Globally exponential synchronization of diffusion recurrent FNNs with time-delays and impulses on time scales

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Abstract: The globally exponential synchronization of diffusion recurrent fuzzy neural networks (FNNs) with time-delays and impulses on time scales is investigated. By applying Lyapunov function and inequality skills, we establish some sufficient conditions to guarantee the globally exponential synchronization of diffusion recurrent FNNs with time-delays and impulses on time scales. One example is given to illustrate the effectiveness of our results.

Key–Words: Globally Exponential synchronization; Diffusion recurrent FNNs; Lyapunov functional; Time scales.

1 Introduction

Artificial neural networks have complex dynamical behaviors such as stability, synchronization, almost periodic attractors, etc. we can refer to [1–40, 43–44] and references cited therein. The study on the neural networks has attracted much attention because of its potential applications such as robust stability, associative memory, image processing, pattern recognition, optimization calculation, information processing, etc..

Synchronization have attracted much attention for the important applications in varies aries after it is proposed by Pecora and Carrol [1–2]. The principle of drive-response synchronization is this: a driver system sent a signal through a channel to a responder system, which uses this signal to synchronize itself with the driver. Namely, the response system is influenced by the behavior of the drive system, but the drive system is independent of the response one. In recently years, many results concerning synchronization problem of time delayed neural networks have been investigated in the literature [1–14].

However, in mathematical modeling of real world problems, we will encounter some other inconvenience, for example, the complexity and the uncertainty or vagueness. Fuzzy theory is considered as a more suitable setting for the sake of taking vagueness into consideration. Based on traditional cellular neural networks (CNNs), T. Yang and L. B. Yang proposed the fuzzy CNNs (FCNNs) [25], which integrates fuzzy logic into the structure of traditional CNNs and maintains local connectedness among cells. Unlike previous CNNs structures, FCNNs have fuzzy logic between its template input and/or output besides the sum of product operation. FCNNs are very useful paradigm for image processing problems, which is a cornerstone in image processing and pattern recognition. In addition, many evolutionary processes in nature are characterized by the fact that their states are subject to sudden changes at certain moments and therefore can be described by impulsive system. Therefore, it is necessary to consider both the fuzzy logic and delay effect on dynamical behaviors of neural networks with impulses.

As is well known, both in biological and manmade neural networks, strictly speaking, diffusion effects can not be avoided in the neural network models when electrons are moving in asymmetric electromagnetic fields, so we must consider that the activations vary in space as well as in time. Many researchers have studied the dynamical properties of continuous time diffusion neural networks (see, for example [26–34]).

Recently, neural networks on time scales have been presented and studied, see, for e.g. [35–40], which can unify the continuous and discrete situations. To the best of our knowledge, few authors have considered the synchronization of time-delayed diffusion recurrent fuzzy neural networks with impulses and Dirichlet boundary conditions on time scales which is a challenging and important problem in theories and applications. Therefore, in this paper, we will investigate the globally exponential synchronization of time-delayed diffusion recurrent fuzzy neural networks (FNNs) with impulses and Dirichlet boundary conditions on time scales as follows:

$$\begin{cases} u_i^{\triangle}(t,x) = \sum\limits_{k=1}^{m} \frac{\partial}{\partial x_k}\big(a_{ik}\frac{\partial u_i}{\partial x_k}\big) - b_i u_i(t,x) \\ \qquad +f_i\bigg(\sum\limits_{j=1}^{n} c_{ij}u_j(t-\tau_{ij},x)+I_i\bigg) \\ \qquad +f_i\bigg(\bigwedge\limits_{j=1}^{n} p_{ij}u_j(t-\tau_{ij},x)+I_i\bigg) \\ \qquad +f_i\bigg(\bigvee\limits_{j=1}^{n} q_{ij}u_j(t-\tau_{ij},x)+I_i\bigg) \\ \qquad +\sum\limits_{j=1}^{n} d_{ij}\upsilon_j + \bigwedge\limits_{j=1}^{n} S_{ij}\upsilon_j \\ \qquad +\bigvee\limits_{j=1}^{n} T_{ij}\upsilon_j, \ \ t\neq t_k, \ \ x\in\Omega, \\ \triangle u_i(t_k,x) = u_i(t_k^+,x)-u_i(t_k^-,x) \\ \qquad = \vartheta_{ik}u_i(t_k,x), \ \ t=t_k, \ \ x\in\Omega, \\ u_i(s,x)=\phi_i(s,x), \ \ (s,x)\in[-\tau,0]_{\mathbb{T}}\times\Omega, \\ u_i(t,x)=0, \ \ (t,x)\in[0,\infty)_{\mathbb{T}}\times\partial\Omega, \end{cases} \tag{1}$$

where $i = 1,2,\dots,n$. n is the number of neurons in the network. $\mathbb{T}\subset\mathbb{R}$ is a time scale and $\mathbb{T}\cap[0,+\infty)\triangleq[0,+\infty)_{\mathbb{T}}$ is unbounded and $\mathbb{T}\cap[-\tau,0]\triangleq[\tau,0]_{\mathbb{T}}\neq\phi$. τ_{ij} is the constant time delay and $\tau = \max_{1\le i,j\le n}\{\tau_{ij}\}$. The impulsive point set $\{t_k\}_{k=0}^{\infty}$ satisfies $0\leqslant t_0 < t_1 < \dots < t_k < \dots$, $t_k\to+\infty$, as $k\to+\infty$, and $x(t_k^+) = \lim_{t\to t_k^+} x(t)$ and $x(t_k^-) = x(t_k)$. $\{\vartheta_{ik}|i = 1,2,...,n, \ k\in\mathbb{N}\}$ denotes impulsive gain set. $x = (x_1,x_2,\dots,x_n)^T \in\Omega\subset\mathbb{R}^m$ and $\Omega = \{x = (x_1,x_2,\dots,x_n)^T : |x_i| < l_i, \ i = 1,2,\dots,m\}$ is a bounded compact set with smooth boundary $\partial\Omega$ in space $\mathbb{R}^m$, $u(t,x) = \big(u_1(t,x),u_2(t,x),\dots,u_n(t,x)\big)^T : \mathbb{T}\times\Omega\to\mathbb{R}^n$ and $u_i(t,x)$ is the state of the ith neurons at time t and in space x. The smooth function $a_{ik} > 0$ corresponds to the transmission diffusion operator along with the ith unit. $b_i > 0$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs. c_{ij} denotes the strength of the jth unit on the ith unit at time t and in space x. d_{ij} is the bias connection strengths of jth unit on the ith unit at time t and in space x. $f_j(\cdot)$ denotes the activation function of the jth unit on the ith unit at time t and in space x. $\bigvee$ and $\bigwedge$ denote the fuzzy AND and fuzzy OR operation, respectively. υ_i and I_i denote input and bias of the ith neuron, respectively. p_{ij}, q_{ij}, S_{ij}, T_{ij} are elements of fuzzy feedback MIN template, fuzzy feedback MAX template, fuzzy feed-forward MIN template and fuzzy feed-forward MAX template, respectively. $\phi(t,x) = \big(\phi_1(t,x),\phi_2(t,x),\dots,\phi_n(t,x),\big)^T : [-\tau,0]_{\mathbb{T}}\times\Omega\to\mathbb{R}^n$ is rd-continuous with respect to $t\in[-\tau,0]_{\mathbb{T}}$ and continuous with respect to $x\in\Omega$, respectively.

The remain of this paper is organized as follows. In Section 2, some notations and basic theorem or lemmas on time scales are given. In Section 3, the main results of globally exponential synchronization is obtained. In Section 4, one example is given to illustrate the effectiveness of our results. Finally, some brief conclusions are presented in Section 5.

2 Preliminaries

In this section, we will firstly state some basic definitions and lemmas are presented on time scales which are used in what follows.

Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operator $\rho,\sigma : \mathbb{T}\to\mathbb{T}$ and the graininess $\mu :\to\mathbb{R}^+$ are defined, respectively, by $\sigma(t) = \inf\{s\in\mathbb{T} : s > t\}$, $\rho(t) = \sup\{s\in\mathbb{T} : s < t\}$, $\mu(t) = \sigma(t)-t$.

A point $t\in\mathbb{T}$ is called left-dense if $t > \inf\mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup\mathbb{T}$ and $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T}\setminus\{m\}$, otherwise $\mathbb{T}^k = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m, then $\mathbb{T}_k = \mathbb{T}\setminus\{m\}$, otherwise $\mathbb{T}_k = \mathbb{T}$.

Definition 1. ([41]) *A function $f : \mathbb{T}\to\mathbb{R}$ is called regulated provided its right-side limits exit (finite) at all right-side points in $\mathbb{T}$ and its left-side limits exist(finite) at all left-side points in $\mathbb{T}$.*

Definition 2. ([41]) *A function $f : \mathbb{T}\to\mathbb{R}$ is called rd-continuous provided it is continuous at right-dense point in $\mathbb{T}$ and its left-side limits exist (finite) at left-dense points in $\mathbb{T}$. The set of rd-continuous function $f : \mathbb{T}\to\mathbb{R}$ will be denoted by $C_{rd} = C_{rd}(\mathbb{T}) = C_{rd}(\mathbb{T},\mathbb{R})$.*

Definition 3. ([41]) *Assume $f : \mathbb{T}\to\mathbb{R}$ and $t\in\mathbb{T}^k$. Then we define $f^{\triangle}(t)$ to be the number (if it exist) with the property that given any $\varepsilon > 0$ there exists a neighborhood* U *of t (i.e.,$U = (t-\Delta,t+\Delta)\cap\mathbb{T}$ for some $\Delta > 0$) such that*

$$|[f(\sigma(t))-f(s)]-f^{\triangle}(t)[\sigma(t)-s]| < \varepsilon|\sigma(t)-s|$$

for all $s\in$ U*. We call $f^{\triangle}(t)$ the delta (or Hilger) derivative of f at t. The set of function $f : \mathbb{T}\to\mathbb{R}$ that is differentiable and whose derivative ia rd-continuous is denote by $C^1_{rd} = C^1_{rd}(\mathbb{T}) = C^1_{rd}(\mathbb{R},\mathbb{T})$. If f is continuous, then f is rd-continuous, If f is rd-continuous, then f is regulated. If f is delta differentiable with region of differentiation D such that $F^{\triangle}(t) = f(t)$ for all $t\in D$.*

Definition 4. ([41]) *Let f be regulated,then there exist a function F which is delta differentiable with region of differentiation D such that $F^{\triangle}(t) = f(t)$ for all $t\in D$.*

Definition 5. ([41]) *Assume $f : \mathbb{T} \to \mathbb{R}$ is a regulated function. Any function F as in Lemma 4 is called a $\triangle$-antiderivative of f.We define the indefinite integral of a regulated function f by*

$$\int f(t)\triangle t = F(t) + C,$$

where C is an arbitrary constant and F is a $\triangle$-antiderivative of f. We define the Cauchy integral by $\int_a^b f(s)\triangle s = F(b) - F(a)$ for all $a, b \in \mathbb{T}$.

A function $F : \mathbb{T} \to \mathbb{R}$ is called an antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^{\triangle}(t) = f(t)$ for all $t \in \mathbb{T}^k$.

Lemma 6. ([41]) *If $a, b \in \mathbb{T}, \alpha, \beta \in \mathbb{R}$ and $f, g \in C(\mathbb{T}, \mathbb{R})$, then*

(i) $\int_a^b [\alpha f(t) + \beta g(t)]\triangle t = \alpha \int_a^b f(t)\triangle t + \beta \int_a^b g(t)\triangle t$,

(ii) *if $f(t) \geq 0$ for all $a \leq t \leq b$, then $\int_a^b f(t)\triangle t \geq 0$,*

(iii) *if $|f(t)| \leq g(t)$ on $[a, b) \triangleq \{t \in \mathbb{T} : a \leq t < b\}$, then $|\int_a^b f(t)\triangle t| \leq \int_a^b g(t)\triangle t$.*

A function $p : \mathbb{T} \to \mathbb{R}$ is called regressive if $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}^k$. The set of all regressive and rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denote by $\mathcal{R} = \mathcal{R}(\mathbb{T}) = \mathcal{R}(\mathbb{T}, \mathbb{R})$. We define the set $\mathcal{R}^+$ of all positive regressive elements of $\mathcal{R}$ by $\mathcal{R}^+ = \mathcal{R}^+(\mathbb{T}, \mathbb{R}) = \{p \in \mathcal{R} : 1 + \mu(t)p(t) > 0$ for all $t \in \mathbb{T}\}$. If p is a regressive function, then the generalized exponential function $e_p(t, s)$ ia define by $e_p(t, s) = \exp\{\int_s^t \xi_{\mu(\tau)}(p(\tau)) \triangle \tau\}$ for all $s, t \in \mathbb{T}$, with the cylinder transformation

$$\xi_h(z) = \begin{cases} \frac{\mathrm{Log}(1+hz)}{h}, & if\ h \neq 0, \\ 0, & if\ h = 0. \end{cases}$$

Let $p, q : \mathbb{T} \to \mathbb{R}$ be two regressive functions, we define

$$p \oplus q = p + q + \mu pq,\ \ominus p = -\frac{p}{1 + \mu p},\ p \ominus q = p \oplus p(\ominus q)$$

If $p \in \mathcal{R}^+$, then $\ominus p \in \mathcal{R}^+$.

The generalized exponential function has the following properties.

Lemma 7. ([41]) *Assume that p, $q : \mathbb{T} \to \mathbb{R}$ are two regressive functions,then*

(i) $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t, s)$;

(ii) $1/e_p(t, s) = e_{\ominus p}(t, s)$;

(iii) $e_p(t, s) = 1/e_p(s, t) = e_{\ominus p}(s, t)$;

(iv) $e_p(t, s)e_p(t, r) = e_p(t, r)$;

(v) $[e_p(t, s)]^{\triangle} = p(t)e_p(t, s)$;

(vi) $[e_p(c, .)]^{\triangle} = -p[e_p(c, .)]^{\sigma}$, *for all* $c \in \mathbb{T}$;

(vii) $\frac{d}{dz}[e_z(t, s)] = \big(\int_s^t (1 + \mu(\tau)z) \triangle \tau\big)e_z(t, s)$.

Lemma 8. ([41]) *Assume that f, $g : \mathbb{T} \to \mathbb{R}$ are Δ-differentiable at $t \in \mathbb{T}^k$. Then*

$$\begin{aligned} (fg)^{\triangle} &= f^{\triangle}(t)g(t) + f(\sigma(t))g^{\triangle} \\ &= g^{\triangle}f(t) + g(\sigma(t))f^{\triangle}(t). \end{aligned}$$

Lemma 9. ([42]) *For each $t \in \mathbb{T}$, let $\mathbb{N}$ be a neighborhood of t. Then, for $\mathrm{V} \in C_{rd}(\mathbb{T}, \mathbb{R}^+)$, define $D^+V^{\triangle}(t)$ to mean that, given $\varepsilon > 0$, there exist a right neighborhood $N_\varepsilon \cap N$ of t such that, for each $s \in N_\varepsilon,\ s > t$,*

$$\frac{1}{u(t)}[V(\sigma(t)) - V(t) - \mu(t)f(t)] < \mathrm{D}^+V^{\triangle}(t) + \varepsilon,$$

where $\mu(t) = \sigma(t) - s$. If t is right-scattered and $V(t)$ is continuous at t, this reduces to $D^+V^{\triangle}(t) = \frac{V(\sigma(t)) - V(t)}{\sigma(t) - t}$.

Next, we introduce the Banach space which is suitable for system (1) and (2). Let $\Omega = \{x = (x_1, x_2, \ldots, x_n)^T : |x_i| < l_i, i = 1, 2, \ldots, m\}$ is an open bounded domain in $\mathbb{R}^m$ with smooth boundary $\partial\Omega$. Let $C_{rd}\big(\mathbb{T} \times \Omega, \mathbb{R}^n\big)$ be the set consisting of all the vector function $u(t, x)$ which is rd-continuous with respect to $t \in \mathbb{T}$ and continuous with respect to $x \in \Omega$, respectively. For every $t \in \mathbb{T}$ and $x \in \Omega$, we define the set $C_{\mathbb{T}}^t = \{u(t, \cdot) : u \in C(\Omega, \mathbb{R}^n)\}$. Then $C_{\mathbb{T}}^t$ is a Banach space with the norm $\|u(t, \cdot)\| = \big(\sum_{i=1}^n \|u_j(t, \cdot)\|_2^2\big)^{1/2}$, where $\|u_i(t, \cdot)\|_2 = \big(\int_\Omega |u_i(t, \cdot)|^2 dx\big)^{1/2}$. Let $C_{rd}[-\tau, 0] \cap \big(\mathbb{T} \times \Omega, \mathbb{R}^n\big)$ consist of all functions $f(t, x)$ which map $[-\tau, 0] \cap \mathbb{T} \times \Omega$ into $\mathbb{R}^n$ and $f(t, x)$ is rd-continuous with respect to $t \in [-\tau, 0] \in \mathbb{T}$ and continuous with respect to $x \in \Omega$, respectively. For every $t \in [-\tau, 0] \cap \mathbb{T}$ and $x \in \Omega$, we define the set $C_{[-\tau,0]\cap\mathbb{T}}^t = \{u(t, \cdot) : u \in C(\Omega, \mathbb{R}^n)\}$. Then $C_{[-\tau,0]\cap\mathbb{T}}^t$ is a Banach space equipped with the norm $\|\phi\|_0 = \big(\sum_{i=1}^n \|\phi_i\|_1^2\big)^{1/2}$, where $\phi(t, x) = \big(\phi_1(t, x), \phi_2(t, x), \ldots, \phi_n(t, x)\big)^T$, $\|\phi_i(t, \cdot)\|_1 = \big(\int_\Omega |\phi_i(\cdot, x)|_\tau^2 dx\big)^{1/2}$, $|\phi_i(\cdot, x)|_\tau = \sup_{s \in \mathbb{T}\cap[-\tau,0]} |\phi_i(s, x)|$.

In order to achieve the globally exponential synchronization, the following system (2) is the controlled slave system corresponding to (1).

$$\begin{cases} \tilde{u}_i^{\triangle}(t,x) = \sum\limits_{k=1}^{m} \frac{\partial}{\partial x_k}\big(a_{ik}\frac{\partial \tilde{u}_i}{\partial x_k}\big) - b_i\tilde{u}_i(t,x) \\ \qquad +f_i\bigg(\sum\limits_{j=1}^{n} c_{ij}\tilde{u}_j(t-\tau_{ij},x)+I_i\bigg) \\ \qquad +f_i\bigg(\bigwedge\limits_{j=1}^{n} p_{ij}\tilde{u}_j(t-\tau_{ij},x)+I_i\bigg) \\ \qquad +f_i\bigg(\bigvee\limits_{j=1}^{n} q_{ij}\tilde{u}_j(t-\tau_{ij},x)+I_i\bigg) \\ \qquad +\sum\limits_{j=1}^{n} d_{ij}\upsilon_j + \bigwedge\limits_{j=1}^{n} S_{ij}\upsilon_j \\ \qquad +\bigvee\limits_{j=1}^{n} T_{ij}\upsilon_j + m_ie_i(t,x),\ t\neq t_k, \\ \triangle\tilde{u}_i(t_k,x) = \tilde{u}_i(t_k^+,x)-\tilde{u}_i(t_k^-,x) \\ \qquad = \vartheta_{ik}\tilde{u}_i(t_k,x),\ \ t=t_k,\ \ x\in\Omega, \\ \tilde{u}_i(s,x)=\psi_i(s,x),\ (s,x)\in[-\tau,0]_{\mathbb{T}}\times\Omega, \\ \tilde{u}_i(t,x)=0,\ (t,x)\in[0,\infty)_{\mathbb{T}}\times\partial\Omega. \end{cases} \tag{2}$$

where $\tilde{u}(t,x) = \big(\tilde{u}_1(t,x),\tilde{u}_2(t,x),\ldots,\tilde{u}_n(t,x)\big)^T$, $e_i(t,x) = \tilde{u}_i(t,x) - u_i(t,x)$. m_i is a positive constant. $\psi(t,x) = \big(\psi_1(t,x),\psi_2(t,x),\ldots,\psi_n(t,x)\big)^T \in C_{rd}\big([-\tau,0]\times\Omega,\mathbb{R}^n\big)$.

On the globally exponential synchronization of coupled neural networks (1) and (2), the following definition is significant.

Definition 10. *Coupled neural network* (1) *and* (2) *is said to be globally exponentially synchronized, if there exist a controlled input* $z(t,x) = (m_1e_1(t,x), m_2e_2(t,x),\ldots,m_ne_n(t,x))^T$ *and a positive constant* $\alpha\in\mathcal{R}^+$ *and* $M\geq 1$ *such that*

$$\|e(t,\cdot)\| = \|\tilde{u}(t,\cdot)-u(t,\cdot)\| \leq Me_{\ominus\alpha}(t,0),\ t\in\mathbb{T}^+,$$

where $\tilde{u}(t,x)$ *and* $u(t,x)$ *are the solutions of system* (1) *and* (2)*, respectively, and satisfy boundary conditions and initial conditions.* α *is called the degree of exponential synchronization on time scales.*

In order to prove the globally exponential synchronization, we need introduce the following two useful lemmas.

Lemma 11. ([31]) *Let* Ω *be a cube* $|x_i| < l_i (i = 1,2,\ldots,m)$ *and assume* $h(x)$ *be a real-valued function belonging to* $C^1(\Omega)$ *which vanish on the boundary* $\partial\Omega$ *of* Ω *, i.e.,* $h(x)|_{\partial\Omega}=0$*. Then*

$$\int_\Omega h^2(x)dx \leq l_i^2\int_\Omega \left|\frac{\partial h}{\partial x_i}\right|^2 dx$$

Lemma 12. ([25]) *Suppose that* $u = (u_1, u_2, \ldots, u_n)^T$ *and* $\tilde{u} = (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_n)^T$ *are the solutions to system* (1) *and* (2)*, respectively, then*

$$\begin{aligned} &\left|\bigwedge_{j=1}^{n} p_{ij}f_j(\tilde{u}_j) - \bigwedge_{j=1}^{n} p_{ij}f_j(u_j)\right| \\ \leq\ & \sum_{j=1}^{n}|p_{ij}||f_j(\tilde{u}_j)-f_j(u_j)|. \\ &\left|\bigvee_{j=1}^{n} q_{ij}f_j(\tilde{u}_j) - \bigvee_{j=1}^{n} q_{ij}f_j(u_j)\right| \\ \leq\ & \sum_{j=1}^{n}|q_{ij}||f_j(\tilde{u}_j)-f_j(u_j)|. \end{aligned}$$

3 Main results

As usual in the theory of impulsive differential equations, at the points of impulse $t_k, k = 1,2,\ldots,,$ we assume that $u_i(t_k,\cdot)\equiv u_i(t_k^-,\cdot)$ and $\dot{u}_i(t_k,\cdot)\equiv \dot{u}_i(t_k^-,\cdot)$.

Inspired by [43], we construct an equivalent theorem between (1) and (3). Then we establish some lemmas which are necessary in the proof of the main results.

Throughout this paper, we always assume that

(H_1) $0 < |\vartheta_{ik}| < 1,\ \ i=1,2,\ldots,n, k\in N, \sum\limits_{k=1}^{\infty}\vartheta_{ik}$ is uniformly absolute convergence.

(H_2) The neurons activation f_i is Lipschitz continuous, that is, there exists a constant $F_i > 0$ such that $|f_i(\xi)-f_i(\eta)| \leq F_i|\xi-\eta|$, for any $\xi,\eta\in\mathbb{R}, i=1,2,\ldots,n$.

(H_3) $-\sum\limits_{k=1}^{m}\frac{2a_{ik}}{l_k^2}+2(m_i-b_i)+K_{ij}+K_{ji}e_{1\oplus1}(\tau_{ji},0) < 0$, where $K_{ij}\triangleq F_i\prod\limits_{k=1}^{\infty}(1-|\vartheta_{ik}|)^{-1}\sum\limits_{j=1}^{n}\prod\limits_{k=1}^{\infty}(1+|\vartheta_{jk}|)\big(|c_{ij}|+|p_{ij}|+|q_{ij}|\big), i=1,2,\ldots,n.$

For the sake of convenience, we will introduce the simple notation by $\kappa_{ik} = \prod\limits_{0\leq t_k\leq t}(1+\vartheta_{ik})\,(k = 1,2,\ldots; i = 1,2,\ldots,n)$. Consider the following non-impulsive system,

$$\begin{aligned} & w_i^{\triangle}(t,x) \\ =\ & \sum_{k=1}^{m}\frac{\partial}{\partial x_k}\big(a_{ik}\frac{\partial w_i}{\partial x_k}\big) - b_iw_i(t,x) \\ & \kappa_{ik}^{-1}\bigg[+f_i\bigg(\sum_{j=1}^{n}c_{ij}\kappa_{jk}w_j(t-\tau_{ij},x)+I_i\bigg) \end{aligned}$$

$$
\begin{aligned}
&+f_i\Big(\bigwedge_{j=1}^{n} p_{ij}\kappa_{jk}w_j(t-\tau_{ij},x)+I_i\Big)\\
&+f_i\Big(\bigvee_{j=1}^{n} q_{ij}\kappa_{jk}w_j(t-\tau_{ij},x)+I_i\Big)\\
&\sum_{j=1}^{n} d_{ij}\upsilon_j+\bigwedge_{j=1}^{n} S_{ij}\upsilon_j+\bigvee_{j=1}^{n} T_{ij}\upsilon_j\Big],\\
&(t,x)\in[0,\infty)_{\mathbb{T}}\times\Omega, \qquad (3)
\end{aligned}
$$

$$w_i(s,x)=\phi_i(s,x),\ \ (s,x)\in[-\tau,0]_{\mathbb{T}}\times\Omega,$$

$$w_i(t,x)=0,\ \ (t,x)\in[0,\infty)_{\mathbb{T}}\times\partial\Omega,$$

We have the following lemma, which shows that system (1) and (3) is equivalent.

Lemma 13. *Suppose (H_1) holds, then we have the following.*

(i) *If $w_i(t,x)$ is a solution of (3), then $u_i(t,x)=\prod\limits_{0\le t_k<t}(1+\vartheta_{ik})w_i(t.x)$ is a solution of (1).*

(ii) *If $u_i(t,x)$ is a solution of (1), then $w_i(t,x)=\prod\limits_{0\le t_k<t}(1+\vartheta_{ik})^{-1}u_i(t,x)$ is a solution of (3).*

Proof: The second result can be proved similarly, so we only proof (i). For any $x\in\Omega$, it is easy to see that $u_i(t,x)=\prod_{0\le t_k<t}(1+\vartheta_{ik})w_i(t,x)$ is absolutely rd-continuous on the interval $(t_k,t_{k+1}]_{\mathbb{T}}$ and for any $x\in\Omega$ and $t\ne t_k$, $k=1,2,\dots,$ we have

$$
\begin{aligned}
&u_i^{\Delta}(t,x)=\kappa_{ik}w_i^{\Delta}(t,x)\\
=\ &\kappa_{ik}\Big\{\sum_{k=1}^{m}\frac{\partial}{\partial x_k}\big(a_{ik}\frac{\partial w_i}{\partial x_k}\big)-b_iw_i(t,x)\\
&+\kappa_{ik}^{-1}\Big[f_i\Big(\sum_{j=1}^{n} c_{ij}\kappa_{jk}w_j(t-\tau_{ij},x)+I_i\Big)\\
&+f_i\Big(\bigwedge_{j=1}^{n} p_{ij}\kappa_{jk}w_j(t-\tau_{ij},x)+I_i\Big)\\
&+f_i\Big(\bigvee_{j=1}^{n} q_{ij}\kappa_{jk}w_j(t-\tau_{ij},x)+I_i\Big)\\
&+\sum_{j=1}^{n} d_{ij}\upsilon_j+\bigwedge_{j=1}^{n} S_{ij}\upsilon_j+\bigvee_{j=1}^{n} T_{ij}\upsilon_j\Big]\Big\}\\
=\ &\sum_{k=1}^{m}\frac{\partial}{\partial x_k}\big(a_{ik}\frac{\partial u_i}{\partial x_k}\big)-b_iu_i(t,x)\\
&+f_i\Big(\sum_{j=1}^{n} c_{ij}u_j(t-\tau_{ij},x)+I_i\Big)\\
&+f_i\Big(\bigwedge_{j=1}^{n} p_{ij}u_j(t-\tau_{ij},x)+I_i\Big)\\
&+f_i\Big(\bigvee_{j=1}^{n} q_{ij}u_j(t-\tau_{ij},x)+I_i\Big)\\
&+\sum_{j=1}^{n} d_{ij}\upsilon_j+\bigwedge_{j=1}^{n} S_{ij}\upsilon_j+\bigvee_{j=1}^{n} T_{ij}\upsilon_j.
\end{aligned}
$$

When $t_k\in\{t_k\}_{k=1}^{\infty}$, for any $x\in\Omega$,

$$
\begin{aligned}
u_i(t_k^+,x)&=\lim_{t\to t_k^+}\prod_{0\le t_j<t}(1+\vartheta_{ij})w_i(t,x)\\
&=\prod_{0\le t_j\le t_k}(1+\vartheta_{ij})w_i(t,x)
\end{aligned}
$$

and

$$u_i(t_k,x)=\prod_{0\le t_j<t_k}(1+\vartheta_{ij})w_i(t,x)$$

so

$$u_i(t_k^+,x)=(1+\vartheta_{ik})u_i(t_k),$$

which implies

$$\Delta u_i(t_k,x)=\vartheta_{ik}u_i(t_k,x).$$

When $(s,x)\in[-\tau,0]_{\mathbb{T}}\times\Omega$,

$$u_i(s,x)=\prod_{0\le t_j<s}(1+\vartheta_{ij})w_i(s,x)=\phi_i(s,x).$$

When $(t,x)\in[0,\infty]_{\mathbb{T}}\times\partial\Omega$, $w_i(t,x)=0$, thus,

$$u_i(t,x)=\prod_{0\le t_j<t}(1+\vartheta_{ij})w_i(t,x)=0.$$

The proof is complete. □

By Lemma 13, we can obtain the equivalent system of the controlled slave system (2) corresponding to (1) as follows:

$$\begin{cases} \tilde{w}_i^{\triangle}(t,x) = \sum\limits_{k=1}^{m} \frac{\partial}{\partial x_k}\big(a_{ik}\frac{\partial \tilde{w}_i}{\partial x_k}\big) - b_i\tilde{w}_i(t,x) \\ \quad +\kappa_{ik}^{-1}\Big[f_i\Big(\sum\limits_{j=1}^{n} c_{ij}\kappa_{jk}\tilde{w}_j(t-\tau_{ij},x)+I_i\Big) \\ \quad +f_i\Big(\bigwedge\limits_{j=1}^{n} p_{ij}\kappa_{jk}\tilde{w}_j(t-\tau_{ij},x)+I_i\Big) \\ \quad +f_i\Big(\bigvee\limits_{j=1}^{n} q_{ij}\kappa_{jk}\tilde{w}_j(t-\tau_{ij},x)+I_i\Big) \\ \quad +\sum\limits_{j=1}^{n} d_{ij}\upsilon_j + \bigwedge\limits_{j=1}^{n} S_{ij}\upsilon_j + \bigvee\limits_{j=1}^{n} T_{ij}\upsilon_j \\ \quad +m_iE_i(t,x)\Big],\ \ t\neq t_k,\ \ x\in\Omega, \\ \triangle\tilde{w}_i(t_k,x) = \tilde{w}_i(t_k^+,x)-\tilde{w}_i(t_k^-,x) \\ \quad = \vartheta_{ik}\tilde{w}_i(t_k,x),\ \ t=t_k,\ \ x\in\Omega, \\ \tilde{w}_i(s,x)=\psi_i(s,x),\ (s,x)\in[-\tau,0]_{\mathbb{T}}\times\Omega, \\ \tilde{w}_i(t,x)=0,\ (t,x)\in[0,\infty)_{\mathbb{T}}\times\partial\Omega. \end{cases} \tag{4}$$

where $\tilde{w}(t,x)=\big(\tilde{w}_1(t,x),\tilde{w}_2(t,x),\ldots,\tilde{w}_n(t,x)\big)^T$, $E_i(t,x)=\tilde{w}_i(t,x)-w_i(t,x)$. m_i is a positive constant. $\psi(t,x)=\big(\psi_1(t,x),\,\psi_2(t,x),\,\ldots,\,\psi_n(t,x)\big)^T \in C_{rd}\big([-\tau,0]\times\Omega,\mathbb{R}^n\big)$.

The following Lemma 14 is useful to prove our main results.

Lemma 14. (see[44]) *Assume that* $0<|a_k|<1$ $(k=0,1,2,\ldots)$, *and the series of number* $\sum_{k=0}^{\infty} a_k$ *is absolute convergence, then the infinite products* $\prod_{k=0}^{\infty}(1-|a_k|)^{-1}$, $\prod_{k=0}^{\infty}(1+a_k)$ *and* $\prod_{k=0}^{\infty}(1+|a_k|)$ *are convergent and* $\prod_{k=0}^{\infty}(1-|a_k|)^{-1}\geq\prod_{k=0}^{\infty}(1+a_k)^{-1}$, $\prod_{k=0}^{\infty}(1+|a_k|)\geq\prod_{k=0}^{\infty}(1+a_k)$.

Theorem 15. *Assume* (H_1)*-*(H_3) *hold. Then the controlled slave system* (2) *is globally exponentially synchronous with the master system* (1).

Proof: From (3) and (4), we obtain the error system (5)-(8) as follows:

$$\begin{aligned} & E_i^{\triangle}(t,x) \\ = \ & \sum_{k=1}^{m}\frac{\partial}{\partial x_k}\big(a_{ik}\frac{\partial E_i}{\partial x_k}\big)+(m_i-b_i)E_i(t,x) \\ & +\kappa_{ik}^{-1}\Big[f_i\Big(\sum_{j=1}^{n}c_{ij}\kappa_{jk}\tilde{w}_j(t-\tau_{ij},x)+I_i\Big) \\ & -f_i\Big(\sum_{j=1}^{n}c_{ij}\kappa_{jk}w_j(t-\tau_{ij},x)+I_i\Big) \\ & +f_i\Big(\bigwedge_{j=1}^{n}p_{ij}\kappa_{jk}\tilde{w}_j(t-\tau_{ij},x)+I_i\Big) \\ & -f_i\Big(\bigwedge_{j=1}^{n}p_{ij}\kappa_{jk}w_j(t-\tau_{ij},x)+I_i\Big) \\ & +f_i\Big(\bigvee_{j=1}^{n}q_{ij}\kappa_{jk}\tilde{w}_j(t-\tau_{ij},x)+I_i\Big) \\ & -f_i\Big(\bigvee_{j=1}^{n}q_{ij}\kappa_{jk}w_j(t-\tau_{ij},x)+I_i\Big)\Big], \\ & t\neq t_k,\ \ x\in\Omega, \end{aligned} \tag{5}$$

$$\begin{aligned} & \Delta E_i(t_k,x)=\vartheta_{ik}\tilde{w}_i(t_k,x)-\vartheta_{ik}w_i(t_k,x) \\ = \ & \vartheta_{ik}E_i(t_k,x),\ \ k=1,2,\ldots,\ \ x\in\Omega, \end{aligned} \tag{6}$$

$$\begin{aligned} & E_i(s,x)=\tilde{w}_i(s,x)-w_i(t,x) \\ = \ & \psi_i(s,x)-\phi_i(s,x), \\ & (s,x)\in[-\tau,0]_{\mathbb{T}}\times\Omega, \end{aligned} \tag{7}$$

and

$$E_i(t,x)=0,\ (t,x)\in[0,\infty)_{\mathbb{T}}\times\partial\Omega. \tag{8}$$

Calculating the delta derivation of $\|E_i(t,\cdot)\|_2^2$ along the solution of (5), we can obtain

$$\begin{aligned} & (\|E_i(t,\cdot)\|_2^2)^{\triangle}=\int_{\Omega}((E_i(t,x))^2)^{\triangle}\mathrm{d}x \\ = \ & \int_{\Omega}\big(E_i(t,x)+E_i(\sigma(t),x)\big)(E_i(t,x))^{\triangle}\mathrm{d}x \\ = \ & \int_{\Omega}\big(2E_i(t,x)+\mu(t)(E_i(t,x))^{\triangle}\big)(E_i(t,x))^{\triangle}\mathrm{d}x \\ = \ & 2\int_{\Omega}E_i(t,x)(E_i(t,x))^{\triangle}\mathrm{d}x \\ & +\mu(t)\int_{\Omega}\big((E_i(t,x))^{\triangle}\big)^2\mathrm{d}x \\ = \ & 2\sum_{k=1}^{m}\int_{\Omega}\frac{\partial}{\partial x_k}E_i(t,x)\big(a_{ik}\frac{\partial E_i}{\partial x_k}\big)\mathrm{d}x \\ & +2\int_{\Omega}(m_i-b_i)(E_i(t,x))^2\mathrm{d}x+2\kappa_{ik}^{-1}\times \\ & \int_{\Omega}E_i(t,x)\Big\{f_i\Big(\sum_{j=1}^{n}c_{ij}\kappa_{jk}\tilde{w}_j(t-\tau_{ij},x)+I_i\Big) \\ & -f_i\Big(\sum_{j=1}^{n}c_{ij}\kappa_{jk}w_j(t-\tau_{ij},x)+I_i\Big) \\ & +f_i\Big(\bigwedge_{j=1}^{n}p_{ij}\kappa_{jk}\tilde{w}_j(t-\tau_{ij},x)+I_i\Big) \\ & -f_i\Big(\bigwedge_{j=1}^{n}p_{ij}\kappa_{jk}w_j(t-\tau_{ij},x)+I_i\Big) \\ & +f_i\Big(\bigvee_{j=1}^{n}q_{ij}\kappa_{jk}\tilde{w}_j(t-\tau_{ij},x)+I_i\Big) \end{aligned}$$

$$-f_i\Big(\bigvee_{j=1}^{n} q_{ij}\kappa_{jk}w_j(t-\tau_{ij},x)+I_i\Big)\Big\}\mathrm{d}x$$
$$+\mu(t)\|(e_i(t,\cdot))^{\triangle}\|_2^2, i=1,2,\ldots,n \tag{9}$$

Employing Green formula [28], Dirichlet boundary condition and Lemma 11, we have for $i=1,2,\ldots,n$,

$$\begin{aligned}&\sum_{k=1}^{m}\int_{\Omega}E_i(t,x)\frac{\partial}{\partial x_k}\big(a_{ik}\frac{\partial E_i}{\partial x_k}\big)\mathrm{d}x\\ =&\sum_{k=1}^{m}\int_{\partial\Omega}a_{ik}\frac{\partial E_i(t,x)}{\partial n_k}E_i(t,x)\mathrm{d}S\\ &-\sum_{k=1}^{m}\int_{\Omega}a_{ik}\big(\frac{\partial E_i(t,x)}{\partial x_k})\big)^2\mathrm{d}x\\ =&-\sum_{k=1}^{m}\int_{\Omega}a_{ik}\big(\frac{\partial E_i(t,x)}{\partial x_k})\big)^2\mathrm{d}x\\ \le&-\sum_{k=1}^{m}\int_{\Omega}\frac{a_{ik}}{l_k^2}\big(E_i(t,x)\big)^2\mathrm{d}x. \end{aligned}\tag{10}$$

Using (9), (10), Lemma 14, conditions (H_1)-(H_3) and Hölder inequality, we get

$$\begin{aligned}&(\|E_i(t,\cdot)\|_2^2)^{\triangle}\\ \le&-\sum_{k=1}^{m}\frac{2a_{ik}}{l_k^2}\|E_i(t,\cdot)\|_2^2+2(m_i-b_i)\|E_i(t,\cdot)\|_2^2\\ &+2F_i\kappa_{ik}^{-1}\sum_{j=1}^{n}\kappa_{jk}\big[|c_{ij}|+|p_{ij}|+|q_{ij}|\big]\\ &\times\|E_i(t,\cdot)\|_2\|E_j(t-\tau_{ij},\cdot)\|_2\\ &+\mu(t)\|(E_i(t,\cdot))^{\triangle}\|_2^2\\ =&-\sum_{k=1}^{m}\frac{2a_{ik}}{l_k^2}\|E_i(t,\cdot)\|_2^2+2(m_i-b_i)\|E_i(t,\cdot)\|_2^2\\ &+2F_i\kappa_{ik}^{-1}\sum_{j=1}^{n}\kappa_{jk}\big[|c_{ij}|+|p_{ij}|+|q_{ij}|\big]\\ &\times\|E_i(t,\cdot)\|_2\|E_j(t-\tau_{ij},\cdot)\|_2\\ &+\mu(t)q(t)\|E_i(t,\cdot)\|_2^2,\end{aligned}\tag{11}$$

where $\|(E_i(t,\cdot))^{\triangle}\|_2^2=q(t)\|E_i(t,\cdot)\|_2^2\ge 0$, $i=1,2,\ldots,n$.

If condition (H_3) holds, we can always choose a positive number $\beta>0$ (may be sufficient small) such that for $1=1,2,\ldots,n$,

$$\begin{aligned}0\;>&\;-\sum_{k=1}^{m}\frac{2a_{ik}}{l_k^2}+2(m_i-b_i)+K_{ij}\\ &+K_{ji}e_{1\oplus 1}(\tau_{ji},0)+\beta.\end{aligned}\tag{12}$$

Consider the following function,

$$\begin{aligned}&q_i(y_i)\\ =&\;y_i\oplus y_i-\sum_{k=1}^{m}\frac{2a_{ik}}{l_k^2}+2(m_i-b_i)+K_{ij}\\ &+K_{ji}e_{1\oplus 1}(\tau_{ji},0)+\frac{\nu(y_i)\mu(t)q(t)}{e_{y_i\oplus y_i}(\sigma(t),0)}\\ &\times\max\big\{e_{(\nu(y_i)-1)\mu(t)q(t)\|E_i(t,\cdot)\|_2^2}(t,0),\\ &e_{y_i\oplus y_i}(\sigma(t),0)\big\},\end{aligned}\tag{13}$$

where $\nu(y_i)=\int_0^{y_i}(e^{y_i-s}/(y_i-s)^2)\mathrm{d}s$, $i=1,2,\ldots,n$. In the light of (12), we get $q_i(0)<-\beta<0$ and $q_i(y_i)$ is continuous for $y_i\in[0,+\infty)$. Moreover, $q_i(y_i)\to+\infty$ as $y_i\to+\infty$, thereby there exist constants $\epsilon_i^*\in(0,+\infty)$ such that $q_i(\epsilon_i^*)=0$ and $q_i(\epsilon_i)<0$, for $\epsilon_i\in(0,\epsilon_i^*)\cap(0,1)$. Choosing $\epsilon=\min\limits_{1\le i\le n}\epsilon_i$, obviously $1>\epsilon>0$, we have for $i=1,2,\ldots,n$,

$$\begin{aligned}&q_i(\epsilon)\\ =&\;\epsilon\oplus\epsilon-\sum_{k=1}^{m}\frac{2a_{ik}}{l_k^2}+2(m_i-b_i)\\ &+K_{ji}e_{1\oplus 1}(\tau_{ji},0)+K_{ij}\\ &+\frac{\nu(\epsilon)\mu(t)q(t)}{e_{\epsilon\oplus\epsilon}(\sigma(t),0)}\times\max\big\{e_{\epsilon\oplus\epsilon}(\sigma(t),0),\\ &e_{(\nu(y_i)-1)\mu(t)q(t)\|E_i(t,\cdot)\|_2^2}(t,0)\big\}\le 0.\end{aligned}\tag{14}$$

Now consider the Lyapunov functional

$$\begin{aligned}&V(t,E(t))\\ =&\sum_{i=1}^{n}\Big\{e_{\epsilon\oplus\epsilon}(\sigma(t),0)\|E_i(t,\cdot)\|_2^2\\ &+K_{ij}\int_{t-\tau_{ij}}^{t}e_{\epsilon\oplus\epsilon}(\sigma(s+\tau_{ij},0))\|E_i(s,\cdot)\|_2^2\Delta s\\ &+e_{(\nu(\epsilon)-1)\mu(t)q(t)\|E_i(t,\cdot)\|_2^2}(t,0)\Big\}.\end{aligned}\tag{15}$$

Calculating $D^+V^{\triangle}(t,E(t))$ along (5) and noting that $\frac{d}{\mathrm{d}z}[e_z(t,s)]=(\int_s^t\frac{1}{1+\mu(\tau)z}\Delta\tau)(e_z(t,s)>0$ if and only if $z\in\mathcal{R}^+$ (that is, $e_z(t,s)$ is increasing with respect to z if and only if $z\in\mathcal{R}^+$), we have

$$\begin{aligned}&D^+V^{\triangle}(t,E(t))\\ =&\sum_{i=1}^{n}\Big\{(\epsilon\oplus\epsilon)e_{\epsilon\oplus\epsilon}(\sigma(t),0)\|E_i(t,\cdot)\|_2^2\\ &+e_{\epsilon\oplus\epsilon}(\sigma(t),0)(\|E_i(t,\cdot)\|_2^2)^{\triangle}\\ &+\big((\nu(\epsilon)-1)\mu(t)q(t)\|E_i(t,\cdot)\|_2^2\big)\\ &\times e_{(\nu(\epsilon)-1)\mu(t)q(t)\|E_i(t,\cdot)\|_2^2}(t,0)\\ &+K_{ij}\Big[e_{\epsilon\oplus\epsilon}(\sigma(t+\tau_{ij},0))\|E_j(t,\cdot)\|_2^2\end{aligned}$$

$$
\begin{aligned}
& -e_{\epsilon\oplus\epsilon}(\sigma(t,0))\|E_j(t-\tau_{ij},\cdot)\|_2^2\Big]\Big\} \\
\leq & \sum_{i=1}^{n}\Big\{(\epsilon\oplus\epsilon)e_{\epsilon\oplus\epsilon}(\sigma(t),0)\|E_i(t,\cdot)\|_2^2 \\
& +e_{\epsilon\oplus\epsilon}(\sigma(t),0)\Big(-\sum_{k=1}^{m}\frac{2a_{ik}}{l_k^2}\|E_i(t,\cdot)\|_2^2 \\
& +2(m_i-b_i)\|E_i(t,\cdot)\|_2^2+K_{ij}\big(\|E_i(t,\cdot)\|_2^2 \\
& +\|E_j(t-\tau_{ij},\cdot)\|_2^2\big)+\mu(t)q(t)\|E_i(t,\cdot)\|_2^2\Big) \\
& +\big((\nu(\epsilon)-1)\mu(t)q(t)\|E_i(t,\cdot)\|_2^2\big) \\
& \times e_{(\nu(\epsilon)-1)\mu(t)q(t)\|E_i(t,\cdot)\|_2^2}(t,0) \\
& +K_{ij}\Big[e_{\epsilon\oplus\epsilon}(\sigma(t+\tau_{ij},0))\|E_j(t,\cdot)\|_2^2 \\
& -e_{\epsilon\oplus\epsilon}(\sigma(t,0))\|E_j(t-\tau_{ij},\cdot)\|_2^2\Big]\Big\} \\
\leq & \sum_{i=1}^{n}\Big\{(\epsilon\oplus\epsilon)e_{\epsilon\oplus\epsilon}(\sigma(t),0)\|E_i(t,\cdot)\|_2^2 \\
& +e_{\epsilon\oplus\epsilon}(\sigma(t),0)\Big(-\sum_{k=1}^{m}\frac{2a_{ik}}{l_k^2}\|E_i(t,\cdot)\|_2^2 \\
& +2(m_i-b_i)\|E_i(t,\cdot)\|_2^2+K_{ij}|E_i(t,\cdot)\|_2^2\Big) \\
& +\mu(t)q(t)\|E_i(t,\cdot)\|_2^2\max\big\{e_{\epsilon\oplus\epsilon}(\sigma(t),0), \\
& e_{(\nu(\epsilon)-1)\mu(t)q(t)\|E_i(t,\cdot)\|_2^2}(t,0)\big\} \\
& +\big((\nu(\epsilon)-1)\mu(t)q(t)\|E_i(t,\cdot)\|_2^2\big) \\
& \times\max\big\{e_{(\nu(\epsilon)-1)\mu(t)q(t)\|E_i(t,\cdot)\|_2^2}(t,0), \\
& e_{\epsilon\oplus\epsilon}(\sigma(t),0)\big\}+e_{\epsilon\oplus\epsilon}(\sigma(t),0)K_{ij}e_{\epsilon\oplus\epsilon}(\tau_{ij},0) \\
& \times\|E_j(t,\cdot)\|_2^2\Big\} \\
\leq & e_{\epsilon\oplus\epsilon}(\sigma(t),0)\sum_{i=1}^{n}\|E_i(t,\cdot)\|_2^2\Big\{\epsilon\oplus\epsilon-\sum_{k=1}^{m}\frac{2a_{ik}}{l_k^2} \\
& +2(m_i-b_i)+K_{ij}+K_{ji}e_{1\oplus 1}(\tau_{ji},0) \\
& +\frac{\nu(\epsilon)\mu(t)q(t)}{e_{\epsilon\oplus\epsilon}(\sigma(t),0)}\times\max\big\{e_{\epsilon\oplus\epsilon}(\sigma(t),0), \\
& e_{(\nu(y_i)-1)\mu(t)q(t)\|E_i(t,\cdot)\|_2^2}(t,0)\big\}\Big\}\leq 0. \qquad (16)
\end{aligned}
$$

Combining (15) and (16), we get for $t\in[0,+\infty)_{\mathbb{T}}$,

$$
\begin{aligned}
& e_{\epsilon\oplus\epsilon}(t,0)\|E(t,.)\|_2^2 \\
= & e_{\epsilon\oplus\epsilon}(t,0)\sum_{i=1}^{n}\|E_i(t,\cdot)\|_2^2 \\
\leq & V(t,E(t))\leq V(0,E(0)) \\
= & \sum_{i=1}^{n}\Big\{\|E_i(0,\cdot)\|_2^2+1 \\
& +K_{ij}\int_{-\tau_{ij}}^{0}e_{\epsilon\oplus\epsilon}(\sigma(s+\tau_{ij},0))\|E_i(s,\cdot)\|_2^2\Delta s\Big\} \\
\leq & \sum_{i=1}^{n}\Big\{\|\psi_i-\phi_i\|_1^2+1+K_{ij}\|\psi_i-\phi_i\|_1^2 \\
& \times\int_{-\tau_{ij}}^{0}e_{\epsilon\oplus\epsilon}(\sigma(s+\tau_{ij},0))\Delta s\Big\} \\
= & \|\psi-\phi\|_0^2+n+\|\psi-\phi\|_0^2 \\
& \times\sum_{i=1}^{n}K_{ij}\int_{-\tau_{ij}}^{0}e_{\epsilon\oplus\epsilon}(\sigma(s+\tau_{ij},0))\Delta s
\end{aligned}
$$

which imply that

$$
\|E(t,\cdot)\|\leq M_1e_{\ominus\epsilon}(t,0), \qquad (17)
$$

where

$$
\begin{aligned}
M_1 = & \Big(\|\psi-\phi\|_0^2+n+\|\psi-\phi\|_0^2\sum_{i=1}^{n}K_{ij} \\
& \times\int_{-\tau_{ij}}^{0}e_{\epsilon\oplus\epsilon}(\sigma(s+\tau_{ij},0))\Delta s\Big)^{\frac{1}{2}}\geq 1.
\end{aligned}
$$

By Lemma 13 and (17), we have

$$
\begin{aligned}
& \|e(t,\cdot)\|_2^2=\sum_{i=1}^{n}\|e_i(t,\cdot)\|_2^2=\sum_{i=1}^{n}\|\tilde{u}_i-u_i\|_2^2 \\
= & \sum_{i=1}^{n}\|\prod_{0\leq t_k\leq t}(1+\vartheta_{ik})(\tilde{w}_i-w_i)\|_2^2 \\
= & \sum_{i=1}^{n}\|\prod_{0\leq t_k\leq t}(1+\vartheta_{ik})E_i(t,\cdot)\|_2^2 \\
\leq & \Pi\sum_{i=1}^{n}\|E_i(t,\cdot)\|_2^2=\Pi^2\|E(t,\cdot)\|^2 \\
\leq & \Pi^2M_1^2(e_{\ominus\epsilon}(t,0))^2
\end{aligned}
$$

which indicate that

$$
\|e(t,\cdot)\|\leq Me_{\ominus\epsilon}(t,0),
$$

here $\Pi=\max_{1\leq i\leq n}\{\prod_{k=1}^{\infty}(1+|\vartheta_{ik}|)\}$. Obviously, $\Pi>1,\ M>1$. According to the Definition 10, we obtain the controlled slave system (2) is globally exponentially synchronous with the master system (1) on time scales. The proof is complete. □

4 Illustrative example

Consider the following two neuron reaction-diffusion FNNs with time-delays and impulses on time scales:

$$\begin{cases} u_i^{\triangle}(t,x) = \sum\limits_{k=1}^{m} \frac{\partial}{\partial x_k}\big(a_{ik}\frac{\partial u_i}{\partial x_k}\big) - b_i u_i(t,x) \\ \qquad +f_i\Big(\sum\limits_{j=1}^{n} c_{ij}u_j(t-\tau_{ij},x)+I_i\Big) \\ \qquad +f_i\Big(\bigwedge\limits_{j=1}^{n} p_{ij}u_j(t-\tau_{ij},x)+I_i\Big) \\ \qquad +f_i\Big(\bigvee\limits_{j=1}^{n} q_{ij}u_j(t-\tau_{ij},x)+I_i\Big) \\ \qquad +\sum\limits_{j=1}^{n} d_{ij}\upsilon_j + \bigwedge\limits_{j=1}^{n} S_{ij}\upsilon_j \\ \qquad +\bigvee\limits_{j=1}^{n} T_{ij}\upsilon_j,\ \ t\neq t_k,\ \ x\in\Omega, \\ \triangle u_i(t_k,x) = u_i(t_k^+,x)-u_i(t_k^-,x) \\ \qquad = \vartheta_{ik}u_i(t_k,x),\ t=t_k,\ x\in\Omega, \\ u_i(s,x)=\phi_i(s,x),\ \ (s,x)\in[-\tau,0]_{\mathbb{T}}\times\Omega, \\ u_i(t,x)=0,\ \ (t,x)\in[0,\infty)_{\mathbb{T}}\times\partial\Omega, \end{cases} \tag{18}$$

the controlled slave system corresponding to (18) can be described as follows:

$$\begin{cases} \tilde{u}_i^{\triangle}(t,x) = \sum\limits_{k=1}^{m} \frac{\partial}{\partial x_k}\big(a_{ik}\frac{\partial \tilde{u}_i}{\partial x_k}\big) - b_i \tilde{u}_i(t,x) \\ \qquad +f_i\Big(\sum\limits_{j=1}^{n} c_{ij}\tilde{u}_j(t-\tau_{ij},x)+I_i\Big) \\ \qquad +f_i\Big(\bigwedge\limits_{j=1}^{n} p_{ij}\tilde{u}_j(t-\tau_{ij},x)+I_i\Big) \\ \qquad +f_i\Big(\bigvee\limits_{j=1}^{n} q_{ij}\tilde{u}_j(t-\tau_{ij},x)+I_i\Big) \\ \qquad +\sum\limits_{j=1}^{n} d_{ij}\upsilon_j + \bigwedge\limits_{j=1}^{n} S_{ij}\upsilon_j \\ \qquad +\bigvee\limits_{j=1}^{n} T_{ij}\upsilon_j + m_i e_i(t,x), \\ \qquad t\neq t_k,\ \ x\in\Omega, \\ \triangle \tilde{u}_i(t_k,x) = \tilde{u}_i(t_k^+,x)-\tilde{u}_i(t_k^-,x) \\ \qquad = \vartheta_{ik}\tilde{u}_i(t_k,x),\ \ t=t_k,\ \ x\in\Omega, \\ \tilde{u}_i(s,x)=\psi_i(s,x),\ \ (s,x)\in[-\tau,0]_{\mathbb{T}}\times\Omega, \\ \tilde{u}_i(t,x)=0,\ \ (t,x)\in[0,\infty)_{\mathbb{T}}\times\partial\Omega. \end{cases} \tag{19}$$

where $f_1(v) = f_2(v) = \frac{e^v-e^{-v}}{e^v+e^{-v}}, \mathbb{T} = \mathbb{T}_1 \bigcup \mathbb{N}_2, \mathbb{T}_1 = \bigcup_{n=0}^{\infty}[n^2+\frac{1}{4},\ (n+1)^2-\frac{1}{4}], \mathbb{T}_2 = \{n^2 : n = 0,1,2,3,\ldots\}, t_k = k^2+k-\frac{1}{2}\ (k=0,1,2,\ldots), \vartheta_{ik} = \frac{(-1)^{i+k}}{2^k}, \Omega = \{x : |x_i| < 1, i=1,2\}, \tau_{ij} = 1 (i,j = 1,2)$, and $I = (I_1, I_2)$ is the constant input vector. Obviously, $\sum_{k=1}^{\infty}\frac{(-1)^{i+k}}{2^k}$ is uniformly absolute convergence. $f_j(v)$ satisfies Lipschitz condition with $F_j = 1$. Let

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 0.7 & 0.4 \\ 0.2 & 0.8 \end{pmatrix},$$

$$\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} 0.4 & 0.5 \\ 0.6 & 0.1 \end{pmatrix},$$

$$\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} 0.1 & 0.2 \\ 0.3 & 0.5 \end{pmatrix},$$

$$\begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix} = \begin{pmatrix} 0.2 & 0.1 \\ 0.7 & 0.8 \end{pmatrix},$$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 11 \\ 10 \end{pmatrix}, \quad \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0.5 \end{pmatrix},$$

Noting that $\ln(1-x) > x$ and $\ln(1+x) < x$ when $0 < x < 1$, we get

$$\begin{aligned} -\sum_{k=1}^{\infty}\ln(1-|\vartheta_{ik}|) &= -\sum_{k=1}^{\infty}\ln(1-\frac{1}{2^k}) \\ &< -\sum_{k=1}^{\infty}\frac{1}{2^k} = -1, \end{aligned}$$

$$\sum_{k=1}^{\infty}\ln(1+|\vartheta_{ik}|) = \sum_{k=1}^{\infty}\ln(1+\frac{1}{2^k}) < \sum_{k=1}^{\infty}\frac{1}{2^k} = 1,$$

$$\prod_{k=1}^{\infty}(1-|\vartheta_{ik}|)^{-1} < \frac{1}{e}, \quad \prod_{k=1}^{\infty}(1+|\vartheta_{ik}|) < e.$$

By simple calculation, we have

$$\sigma(t) = \begin{cases} t, & t\in\mathbb{T}_1; \\ (\sqrt{t}+1)^2, & t\in\mathbb{T}_2. \end{cases}$$

$$\mu(t) = \begin{cases} 0, & t\in\mathbb{T}_1; \\ 2\sqrt{t}+1, & t\in\mathbb{T}_2. \end{cases}$$

$$e_1(t,0) = \begin{cases} e^t, & t\in\mathbb{T}_1; \\ 2^{\sqrt{t}}(\sqrt{t})!, & t\in\mathbb{T}_2. \end{cases}$$

$$e_{1\oplus 1}(t,0) = (e_1(t,0))^2 = \begin{cases} e^{2t}, & t\in\mathbb{T}_1; \\ 2^t[(\sqrt{t})!]^2, & t\in\mathbb{T}_2. \end{cases}$$

$$\begin{aligned} K_{1j} &= F_1\prod_{k=1}^{\infty}(1-|\vartheta_{1k}|)^{-1}\sum_{j=1}^{n}\prod_{k=1}^{\infty}(1+|\vartheta_{jk}|) \\ &\quad \times\big(|c_{1j}|+|p_{1j}|+|q_{1j}|\big) < 1.5, \end{aligned}$$

$$\begin{aligned} K_{2j} &= F_2\prod_{k=1}^{\infty}(1-|\vartheta_{2k}|)^{-1}\sum_{j=1}^{n}\prod_{k=1}^{\infty}(1+|\vartheta_{jk}|) \\ &\quad \times\big(|c_{2j}|+|p_{2j}|+|q_{2j}|\big) < 3, \end{aligned}$$

$$K_{j1} = F_j \prod_{k=1}^{\infty}(1-|\vartheta_{jk}|)^{-1} \sum_{j=1}^{n} \prod_{k=1}^{\infty}(1+|\vartheta_{1k}|) \times \big(|c_{j1}|+|p_{j1}|+|q_{j1}|\big) < 2.3,$$

$$K_{j2} = F_2 \prod_{k=1}^{\infty}(1-|\vartheta_{jk}|)^{-1} \sum_{j=1}^{n} \prod_{k=1}^{\infty}(1+|\vartheta_{2k}|) \times \big(|c_{j2}|+|p_{j2}|+|q_{j2}|\big) < 2.2,$$

$$\begin{aligned} & -\sum_{k=1}^{2}\frac{2a_{1k}}{l_k^2}+2(m_1-b_1)+K_{1j} \\ & +K_{j1}e_{1\oplus 1}(\tau_{j1},0) \\ \leq & -\sum_{k=1}^{2}\frac{2a_{1k}}{l_k^2}+2(m_1-b_1)+K_{1j} \\ & +K_{j1}\max\{e^2,2\} < -18.7+2.3e^2 \\ \approx & -1.67 < 0, \end{aligned}$$

$$\begin{aligned} & -\sum_{k=1}^{2}\frac{2a_{2k}}{l_k^2}+2(m_2-b_2)+K_{2j} \\ & +K_{j2}e_{1\oplus 1}(\tau_{j2},0) \\ \leq & -\sum_{k=1}^{2}\frac{2a_{2k}}{l_k^2}+2(m_2-b_2)+K_{2j} \\ & +K_{j2}\max\{e^2,2\} < -18+2.2e^2 \\ \approx & -0.42 < 0. \end{aligned}$$

Therefore we verified that the conditions (H_1)-(H_3) of Theorem 15 hold. Thus It follows from Theorem 15 that system (18) and system (19) are globally exponentially synchronized.

5 Conclusions

Artificial neural networks have complex dynamical behaviors such as stability, synchronization, almost periodic attractors, etc. The study on the neural networks has attracted much attention because of its potential applications such as robust stability, associative memory, image processing, pattern recognition, optimization calculation, information processing, etc.. Specially, Synchronization have attracted much attention for the important applications in varies aries. The principle of drive-response synchronization is this: a driver system sent a signal through a channel to a responder system, which uses this signal to synchronize itself with the driver. In this paper, we study the globally exponential synchronization of delayed reaction-diffusion static recurrent FNNs with Dirichlet boundary conditions in the continuous and discrete conditions uniformly. For example, If choose $\mathbb{T}=\mathbb{R}$, then $\sigma(t)=t$, $\mu(t)=0$. In this case, system (1) is the continuous delayed static FNNs. If $\mathbb{T}=\mathbb{Z}$, then $\mu(t)=1$, system (1) is the discrete delayed static FNNs. what's more, system (1) is good model which handle many problems such as predator-prey forecast or optimizing of goods output. In addition, the our results obtained are new and interesting and the our methods can be used to study the synchronization for other types of neural network system.

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References:

[1] T. L. Carroll, L. M. Pecora, Cascading synchronized chaotic systems, *Physica D: Nonlinear Phenomena,* Vol. 67, No. 1-3, 1993, pp. 126–140.

[2] T. L. Carroll, J. Heagy, L. M. Pecora, Synchronization and desynchronization in pulse coupled relaxation oscillators, *Physics Letters A,* Vol. 186, No. 3, 1994, pp. 225–229.

[3] Y. Tang, J. A. Fang, Robust synchronization in an array of fuzzy delayed cellular neural networks with stochastic ally hybrid coupling, *Neurocomputing,* Vol. 72, 2009, pp. 3253–3262.

[4] W. W. Yu, J. D. Cao, Adaptive synchronization and lag synchronization of uncertain dynamical system with time delay based on parameter identification, *Physica A,* Vol. 375, 2007, pp. 467–482.

[5] K. Wang, Z. D. Teng , H. J. Jiang, Adaptive synchronization in an array of linearly coupled neural networks with reaction–diffusion terms and time delays, *Commun. Nonlinear Sci. Numer. Simulat.,* Vol. 17, 2012, pp. 3866–3875 .

[6] J. H. Ge, J. Xu, Synchronization and synchronized periodic solution in a simplified five–neuron BAM neural network with delays, *Neurocomputing,* Vol. 74, 2011, pp. 993–999.

[7] P. Yan, T. Lv, Exponential synchronization of fuzzy cellular neural networks with mixed delays and general boundary conditions, *Commun.*

Nonlinear Sci. Numer. Simulat., Vol. 17, 2012, pp. 1003–1011.

[8] Q. T. Gan, Exponential synchronization of stochastic Cohen–Grossberg neural networks with mixed time–varying delays and reaction–diffusion via periodically intermittent control, *Neural Networks,* Vol. 31, 2012, pp. 12–21.

[9] Y. J. Zhang, S. Y. Xu, Robust global synchronization of complex networks with neutral–type delayed nodes, *Applied Mathematics and Computation,* Vol. 216, 2010, pp. 768–778.

[10] M. Luo, J. Xu, Suppression of collective synchronization in a system of neural groups with washout–filter–aided feedback, *Neural Networks,* Vol. 24, 2011, pp. 538–543.

[11] X. S. Yang, C. X. Huang, Q. X. Zhu, Synchronization of switched neural networks with mixed delays via impulsive control, *Chaos, Solitons & Fractals,* Vol. 44, 2011, pp. 817–826.

[12] T. Lia, S. M. Fei, etc, Exponential synchronization of chaotic neural networks with mixed delays, *Neurocomputing,* Vol. 71, 2008, pp. 3005–3019.

[13] L. Sheng, H. Z. Yang, Exponential synchronization of a class of neural networks with mixed time–varying delays and impulsive effects, *Neurocomputing,* Vol. 71, 2008, pp. 3666–3674.

[14] F. Y. Zhou, Global Exponential synchronization of a class of BAM neural networks with time–varying delays, *WSEAS Transactions on Mathematics,* Vol. 12, No. 2, 2013, pp. 138–148.

[15] X. R. Wei, Exponential stability of periodic solutions for inertial Cohen–Grossberg–type BAM neural networks with time delays, *WSEAS Transactions on Mathematics,* Vol. 12, No. 2, 2013, pp. 159–169.

[16] Y. Q. Ke, C. F. Miao, Stability analysis of BAM neural networks with inertial term and time delay, *WSEAS Transactions on Mathematics,* Vol. 10, No. 12, 2011, pp. 425–438.

[17] Q. H. Zhang, L. H. Yang, Exponential p–stability of impulsive stochastic fuzzy cellular neural networks with mixed delays, *WSEAS Transactions on Mathematics,* Vol. 10, No. 12, 2011, pp. 490–499.

[18] H. Y. Yin, Q. H. Zhang, L. H. Yang, Analysis of global exponential stability of fuzzy BAM neural networks with delays, *WSEAS Transactions on Signal Processing,* Vol. 9, No. 4, 2013, pp. 195–202.

[19] B. Z. Du, L. James, Stability analysis of static recurrent neural networks using delay–partitioning and projection, *Neural Networks,* Vol. 22, 2009, pp. 343–347.

[20] M. Gilli, Stability of cellular neural networks with nonpositive templates and nonmonotonic output functions, *IEEE Trans. Circ. Syst.,* Vol. 41, 1994, pp. 518–528.

[21] K. Gopalsamy, X. Z. He, Stability in asymmetric Hopfield nets with transmission delays, *Physica D,* Vol. 76, 1994, pp. 344–358.

[22] Z. Zeng, J. Wang, Improved conditions for global exponential stability of recurrent neural networks with time–varying delays, *Chaos, Solitons & Fractals,* Vol. 23, No. 3, 2006, pp. 623–635.

[23] Y. F. Shao, Exponential stability of periodic neural networks with impulsive effects and time–varying delays, *Applied Mathematics and Computation,* Vol. 217, 2011, pp. 6893–6899

[24] W. Ding, L. H. Wang, 2^N almost periodic attractors for Cohen–Grossberg–type BAM neural networks with variable coefficients and distributed delays, *J. Math. Anal. Appl.,* Vol. 373, 2011, pp. 322–342.

[25] T. Yang, L. B. Yang, The global stability of fuzzy cellular neural networks, *IEEE Trans. Ciruits Syst.,* Vol. 43 No. 1, 1996, pp. 880–883.

[26] F. Y. Zhou, Exponential stability of reaction–diffusion Cohen–Grossberg–type BAM neural networks with time delays, *WSEAS Transactions on Mathematics,* Vol. 11, No. 12, 2012, pp. 1063–1075.

[27] C. R. Ma, F. Y. Zhou, Global exponential stability of high–order BAM neural networks with S–type distributed delays and reaction diffusion terms, *WSEAS Transactions on Mathematics,* Vol. 10, No. 10, 2011, pp. 333–345.

[28] P. C. Liu, F. Q. Yi, Q. Guo, etc, Analysis on global exponential robust stability of reaction–diffusion neural networks with S–type distributed delays, *Physica D,* Vol. 237, 2008, pp. 475–485.

[29] Z. Q. Zhang, Y. Yang, Y. S. Huang, Global exponential stability of interval general BAM neural networks with reaction–diffusion terms and multiple time–varying delays, *Neural Networks,* Vol. 24, 2011, pp. 457–465.

[30] L. S. Wang, Y. Y. Gao, Global exponential robust stability of reaction–diffusion interval neural networks with time–varying delays and terms, *Phys. Lett. A,* Vol. 350 No. 5–6, 2006, pp. 342–348.

[31] J. G. Lu, Global exponential stability and periodicity of reaction–diffusion delayed recurrent neural networks with Dirichlet boundary conditions, *Chaos Solitons & Fractals,* Vol. 35, No. 1, 2008, pp. 116–125.

[32] A. Wu, C. Fu, Global exponential stability of non–autonomous FCNNs with Dirichlet boundary conditions and reaction–diffusion terms, *Appl. Math. Modelling,* Vol. 34, 2010, pp. 3022–3029.

[33] Y. Lv, W. Lv, J. Sun, Convergence dynamics of stochastic reaction–diffusion recurrent neural networks with continuously distributed delays, *Nonlinear Anal. Real World Appl.,* Vol. 9, 2008, pp. 1590–1606.

[34] P. Balasubramaniam, C. Vidhya, Global asymptotic stability of stochastic BAM neural networks with distributed delays and reaction–diffusion terms, *J. Comput. Appl. Math.,* Vol. 234, 2010, pp. 3458–3466.

[35] Y. K. Li, K. H. Zhao, Robust stability of delayed reaction–diffusion recurrent neural networks with Dirichlet boundary conditions on time scales, *Neurocomputing,* Vol. 74, 2011, pp. 1632–1637.

[36] K. H. Zhao, Y. K. Li, Existence and global exponential stability of equilibrium solution to reaction–diffusion recurrent neural networks on time scales, *Discrete Dynamics in Nature and Society,* Vol. 2010, 2010: (Art. ID 624619).

[37] Y. K. Li, K. H. Zhao, Y. Ye, Stability of reaction–diffusion recurrent neural networks with distributed delays and Neumann boundary conditions on time scales, *Neural Process Lett.,* Vol. 36, 2012, pp. 217–234.

[38] Y. K. Li, L. Zhao, P. Liu, Existence and exponential stability of periodic solution of high-order Hopfield neural network with delays on time scales, *Discrete Dyn. Nat. Soc.,* Vol. 2009, 2009: (Art. ID 573534).

[39] Y. K. Li, X. R. Chen, L. Zhao, Stability and existence of periodic solutions to delayed Cohen–Grossberg BAM neural networks with impulses on time scales, *Neurocomputing,* Vol. 72, 2009, pp. 1621–1630.

[40] Y. K. Li, S. Gao, Global exponential stability for impulsive BAM neural networks with distributed delays on time scales, *Neural Process. Lett.,* Vol. 31, No. 1, 2010, pp. 65–91.

[41] M. Bohner and A. Peterson, *Dynamic Equations on Time Scales: An Introduction with Applications,* Birkhäuser, Boston, Mass, USA 2001.

[42] V. Lakshmikantham, A. S. Vatsala, Hybrid system on time scales, *J. Comput. Appl. Math,* Vol. 141, 2002, pp. 227–235.

[43] Y. K. Li, Global exponential stability of BAM neural networks with delays and impulses, *Chaos, Soliton* & *Fractals,* Vol. 24, No. 1, 2005, pp. 279–285.

[44] K. H. Zhao, L. W. J. Wang, J. Q. Liu, Global robust attractive and invariant sets of fuzzy neural networks with delays and impulses, *J. of Applied Mathematics,* Vol. 2013, 2013: (Art. ID 935491).