Research on the revenue-sharing mechanism based on the price game of retailers

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Abstract: We set up a two level supply chain model which consists of one manufacturer and two retailers in this paper. Necessities as the main research object, we develop the nonlinear demand function of products which is more related to the reality. In order to increase the performance of whole supply chain in the decentralized decision, we introduce revenue sharing mechanism in this paper. Through numerical simulations, we analyze the optimal retail prices of products in the basic model and revenue-sharing model respectively, study the impacts of price sensitive coefficient and price adjustment speed parameter on the performance of supply chain. Investigation results from the analysis reveal that, the revenue-sharing mechanism can effectively improve the profits of retailers, and make the competitive market environment more stable.

Key–Words: revenue sharing mechanism, game, equilibrium state, price sensitive coefficient, price adjustment speed parameter

1 Introduction

With the development of economic globalization, the enterprisers meet more and more challenges during the production and management. These challenges are not only from the competitions among domestic companies, but also from the risks of International market. The enterpriser cannot deal with these challenges effectively only by their own, they need to develop cooperation relationship with supply chain members so that they can cope with the fierce competition in the product market and achieve long-term development. Therefore, the competition among enterprisers has gradually transformed into competition between supply chains.

Compared with the decentralized decision, the supply chain can achieve good performance and earn more profits in the centralized decision. However, because the enterpriser doesn’t completely trust each other and the reason for protection of confidential information, they cannot achieve the performance of centralized decision. We need to develop a coordination mechanism, which makes the performance of decentralized decision achieved the level of centralized decision’s.

Up to now, the research on the supply chain coordination is very extensive. There are lots of researchers who have got meaningful results in the field of revenue sharing mechanism. Giannoccaro, Pontrandolfo [1] set up a three-level supply chain model with revenue sharing policy, and obtained the contract parameters which make the profits of supply chain members optimal. Yao, Leung et al.[2] found that the performance of revenue sharing model is better than that of the wholesale contract, and the demand changes and competition can affect the optimal pricing, order quantity and supply chains profit. Li, Zhu et al. (2009)[3] set up revenue sharing models under two kinds of demand functions respectively, and analyzed the influences of price elasticity index on the optimal sale strategy and performance of supply chain. Hsieh, Wu [4] studied the performances of revenue-sharing model, return policy model and combination of revenue-sharing and return policy model, and they examined the influences of retailers attitude toward risk, product substitutable, demand and supply uncertainty on the performance of supply chain. Pan, Lai et al. [5] analyzed whether the performance of revenue sharing model is always better than that of wholesale contract or not under different channel structures. Rhee, Veen et al.[6] found that the revenue sharing mechanism can effectively encourage the related companies to decrease the wholesale price and improve the performance of whole supply chain through sharing profits. Zhang, Fu et al. [7] mainly studied on the effect of demand fluctuation on the performance.
of revenue sharing model. Cao, Wan et al.[8] developed a revenue sharing model which consists of one manufacturer and n retailers. They mainly examined the influences of demand and cost changes on the optimal strategy of retailers. Besides that, some scholars introduce the knowledge of dynamical system into supply chain. They research on the process of repeated game. Guo, Ma [9] studied the repeated game between manufacturer and retailer in the close-loop supply chain, and analyzed the recycling price and profit under the decentralized and centralized decisions respectively. Ma, Sun [10] considered the pricing strategy when new oligarch access to the two oligarch market under different demand patterns. And Ma, Zhang [11] introduced the time-delay decision into the insurance market. They also came up with effective control methods for system chaos. So did Ma, Pu [12]. They studied the dynamic competition between two companies with output and price as the decision variables respectively, and used state feedback method and parameter variation control method to control the effects of decision variables adjustment parameters on the chaotic state of market. Guan and Ma[13] studied the output gaming analysis and chaos control among enterprisers of rational difference in a two-level supply chain. Sun and Ma [14] introduced the bifurcation theory into reality, and analyzed the dynamic game in the Chinese cold rolled steel market. Wu and Ma [15] studied the dynamic game between imported luxury car and domestic car and competition strategy comparison in Chinese car market.

According to the researches on the relative literatures, we develop a revenue sharing model in this paper, which the manufacturer is leader. The manufacturer shares sale profits with two retailers according to certain proportion respectively, and he also decreases the wholesale price under the product cost to motivate the enthusiasm of retailers and earn more sale profits. Different from previous studies, necessities as the main research object, we develop the nonlinear demand function based on the linear demand function, which is more closed to the reality. Then, we respectively analyze the optimal retail prices in the basic and revenue sharing model through numerical simulation, and study the effects of price sensitive coefficient and price adjustment speed parameter on the supply chain’s performance. We find that the revenue sharing mechanism can effectively improve the profits of retailers, and expand the stability range of dynamical system in the process of price game.

2 Model description

We set up a two level supply chain model which consists of a manufacturer and two retailers in this paper. The manufacturer sells products to customers through two competing retailers. Two retailers firstly forecast the market demand for products by their own respectively. Then, they order products from the manufacturer to sell. We assume that the actual demand for product is less than retailers expectation. Therefore, the manufacturer and retailers should make the optimal sale strategies to be more competitive and earn more profits in the product market.

2.1 Notations

\( w \): wholesale price of product per unit, 
\( D_i(\ i = 1, 2) \): expected demand for product of Retailer \( i(\ i = 1, 2) \), 
\( d_i(\ i = 1, 2) \): actual demand for product of Retailer \( i(\ i = 1, 2) \), 
\( c_m \): production cost of product per unit, 
\( p_i \): retail price of Retailer \( i \) per unit \( (\ i = 1, 2) \), 
\( c_i \): sale cost of product \( i(\ i = 1, 2) \), 
\( u \): residual value of product, 
\( \Pi_{R_i} \): expected profit of Retailer \( i(\ i = 1, 2) \), 
\( \Pi_{M} \): expected profit of manufacturer, 
\( \Pi_{S} \): expected profit of supply chain.

2.2 Assumptions

Assumptions 1. The manufacturer satisfies the demand for products of the two retailers, and the quantity supplied does not go beyond the quantity demanded as well as the situation of shortages is prevented.

Assumptions 2. We did not consider the personal preferences of customers, and the demand for product is only related to the market share and the retail price.

Assumptions 3. According to the history data accumulated previously and experiences they have gained, two retailers forecasted the demand for products firstly. Considering the reality, the market demand of product doesn’t present a linear change trend. Especially for necessities, the market demand does not change a lot as price is increasing. But the price effects between similar necessities will be relatively large. Necessities as the main research object, we assume that the expected product demands of two retailers satisfy 
\( D_1 = \alpha_1 - \beta_1 \sqrt{p_1} + \gamma_1 p_2 \) and 
\( D_2 = \alpha_2 - \beta_2 \sqrt{p_2} + \gamma_2 p_1 \). Where \( \alpha_i(\ i = 1, 2) \) is the maximum demand for product which Retailer \( i \) estimates, and \( \alpha_i > 0 \). \( \beta_i \) and \( \gamma_i \) are own-price sensitive coefficient and cross-price sensitive coefficient respectively which Retailer \( i \) set up. And the demand for product is more sensitive to
its own price which satisfies \( \beta_i > \gamma_i > 0 \) \((i = 1, 2)\). After that, they order products from the manufacturer. However, due to their optimistic estimation or market fluctuation, the actual demands for products in the market are less than that they expected. We assume that the actual demand for product is \( d_i = D_i - \Delta \alpha_i \) \((i = 1, 2)\), where \( \Delta \alpha_i \) is the market share that Retailer \( i \) lost.

3 Modeling and analysis

3.1 The decentralized models

In the basic model, the manufacturer and two retailers make decisions independently. We assume that the manufacturer is leader in the market, he decides the wholesale price of product firstly. Then, two retailers determine the retail prices based on the goal of maximum profit respectively, and change them according to the manufacturer’s adjustments.

The expected profit of manufacturer is formulated as

\[
\Pi_M = (w - c_m)(D_1 + D_2)
\]

The expected profit functions of two competing retailers are as follows:

\[
\Pi_{R_1} = p_1 d_1 - (w + c_1)D_1 + u(D_1 - d_1),
\]

\[
\Pi_{R_2} = p_2 d_2 - (w + c_2)D_2 + u(D_2 - d_2).
\]

We can get the relational of \( p_1 \) and \( p_2 \) based on the goal of maximum two retailers’ profits respectively:

\[
\left\{ \begin{array}{l}
3\sqrt{p_1} - \frac{w + c_1}{\sqrt{p_1}} = \frac{2}{\beta_1} (\alpha_1 - \Delta \alpha_1 + \gamma_1 p_2) \\
3\sqrt{p_2} - \frac{w + c_2}{\sqrt{p_2}} = \frac{2}{\beta_2} (\alpha_2 - \Delta \alpha_2 + \gamma_2 p_1)
\end{array} \right. \tag{1}
\]

In the reality, two retailers make decisions on the pricing strategy not only depend on the decision of manufacturer, but also rely on the sales of product at previous time period. It is a dynamic game process. If the profit of retailer is negative at previous time period, he will change his sale strategy at this time. And if the profit is positive, he will continue to use this sale strategy to earn more profits. In this paper, we assume that the pricing strategies of two retailers satisfy:

\[
\left\{ \begin{array}{l}
p_1(t + 1) = p_1(t) + k_1 p_1(t) \frac{\partial \Pi_{R_1}(t)}{\partial p_1(t)} \\
p_2(t + 1) = p_2(t) + k_2 p_2(t) \frac{\partial \Pi_{R_2}(t)}{\partial p_2(t)}
\end{array} \right.
\]

where \( k_i \) \((i = 1, 2)\) denotes the price adjustment speed parameter in the basic model, which satisfies \( k_i > 0 \). It is the speed that the retailer adjust the product’s price during the price game. The dynamical system in the decentralized decision can be described by:

\[
\left\{ \begin{array}{l}
p_1(t + 1) = p_1(t) + k_1 p_1(t) \frac{\partial \Pi_{R_1}(t)}{\partial p_1(t)} - 1.5 \beta_1 \sqrt{p_1(t)} + \frac{(w + c_1) \beta_1}{2 \sqrt{p_1(t)}} - \Delta \alpha_1 + \gamma_1 p_2(t) \\
p_2(t + 1) = p_2(t) + k_2 p_2(t) \frac{\partial \Pi_{R_2}(t)}{\partial p_2(t)} - 1.5 \beta_2 \sqrt{p_2(t)} + \frac{(w + c_2) \beta_2}{2 \sqrt{p_2(t)}} - \Delta \alpha_2 + \gamma_2 p_1(t)
\end{array} \right.
\]

3.2 The revenue sharing model

In order to improve the performance of whole supply chain in the decentralized decision, we introduce the revenue sharing mechanism. The manufacturer decreases the wholesale price under the production cost, and respectively shares the sale profits with two retailers according to certain proportion. The sale profits of two retailers are described as follows.

\[
\Phi_{R_1} = p_1 d_1 + u(D_1 - d_1)
\]

\[
\Phi_{R_2} = p_2 d_2 + u(D_2 - d_2)
\]

We assume that the revenue sharing proportion of Retailer 1 and Retailer 2 is \( \rho_1 \) and \( \rho_2 \) respectively, so the expected profits of two retailers and the manufacturer can be expressed as

\[
\Pi_{R_1} = \rho_1 \Phi_{R_1} - (c_1 + w)D_1
\]

\[
\Pi_{R_2} = \rho_2 \Phi_{R_2} - (c_2 + w)D_2
\]

\[
\Pi_M = (1 - \rho_1) \Phi_{R_1} + (1 - \rho_2) \Phi_{R_2} + (w - c_m)(D_1 + D_2)
\]

The relational expressions of \( p_1 \) and \( p_2 \) which based on the goal of maximum two retailers’ profits are formulated as

\[
\left\{ \begin{array}{l}
\alpha_1 - \Delta \alpha_1 - \beta_1 \sqrt{p_1} - \frac{\beta_1 p_1 p_2 - c_1 - w}{2 \sqrt{p_1}} = \gamma_1 p_2 = 0 \\
\alpha_2 - \Delta \alpha_2 - \beta_2 \sqrt{p_2} - \frac{\beta_2 p_1 p_2 - c_2 - w}{2 \sqrt{p_2}} = \gamma_2 p_1 = 0
\end{array} \right. \tag{2}
\]

Under the condition of revenue sharing mechanism, the dynamical system of two retailers satisfies:

\[
\left\{ \begin{array}{l}
p_1(t + 1) = p_1(t) + k_1 p_1(t) \frac{\partial \Pi_{R_1}(t)}{\partial p_1(t)} \alpha_1 - \Delta \alpha_1 - \beta_1 \sqrt{p_1(t)} - \frac{\beta_1 p_1(t)p_2(t) - c_1 - w}{2 \sqrt{p_1(t)}} + \gamma_1 p_2(t) \\
p_2(t + 1) = p_2(t) + k_2 p_2(t) \frac{\partial \Pi_{R_2}(t)}{\partial p_2(t)} \alpha_2 - \Delta \alpha_2 - \beta_2 \sqrt{p_2(t)} - \frac{\beta_2 p_1(t)p_2(t) - c_2 - w}{2 \sqrt{p_2(t)}} + \gamma_2 p_1(t)
\end{array} \right.
\]

where \( k_i \) \((i = 3, 4)\) denotes the price adjustment speed parameter in the revenue sharing model, which satisfies \( k_i > 0 \).\((i = 3, 4)\).
4 Numerical simulation

In this section, we use numerical simulations to study the optimal retail price in the basic model and revenue sharing model respectively. And we also analyze the influences of price sensitive coefficient and price adjustment speed parameter on the supply chain’s performance. The relative parameters are setting as follows:

\[
\begin{align*}
\alpha_1 &= 300, \quad \alpha_2 = 320, \quad \Delta \alpha_1 = 70, \quad \Delta \alpha_2 = 60, \\
\beta_1 &= 48, \quad \beta_2 = 51, \quad \gamma_1 = 3, \quad \gamma_2 = 2, \\
c_1 &= 2, \quad c_2 = 3, \quad c_m = 10, \quad u = 10, \\
w &= 15, \quad \rho_1 = 0.3, \quad \rho_2 = 0.25.
\end{align*}
\]

4.1 The basic model

We can observe the intersection of two function graphic that are in formula (1) from the three dimensional coordinate in Fig.1. And we get the crossover points of two curves in the zero position of equipotential line, which are shown in Fig.2 and Fig.3.

Figure 1: The three dimensional intersection graph in the basic model

Figure 2: The zero equipotential line intersection point in the basic model

Figure 3: The zero equipotential line intersection point in the basic model

The two retailers conduct dynamic game in the market competition based on the goal of maximum profit. When the retail prices are 29.3 and 28.2, they achieve the state of equilibrium. If one of them improve the price, the other one will also changes his pricing strategy. They conduct price game until the price reach another equilibrium point (793.6, 595.1). This equilibrium state fits the condition of luxury goods. Because we mainly consider the competition of necessities in this paper, we don’t discuss the second point deeply. So, in the basic model, the optimal prices of two retailers are 29.3 and 28.2 respectively.

Figure 4: The effect of \(\beta_1\) on retail price in basic model

Figure 5: The effect of \(\gamma_1\) on retail price in basic model
Fig. 4 and Fig. 5 illustrate the influences of own-price sensitive coefficient $\beta_1$ and cross-price sensitive coefficient $\gamma_1$ on the retail prices respectively. When $\beta_1$ increases from 10 to 55, retail price $p_1$ and $p_2$ rise firstly and then drop sharply. The fold point, which is shown in Fig.4, is a cut-off point. It denotes whether there is a price equilibrium point between two retailers. When the own-price sensitive coefficient $\beta_1$ is smaller than the fold point, the sensitivity of price for product is lower. There doesn’t exist an equilibrium point at this time, which can satisfiy the maximum profit of two retailers together. And the retail price is higher with the increasing of $\beta_1$. When the value of $\beta_1$ is higher than the fold point, there exist equilibrium state between two retailers. And the retail price is lower with the increasing of $\beta_1$. 

When the cross-price sensitive coefficient $\gamma_1$ changes, we can observe similar situations in Fig.5. When $\gamma_1$ is smaller, the substitutability between products is lower. There exist an equilibrium point between two retailers at this time, which can make the two retailers’ profit maximization. And the retail price is higher with the increasing of $\gamma_1$. When the substitutability between products is beyond the fold point, the competition between substitutable products are fierce. And there doesn’t exist equilibrium state between two retailers. The price is lower with the increasing of $\gamma_1$ at this time.

We assume that the price adjustment speed parameter of Retailer 2 is 0.005 in the decentralized decision. From Fig.6, we can find that the dynamical system is stable when $0 < k_1 < 0.00744$. The two retailers achieve the optimal retail prices through price game, which are basically identical with the above simulation results. When $k_1 = 0.00744$, the system appears the first bifurcation. When the value of $k_1$ is increased to 0.00984, the system appears the second bifurcation. And the dynamical system gradually enters into the state of chaos as price adjustment speed parameter $k_1$ is increasing. Fig.7 illustrates the effect of $k_1$ on the profits of manufacturer and two retailers. We can find that the optimal profit of manufacturer is not maximum in the stable situation, the maximum profit appears in the second bifurcation. So the profit of manufacturer is not optimal under the situation of Stackelberg game.

![Figure 6: The effect of $k_1$ on retail price in basic model](image6)

![Figure 7: The effect of $k_1$ on expected profits in basic model](image7)

![Figure 8: The effect of $k_1$ on price in basic model ($\beta_1 = 55$)](image8)
As the own-price sensitive coefficient $\beta_1$ and $\beta_2$ are increasing, we can find that the optimal retail prices are smaller, and the system appears the first bifurcation when $k_1$ is 0.00708 and 0.00768 respectively from Fig.8 and Fig.9. When the own-price sensitive coefficient is higher, Retailer 1 increases the price adjustment speed resulted in the decreasing of prices. The stable range of system is smaller under the influence of $\beta_1$, and it is larger under the effect of $\beta_2$.

4.2 The revenue sharing model

In the revenue sharing model, according to the certain proportion, the manufacturer shares the sale profits with two retailers respectively. And he also decreases the wholesale price under the production cost, which motivates the enthusiasms of retailers. In this section, we assume $w = 1$. 

Fig.10 and Fig.11 show that, as cross-price sensitive coefficient $\gamma_1$ and $\gamma_2$ decrease, the optimal retail prices are smaller, and the system appears the first bifurcation when $k_1$ is 0.00888 and 0.00804 respectively. The decreasing of cross-price sensitive coefficient makes the competition in retailers weaken. So high-speed increasing in price adjustment makes the retail prices decrease and the stable range of system larger. If two products are similar, the optimal prices tend to rise, and the fierce competition would weaken the stability of product market.
in the zero position of equipotential line. The two competing retailers achieve the price equilibrium point \((797.8, 598.6)\) through persistent price game. And if one of them changes the price of product, two retailers will continue to game until they reach the equilibrium point \((24.5, 26.1)\). So, in the revenue sharing model, the optimal retail prices of two retailers are 24.5 and 26.1 respectively. Compared with the basic model, the optimal retail prices are lower in the revenue sharing model. The retailers can be more competitive in the product market. Similar to the basic model, the curves of retail prices appear the situation of break point fluctuation in the revenue sharing model when price sensitive coefficient \(\beta_1\) and \(\gamma_1\) are changing, which are shown in Fig.15 and Fig.16.

In the revenue sharing model, the price adjustment speed parameter of Retail 2 which is denoted by \(k_4\) is 0.005. We can find the influence of price adjustment speed parameter \(k_3\) on the retail prices through numerical simulation. When \(0 < k_3 < 0.03195\), the dynamical system is stable in Fig.17. And the optimal prices of retailers are 24.47 and 25.88 respectively, which are identical with above simulation results. When \(k_3 = 0.03195\), the system appears the first bifurcation. Compared with the situation of decentralized decision, the stable range of system increases significantly in the revenue sharing model. And the market is more stable during the process of dynamic price game.

From Fig.18, we can observe the optimal profits of three supply chain members in the revenue sharing model. Although the expected profit of manufacturer decreases, the profits of two retailers are higher. The revenue sharing mechanism can transfer part of supply chain’s profit from manufacturer to retailers, effectively improve the competitions and enthusiasms of retailers in the market. And it also enhances the stability of product market.

Similar to the condition of decentralized decision, when the price adjustment speed of Retail 1 increases, the increasing of own-price sensitive coefficient \(\beta_1\) makes the optimal retail prices lower and the stable range of system smaller in the revenue sharing model. When the own-price sensitive coefficient \(\beta_2\) increases and cross-price sensitive coefficient \(\gamma_1\), \(\gamma_2\) decrease, the optimal retail prices are lower and the stable range of system is larger, which are shown from Fig.19 to Fig.22. So if the demand function of product is nonlinear, when retailer increases the price...
adjustment speed, the increasing of his product’s own-price sensitive coefficient can lower both the product’s price and the stable range of product market. The decreasing of cross-price sensitive coefficient and increasing of substitutable product’s own-price sensitive coefficient will lower the price, but expand the stable range of market.

Figure 19: The effect of $k_3$ on price in revenue sharing model ($\beta_1 = 55$)

Figure 20: The effect of $k_3$ on price in revenue sharing model ($\beta_2 = 58$)

Figure 21: The effect of $k_3$ on price in revenue sharing model ($\gamma_1 = 1$)

5 Conclusion

We develop a two level supply chain model which consists of one manufacturer and two competing retailers in this paper. Two retailers forecast the market demands for products firstly, then order from manufacturer for sale. Because of the uncertainty of product market, the actual demands for products are less than retailers’ expectations. Different from previous studies, necessities as the main research object, we change the demand function of product in this paper, which is more closed to the reality. In order to improve the performance of whole supply chain, we introduce revenue sharing mechanism into model. The manufacturer decreases the wholesale price under the production cost, and respectively shares sale profits with two retailers according to certain proportion. With numerical simulations, we analyze the optimal retail prices in the basic and revenue sharing model respectively, and study the influences of price sensitive coefficient and price adjustment speed parameter on the performance of whole supply chain. We draw several practical conclusions from the analysis. Firstly, the equilibrium state of retailer that in the process of repeated game is affected by the price sensitive coefficient. If there doesn’t exist equilibrium state, the retail price rises with the increasing of own-price sensitive coefficient, and reduces with the increasing of cross-price sensitive coefficient. If the equilibrium state exists, the situation is opposite. Secondly, the system will gradually enter into chaos when the price adjustment speed parameter is higher. When the demand function of product is nonlinear, if retailer changes the speed of price adjustment, the increasing of his product’s own-price sensitive coefficient will weaken the stability of market, and the decreasing of cross-price sensitive coefficient and increasing of substitutable product’s own-price sensitive coefficient will make the market more stable. Thirdly, revenue sharing mechanism can transfer
part of supply chain’s profit from manufacturer to retailers, which motivates the enthusiasms of retailers and improves their profits. Lastly, compared with the basic model, revenue sharing mechanism can effectively expand the stable range of dynamical system, and make the product market more stable in the process of retailers’ repeated game.

There are several extensions of this paper that could be considered for future research. First of all, the revenue sharing proportions can be related to the efforts of retailers. Then, we recommend other researchers to investigate the supply chain coordination under the information asymmetry. And, it would also be meaningful to examine the coordination mechanism in a multi-retailers or multi-manufacturers supply chain system.

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