x(t),x(t-\tau(t))).$$

Since neutral neural networks contain some very important information about the derivative of the past state, thus it is very important for us to study such complicated system. And so far, the stability analysis for neural networks of neutral type has been investigated [8-13]. For example, in [12] the authors investigate the existence of periodic solutions for a class of Cohen-Grossberg type neural networks with neutral delays. Zhang et al.[13] investigated the global asymptotic stability to a generalized Cohen-Grossberg BAM neural networks of neutral type delays.

This paper is devoted to presenting a theoretical stability analysis for second-order neutral type Cohen-Grossberg neural networks with time-varying delays, they can provide the theoretical basis of the practical application.

We consider the following class second-order neutral type Cohen-Grossberg neural networks with time-varying delays

$$\begin{aligned}\ddot{x}_i(t) &= -d_i\dot{x}_i(t)-\alpha_i(x_i(t))[h_i(x_i(t)) \\ &-\sum_{j=1}^{n}a_{ij}f_j(x_j(t))-\sum_{j=1}^{n}b_{ij}f_j(x_j(t-\tau_{ij}(t))) \\ &-\sum_{j=1}^{n}c_{ij}\dot{x}_j(t-\tau_{ij}(t))+I_i], \qquad (1)\end{aligned}$$

where $i=1,2,\ldots,n$, $d_i>0$ is constant, $x_i(t)$ denotes the states variable of the ith neuron at the time

*Corresponding author: Yunquan Ke

t, $\alpha_i(\cdot)$ denotes an amplification function; $h_i(\cdot)$ is the behaved function, a_{ij} and b_{ij} are connection weights of the neural networks; f_j denotes the activation function of jth neuron at the time t; $\tau_{ij}(t)$ is time delay, $0 < \tau_{ij}(t) < \tau$ and $0 < \dot{\tau}_{ij}(t) \leq \eta < 1$, where τ, η are constants; c_{ij} is coefficient of the time derivative of the delayed states; I_i denotes the external inputs on the ith neuron at the time t.

When $n = 1$, $\alpha(x(t)) = q(t)$, $h(x(t)) = x(t)$, then system (1) is the one of [1][2].

The initial values of system (1) are

$$x_i(s) = \varphi_i(s), \ \ \dot{x}_i(s) = \psi_i(s), \tag{2}$$

where $-\tau < \tau_{ij}(t) \leq s \leq 0, \varphi_i(s), \psi_i(s)$ are bounded and continuous functions.

This paper is organized as follows. Some preliminaries are given in Section 2. In Section 3, the sufficient conditions are derived to ensure the existence and global asymptotic stability of equilibrium point for second-order neutral type neural networks with time delays. In Section 4, an illustrative example is given to show the effectiveness of the proposed theory.

2 Preliminaries

For the sake of convenience, we introduce some notations and definitions as follows. E denotes the unit matrix, for any matrix A, A^T stands for the transpose of A, A^{-1} denotes the inverse of A. If A is a symmetric matrix, $A > 0 (A \geq 0)$ means that A is positive definite (nonnegative definite). Similarly, $A < 0 (A \leq 0)$ means that A is negative definite (negative semidefinite). $\lambda_M(A)$, and $\lambda_m(A)$ denote the maximum and minimum eigenvalue of a square matrix A.

Let $\mathbb{R}^m$ be an $m-$dimensional Euclidean space, which is endowed with a norm $\|.\|$ and inner product $(.,.)$, respectively. Given column vector $x = (x_1, x_2, \ldots, x_m) \in \mathbb{R}^m$, the Euclidean vector norm is $\|x\| = (\sum\limits_{i=1}^{m} x_i^2)^{1/2}$.

Definition 1 *[14] A map $H : \mathbb{R}^n \to \mathbb{R}^n$ is a homeomorphism of R^n onto itself if H is continuous and one-to -one and its inverse map H^{-1} is also continuous.*

Lemma 2 *[14] If $H(u) \in C^0$, and it satisfies the following conditions*

1) $H(u)$ is injective on $\mathbb{R}^n$,

2) $\|H(u)\| \to +\infty$, as $\|u\| \to +\infty$,

then $H(u)$ is a homeomorphism of $\mathbb{R}^n$.

Lemma 3 *If $x, y \in \mathbb{R}^n$ are two vectors, for any $\varepsilon > 0$, then*

$$2X^T Y \leq \varepsilon X^T X + \varepsilon^{-1} Y^T Y.$$

Throughout this paper, we make the following assumptions.

(H_1) : For each $i = 1, 2, \ldots, n$, $\alpha_i(x)$ is continuous bounded and satisfies inequality $0 < \underline{\alpha}_i \leq \alpha_i(x) \leq \bar{\alpha}_i$, for all $x \in \mathbb{R}$.

(H_2) : For each $i = 1, 2, \ldots, n$, $h_i(x)$ is differentiable and satisfies condition $0 < \underline{h}_i \leq h_i'(x) \leq \bar{h}_i$, for all $x \in \mathbb{R}$.

(H_3) : f_j satisfies Lipschitz condition, i.e., there exists constant $l_j > 0$, such that

$$|f_j(v_1) - f_j(v_2))| \leq l_j |v_1 - v_2|,$$

for $j = 1, 2, \cdots, n, v_1, v_2 \in \mathbb{R}$.

Introducing variable transformation:

$$y_i(t) = \dot{x}_i(t) + x_i(t), \quad i = 1, 2, \ldots, n,$$

then (1) and (2) can be rewritten as

$$\begin{cases} \dot{x}_i(t) = -x_i(t) + y_i(t), \\ \dot{y}_i(t) = (d_i - 1)x_i(t) - (d_i - 1)y_i(t) \\ -\alpha_i(x_i(t))[h_i(x_i(t)) - \sum\limits_{j=1}^{n} a_{ij} f_j(x_j(t)) \\ - \sum\limits_{j=1}^{n} b_{ij} f_j(x_j(t - \tau_{ij}(t))) \\ - \sum\limits_{j=1}^{n} c_{ij} \dot{x}_j(t - \tau_{ij}(t)) + I_i] \end{cases} \tag{3}$$

for $i = 1, 2, \ldots, n$, and

$$\begin{cases} x_i(s) = \varphi_i(s), \quad \dot{x}_i(s) = \psi_i(s), \\ y_i(s) = \varphi_i(s) + \psi_i(s) \doteq \varphi_i^*(s), \end{cases} \tag{4}$$

for $i = 1, 2, \ldots, n$, $-\tau < \tau_{ij}(t) \leq s \leq 0$.

System (3) can be written in the vector-matrix form as follows:

$$\begin{cases} \dot{X}(t) = -X(t) + Y(t), \\ \dot{Y}(t) = RX(t) - RY(t) \\ \quad -\alpha(X(t)) \Big[H(X(t)) - Af(X(t)) \\ \qquad -Bf((X(t - \tau_{ij}(t))) \\ \qquad -C\dot{X}(t - \tau_{ij}(t)) + I \Big], \end{cases} \tag{5}$$

where

$$
\begin{aligned}
&X(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T,\\
&Y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T,\\
&f(t) = (f(x_1(t)), f(x_2(t)), \ldots, f(x_n(t)))^T,\\
&R = diag(d_1 - 1, d_2 - 1, \ldots, d_n - 1),\\
&\alpha(X(t)) = diag(\alpha_1(x_1(t)), \ldots, \alpha_n(x_n(t))),\\
&H(X(t)) = (h_1(x_1(t)), \ldots, h_n(x_n(t)))^T,\\
&A = (a_{ij})_{n\times n},\ B = (b_{ij})_{n\times n},\ C = (c_{ij})_{n\times n},\\
&I = (I_1, I_2, \ldots, I_n)^T.
\end{aligned}
$$

Definition 4 *A point $X^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ is called equilibrium point of system (1) if*

$$
\begin{aligned}
&-h_i(x_i^*) + \sum_{j=1}^{n} a_{ij} f_j(x_j^*) + \sum_{j=1}^{n} b_{ij} f_j(x_j^*) - I_i = 0,\\
&i = 1, 2, \ldots, n.
\end{aligned}
\tag{6}
$$

or

$$
-H(X^*) + Af(X^*) + Bf(X^*) - I = 0. \tag{7}
$$

3 Main results

In this section, we can derive some sufficient conditions which can ensure the existence and global asymptotic stability of equilibrium point for the system (1).

Theorem 5 *Under the hypotheses $(H_1) - (H_3)$, if $1 + \underline{h}_i\underline{\alpha}_i - d_i > 0$, and there exists positive constants β, δ, γ and a positive diagonal matrix $Q = diag(q_1, q_2, \cdots, q_n)$, and $0 < q_i < \sqrt{d_i - \bar{h}_i\bar{\alpha}_i} < 1$, $(i = 1, 2, \cdots, n)$ such that*

$$
\begin{aligned}
&P_1 = 2\underline{H} - (\beta + \delta + 1)E > 0,\\
&P_2 = (E - Q^2 - G)L^{-2} - \tfrac{1}{\beta}A^T A > 0,\\
&P_3 = 2R - E - G - (\beta + \delta + \gamma)(\bar{\alpha})^2 > 0,\\
&P_4 = (1 - \eta)Q^2 L^{-2} - \tfrac{1}{\delta}B^T B > 0,\\
&P_5 = (1 - \eta)E - \tfrac{1}{\gamma}C^T C > 0,
\end{aligned}
$$

then system (1) has a unique equilibrium point, which is globally asymptotically stable, where

$$
\begin{aligned}
&L = diag(l_1, l_2, \cdots, l_n),\\
&\underline{H} = diag(\underline{h}_1, \underline{h}_2, \cdots, \underline{h}_n),\\
&\bar{\alpha} = diag(\bar{\alpha}_1, \bar{\alpha}_2, \cdots, \bar{\alpha}_n),\\
&G = diag(1 + \bar{h}_1\bar{\alpha}_1 - d_1, \cdots, 1 + \bar{h}_n\bar{\alpha}_n - d_n),\\
&0 < \dot{\tau}_{ij}(t) \le \eta < 1,
\end{aligned}
$$

$l_i, \underline{h}_i, \bar{\alpha}_i, d_i$ are given by hypotheses $(H_1) - (H_3)$.

Proof. In order to prove the existence and uniqueness of the equilibrium point, we consider the following mapping associated with system (1):

$$
W(X) = -H(X) + Af(X) + Bf(X) - I,
$$

If $X^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ is an equilibrium point of (1), then X^* satisfies the following equation:$W(X^*) = 0$.

It is obvious that $W(X) = 0$ is an equilibrium point of (1). Therefore, we can directly conclude from Lemma 2 that for the system defined by (1), there exists a unique equilibrium point for every input vector I if $W(x)$ is homeomorphism of $\mathbb{R}^n$.

Now we will show that $W(x)$ is a homeomorphism of $\mathbb{R}^n$ under the conditions of Theorem 5.

First, we prove that $W(x)$ is an injective map on $\mathbb{R}^n$. Let us choose two vectors $X, Z \in \mathbb{R}^n$ such that $X \ne Z$, we have

$$
\begin{aligned}
&W(X) - W(Z) = -[H(X) - H(Z)]\\
&+A[f(X) - f(Z)] + B[f(X) - f(Z)].
\end{aligned}
\tag{8}
$$

$2(X - Z)^T$ is multiplied by both sides of (8), we obtain

$$
\begin{aligned}
& 2(X - Z)^T(W(X) - W(Z))\\
= \ & 2(X - Z)^T\{-[H(X) - H(Z)]\\
& +A[f(X) - f(Z)] + B[f(X) - f(Z)]\}\\
= \ & -2(X - Z)^T(H(X) - H(Z))\\
& +2(X - Z)^T A[f(X) - f(Z)]\\
& +2(X - Z)^T B[f(X) - f(Z)]\\
= \ & -2(X - Z)^T(H(X) - H(Z))\\
& +2(X - Z)^T A[f(X) - f(Z)]\\
& +2(X - Z)^T B[f(X) - f(Z)]\\
& (X - Z)^T(E - Q^2)(X - Z)\\
& -(X - Z)^T Q^2(X - Z)\\
& +(X - Z)^T(X - Z).
\end{aligned}
\tag{9}
$$

By Lemma 3, there exists positive constants β, δ, such that

$$
\begin{cases}
2(X - Z)^T A[f(X) - f(Z)]\\
\le \beta(X - Z)^T(X - Z)\\
\qquad +\frac{1}{\beta}[f(X) - f(Z)]^T\\
\qquad \times A^T A[f(X) - f(Z)]\\
2(X - Z)^T B[f(X) - f(Z)]\\
\le \delta(X - Z)^T(X - Z)\\
\qquad +\frac{1}{\delta}[f(X) - f(Z)]^T\\
\qquad \times B^T B[f(X) - f(Z)]
\end{cases}
\tag{10}
$$

On the other hand, when $0 < q_i < 1$ we have

$$
(X - Z)^T(E - Q^2)(X - Z)
$$

$$\begin{aligned} &= \sum_{i=1}^{n}(1-q_i^2)(x_i-z_i)^2 \\ &\geq \sum_{i=1}^{n}\frac{1-q_i^2}{l_i^2}(f(x_i)-f(z_i))^2 \\ &= [f(X)-f(Z)]^T(E-Q^2)L^{-2} \\ &\quad \times[f(X)-f(Z)]. \end{aligned} \tag{11}$$

$$\begin{aligned} &(X-Z)^TQ^2(X-Z) = \sum_{i=1}^{n}q_i^2(x_i-z_i)^2 \\ \geq\ & \sum_{i=1}^{n}q_i^2l_i^{-2}(f(x_i)-f(z_i))^2 \\ =\ & [f(X)-f(Z)]^TQ^2L^{-2}[f(X)-f(Z)]. \end{aligned} \tag{12}$$

From assumption (H_2), $0 < \underline{h}_i \leq h_i'(x) \leq \bar{h}_i$, for any finite values x, y, let $x > y$, using differential mean value theorem, we have

$$h_i(x)-h_i(y) = h_i'(y+\theta_i(x-y))(x-y), 0 < \theta_i < 1,$$

we can get

$$\begin{aligned} & (X-Z)^T(H(X)-H(Z)) \\ =\ & \sum_{i=1}^{n}(h_i(x_i)-h_i(z_i))(x_i-z_i) \\ =\ & \sum_{i=1}^{n}h'(z_i+\theta_i(x_i-z_i))(x_i-z_i)^2 \\ \geq\ & \sum_{i=1}^{n}\underline{h}_i(x_i-z_i)^2 \\ =\ & (X-Z)^T\underline{H}(X-Z). \end{aligned} \tag{13}$$

From (9)-(13), we can obtain

$$\begin{aligned} & 2(X-Z)^T(W(X)-W(Z)) \\ \leq\ & -2(X-Z)^T\underline{H}(X-Z) \\ & +\beta(X-Z)^T(X-Z) \\ & +\frac{1}{\beta}[f(X)-f(Z)]^TA^TA[f(X)-f(Z)] \\ & +\delta(X-Z)^T(X-Z) \\ & +\frac{1}{\delta}[f(X)-f(Z)]^TB^TB[f(X)-f(Z)] \\ & -[f(X)-f(Z)]^T(E-Q^2)L^{-2}\times \\ & \times[f(X)-f(Z)] \\ & -[f(X)-f(Z)]^TQ^2L^{-2}[f(X)-f(Z)] \\ & +(X-Z)^T(X-Z) \\ \leq\ & -(X-Z)^T[2\underline{H}-(\beta+\delta+1)E](X-Z) \\ & -[f(X)-f(Z)]^T[(E-Q^2)L^{-2} \\ & -\frac{1}{\beta}A^TA][f(X)-f(Z)] \\ & -[f(X)-f(Z)]^T[Q^2L^{-2} \\ & -\frac{1}{\delta}B^TB][f(X)-f(Z)] \\ \leq\ & -(X-Z)^TP_1(X-Z) \\ & -[f(X)-f(Z)]^TP_1^*[f(X)-f(Z)] \\ & -[f(X)-f(Z)]^TP_4[f(X)-f(Z)]^T, \end{aligned} \tag{14}$$

where $P_1^* = (E-Q^2)L^{-2} - \frac{1}{\beta}A^TA$.

Since $P_2 = (E-Q^2-G)L^{-2} - \frac{1}{\beta}A^TA > 0$, we have

$$P_1^* = (E-Q^2)L^{-2} - \frac{1}{\beta}A^TA > GL^{-2} > 0.$$

When $X \neq Z$, $P_1 > 0$, $P_1^* > 0$, $P_4 > 0$, (14) implies that $2(X-Z)^T(W(X)-W(Z)) < 0$, we conclude that $W(X) \neq W(Z)$ for all $X \neq Z$. So $W(X)$ is an injective on $\mathbb{R}^n$.

Next, we prove that $\|W(X)\| \to +\infty$ as $\|X\| \to +\infty$. From (14), we have

$$\begin{aligned} & 2X^T(W(X)-W(0)) \\ \leq\ & -X^TP_1X \\ & -[f(X)-f(0)]^TP_1^*[f(X)-f(0)] \\ & -[f(X)-f(0)]^TP_4[f(X)-f(0)]. \end{aligned} \tag{15}$$

Using Schwartz inequality

$$-X^TY \leq \|X^TY\| \leq \|X\|\|Y\|, X, Y \in \mathbb{R}^n,$$

from (15), we get

$$\begin{aligned} & 2\|X\|\|W(X)-W(0)\| \\ \geq\ & X^TP_1X \\ & +[f(X)-f(0)]^TP_1^*[f(X)-f(0)] \\ & +[f(X)-f(0)]^TP_4[f(X)-f(0)] \\ \geq\ & \lambda_m(P_1)\|X\|^2 + \lambda_m(P_1^*)\|f(X)-f(0)\|^2 \\ & +\lambda_m(P_4)\|f(X)-f(0)\|^2 \\ \geq\ & \lambda_m(P_1)\|X\|^2. \end{aligned} \tag{16}$$

From (16), we obtain

$$\|W(X)-W(0)\| \geq \frac{1}{2}\lambda_m(P_1)\|X\|. \tag{17}$$

Since $\|W(X)-W(0)\| \leq \|W(X)\| + \|W(0)\|$, from (17) we have

$$\|W(X)\| \geq \frac{1}{2}\lambda_m(P_1)\|X\| - |W(0)\|. \tag{18}$$

Therefore $\|W(X)\| \to +\infty$ as $\|X\| \to +\infty$. From Lemma 2, we know that $W(x)$ is a homeomorphism on $\mathbb{R}^n$. Thus, system (1) has a unique equilibrium point.

If x^* is equilibrium point of equation (1), then we can obtain $({x^*}^T, {y^*}^T)$ is also equilibrium point of system (3), and vice versa. In the following we only prove the unique equilibrium point $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T$ of system (3) is globally asymptotically stable.

Let $u_i(t) = x_i(t) - x_i^*$, $z_i(t) = y_i(t) - y_i^*$, $i = 1, 2, \ldots, n$. From (5), (7), we can obtain

$$\begin{cases} \dot{U}(t) = -U(t) + Z(t), \\ \dot{Z}(t) = RU(t) - RZ(t) \\ -\alpha(U(t))[H(U(t)) - Af(U(t)) \\ \quad -Bf(U(t - \tau_{ij}(t))) \\ \quad -C\dot{U}(t - \tau_{ij}(t))], \end{cases} \tag{19}$$

where

$$\begin{aligned} & U(t) = (x_1(t) - x_1^*, \cdots, x_n(t) - x_n^*)^T, \\ & Z(t) = (y_1(t) - y_1^*, \cdots, y_n(t) - y_n^*)^T, \\ & \alpha(U(t)) = diag(\alpha_1(u_1(t) + x_1^*), \\ & \qquad \cdots \alpha_n(u_n(t) + x_n^*)), \\ & H(U(t)) = (h_1(u_1(t) + x_1^*) - h_1(x_1^*), \\ & \qquad \cdots, h_n(u_n(t) + x_n^*) - h_n(x_1^*))^T, \\ & f(U(t)) = (f_1(x_1(t)) - f_1(x_1^*), \\ & \qquad \cdots, f_n(x_n(t)) - f_n(x_n^*))^T. \end{aligned}$$

Define the following positive definite Lyapunov functional

$$\begin{aligned} & V(t) = U^T(t)U(t) + Z^T(t)Z(t) \\ & + \int_{t-\tau_{ij}(t)}^{t} U^T(t)Q^2U(t)dt \\ & + \int_{t-\tau_{ij}(t)}^{t} \dot{U}^T(t)\dot{U}(t)dt. \end{aligned} \tag{20}$$

Calculating the upper right Dini-derivative $D^+V(t)$ of $V(t)$ along the solution of (19), we can obtain as follows

$$\begin{aligned} & D^+V(t) = 2\dot{U}^T(t)U(t) + 2\dot{Z}^T(t)Z(t) \\ & +U^T(t)Q^2U(t) + \dot{U}^T(t)\dot{U}(t) \\ & -(1 - \dot{\tau}_{ij}(t))U^T(t - \tau_{ij}(t))Q^2U(t - \tau_{ij}(t)) \\ & -(1 - \dot{\tau}_{ij}(t))\dot{U}^T(t - \tau_{ij}(t))\dot{U}(t - \tau_{ij}(t)) \\ & = \dot{U}^T(t)[2U(t) + \dot{U}(t)] + 2Z^T(t)\dot{Z}(t) \\ & +U^T(t)Q^2U(t) \\ & -(1 - \dot{\tau}_{ij}(t))U^T(t - \tau_{ij}(t))Q^2U(t - \tau_{ij}(t)) \\ & -(1 - \dot{\tau}_{ij}(t))\dot{U}^T(t - \tau_{ij}(t))\dot{U}(t - \tau_{ij}(t)) \\ & = [-U^T(t) + Z^T(t)][U(t) + Z(t)] \\ & +2Z^T(t)[RU(t) - RZ(t) \\ & -\alpha(U(t))[H(U(t)) - Af(U(t)) \\ & -Bf((U(t - \tau_{ij}(t))) - C\dot{U}(t - \tau_{ij}(t))]] \\ & +U^T(t)Q^2U(t) \\ & -(1 - \dot{\tau}_{ij}(t))U^T(t - \tau_{ij}(t))Q^2U(t - \tau_{ij}(t)) \\ & -(1 - \dot{\tau}_{ij}(t))\dot{U}^T(t - \tau_{ij}(t))\dot{U}(t - \tau_{ij}(t)) \\ & = -U^T(t)U(t) + Z^T(t)Z(t) \\ & +2Z^T(t)RU(t) - 2Z^T(t)RZ(t) \\ & -2Z^T(t)\alpha(U(t))H(U(t)) \\ & +2Z^T(t)\alpha(U(t))Af(U(t)) \\ & +2Z^T(t)\alpha(U(t))Bf((U(t - \tau_{ij}(t))) \\ & +2Z^T(t)\alpha(U(t))C\dot{U}(t - \tau_{ij}(t)) \\ & +U^T(t)Q^2U(t) \\ & -(1 - \dot{\tau}_{ij}(t))U^T(t - \tau_{ij}(t))Q^2U(t - \tau_{ij}(t)) \\ & -(1 - \dot{\tau}_{ij}(t))\dot{U}^T(t - \tau_{ij}(t))\dot{U}(t - \tau_{ij}(t)). \end{aligned} \tag{21}$$

From assumption (H_2), we have

$$h_i(x) - h_i(y) = h_i'(y + \theta_i(x - y))(x - y),$$

for $0 < \theta_i < 1$.

Let

$$\begin{aligned} H'(t) \; = \; & diag(h_1'(x_1 + \theta_1(x_1 - x_1^*))(x_1 - x_1^*), \\ & h_2'(x_2 + \theta_2(x_2 - x_2^*))(x_2 - x_2^*), \cdots, \\ & h_n'(x_n + \theta_n(x_n - x_n^*))(x_n - x_n^*)). \end{aligned}$$

Since $0 < \underline{h}_i \le h_i'(x) \le \bar{h}_i$, we can obtain

$$H(U(t)) = H'(t)U(t),$$

and

$$\begin{aligned} & 1 + \bar{h}_i\bar{\alpha}_i - d_i \\ & \ge 1 + \alpha_i h_i'(x_i + \theta_i(x_i - x_i^*)) - d_i \\ & \ge 1 + \underline{h}_i\underline{\alpha}_i - d_i > 0; \\ & 2Z^T(t)RU(t) - 2Z^T(t)\alpha(U(t))H(U(t) \\ & = 2Z^T(t)RU(t) - 2Z^T(t)\alpha(U(t))H'(t)U(t) \\ & = -2Z^T(t)[\alpha(U(t))H'(t) - R]U(t) \\ & = -2\sum_{i=1}^{n}[1 + \alpha_i h_i'(x_i + \theta_i(x_i - x_i^*)) - d_i]z_i u_i \\ & \le \sum_{i=1}^{n}[1 + \alpha_i h_i'(x_i + \theta_i(x_i - x_i^*)) - d_i](z_i^2 + u_i^2) \\ & \le \sum_{i=1}^{n}[1 + \bar{\alpha}_i\bar{h}_i - d_i](z_i^2 + u_i^2) \\ & = U^T(t)GU(t) + Z^T(t)GZ(t); \\ & Z^T(t)\alpha(U(t)\alpha^T(U(t))Z(t) \end{aligned}$$

$$
\begin{aligned}
&= \sum_{i=1}^{n} \alpha_i^2(u_i(t)+x_i^*) z_i^2 \\
&\le Z^T(t)\bar{\alpha}^2 Z(t).
\end{aligned}
$$

By Lemma 3, there exists positive constants β, γ, δ, such that

$$
\begin{aligned}
&2Z^T(t)\alpha(U(t)Af(U) \\
&\le \beta Z^T(t)\alpha(U(t))\alpha^T(U(t))Z(t) \\
&\quad + \frac{1}{\beta} f^T(U)A^TAf(U) \\
&\le \beta Z^T(t)\bar{\alpha}^2 Z(t) + \frac{1}{\beta} f^T(U)A^TAf(U), \\
&2Z^T(t)\alpha(U(t))Bf(U(t-\tau_{ij}(t))) \\
&\le \delta Z^T(t)\alpha(U(t))\alpha^T(U(t))Z(t) \\
&\quad + \frac{1}{\delta} f^T(U(t-\tau_{ij}(t))) \\
&\quad \times B^TBf(U(t-\tau_{ij}(t))) \\
&\le \delta Z^T(t)\bar{\alpha}^2 Z(t) \\
&\quad + \frac{1}{\delta} f^T(U(t-\tau_{ij}(t)))B^TBf(U(t-\tau_{ij}(t))), \\
&2Z^T(t)\alpha(U(t))C\dot{U}(t-\tau_{ij}(t))) \\
&\le \gamma Z^T(t)\alpha(U(t))\alpha^T(U(t))Z(t) \\
&\quad + \frac{1}{\gamma}\dot{U}^T(t-\tau_{ij}(t))C^TC\dot{U}(t-\tau_{ij}(t)) \\
&\le \gamma Z^T(t)\bar{\alpha}^2 Z(t) \\
&\quad + \frac{1}{\gamma}\dot{U}^T(t-\tau_{ij}(t))C^TC\dot{U}(t-\tau_{ij}(t)).
\end{aligned} \tag{22}
$$

On the other hand,

$$
\begin{aligned}
&U^T(t)(E-Q^2-G)U(t) \\
&= \sum_{i=1}^{n}(d_i - q_i^2 - \bar{\alpha}_i\bar{h}_i)(x_i - x_i^*)^2 \\
&\ge \sum_{i=1}^{n}(d_i - q_i^2 - \bar{\alpha}_i\bar{h}_i) l_i^{-2}(f_i(x_i(t)) - f_i(x_i^*))^2 \\
&= f^T(U(t))[E-Q^2-G]L^{-2}f(U(t)).
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
&U(t-\tau_{ij}(t))^TQ^2U(t-\tau_{ij}(t)) \\
&= \sum_{i=1}^{n} q_i^2(x_i(t-\tau_{ij}(t)) - x_i^*)^2 \\
&\ge \sum_{i=1}^{n} q_i^2 l_i^{-2}(f(x_i(t-\tau_{ij}(t))) - f(x_i^*))^2 \\
&= f^T(U(t-\tau_{ij}(t)))Q^2L^{-2}f(U(t-\tau_{ij}(t))
\end{aligned} \tag{24}
$$

From (21)-(24), we can obtain

$$
\begin{aligned}
&D^+V(t) \\
\le\; & -U^T(t)U(t) + Z^T(t)Z(t) + U^T(t)Q^2U(t) \\
& -(1-\eta)U^T(t-\tau_{ij}(t))Q^2U(t-\tau_{ij}(t)) \\
& -(1-\eta)\dot{U}^T(t-\tau_{ij}(t))\dot{U}(t-\tau_{ij}(t)) \\
& +U^T(t)GU(t) + Z(t)^TGZ(t) - 2Z^T(t)RZ(t) \\
& +(\beta+\delta+\gamma)Z^T(t)\bar{\alpha}^2Z(t) + \frac{1}{\beta}f^T(U)A^TAf(U) \\
& +\frac{1}{\delta}f^T(U(t-\tau_{ij}(t)))B^TBf(U(t-\tau_{ij}(t))) \\
& +\frac{1}{\gamma}\dot{U}^T(t-\tau_{ij}(t))C^TC\dot{U}(t-\tau_{ij}(t)) \\
\le\; & -U^T(t)[E-Q^2-G]U(t) \\
& -Z^T(t)[2R-E-G-(\beta+\delta+\gamma)\bar{\alpha}^2]Z(t) \\
& -(1-\eta)U(t-\tau_{ij}(t))^TQ^2U(t-\tau_{ij}(t)) \\
& -\dot{U}^T(t-\tau_{ij}(t))[(1-\eta)E \\
& -\frac{1}{\gamma}C^TC]\dot{U}(t-\tau_{ij}(t)) + \frac{1}{\beta}f^T(U)A^TAf(U) \\
& +\frac{1}{\delta}f^T(U(t-\tau_{ij}(t)))B^TBf(U(t-\tau_{ij}(t))) \\
\le\; & -f^T(U(t))[(E-Q^2-G)L^{-2} - \frac{1}{\beta}AA^T]f(U(t)) \\
& -Z^T(t)[2R-E-G-(\beta+\delta+\gamma)\bar{\alpha}^2]Z(t) \\
& -f^T(U(t-\tau_{ij}(t)))[(1-\eta)Q^2L^{-2} \\
& -\frac{1}{\delta}B^TB]f(U(t-\tau_{ij}(t))) \\
& -\dot{U}^T(t-\tau_{ij}(t))[(1-\eta)E - \frac{1}{\gamma}C^TC]\dot{U}(t-\tau_{ij}(t)) \\
\le\; & -f^T(U(t))P_2f(U(t)) - Z^T(t)P_3Z(t) \\
& -f^T(U(t-\tau_{ij}(t)))P_4f(U(t-\tau_{ij}(t))) \\
& -\dot{U}^T(t-\tau_{ij}(t))P_5\dot{U}(t-\tau_{ij}(t)).
\end{aligned} \tag{25}
$$

Since $P_2 > 0, P_3 > 0, P_4 > 0, P_5 > 0$, (25) implies that $D^+V(t) < 0$. Then, according to the standard Lyapounov theory, it concludes that the unique equilibrium point of system (1) is globally asymptotically stable. This completes the proof of Theorem 5. □

When $\alpha_i(x_i(t)) = \alpha_i, h_i(x_i(t)) = x_i(t)$, the system (1) becomes the following second-order neutral type neural networks with time delays

$$
\begin{aligned}
\ddot{x}_i(t) &= -d_i\dot{x}_i(t) - \alpha_i x_i(t) \\
&+ \sum_{j=1}^{3} a_{ij}f_j(x_j(t)) + \sum_{j=1}^{3} b_{ij}f_j(x_j(t-\tau_{ij}(t))) \\
&+ \sum_{j=1}^{3} c_{ij}\dot{x}_j(t-\tau_{ij}(t)) + I_i.
\end{aligned} \tag{26}
$$

From Theorem 5, we can get the following Theorem

Theorem 6 *Under the hypotheses* (H_3)*, if* $1+\alpha_i - d_i > 0$*, and there exists positive constants* β, δ, γ *and* *a positive diagonal matrix* $Q = diag(q_1, q_2, \cdots, q_n)$, *and*

$$0 < q_i < \sqrt{d_i - \alpha_i} < 1, \quad (i = 1, 2, \cdots, n),$$

such that

$$\begin{aligned}
&P_1 = (1 - \beta - \delta)E > 0, \\
&P_2 = (E - Q^2 - G)L^{-2} - \frac{1}{\beta}A^T A > 0, \\
&P_3 = 2R - E - G - (\beta + \delta + \gamma)(\bar{\alpha})^2 > 0, \\
&P_4 = (1 - \eta)Q^2 L^{-2} - \frac{1}{\delta}B^T B > 0, \\
&P_5 = (1 - \eta)E - \frac{1}{\gamma}C^T C > 0,
\end{aligned}$$

then system (26) has a unique equilibrium point, which is globally asymptotically stable, where

$$L = diag(l_1, l_2, \ldots, l_n),\ \bar{\alpha} = diag(\alpha_1, \alpha_2, \ldots, \alpha_n),$$
$$G = diag(1 + \alpha_1 - d_1, 1 + \alpha_2 - d_2, \cdots, 1 + \alpha_n - d_n)$$

$0 < \dot{\tau}_{ij}(t) \le \eta < 1$,$l_i$ *is given by hypotheses* (H_3).

4 Numerical Example

In this Section, we give two examples to show our results.

Example 7 *We consider the following second-order neutral type Cohen-Grossberg neural networks with time delays*

$$\begin{aligned}
\ddot{x}_i(t) &= -d_i \dot{x}_i(t) - \alpha_i(x_i(t))[h_i(x_i(t)) \\
&- \sum_{j=1}^{3} a_{ij} f_j(x_j(t)) - \sum_{j=1}^{3} b_{ij} f_j(x_j(t - \tau_{ij}(t))) \\
&- \sum_{j=1}^{3} c_{ij} \dot{x}_j(t - \tau_{ij}(t)) + I_i],
\end{aligned} \tag{27}$$

where $i = 1, 2, 3,\ d_1 = 2.3, d_2 = 2.1, d_3 = 2.3,$

$$\begin{aligned}
&\alpha_1(x_1) = 2 + \frac{1}{1 + x_1^2}, \qquad h_1(x_1) = \tfrac{2}{3}x_1, \\
&\alpha_2(x_2) = 2.5 - \frac{1}{1 + x_2^2}, \qquad h_2(x_2) = \tfrac{3}{4}x_2, \\
&\alpha_3(x_3) = 1.6 + \frac{1}{2(1 + x_3^2)}, \qquad h_3(x_3) = \tfrac{5}{6}x_3, \\
&\tau_{ij}(t) = \frac{2 - e^{-t}}{3}, \quad t > 0, \quad i, j = 1, 2, 3. \\
&f_j(x_j) = \tanh(x_j), \quad I_1 = -\tfrac{2}{3}\ln 2 + \tfrac{31}{2340}, \\
&I_2 = -0.75\ln 3 - \frac{31}{2340}, \quad I_3 = -\tfrac{5}{6}\ln 5 + \tfrac{31}{2340}.
\end{aligned}$$

$$A = \frac{1}{72}\begin{bmatrix} 1 & -1 & 1 \\ -1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}, B = \frac{1}{72}\begin{bmatrix} 1 & -1 & 1 \\ -1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix},$$

$$C = \frac{1}{36}\begin{bmatrix} 1 & -1 & 1 \\ -1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}.$$

We have

$$\begin{aligned}
&2 \le \alpha_1(x_1) \le 3, \qquad 1.5 \le \alpha_2(x_2) \le 2.5, \\
&1.6 \le \alpha_3(x_3) \le 2.1, \qquad h_1'(x_1) = \frac{2}{3} \\
&h_2'(x_2) = \tfrac{3}{4}, \qquad h_3'(x_3) = \frac{5}{6}, \\
&\dot{\tau}_{ij}(t) = \tfrac{e^{-t}}{3}, t > 0, \qquad i, j = 1, 2, 3 \\
&|f_j(x) - f_j(y)| \le |x - y|.
\end{aligned}$$

By assumptions $(H_1) - (H_3)$, we select

$$\begin{aligned}
&\underline{\alpha}_1 = 2, \quad \bar{\alpha}_1 = 3, \quad \underline{\alpha}_2 = 1.5, \quad \bar{\alpha}_2 = 2.5, \\
&\underline{\alpha}_3 = 1.6, \quad \bar{\alpha}_3 = 2.1, \qquad \underline{h}_1 = \bar{h}_1 = \tfrac{2}{3}, \\
&\underline{h}_2 = \bar{h}_2 = \frac{3}{4}, \qquad \underline{h}_3 = \bar{h}_3 = \tfrac{5}{6}, \\
&l_1 = l_2 = l_3 = 1, \tau = \frac{2}{3}, \qquad \eta = \tfrac{1}{3}, \\
&\beta = \frac{1}{81}, \quad \delta = \frac{1}{27}, \qquad \gamma = \tfrac{1}{81}, \\
&Q = diag(0.4, 0.3, 0.2), \qquad L = diag(1, 1, 1).
\end{aligned}$$

We have the following results by simple calculation

$$R = \begin{bmatrix} d_1 - 1 & 0 & 0 \\ 0 & d_2 - 1 & 0 \\ 0 & 0 & d_3 - 1 \end{bmatrix} = \begin{bmatrix} 1.3 & 0 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 1.3 \end{bmatrix}$$

$$\underline{H} = \begin{bmatrix} \underline{h}_1 & 0 & 0 \\ 0 & \underline{h}_2 & 0 \\ 0 & 0 & \underline{h}_3 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & 0 & 0 \\ 0 & \frac{3}{4} & 0 \\ 0 & 0 & \frac{5}{6} \end{bmatrix}.$$

$$\bar{\alpha} = \begin{bmatrix} \bar{\alpha}_1 & 0 & 0 \\ 0 & \bar{\alpha}_2 & 0 \\ 0 & 0 & \bar{\alpha}_3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 2.1 \end{bmatrix}.$$

$$G = \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix} = \begin{bmatrix} 0.7 & 0 & 0 \\ 0 & 0.775 & 0 \\ 0 & 0 & 0.45 \end{bmatrix}.$$

where $g_{ii} = 1 + \bar{h}_i\bar{\alpha}_1 - d_i, i = 1, 2, 3.$

$$
\begin{aligned}
&1 + \underline{h}_1\underline{\alpha}_1 - d_1 = \frac{1}{30} > 0,\\
&1 + \underline{h}_2\underline{\alpha}_2 - d_2 = \frac{1}{40} > 0,\\
&1 + \underline{h}_3\underline{\alpha}_3 - d_3 = \frac{1}{30} > 0,\\
&0 < q_1 < \sqrt{d_1 - \bar{h}_1\bar{\alpha}_1} = \sqrt{0.3} < 1,\\
&0 < q_2 < \sqrt{d_2 - \bar{h}_2\bar{\alpha}_2} = \sqrt{0.225} < 1,\\
&0 < q_3 < \sqrt{d_3 - \bar{h}_3\bar{\alpha}_3} = \sqrt{0.55} < 1.
\end{aligned}
$$

$$P_1 = \frac{1}{162}\begin{bmatrix} 46 & 0 & 0 \\ 0 & 73 & 0 \\ 0 & 0 & 100 \end{bmatrix} > 0,$$

$$P_2 = \frac{1}{64}\begin{bmatrix} 5.96 & -1 & 1 \\ -1 & 36.04 & 3 \\ 1 & 3 & 26.44 \end{bmatrix} > 0,$$

$$P_3 = \frac{1}{81}\begin{bmatrix} 27.9 & 0 & 0 \\ 0 & 5.2 & 0 \\ 0 & 0 & 71.1 \end{bmatrix} > 0,$$

$$P_4 = \frac{1}{192}\begin{bmatrix} 17.48 & -1 & 1 \\ -1 & 8.52 & 3 \\ 1 & 3 & 2.21 \end{bmatrix} > 0,$$

$$P_5 = \frac{1}{48}\begin{bmatrix} 23 & -3 & 3 \\ -3 & 23 & 9 \\ 3 & 9 & 23 \end{bmatrix} > 0.$$

From (27), we can get the equation of the equilibriums

$$
\begin{cases}
\frac{2}{3}x_1 - \frac{1}{72}\tanh(x_1) + \frac{1}{72}\tanh(x_2) \\
-\frac{1}{72}\tanh(x_3) - \frac{2}{3}\ln 2 + \frac{47}{2680} = 0, \\
\frac{3}{4}x_2 + \frac{1}{72}\tanh(x_1) + \frac{1}{72}\tanh(x_2) \\
-\frac{1}{72}\tanh(x_3) - \frac{3}{4}\ln 3 - \frac{31}{2680} = 0, \\
\frac{5}{6}x_3 - \frac{1}{72}\tanh(x_1) - \frac{1}{72}\tanh(x_2) \\
+\frac{1}{72}\tanh(x_3) - \frac{5}{6}\ln 5 + \frac{31}{2680} = 0.
\end{cases}
\tag{28}
$$

By calculation, there exists a unique equilibrium point of (28),i.e.,

$$(x_1^*, x_2^*, x_3^*)^T = (\ln 2, \ln 3, \ln 5)^T.$$

Then, the conditions of Theorem 5 hold. Using Theorem 5 there exists a unique equilibrium point of system (27), which is globally asymptotically stable.

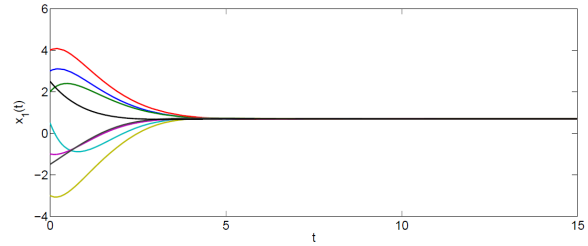

Figure 1: Transient response of state variables $x_1(t)$ of Example 7.

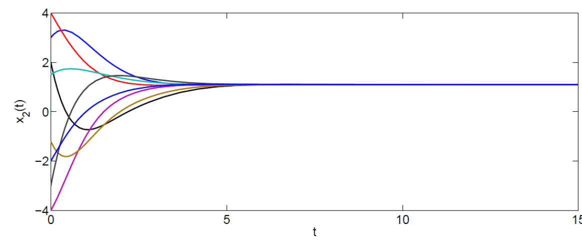

Figure 2: Transient response of state variables $x_2(t)$ of Example 7.

On the other hand, we give any eight groups initial conditions

$$
\begin{aligned}
&[\varphi_1(0), \varphi_2(0), \varphi_3(0), \psi_1(0), \psi_2(0), \psi_3(0))] =: \\
&[3, 3, 3, 4, 5, 6]; [2, 2, 4, 4, -5, -6]; \\
&[4, 4, 2.5, 5, 1.5, 1.6]; \\
&[0.5, 1.5, 1, -4, 2.5, -1.6]; \\
&[-1, -4, 1.2, -1.4, -1.5, 3.6]; \\
&[-3, -1.2, 2, -4, -4.5, -3.6]; \\
&[-1.5, -3, -0.5, -0.4, 4.5, -0.6]; \\
&[2.5, -2, 0.5, 0.4, 0.5, 0.6].
\end{aligned}
$$

Numerical results are presented in Figs.1-Figs.3 using Eqs.(27). Its depict the time responses of state variables of $x_1(t), x_2(t), x_3(t)$ of system in Example7, respectively. Evidently, this consequence is coincident with the result of Theorem 5.

Example 8 *For system (27), let*

$$\alpha_1 = 2, \ \alpha_2 = 1.5, \ \ \alpha_3 = 1.6,$$

$$h_i(x_i(t)) = x_i(t), \quad I_1 = 2\ln 2 - \frac{47}{2340}$$

$$I_2 = 1.5\ln 3 + \frac{31}{2340}, \quad I_3 = 1.6\ln 5 - \frac{31}{2340}.$$

the other parameters are the same as that in Example 7.

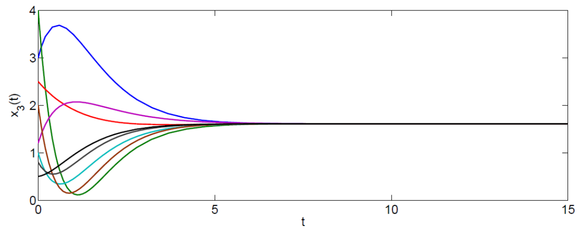

Figure 3: Transient response of state variables $x_3(t)$ of Example 7.

We have the following results by simple calculation

$$R=\begin{bmatrix} d_1-1 & 0 & 0 \\ 0 & d_2-1 & 0 \\ 0 & 0 & d_3-1 \end{bmatrix}=\begin{bmatrix} 1.3 & 0 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 1.3 \end{bmatrix}$$

$$\bar{\alpha}=\begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix}=\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1.6 \end{bmatrix}.$$

$$G=\begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix}=\begin{bmatrix} 0.7 & 0 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}.$$

where $g_{ii}=1+\alpha_1-d_i, i=1,2,3$.

$$\begin{array}{c} 1+\alpha_1-d_1=0.7>0, \\ 1+\alpha_2-d_2=0.4>0, \\ 1+\alpha_3-d_3=0.3>0, \\ 0<q_1<\sqrt{d_1-\alpha_1}=\sqrt{0.3}<1, \\ 0<q_2<\sqrt{d_2-\alpha_2}=\sqrt{0.6}<1, \\ 0<q_3<\sqrt{d_3-\alpha_3}=\sqrt{0.7}<1. \end{array}$$

$$P_1=\frac{77}{81}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}>0,$$

$$P_2=\frac{1}{64}\begin{bmatrix} 6.96 & -1 & 1 \\ -1 & 29.64 & 3 \\ 1 & 3 & 39.24 \end{bmatrix}>0,$$

$$P_3=\frac{5}{81}\begin{bmatrix} 10.58 & 0 & 0 \\ 0 & 10.71 & 0 \\ 0 & 0 & 18.51 \end{bmatrix}>0,$$

$$P_4=\frac{1}{192}\begin{bmatrix} 17.48 & -1 & 1 \\ -1 & 8.52 & 3 \\ 1 & 3 & 2.21 \end{bmatrix}>0,$$

$$P_5=\frac{1}{48}\begin{bmatrix} 23 & -3 & 3 \\ -3 & 23 & 9 \\ 3 & 9 & 23 \end{bmatrix}>0.$$

From (27), we can get the equation of the equilibriums

$$\begin{cases} 2x_1-\frac{1}{36}\tanh(x_1)+\frac{1}{36}\tanh(x_2) \\ -\frac{1}{36}\tanh(x_3)-2\ln 2+\frac{47}{2340}=0, \\ 1.5x_2+\frac{1}{36}\tanh(x_1)+\frac{1}{36}\tanh(x_2) \\ -\frac{1}{36}\tanh(x_3)-1.5\ln 3-\frac{31}{2340}=0, \\ 1.6x_3-\frac{1}{36}\tanh(x_1)-\frac{1}{36}\tanh(x_2) \\ +\frac{1}{36}\tanh(x_3)-1.6\ln 5+\frac{31}{2340}=0. \end{cases} \quad (29)$$

By calculation, there exists a unique equilibrium point of (29), i.e.,

$$(x_1^*,x_2^*,x_3^*)^T=(\ln 2,\ln 3,\ln 5)^T.$$

Then, the conditions of Theorem 2 hold. Using Theorem 6 there exists a unique equilibrium point of system (27), which is globally asymptotically stable.

On the other hand, we give any three groups initial condition

$$\begin{array}{l} [\varphi_1(0),\varphi_2(0),\varphi_3(0),\psi_1(0),\psi_2(0),\psi_3(0))]= \\ [2,3,1,4,5,6]; \\ [1.2,-0.5,4,4,-5,-6]; \\ [-1.5,2,2.5,5,1.5,1.6]. \end{array}$$

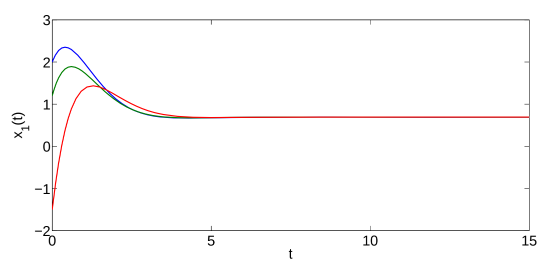

Figure 4: Transient response of state variables $x_1(t)$ of Example 8.

Numerical results are presented in Fig.4-Fig.6 using Eqs. (27). Its depict the time responses of state variables of $x_1(t), x_2(t), x_3(t)$ of system in Example 8, respectively. Evidently, this consequence is coincident with the result of Theorem 6.

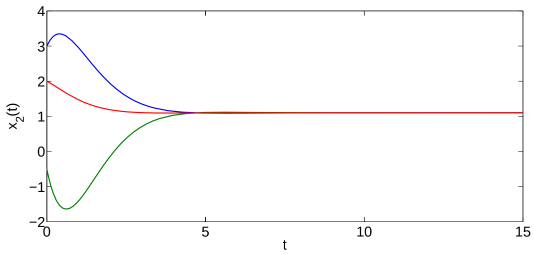

Figure 5: Transient response of state variables $x_2(t)$ of Example 8.

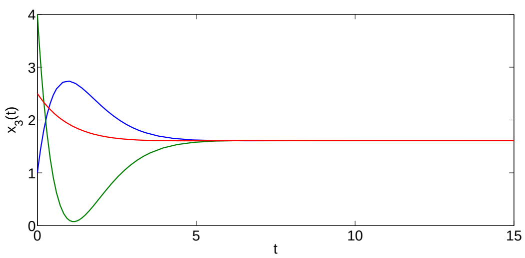

Figure 6: Transient response of state variables $x_3(t)$ of Example8.

5 Conclusions

In this paper, global asymptotic stability for a class second order neutral-type Cohen-Grossberg neural networks with time-varying delays was considered. Some effective criteria which can ensure the existence and stability of equilibrium point for this class of second-order neutral-type systems were derived by using homeomorphism and the standard Lyapounov theory, constructing suitable Lyapunov functional and applying the inequality technique. Novel existence and stability conditions are stated in simple algebraic forms and their verification and applications are straightforward and convenient. Finally, an illustrative example for this class of system was presented.

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