Models and methods of multiobjective optimization in problems of quarry design and planning

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Abstract: - In the paper existing approaches are practice of quarry design and long-term planning are analyzed. Mathematical model of a quarry contour represented with a set of curves, namely bench edges (a SC model) is formulated for a quarry on a flat territory and an on-slope quarry; its likelihood to dynamic systems is shown. Most attention is given to optimization problems setup and solution on the basis of continuous-valued models resulting from piecewise-linear approximation of SC models. Sectoral and contour models aimed at solution of problems in question are presented; they are compared from viewpoints of adequacy and accuracy. Characteristic features of optimization problems based on presented models are shown resulting in specific features of direct methods application. Practical significance of multiobjective optimization for problems in question is substantiated. The solution of that type of problems is exemplified by multiobjective optimization of mining works for "Neriungrinskii" coal quarry as well as by a design stage contour optimization for Ekibastuz coal quarry. Main features of intelligent optimization software developed by the author and used in these computations are presented as well.

Key-Words: - Multiobjective optimization, mining work development, mathematical modelling, feasible direction method, gradient-restoration method, decomposition, intelligent software

1 Introduction

Optimization methods are undoubtedly a powerful tool for engineering systems design and business processes improvement [1-3]. It is taken for granted that mineral industry is a very important part of world economy. New achievements of IT applications to coal mining take place in many areas, i.e. in excavation and transportation processes control [4] with the use of GPS technologies as well as production processes simulation [5]. But as to development of optimization methods for opencast mining, their development stagnates for decades. This situation would be easily explained if general methods of nonlinear programming or optimum control were easily applicable to quarry design and planning problems. If fact, we have just opposite case. Specific features in mathematical description of production processes of opencast mining and therefore in optimization problems for them are numerous, moreover, all of them are unfavourable for application of most reliable methods.

As we see below, optimization problems of open-pit design may be formulated either in functional space of curves or in a finite space. But newest achievements in numerical solution of optimization problems in abstract functional spaces [6], optimum control [7] or even mathematical programming [8–10] are related mostly to convex optimization or some other narrow classes of problems to which problems of question do not belong. It is very important, however, that contemporary mathematicians learned to cope with non-smooth problems, since non-smooth dependencies emerge frequently in the area of mining processes.

The most widely known and practically used method, namely the method of optimal ultimate pit limits establishment by Lerchs and Grossman [11, 12] was proposed almost fifty years ago. It represents both mined-out space of a quarry and a deposit as sets of blocks of regular shape. Based on correct use of dynamic programming, the method belongs to a very scarce set of optimization methods for mining industry problems that have strict mathematical substantiation. Since that time there were many perfections and similar methods [13– 15], but all of them do not change the original setup of the problem. However, it is known for many years that this problem setup reflects the property of a real quarry in a very limited way, so the solution may give only preliminary recommendation as to quarry final contour and depth. It is practically useless for determination of stage quarry contours and mining work regime [16, pp. 412–437], what are the main problems of feasibility study of opencast mining design.

On the contrary, there are many methods of industrial mathematics with much more narrow field of application of each. They are not more than heuristics. Worst of all, usually we have not only the absence of any proof of their convergence, but even the exact formulation of the problem being solved is absent. It is sometimes stated that practical results of their application turned to be satisfactory but no analysis confirming such assertions can be found.

So we have a strange situation. There are a lot of strict methods for a problem with a clear formulation that, however, cannot give more than a crude assessment of a quarry design as a whole and hardly can be spread to assessment of stages of mining works. On the other hand, here are a lot of methods without any substantiation for various design and planning problems. And, after all, several widely used software packages aimed at design and planning problems solution with very little (if any) application of optimization methods, with which most decisions are made by usersengineers with a trial-and-error method in a tedious low-level man-machine dialogue. Another question is whether general methods of linear, nonlinear or integer programming may be applied to problems in question without any adaptation to its specific features. My experience convinces me that more usual applications of linear programming require oversimplified models. On the contrary, more adequate models of nonlinear and mixed integer programming belong to the area where methods are less universal and the success of their application depends usually on taking into account specific features of a problem. In the regarded domain it is not likely to encounter problems with favourable smooth convex programming features, e.g., problems.

For this reason, choices of more adequate model and more efficient method in the regarded area are interrelated and often must be performed simultaneously. My observations for the period of more than 20 years, discussion on several most important international scientific conferences — 3rd, 5th and 20th International Symposium on Mine Planning and Equipment Selection (1994, 1996, 2011}, 1st and 2nd regional APCOM Symposium on Application of Computers and Operations Research in the Mineral Industries (1994, 1997) — show that there are more difficulties than successes on this way [17].

After all, necessity to choose design solutions on the basis of several criteria makes the problem of optimization method application to these choices much more complicated. In Russia, the principal problem exist in balancing interests of the state the owner of mineral resources — and manufacturers that running their business of mining them [18]. Interests of both cannot be reduced to the sole criterion as well.

The paper presents author's approach to setup and solution of the quarry stage contour and some examples of its practical application as well as author's understanding of ways of its further development.

2 Surface Mining Modelling

During the process of surface mining the rock massif is permanently excavated and pit mined-out space grows, part of which can be used as overburden dumps. If a quarry is represented in more detail, we must distinguish collapses of a blasted rock that quickly change their shapes and sizes during excavation and slopes of rock massif sufficiently changing almost instantaneously by blasts separated in time with periods of several days or even weeks. In design and long-time planning problems it is sufficient to reduce consideration to shapes of these slopes as well as horizontal grounds (berms) used to allocate extraction machines and provide freight traffic by trucks or trains. The whole surface of mined-out space is named quarry contour. Its stepwise shape reflects separation of mining work places into a set of benches, each corresponding to a certain layer.

Supposing that slope inclination does not change from the bottom to the top of one bench we fully represent an open pit contour with lower and higher edges of bench slopes. Moreover, for known slope angles depending on local rock properties it is sufficient to establish only the lower edge of each bench, the higher edge being wholly determined with the lower one. To guarantee the possibility of road allocation on grounds we assume that these edges are smooth lines on planes which curvature does not exceeds where R_{min} is the minimum radius of road turns. This representation of a quarry contour in plan with bench edges is shown on Fig.1.

So the mined-out space bounded with a certain pit contour may be approximately treated as an irregular step pyramid. Its representation as a combination of the same rectangular blocks or hexagonal prisms is universal but crude. Horizontal grounds used for excavators and dumpers allocation and horizontal segments of truck or train roads must be composed of block upper bases. So to obtain high accuracy very small blocks must be used. There is no need for high accuracy in the design of ultimate pit contour. Moreover, broad bases of large blocks constitute grounds enabling to locate roads with relatively small curvature that may be practically used. However, in design of stage pit contours so low accuracy is unsatisfactory...



Fig. 1. The general view of a quarry in horizontal projection (as in design drawings)

However, it was understood many years ago that the model in use expresses the only principal property of a quarry shape: it becomes wider from its bottom to top and inclination of its slopes corresponds to rock properties. Blocks models do not include any conditions for transport access that is the next practical requirement for quarry design and its implementation. So these methods may give satisfactory shape for upper benches, but for lower ones often quite non-implementable edges are obtained (see Fig. 2).



Fig.2. An example of ultimate pit contour generated by Korobov's method with MINEFRAME software package [19]

In turn, we must emphasize, after all, that methods of this type admit the only criterion, namely the total profit of mining the deposit. Moreover, this profit is calculated in a conditional way, without possibility of taking into account discounting of the income and expenses.

Bench edges must have no self-intersections and narrow loops (the minimum loop width is $g \ge 2R_{\min}$). We consider edges in their projection on the same horizontal plane. Then between the *l*th and (l+1)th edge must be a ground with minimum width (where b is the minimum width of each bench ground, H_l is the absolute height of the *l*th bench bottom and $d_l = b + (H_l - H_{l+1}) / \tan \alpha_{l+1}$ is the inclination angle for the *l*th bench slope). Here we consider a simplified model with uniform rock on the entire stretch of each bench. Using representation of each curve with dependencies of its points coordinates from natural parameter s (the length of the curve between point and conditional initial point) we formulate the model of a quarry shape in such a way

 $dx_{l} / ds = \cos \varphi_{l}, \ dy_{l} / ds = \sin \varphi_{l}, \ d\varphi_{l} / ds = k_{l}, (1)$ $x_{l} (S_{l}) = x_{l} (0), \ y_{l} (S_{l}) = y_{l} (0), \ \varphi_{l} (S_{l}) = \varphi_{l} (0), (2)$

$$|k_l| \le R_{\min}^{-1}, \tag{3}$$

$$\rho(x_{l}(s), y_{l}(s); x_{l}(s'), y_{l}(s')) \ge g,$$
(4)

$$s, s \in [0, S_l], |s - s| \ge \pi \kappa_{\min},$$

 $Q(x, (s), y, (s); x, (s'), y, (s')) \ge d,$

$$s \in [0, S_l], s' \in [0, S_{l+1}], l = 1, \dots, L-1.$$
(5)

Relationships (1)–(5) describe a typical quarry having the only bottom and located on the territory with horizontal surface and may be modified in a natural way for other cases: for quarry on a mountain slope or between hills, for a quarry which lower area consists of several separate pits etc. For a simple case of an on-slope quarry we must determine its surface with equation F(x, y, z) = 0meaning that points of the rock massif satisfy

$$F(x, y, z) \leq 0$$

Then we have, for initial and terminal points of open edge curves and intermediate points of these curves, resp.

$$F(x_{l}(0), y_{l}(0), H_{l}) = 0, \ F(x_{l}(S_{l}), y_{l}(S_{l}), H_{l}) = 0,$$

$$F(x_{l}(s), y_{l}(s), H_{l}) \le 0, \ s \in [0, S_{l}].$$
(6)

In more complicated cases extra lines representing contours of separate pits must be introduced. We name a model of this type represented with a *set of curves* a *SC model*.

One can see that the model (1)–(5) and more general SC model containing restriction (6) etc resembles a dynamic system described with ODE. So optimization problems for SC models must be a kind of optimum control problems. For example, a problem of optimization the entire value of minerals within the quarry bound corresponds to the criterion

$$\sum_{l=1}^{L} H_l \iint_{Q_l} C_l(x, y) dx dy =$$
$$= \sum_{l=1}^{L} H_l \iint_{0}^{S_l} E(x_l(s), y_l(s)) \cos \varphi_l(s) ds \to \max$$
(7)

where $C_l(x, y)$ characterizes value of minerals in an elementary column of the *l*-th layer with base coordinates x, y (including cost of extraction, transportation and overburden dumping) and

$$E(x, y) = \int_{0}^{y} C(x, Y) dY$$

However, restrictions (4)-(5) are very untypical and look like restrictions in semi-infinite programming problems but are imposed on continuum sets of curves points. There is no hope to solve the problem of this type analytically. Recent work [7] in the area of numerical solution of optimum control problem show that reliable strictly substantiated methods enable to solve more simple problems only. On the contrary, methods of optimum control problems reduction to mathematical programming show their efficiency three decades ago still [20, pp. 284-371]. A similar approach by author based on discrete-time approximation of optimum control problems with constraints on state coordinates showed its efficiency too [21].

So it was proposed to approximate a quasidynamic SC model with a finite-dimensional *SPL model* in which curves are approximated with *polylines* (i.e., broken lines). Variables in a SPL model are coordinates of points — straight segments of polylines. They are denoted as x_{il} , y_{il} , and segment lengths as

$$s_{il} = \rho(x_{il}, y_{il}, x_{i+1l}, y_{i+1l}).$$

To guarantee that the maximum approximation error has the order $\varepsilon^2 R_{\min}$ for a given value ε a condition is introduced

$$s_{i+1} \leq \varepsilon R_{\min}$$
 (8)

Then (3) is approximated with

$$2 \operatorname{tg}(\psi_l/2)/s_{il} \leq (R_{\min})^{-1}, \quad 2 \operatorname{tg}(\psi_l/2)/s_{i+1l} \leq (R_{\min})^{-1}, \quad (9)$$

(4) and (5) — respectively with

 $\rho_1(x_{il}, y_{il}, x_{jl}, y_j, x_{j+1l}, y_{j+1l}) \ge g,$ (10)

$$\rho_1(x_{i+1l}, y_{i+1l}, x_j, y_{j+1l}, x_{j+1l}, y_{j+1l}) \ge g, \qquad (11)$$

$$\rho_1(x_{il}, y_{il}, x_{il}, y_{il}, x_{i+1l}, y_{i+1l}) \ge g, \tag{12}$$

$$\rho_1(x_{j+1l}, y_{j+1l}, x_{il}, y_{il}, x_{i+1l}, y_{i+1l}) \ge g$$
(13)

for

$$i < j, \ s_{i+1l} + \ldots + s_{j-1l} \le \pi R_{\min}, \ s_{j+1l} + \ldots + s_{n_ll} + s_{1l} + \ldots + s_{i-1l} \le \pi R_{\min}.$$
 (14)

and

$$\rho_1(x_{il}, y_{il}, x_{jl+1}, y_{jl+1}, x_{j+1l+1}, y_{j+1l+1}) \ge d_{\min l}, \quad (15)$$

$$\rho_1(x_{i+1l}, y_{i+1l}, x_{jl+1}, y_{j+1l}, x_{j+1l+1}, y_{j+1l+1}) \ge d_{\min l}, \quad (16)$$

$$\rho_1(x_{jl+1}, y_{jl+1}, x_{il}, y_{il}, x_{i+1l}, y_{i+1l}) \ge d_{\min l}, \quad (17)$$

$$\rho_1(x_{j+1l+1}, y_{j+1l+1}, x_{il}, y_{il}, x_{i+1l}, y_{i+1l}) \ge d_{\min l}, \quad (18)$$

The listed restrictions are posed: (8), (9) —on all vertices of all polylines; (10)–(13) on all pairs (a vertex, a segment) of each polyline satisfying (14), (15)–(18) — on all pairs (a vertex, a segment of polyline of an adjacent level).

Here $\rho_1(x_0, y_0, x_1, y_1, x_2, y_2)$ denotes the distance between a point with coordinates x_0, y_0 and a straight segment which ends have coordinates x_1, y_1 and x_2, y_2 . Thus relationships (8)–(18) constitute a SPL model, i.e., a finite-dimensional contour model of a quarry. We shall the following notation: u_{il} for a pair (x_{il}, y_{il}), u_l for a vector comprising u_{il} for all vertices of the *l*-th polyline; *u* for an *N*-dimensional vector comprising components of all u_l .

3 Optimization Problems of Quarry Design on a Finite-Dimensional Contour Model

If an optimization problem of quarry design is formulated on the basis of a SPL model, then constraints (8)–(18) constitute most of relationships in it. Beside them, a problem may contain a relative small number of additional restrictions depending either on all variables or on variables representing a certain polyline or a pair of polylines; these restrictions may be equalities as well as inequalities. This fact is the most important in the choice of the proper optimization method:

- 1. The number of all constraints is great, much more than *N*.
- 2. Nevertheless, on each *u* satisfying (8)–(18) the number of active and ε -active constraints has the same order that *N* (an inequality constraint $F(u) \le 0$ is named *active* on vector *v* if F(v) = 0 and ε -*active* if $-\varepsilon \le F(v) \le 0$).
- 3. For constraints (8)-(18) its residual

F(u) depends on a limited of components of u which number is not greater than 6.

- 4. Therefore, matrix $M_{\varepsilon}(u)$ which rows are gradients of F(u) for ε -active constraints is a band-cellwise matrix. Besides, each cell is a sparse and usually triangular matrix resembling analogous matrices for discrete-time optimum control problems.
- 5. A vector *u* satisfying (8)–(18) may be easily found in many ways. It suffices to determine a succession of concentric circles with proper radii and determine each polyline as an arc polygon for respective circle. Then there are many ways of deformation of such polygons retaining (8)–(18) validity. As to additional constraints, it is much more difficult to satisfy them (it may be impossible, if the problem setup is unreasonable).

For this reason, non-direct methods like modified Lagrange multipliers method [22] in its standard form seem to be inefficient since modified Lagrange function must be a sum of a too large number of terms. More crude methods like penalty functions methods may useful to find an admissible u satisfying all additional constraints [23]. Recent years showed new achievements in convex programming, including nonsmooth optimization [8-10]. It must be noted, however, that some of these constraints must reflect relations between quarry and deposit shapes and so are very unlike to be expressed with convex functions. So a penalty or modified Lagrange function is very likely to have many local minima that are useless for obtaining the optimum (and even feasible) solution. On the other hand, global minimization methods obviously cannot cope with dimensions and complex structure of existing problems. It looks more practical to use local optimization methods but repeat calculations, if necessary, from different and probably randomly generated initial points.

Feasible directions method (FDM) and direct methods combining features of FDM and gradient-restoration method (for problems with equality constraints) have some preferences. Starting from a satisfying (8)–(18) as an initial point, these methods enable to retain their validity. It is necessary to take into account on each iteration only *ɛ*-active constraints. Search of descent direction may be performed efficiently taking into account sparsity of matrix determining the respective LP problem. More efficiency of the above mentioned direct methods is gained with the use of decomposition techniques.

The general form of optimization problems in question (without equality constraints) is the following

$$F_0(u_1, \dots, u_n) \to \min, \tag{19}$$

$$F_j(u_l) \le 0, \ j \in I_{1l}, l = 1, ..., n,$$
 (20)

$$F_i(u_l, u_{l-1}) \le 0, \ j \in I_{2l}, l = 2, ..., n.$$
 (21)

$$F_{i}(u_{1},...,u_{n}) \leq 0, \ j \in I_{0}.$$
 (22)

With a formal equation of dynamics

$$x_l = u_{l-1}, l = 2, ..., n,$$

it is transformed into a problem of discrete-time optimum control for which a FDM with decomposition determination of descent direction was proposed in [21] and evolved later in [24]. Principally it is based on representation of control vector variation in the form

$$w = H_1 y_1 + H_2 y_2 + \ldots + H_{n+1} y_{n+1}$$
(23)

where the set of ε -active constraints is divided into 2n+1 subsets

$$I_{\varepsilon}(u) = J_{11} \cup J_{21} \cup J_{21} \cup ... \cup J_{2n} \cup J_{n+1}$$
(24)
where

$$J_{1i} \subseteq I_{1l\varepsilon}(u), \ J_{2i} \subseteq I_{2l\varepsilon}(u_l, u_{l-1}),$$
 (25)

and matrices H_1, \ldots, H_{n+1} are determined from the condition: for any *w*

$$(F_{ju}(u_i), w) = (F_{ju}(u_i), H_i y_i), \ j \in J_{1l}, \quad (26)$$

$$(F_{ju}(u_i, u_{i-1}), w) = (F_{ju}(u_i), H_i y_i), \ j \in J_{2l}, \ (27)$$

$$(F_{ju}(u), w) = (F_{ju}(u), H_{n+1}y_{n+1}), \ j \in J_{n+1}.$$
 (28)

In fact, formally representation (23) satisfying (24)– (28) may be applied to any regular nonlinear programming problem, but is efficient only for dynamical or quasi-dynamical problems like (19)– (22). Then the problems of descent directions determination for given $\delta \leq \varepsilon$ are

$$s_l \rightarrow \min; \quad (F_{0u}(u), H_i y_i) \le -s_l, \quad (29)$$

$$(F_{ju}(u_l), H_l y_l) \le -K_{lj} s_l, \ j \in I_{1l\delta}(u_i) \cap J_l,$$
 (30)

$$(F_{ju}(u_l, u_{l-1}), H_l y_l) \le -K_{lj} s_l, \ j \in I_{2l\delta}(u_i, u_{l-1}) \cap J_l, (31)$$

$$-1 \le y_{il} \le 1, i=1,..., M_l.$$
 (32)

and the analogous problem for l=n+1 determined by (29), (32) and

$$(F_{ju}(u), H_l y_l) \le -K_{lj} s_l, \ j \in I_{\delta}(u_i) \cap J_l.$$
 (33)

It is proved that for regular ways of determination of $H_1(u)$, $H_2(u)$, ..., $H_{n+1}(u)$ necessary condition of optimality is equivalent to the condition that for solutions of all the problems (29)–(32) and (29), (32), (33) with $\delta = 0$ takes place $s_l = 0$. So the FDM with decomposition is equivalent to the original FDM by requires several times less computations.

This approach was applied to optimization of stage contours of Ekibastuz quarry for variant of its mining with a sole quarry that was earlier put forward and developed with engineering methods by a group of scientists from Moscow State Mining University. With the proposed method, starting from their design contours it was achieved a significant improvement in main parameters. E.g., for the 7-th stage (see Fig. 3) stripping ratio was reduced from 2.77 to 2.72 m³/ton.



Fig. 3. A fragment of general view of design contour of Ekibastuz quarry for depth 480 m before and after optimization

These results seem promising, but many serious difficulties arise. Most of them result from calculation of quantitative and qualitative indices of mineral production for design contours obtained during optimization process. Deposit is usually consists of a lot of separate seams (orebodies) with a complicated shape of bounds. Besides, these parts of a deposit are not uniform. Note, that in optimization process it is necessary calculate not only required indices but as well their derivatives with respect to x_{il}, y_{il} . Representing orebody bounds for a certain horizontal section with polylines and performing preliminary triangulation of its internal domain we reach moderate computational complexity of required calculations. Nevertheless, in many cases non-differentiable dependences take place; and worst feature of non-smoothness is that it is most likely in the vicinity of optimum contours.

Fig. 4 shows cases in which the square of intersection of polygonal domains is continuously differentiable with respect to their vertices coordinates or not. One can see that the square of a polygon domain which bound includes polyline

ABC is continuously differentiable with respect to $x_{\rm B}$, $x_{\rm C}$ and $y_{\rm C}$ and not continuously differentiable with respect to $x_{\rm A}$, $y_{\rm A}$ and $y_{\rm B}$. Formula (7) displays the same properties. So regularization techniques are required for optimization reliability.



Fig. 4. Mutual disposition of a bench edge and a seam

4 Sectoral Representation of a Quarry Contour in Optimization Problems

Sectoral representation was proposed by I. B. Tabakman [25, pp. 64–72] for the sake of annual planning for quarries. The main idea consists in representation of each edge with points of its Thersection with a system of lines. These lines may be either parallel or forming an acute angle between neighbouring lines. Domain between two lines is named sector. Dependencies of qualitative and quantitative characteristics of mineral along the sector axis are determined before solution of any optimization problems and used in further computations. For rounded columnar deposits it was sufficient to use the same system of sectors for all benches [25], in general it is impossible (see Fig. 5).

It must be emphasized that sectoral representation is much less universal than (8)–(18) and enable to represent a subset of possible variants, their main property is that each edge intersects each sector axis one time. It would be better if intersection angles are close to the right angle. Nevertheless, on deep quarries with regular technology directions of mining works change slowly, and for periods of several years sectoral representation is satisfactory.

In works by I. B. Tabakman and his followers [26] constraints expressing technological requirements on shape of a quarry were represented with linear inequality constraints. Slightly modified original model is expressed with sets of linear inequalities posed on positions of points of benches intersections with sector axes x_{ij} measured as distances from sector origin points:

$$x_{ij} \le x_{i-1\,l} + d_{ijl} - D_{i-1},\tag{34}$$

$$|x_{ij} - (x_{ij-1} - L_{ij})| \le D_0, \qquad (35)$$

They link, resp., the above points for adjacent benches in overlapping sectors to provide necessary width of working grounds (34) and in adjacent sectors of the same bench (35) to provide transport access. This representation turns to be crude. Another form of a sectoral model is based on the use of relationships (8)–(18) supposing that each polyline vertex lie on border line of a corresponding sector. Additional relationship is the constraint on angles between sector border lines and edge polylines in vertices of the latter.

It was developed, rather occasionally, a research software package aimed at solution of quarry contour optimization problems on the basis of sectoral models [27]. The initial objective was to work out computerized means for investigation of variants of mining work development on coal quarry "Neriungrinskii" in connection with equipment selection. Practical requirements resulted in multiobjective optimization implementation in the software package. In practice, it meant that a simple language of problem determination was proposed and each problem, including format and order of its inputted and resulting data is expressed not with program modules, but with special text files treated with program modules, as it is described in [28].



Fig. 5. Sectoral representation of a quarry contour

See below the fragment of language representation of a planning problem:

1pit

pit_name c15:0:input coal_dens r:0:input overburden_dens r:0:input yr_advance r:0:input CC_mass r:2:sumobj(zone.CC_mass) K9_mass r:2:sumobj(zone.K9_mass) ash_mass r:2:sumobj(zone.coal_mass) coal_mass r:2:sumobj(zone.coal_mass)

In this fragment we see description of both data structure and problem relationships expressed with arithmetic operators and a set of functions (here it is "sumobj" for the sum of an array components).

5 Experience for Coal Quarry "Neriungrinskii"

In complex study of prospects of "Neriungrinskii" coal quarry production the use of our software package played a very important role. Main aim of the study consisted in the choice of transportation fleet for the next four-year period, but this choice demanded elaboration and comparison of mining work development variants. These variants were generated with the package as successions of optimum year plans with respect to given criteria and restrictions sets.

Characteristic feature of this quarry is the presence of resources of two types of coal, coking coal K9 and coal for power generation CC. Production of both types plays important role and is run simultaneously. Quarry is very deep; the number of benches for the period in question was 15. To represent mining work development a non-linear model was elaborated and prepared for processing by our software package.

Mining work modelling was effected in the following way: first, four-year plan was calculated by optimization with respect to indices for the entire four-year period P, then step-wise optimization was run for succession of years within a period with posing restrictions that guarantee achievement of the optimum values of indices for the entire period.

Table 1. Results of solution of some optimizationproblemsincomplexinvestigationof"Neriungrinskii" coal quarry

Problem objective	Total coal, 10 ⁶ ton	outp	iding ut of sorts	Over- burden, 1000 m ³	Ash con- tent, %	Strip- ping ratio, m ³ /ton
		CC	К9			
Maximi- zation of coking	41.8	21.2	20.6	36.4	17.38	8.71
coal						
output for						
period P						
Minimi- zation of	35.8	18.2	17.5	17.0	17.2	4.74
stripping	55.0	10.2	17.5	17.0	17.2	т./т
ratio for						
period P						
Minimi-						
zation of	38.7	19.6	19.1	23.7	17.29	6.13
over-						
burden						
volume						
for fixed						
coal						
output						

Table 2. Minimization of year stripping ratio deviation from its mean value in the contours of the four-year period P

Total coal, 1000 ton	Inclu CC	uding K9	Overburden, 1000 m ³	Ash content, %	Mining operation advance, m
7730	3511	4218	43288	17.16	97
7708	3421	4286	43167	17.51	87
7817	3507	4309	43779	17.60	98
7496	3257	4239	41983	17.50	98

In the second cycle of research modelling was effected under condition that stripping ratio for each year is equal to the fixed value (its variants were 5.6 and 5.8 m3 per ton) that gave the chance of maximum possible achievement of demands for quarry production of both types. Possible proportion

between K9 and CC production were established by maximization of production of any of these types. It was shown that stable production of CC coal in the range from 4.0 to 4.2 million ton from year to year and minimization of deflection of stripping ratio for a year from its average value yields stabilization of total coal production as well. This variant demands, however, purchase of all new trucks in the beginning of the period. In comparison, another variant was calculated with step-wise minimization of overburden extraction and given range of coal production. Some general results of the above mentioned computations are presented in tables 1–2, the first presenting results of optimization for period P and the second for subsequent years.

6 Conclusion

In the paper the author tried to show practical significance of the problem in question, some particular achievements and still existing problems. In particular, author's approach based on domainoriented modifications of direct optimization methods vielded practical results. some Nevertheless, use of optimization methods, outside a very narrow domain of ultimate pit limits determination, stays occasional. One of the reasons is that they are not integrated in practically used software packages. On the other hand, to perform this integration it is necessary to persuade software developers in reliability of these methods. It must be emphasized that reliable methods are required for solution of complicated problems, nonconvex and as a rule nonsmooth..

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