# Anti-periodic Solutions for A Shunting Inhibitory Cellular Neural Networks with Distributed Delays and Time-varying Delays in the Leakage Terms

CHANGJIN XU
Guizhou University of Finance and Economics
Guizhou Key Laboratory of Economics System Simulation
Longchongguan Street 276, 550004 Guiyang
CHINA
xcj403@126.com

QIMING ZHANG
Hunan University of Technology
College of science
Beijing Street 42, 412007 Zhuzhou
CHINA
zhqm20082008@sina.com

*Abstract:* This paper is concerned with the existence and exponential stability of anti-periodic solutions of a class of shunting inhibitory cellular neural networks with continuously distributed delays and time-varying delays in the leakage terms. By using differential inequality techniques, some verifiable and practical delay-dependent criteria on the existence and exponential stability of anti-periodic solutions for the model are obtained. Numerical simulations are carried out to illustrate the theoretical findings.

*Key–Words:* Shunting inhibitory cellular neural networks, Anti-periodic solution, Exponential stability, Distributed delay, Time-varying delay, Leakage term

## 1 Introduction

The shunting inhibitory cellular neural networks (SICNNs) which were introduced by Bouzerdoum and Pinter [1-3] have been extensively applied in psychophysics, speech, perception, robotics, adaptive pattern recognition, vision, and image processing. During the past decades, there have been intensive results on the problem of the existence and stability of periodic and almost periodic solutions of SICNNs, for example, Fan and Shao [4] considered the positive almost periodic solutions of shunting inhibitory cellular neural networks with time-varying and continuously distributed delays, Li and Huang [5] addressed the exponential convergence behavior of solutions to shunting inhibitory cellular neural networks with delays and time-varying coefficients, Cai et al. [6] analyzed the positive almost periodic solutions for shunting inhibitory cellular neural networks with time-varying delays. For more related work, on can see [7-14]. Many scholars [15-18] have argued that the existence of anti-periodic solutions plays a key role in characterizing the behavior of nonlinear differential equations. However, only very few results are available on a generic, in depth, existence and exponential stability of anti-periodic solutions of SICNNs, for example, Aftabizadeh et al. [19] investigated a class of second-order anti-periodic boundary value problems, Aizicoviciet al. [20] discussed the anti-periodic solutions to nonmonotone evolution equations with discontinuous nonlinearities, Gong [21] focused on the anti-periodic solutions for a class of Cohen-Grossberg neural networks. In details, one can see [22-47]. In recent years, there are some papers which focus on the existence and stability of anti-periodic solutions for neural networks with delays. In 1992, Gopalsamy [48] pointed out that in real nervous systems, time delay in the stabilizing negative feedback terms has a tendency to destabilize a system (This kind of time delays are known as leakage delays or “forgetting” delays). Moreover, sometimes it has more significant effect on dynamical behaviors of neural networks than other kinds of delays. Hence, it is of significant importance to consider the leakage delay effects on dynamics of neural networks. However, due to some theoretical and technical difficulties, there has been very little existing work on neural networks with leakage delays [49-53]. Motivated by the above arguments, In this paper, we will investigate a shunting inhibitory cellular neural network with continuously distributed delays and time-varying delays in the leakage terms which takes the form

$$\dot{x}_{ij}(t) = -a_{ij}(t)x_{ij}(t-\tau_{ij}(t))$$
$$-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)\int_0^{\infty} K_{ij}(u)f(x_{kl}(t-u))du$$

$$\times x_{ij}(t) + L_{ij}(t), \tag{1}$$

where $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$, $C_{ij}$ denotes the cell at the $(i, j)$ position of the lattice, the $r$-neighborhood $N_r(i, j)$ of $C_{ij}$ is given by

$$N_r(i,j) = \{C_{kl} : \max(|k-i|, |l-j|) \le r,$$
$$1 \le k \le m, 1 \le l \le n\}.$$

$x_{ij}$ acts as the activity of the cell $C_{ij}$, $L_{ij}(t)$ is the external input to $C_{ij}$, the constant $a_{ij} > 0$ represents the passive decay rate of the cell activity, $C_{ij}^{kl} \ge 0$ is the connection or coupling strength of postsynaptic activity of the cell transmitted to the cell $C_{ij}$, and the activity function $f(.)$ is a function representing the output or firing rate of the cell $C_{kl}$, and $\alpha(.)$ is a continuous function, $\tau_{ij}(t) \ge 0$ correspond to the transmission delays and $t - \tau_{ij}(t) > 0$ for all $t > 0$.

The purpose of this paper is to present sufficient conditions of existence and exponential stability of anti-periodic solution of system (1). Some new sufficient conditions ensuring the existence, unique and exponential stability of anti-periodic solutions of system (1) are established. Our results not only can be applied directly to many concrete examples of cellular neural networks, but also extend, to a certain extent, some previously known ones. In addition, an example is presented to illustrated the effectiveness of our main results.

The rest of this paper is organized as follows. In Section 2, we give some notations and preliminary knowledge. In Section 3, we present our main results. In Section 4, we present an example with its numerical simulations to illustrate the effectiveness of our main results.

## 2 Preliminary Results

A continuous function $g : R$ to $R$ is said to be $T$-anti-periodic on $R$, if

$$g(t+T) = -g(t), \text{for all} \quad t \in R.$$

Define $ij \in \Omega = \{11, 12, \ldots, 1n, \ldots, m1, m2, \ldots, mn\}$ and let

$$L_{ij}^{+} = \sup_{t\in R} |L_{ij}(t)|, \quad a_{ij}^{+} = \sup_{t\in R} a_{ij}(t),$$

$$\tau_{ij}^{+} = \sup_{t\in R} \tau_{ij}(t), \quad \bar{C}_{ij}^{kl} = \sup_{t\in R} C_{ij}^{kl}(t).$$

We consider (1) under the following assumptions: $a_{ij}, C_{ij} : R \to [0, +\infty), K_{ij} : [0, +\infty) \to R$ and $\alpha, L_{ij} : R \to R$ are continuous functions and

$$a_{ij}(t+T)\alpha(u) = -a_{ij}(t)\alpha(-u),$$

$$L_{ij}(t+T) = -L_{ij}(t), \quad \tau_{ij}(t+T) = \tau_{ij}(t)$$
$$C_{ij}(t+T) = -C_{ij}(t), \quad f(-u) = f(u),$$
$$(\text{or } C_{ij}(t+T) = C_{ij}(t), \quad f(u) = -f(-u)),$$

for all $t, u \in R$. In order to obtain our main results in this paper, we make the assumptions as follows:

(H1) There exist constants $L > 0$ and $M > 0$ such that for all $u, v \in R$,

$$|f(u) - f(v)| \le L|u - v|, |f(u)| \le M.$$

(H2) There exist a constant $\gamma > 0$ such that for all $t > 0$,

$$-a_{ij}(t)\gamma + |a_{ij}(t)|\tau_{ij}(t)\Big[a_{ij}^{+}\gamma + \sum_{C_{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl} \int_0^\infty |K_{ij}(u)| du (L\gamma + |f(0)|)\gamma + L_{ij}^{+}\Big] + \sum_{C_{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl}(t) \int_0^\infty |K_{ij}(u)| du \times (L\gamma + |f(0)|)\gamma + L_{ij}^{+} < 0.$$

(H3) There exist constants $\eta > 0$ and $\xi_{ij} > 0, ij \in \Omega$, such that for all $t > 0$,

$$-a_{ij}(t)\xi_{ij} + a_{ij}(t)\xi_{ij}\tau_{ij}^{+}\Bigg[a_{ij}^{+}\xi_{ij} + \gamma \times \sum_{C_{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl} \int_0^\infty |K_{ij}(u)| L\xi_{kl} du + \sum_{C_{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl} \int_0^\infty |K_{ij}(u)| M\xi_{ij} du\Bigg] + \gamma \sum_{C_{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl} \int_0^\infty |K_{ij}(u)| L\xi_{kl} du + \sum_{C_{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl} \int_0^\infty |K_{ij}(u)| M\xi_{ij} du < -\eta.$$

**Definition 1** *Let $x^*(t) = \{x_{ij}^*(t)\}$ be an anti-periodic solution of (1) with initial value $\varphi^* = \{\varphi_{ij}^*(t)\}$. If there exist constants $\lambda > 0$ and $M_\varphi > 1$ such that for every solution $x(t) = \{x_{ij}(t)\}$ of (1) with an initial value $\varphi = \{\varphi_{ij}(t)\}$,*

$$|x_{ij}(t) - x_{ij}^*(t)| \le M_\varphi \|\varphi - \varphi^*\| e^{-\lambda t}, \ \forall\, t > 0,$$

*where $ij \in \Omega$ and*

$$\|\varphi - \varphi^*\| = \sup_{-\infty \le s \le 0} \max_{ij\in\Omega} |\varphi_{ij}(s) - \varphi_{ij}^*(s)|.$$

*Then $x^*(t)$ is said to be globally exponentially stable.*

Next, we present two important lemmas which are used in proof of our main results in Section 3.

**Lemma 2** *Let* (H1) *and* (H2) *hold. Suppose that* $\widetilde{x}(t) = \{\widetilde{x}_{ij}(t)\}$ *is a solution of (1) with initial conditions*

$$\widetilde{x}_{ij}(s) = \widetilde{\varphi}_{ij}(s), \quad |\widetilde{\varphi}_{ij}(s)| < \gamma, \quad s \in [-\infty, 0]. \tag{2}$$

*Then*

$$|\widetilde{x}_{ij}(t)| < \gamma, \text{for all } t \geq 0, \tag{3}$$

*where* $ij \in \Omega$.

**Proof.** For the given initial condition, the assumption (H1) guarantees the existence and uniqueness, it follows from the theory of functional differential equations that the interval of existence of solution is $(0, +\infty)$. By way of contradiction, we assume that (3) does not hold. Then there must exist $ij \in \Omega$ and $\sigma > 0$ such that

$$|\widetilde{x}_{ij}(\sigma)| = \gamma, \text{and } |\widetilde{x}_{ij}(t)| < \gamma \ \forall\, t \in (-\tau, \sigma). \tag{4}$$

From system (1), we have

$$\begin{aligned}
\dot{x}_{ij}(t) &= -a_{ij}(t)x_{ij}(t-\tau_{ij}(t)) \\
&\quad - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)\int_0^{\infty} K_{ij}(u) \\
&\quad \times f(x_{kl}(t-u))du x_{ij}(t) + L_{ij}(t) \\
&= -a_{ij}(t)x_{ij}(t) + a_{ij}(t)[x_{ij}(t) \\
&\quad -x_{ij}(t-\tau_{ij}(t))] - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t) \\
&\quad \times \int_0^{\infty} K_{ij}(u)f(x_{kl}(t-u))du x_{ij}(t) \\
&\quad +L_{ij}(t) \\
&= -a_{ij}(t)x_{ij}(t) + a_{ij}(t)\int_{t-\tau_{ij}(t)}^{t} x_{ij}^{'}(s)ds \\
&\quad - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)\int_0^{\infty} K_{ij}(u) \\
&\quad \times f(x_{kl}(t-u))du x_{ij}(t) + L_{ij}(t).
\end{aligned} \tag{5}$$

Computing the upper left derivative of $|\widetilde{x}_{ij}(t)|$, together with the assumptions (H1) and (H2) and (4) and (5), we deduce that

$$\begin{aligned}
0 &\leq D^{-}(|\widetilde{x}_{ij}(\sigma)|) \\
&\leq -a_{ij}(\sigma)|x_{ij}(\sigma)| + \Big|a_{ij}(\sigma)\int_{\sigma-\tau_{ij}(\sigma)}^{\sigma} x_{ij}^{'}(s)ds \\
&\quad - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(\sigma)\int_0^{\infty} K_{ij}(u) \\
&\quad \times f(x_{kl}(\sigma-u))du x_{ij}(\sigma) + L_{ij}(\sigma)\Big| \\
&\leq -a_{ij}(\sigma)|x_{ij}(\sigma)| + |a_{ij}(\sigma)|\int_{\sigma-\tau_{ij}(\sigma)}^{\sigma} |x_{ij}^{'}(s)|ds \\
&\quad + \sum_{C_{kl}\in N_r(i,j)} |C_{ij}^{kl}(\sigma)|\int_0^{\infty} |K_{ij}(u)| \\
&\quad \times(|f(x_{kl}(\sigma-u)) - f(0)| + |f(0)|)du|x_{ij}(\sigma)| \\
&\quad +|L_{ij}(\sigma)| \\
&\leq -a_{ij}(\sigma)|x_{ij}(\sigma)| + |a_{ij}(\sigma)| \\
&\quad \times \int_{\sigma-\tau_{ij}(\sigma)}^{\sigma} |-a_{ij}(s)x_{ij}(s-\tau_{ij}(s)) \\
&\quad - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s)\int_0^{\infty} K_{ij}(u) \\
&\quad \times f(x_{kl}(s-u))du x_{ij}(s) + L_{ij}(s)|ds \\
&\quad + \sum_{C_{kl}\in N_r(i,j)} |C_{ij}^{kl}(\sigma)|\int_0^{\infty} |K_{ij}(u)| \\
&\quad \times(|f(x_{kl}(\sigma-u)) - f(0)| + |f(0)|)du|x_{ij}(\sigma)| \\
&\quad +|L_{ij}(\sigma)| \\
&\leq -a_{ij}(\sigma)|x_{ij}(\sigma)| + |a_{ij}(\sigma)| \\
&\quad \times \int_{\sigma-\tau_{ij}(\sigma)}^{\sigma} |a_{ij}(s)||x_{ij}(s-\tau_{ij}(s))| \\
&\quad + \sum_{C_{kl}\in N_r(i,j)} |C_{ij}^{kl}(s)|\int_0^{\infty} |K_{ij}(u)| \\
&\quad \times(f(x_{kl}(s-u)) - f(0)| + |f(0)|)du|x_{ij}(s)| \\
&\quad +|L_{ij}(s)|ds + \sum_{C_{kl}\in N_r(i,j)} |C_{ij}^{kl}(\sigma)| \\
&\quad \times \int_0^{\infty} |K_{ij}(u)|(|f(x_{kl}(\sigma-u)) - f(0)| \\
&\quad +|f(0)|)du|x_{ij}(\sigma)| + |L_{ij}(\sigma)| \\
&\leq -a_{ij}(\sigma)\gamma + |a_{ij}(\sigma)|\tau_{ij}(\sigma)\Big[a_{ij}^{+}\gamma \\
&\quad + \sum_{C_{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl}\int_0^{\infty} |K_{ij}(u)|du(L\gamma \\
&\quad +|f(0)|)\gamma + L_{ij}^{+}\Big] \\
&\quad + \sum_{C_{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl}(\sigma)\int_0^{\infty} |K_{ij}(u)|du(L\gamma \\
&\quad +|f(0)|)\gamma + L_{ij}^{+} < 0,
\end{aligned} \tag{6}$$

which is a contradiction and implies that (3) holds. This completes the proof. □

**Lemma 3** *Suppose that* (H1)–(H3) *hold. Let* $\widetilde{x}(t) = \{\widetilde{x}_{ij}(t)\}$ *be the solution of (1) with initial value* $\varphi^{*} = \{\varphi_{ij}^{*}(t)\}$*, and* $x(t) = \{x_{ij}(t)\}$ *be the solution of (1) with initial value* $\varphi = \{\varphi_{ij}(t)\}$*. Then there exists a constant* $M_{\varphi} > 1$ *such that*

$$|x_{ij}(t) - x_{ij}^{*}(t)| \leq M_{\varphi}\|\varphi - \varphi^{*}\|e^{-\lambda t}, \ \forall\, t > 0, ij \in \Omega.$$

**Proof.** Let $y(t) = \{y_{ij}(t)\} = \{x_{ij}(t) - x^*_{ij}(t)\} = x(t) - x^*(t)$. Then

$$\begin{aligned} y'_{ij}(t) &= -a_{ij}(t)y_{ij}(t-\tau_{ij}(t)) \\ &\quad - \sum_{C_{kl}\in N_r(i,j)} C^{kl}_{ij}(t)\int_0^\infty K_{ij}(u) \\ &\quad \times[f(x_{kl}(t-u)) - f(x^*_{kl}(t-u))]du \\ &\quad \times[x_{ij}(t) - x^*_{ij}(t)], \qquad (7) \end{aligned}$$

where $ij \in \Omega$. By Lemma 2, it follows that

$$|x_{ij}(t)| < \gamma, \ \forall\, t > 0.$$

Define continuous functions $\rho_i(\theta)$ as follows:

$$\begin{aligned} \rho_i(\theta) &= \left(\theta - a_{ij}(t)e^{\theta\tau_{ij}(t)}\right)\xi_{ij} + a_{ij}(t)e^{\theta\tau_{ij}(t)}\xi_{ij} \\ &\quad \times\tau^+_{ij}\Bigg[\left(\theta + a^+_{ij}e^{\theta\tau^+_{ij}}\right)\xi_{ij} \\ &\quad +\gamma\sum_{C_{kl}\in N_r(i,j)}\bar{C}^{kl}_{ij}\int_0^\infty |K_{ij}(u)|Le^{\theta u}\xi_{kl}du \\ &\quad +\sum_{C_{kl}\in N_r(i,j)}\bar{C}^{kl}_{ij}\int_0^\infty |K_{ij}(u)|M\xi_{ij}du\Bigg] \\ &\quad +\gamma\sum_{C_{kl}\in N_r(i,j)}\bar{C}^{kl}_{ij}\int_0^\infty |K_{ij}(u)|Le^{\theta u}\xi_{kl}du \\ &\quad +\sum_{C_{kl}\in N_r(i,j)}\bar{C}^{kl}_{ij}\int_0^\infty |K_{ij}(u)|M\xi_{ij}du, \end{aligned}$$

where $\theta \ge 0$, and $t \ge 0, ij \in \Omega$. Then

$$\begin{aligned} \rho_i(0) &= -a_{ij}(t)\xi_{ij} + a_{ij}(t)\xi_{ij}\tau^+_{ij}\Bigg[a^+_{ij}\xi_{ij} \\ &\quad +\gamma\sum_{C_{kl}\in N_r(i,j)}\bar{C}^{kl}_{ij}\int_0^\infty |K_{ij}(u)|L\xi_{kl}du \\ &\quad +\sum_{C_{kl}\in N_r(i,j)}\bar{C}^{kl}_{ij}\int_0^\infty |K_{ij}(u)|M\xi_{ij}du\Bigg] \\ &\quad +\gamma\sum_{C_{kl}\in N_r(i,j)}\bar{C}^{kl}_{ij}\int_0^\infty |K_{ij}(u)|L\xi_{kl}du \\ &\quad +\sum_{C_{kl}\in N_r(i,j)}\bar{C}^{kl}_{ij}\int_0^\infty |K_{ij}(u)|M\xi_{ij}du, \end{aligned}$$

where $t \ge 0, ij \in \Omega$. In view of the continuity of $\rho_i(\theta)$, we can choose sufficiently small $\lambda$ satisfying $a_{ij}(t) > \lambda > 0$ and $\sigma > 0$ such that

$$\begin{aligned} -\sigma &> \rho_i(\lambda) \\ &= \left(\lambda - a_{ij}(t)e^{\lambda\tau_{ij}(t)} + a_{ij}(t)e^{\lambda\tau_{ij}(t)}\right)\xi_{ij} \\ &\quad \times\tau^+_{ij}\Bigg[\left(\theta + a^+_{ij}e^{\theta\tau^+_{ij}}\right)\xi_{ij} \\ &\quad +\gamma\sum_{C_{kl}\in N_r(i,j)}\bar{C}^{kl}_{ij}\int_0^\infty |K_{ij}(u)|Le^{\lambda u}\xi_{kl}du \\ &\quad +\sum_{C_{kl}\in N_r(i,j)}\bar{C}^{kl}_{ij}\int_0^\infty |K_{ij}(u)|M\xi_{ij}du\Bigg] \\ &\quad +\gamma\sum_{C_{kl}\in N_r(i,j)}\bar{C}^{kl}_{ij}\int_0^\infty |K_{ij}(u)|Le^{\lambda u}\xi_{kl}du \\ &\quad +\sum_{C_{kl}\in N_r(i,j)}\bar{C}^{kl}_{ij}\int_0^\infty |K_{ij}(u)|M\xi_{ij}du, \end{aligned}$$

where $t \ge 0, ij \in \Omega$.

Next, we define a Lyapunov functional as

$$V_{ij}(t) = y_{ij}(t)e^{\lambda t}, \ \ ij \in \Omega. \qquad (8)$$

It follows from (7) that

$$\begin{aligned} &V'_{ij}(t) = \lambda V_{ij}(t) - a_{ij}(t)e^{\lambda t}y_{ij}(t-\tau_{ij}(t)) \\ &+\Bigg[\sum_{C_{kl}\in N_r(i,j)} C^{kl}_{ij}(t)\int_0^\infty K_{ij}(u) \\ &\times f(x_{kl}(t-u))du x_{ij}(t) - \sum_{C_{kl}\in N_r(i,j)} C^{kl}_{ij}(t) \\ &\times\int_0^\infty K_{ij}(u)f(x^*_{kl}(t-u))du x^*_{ij}(t)\Bigg]e^{\lambda t} \\ &= \lambda V_{ij}(t) - a_{ij}(t)e^{\lambda\tau_{ij}(t)}V_{ij}(t) + a_{ij}(t)e^{\lambda\tau_{ij}(t)} \\ &\times[V_{ij}(t) - V_{ij}(t-\tau_{ij}(t))] \\ &+\Bigg[\sum_{C_{kl}\in N_r(i,j)} C^{kl}_{ij}(t)\int_0^\infty K_{ij}(u) \\ &\times f(x_{kl}(t-u))du x_{ij}(t) - \sum_{C_{kl}\in N_r(i,j)} C^{kl}_{ij}(t) \\ &\int_0^\infty K_{ij}(u)f(x^*_{kl}(t-u))du x^*_{ij}(t)\Bigg]e^{\lambda t} \\ &= \lambda V_{ij}(t) - a_{ij}(t)e^{\lambda\tau_{ij}(t)}V_{ij}(t) \\ &+a_{ij}(t)e^{\lambda\tau_{ij}(t)}\int_{t-\tau_{ij}(t)}^t V'_{ij}(s)ds \\ &+\Bigg[\sum_{C_{kl}\in N_r(i,j)} C^{kl}_{ij}(t)\int_0^\infty K_{ij}(u) \\ &\times f(x_{kl}(t-u))du x_{ij}(t) - \sum_{C_{kl}\in N_r(i,j)} C^{kl}_{ij}(t) \\ &\int_0^\infty K_{ij}(u)f(x^*_{kl}(t-u))du x^*_{ij}(t)\Bigg]e^{\lambda t} \\ &= \lambda V_{ij}(t) - a_{ij}(t)e^{\lambda\tau_{ij}(t)}V_{ij}(t) + a_{ij}(t)e^{\lambda\tau_{ij}(t)} \\ &\times\int_{t-\tau_{ij}(t)}^t\Big\{\lambda V_{ij}(s) - a_{ij}(s)e^{\lambda s}y_{ij}(s-\tau_{ij}(s)) \end{aligned}$$

$$
\begin{aligned}
&+\Bigg[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s)\int_0^\infty K_{ij}(u)\\
&\times f(x_{kl}(s-u))dux_{ij}(s)-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s)\\
&\times\int_0^\infty K_{ij}(u)f(x_{kl}^*(s-u))dux_{ij}^*(s)\Bigg]e^{\lambda s}\Bigg\}ds\\
&+\Bigg[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)\int_0^\infty K_{ij}(u)\\
&\times f(x_{kl}(t-u))dux_{ij}(t)-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)\\
&\times\int_0^\infty K_{ij}(u)f(x_{kl}^*(t-u))dux_{ij}^*(t)\Bigg]e^{\lambda t}, \qquad (9)
\end{aligned}
$$

where $ij\in\Omega$.

Define a positive constant $M$ as follows:

$$M=\max_{ij\in\Omega}\{\sup_{s\in(-\infty,0]}|V_{ij}(s)|\}.$$

Let $K$ be a positive number such that

$$|V_{ij}(t)|\le M<K\xi_{ij}\text{for all } t\in(-\infty,0],ij\in\Omega. \qquad (10)$$

We claim that

$$|V_{ij}(t)|<K\xi_{ij}\text{for all } t>0,ij\in\Omega. \qquad (11)$$

Otherwise, there must exist $ij\in\Omega$ and $t_{ij}>0$ such that one of the two cases occurs.

$$(1)\ V_{ij}(t_{ij})=K\xi_{ij},|V_{ij}(t)|<K\xi_{ij}, \qquad (12)$$

$$(2)\ V_{ij}(t_{ij})=-K\xi_{ij},|V_{ij}(t)|<K\xi_{ij}, \qquad (13)$$

where $t\in[-\infty,t_{ij}),ij\in\Omega$.

We discuss the two cases.

**Case 1.** If (12) holds, then from (9) and (H1)–(H3), we get

$$
\begin{aligned}
&0\le V_{ij}'(t_{ij})\\
=\ &\lambda V_{ij}(t_{ij})-a_{ij}(t_{ij})e^{\lambda\tau_{ij}(t_{ij})}V_{ij}(t_{ij})\\
&+a_{ij}(t_{ij})e^{\lambda\tau_{ij}(t_{ij})}\int_{t_{ij}-\tau_{ij}(t_{ij})}^{t_{ij}}\Bigg\{\lambda V_{ij}(s)\\
&-a_{ij}(s)e^{\lambda s}y_{ij}(s-\tau_{ij}(s))+\Big[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s)\\
&\int_0^\infty K_{ij}(u)f(x_{kl}(s-u))dux_{ij}(s)\\
&-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s)\int_0^\infty K_{ij}(u)f(x_{kl}^*(s-u))du\\
&\times x_{ij}^*(s)\Big]e^{\lambda s}\Bigg\}ds+\Big[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t_{ij})\\
&\times\int_0^\infty K_{ij}(u)f(x_{kl}(t_{ij}-u))dux_{ij}(t)\\
&-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t_{ij})\int_0^\infty K_{ij}(u)\\
&\times f(x_{kl}^*(t_{ij}-u))dux_{ij}^*(t_{ij})\Big]e^{\lambda t_{ij}}\\
\le\ &\lambda V_{ij}(t_{ij})-a_{ij}(t_{ij})e^{\lambda\tau_{ij}(t_{ij})}V_{ij}(t_{ij})\\
&+a_{ij}(t_{ij})e^{\lambda\tau_{ij}(t_{ij})}\int_{t_{ij}-\tau_{ij}(t_{ij})}^{t_{ij}}\Bigg\{\lambda V_{ij}(s)\\
&-a_{ij}(s)e^{\lambda s}y_{ij}(s-\tau_{ij}(s))+\Big[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s)\\
&\times\int_0^\infty K_{ij}(u)f(x_{kl}(s-u))dux_{ij}(s)\\
&-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s)\int_0^\infty K_{ij}(u)\\
&\times f(x_{kl}^*(s-u))dux_{ij}^*(s)\Big]e^{\lambda s}\Bigg\}ds\\
&+\Big[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t_{ij})\int_0^\infty K_{ij}(u)\\
&\times f(x_{kl}(t_{ij}-u))dux_{ij}(t)\\
&-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t_{ij})\int_0^\infty K_{ij}(u)\\
&\times f(x_{kl}^*(t_{ij}-u))dux_{ij}^*(t_{ij})\Big]e^{\lambda t_{ij}}\\
\le\ &\lambda V_{ij}(t_{ij})-a_{ij}(t_{ij})e^{\lambda\tau_{ij}(t_{ij})}V_{ij}(t_{ij})\\
&+a_{ij}(t_{ij})e^{\lambda\tau_{ij}(t_{ij})}\int_{t_{ij}-\tau_{ij}(t_{ij})}^{t_{ij}}\Bigg\{\lambda V_{ij}(s)\\
&-a_{ij}(s)e^{\lambda s}y_{ij}(s-\tau_{ij}(s))\\
&+\Big[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s)\int_0^\infty K_{ij}(u)\\
&\times f(x_{kl}(s-u))dux_{ij}(s)\\
&-\Big[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s)\int_0^\infty K_{ij}(u)\\
&\times f(x_{kl}^*(s-u))dux_{ij}(s)+\Big[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s)\\
&\int_0^\infty K_{ij}(u)f(x_{kl}^*(s-u))dux_{ij}(s)\\
&-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(s)\int_0^\infty K_{ij}(u)\\
&\times f(x_{kl}^*(s-u))dux_{ij}^*(s)\Big]e^{\lambda s}\Bigg\}ds\\
&+\Big[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t_{ij})\int_0^\infty K_{ij}(u)
\end{aligned}
$$

$$
\begin{aligned}
&\times f(x_{kl}(t_{ij}-u))dux_{ij}(t)-\Big[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t_{ij})\\
&\int_0^\infty K_{ij}(u)f(x_{kl}^*(t_{ij}-u))dux_{ij}(t)\\
&+\Big[\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t_{ij})\int_0^\infty K_{ij}(u)\\
&\times f(x_{kl}^*(t_{ij}-u))dux_{ij}(t)\\
&-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t_{ij})\int_0^\infty K_{ij}(u)\\
&\times f(x_{kl}^*(t_{ij}-u))dux_{ij}^*(t_{ij})\Big]e^{\lambda t_{ij}}\\
\le\ & \lambda V_{ij}(t_{ij})-a_{ij}(t_{ij})e^{\lambda\tau_{ij}(t_{ij})}V_{ij}(t_{ij})\\
&+a_{ij}(t_{ij})e^{\lambda\tau_{ij}(t_{ij})}\int_{t_{ij}-\tau_{ij}(t_{ij})}^{t_{ij}}\Bigg\{\lambda V_{ij}(s)\\
&+a_{ij}^+e^{\lambda\tau_{ij}(s)}|V_{ij}(s-\tau_{ij}(s))|\\
&+\gamma\sum_{C_{kl}\in N_r(i,j)}\bar{C}_{ij}^{kl}\int_0^\infty |K_{ij}(u)|Le^{\lambda u}\\
&\times|V_{kl}(s-u)|du+\sum_{C_{kl}\in N_r(i,j)}\bar{C}_{ij}^{kl}\\
&\times\int_0^\infty |K_{ij}(u)|M|V_{ij}^*(s)|du\Bigg\}ds\\
&+\gamma\sum_{C_{kl}\in N_r(i,j)}\bar{C}_{ij}^{kl}\int_0^\infty |K_{ij}(u)|\\
&\times Le^{\lambda u}|V_{kl}(s-u)|du\\
&+\sum_{C_{kl}\in N_r(i,j)}\bar{C}_{ij}^{kl}\int_0^\infty |K_{ij}(u)|M|V_{ij}^*(s)|du\\
\le\ & \Bigg\{\Big(\lambda-a_{ij}(t_{ij})e^{\lambda\tau_{ij}(t_{ij})}\Big)\xi_{ij}\\
&+a_{ij}(t_{ij})e^{\lambda\tau_{ij}(t_{ij})}\xi_{ij}\tau_{ij}^+\Bigg[\Big(\lambda+a_{ij}^+e^{\lambda\tau_{ij}^+}\Big)\xi_{ij}\\
&+\gamma\sum_{C_{kl}\in N_r(i,j)}\bar{C}_{ij}^{kl}\int_0^\infty |K_{ij}(u)|Le^{\lambda u}\xi_{kl}du\\
&+\sum_{C_{kl}\in N_r(i,j)}\bar{C}_{ij}^{kl}\int_0^\infty |K_{ij}(u)|M\xi_{ij}du\Bigg]\\
&+\gamma\sum_{C_{kl}\in N_r(i,j)}\bar{C}_{ij}^{kl}\int_0^\infty |K_{ij}(u)|Le^{\lambda u}\xi_{kl}du\\
&+\sum_{C_{kl}\in N_r(i,j)}\bar{C}_{ij}^{kl}\int_0^\infty |K_{ij}(u)|M\xi_{ij}du\Bigg\}K\\
<\ & -\bar{\eta}K<0. \qquad (14)
\end{aligned}
$$

which is a contradiction which implies (12) do not hold.

**Case 2.** If (13) holds, then, together with (9) and (H1)–(H3), using a similar argument as in case 1, we can show that (13) is not true. Thus (13) holds. Consequently, we know that

$$|V_{ij}(t)|=|y_{ij}|e^{\lambda t}<K\xi_{ij}, \text{for all } t>0, ij\in\Omega.$$

Then

$$|x_{ij}(t)-x_{ij}(t)|\le K\xi_{ij}e^{-\lambda t}, \text{for all } t>0, ij\in\Omega.$$

This completes the proof of Lemma 3. □

**Remark 4** *If $x^*(t)=\{x_{ij}^*(t)\}$ is the $T$-anti-periodic solution of (1), it follows from Lemma 3 and the Definition 1 that $x^*(t)$ is globally exponentially stable.*

# 3 Main results

In this section,we present our main result that there exists the exponentially stable anti-periodic solution of (1).

**Theorem 5** *Assume that* (H1)–(H3) *are satisfied. Then (1) has exactly one $T$-anti-periodic solution $x^*(t)$. Moreover, this solution is globally exponentially stable.*

**Proof.** Let $v(t)=(v_{11}(t),v_{12}(t),\ldots, v_{mn}(t))^T$ is a solution of (1) with initial conditions

$$v_{ij}(s)=\varphi_{ij}^v(s), |\varphi_{ij}^v(s)|<\gamma, s\in(-\infty,0], ij\in\Omega. \qquad (15)$$

Thus according to Lemma 2, the solution $v(t)$ is bounded and

$$|v_{ij}(t)|<\gamma, \text{for all } t\in R, ij\in\Omega. \qquad (16)$$

From (1), we obtain

$$
\begin{aligned}
&[(-1)^{k+1}v_{ij}(t+(k+1)T)]'\\
&=(-1)^{k+1}v_{ij}'(t+(k+1)T)\\
&=(-1)^{k+1}\Big\{-a_{ij}(t+(k+1)T)\\
&v_{ij}(t+(k+1)T-\tau_{ij}(t+(k+1)T))\\
&-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t+(k+1)T)\\
&\times\int_0^\infty K_{ij}(u)f(v_{kl}(t+(k+1)T-u))du\\
&\times v_{ij}(t+(k+1)T)+L_{ij}(t+(k+1)T)\Big\}\\
&=-a_{ij}(t+(k+1)T)(-1)^{k+1}\\
&\times v_{ij}(t+(k+1)T-\tau_{ij}(t))\\
&-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t+(k+1)T)
\end{aligned}
$$

$$\times \int_0^\infty K_{ij}(u)f((-1)^{k+1}v_{kl}(t+(k+1)T \\ -u))du\times(-1)^{k+1}v_{ij}(t+(k+1)T) \\ +L_{ij}(t+(k+1)T), \qquad (17)$$

where $ij \in \Omega$. Thus $(-1)^{k+1}v_{ij}(t+(k+1)T)$ are the solutions of (1) on $R^{m+n}$ for any natural number $k$. Then, from Lemma 3, there exists a constant $K > 0$ such that

$$|(-1)^{k+1}v_{ij}(t+(k+1)T)-(-1)^k v_{ij}(t+kT)| \\ = |v_{ij}(t+(k+1)T)+v_{ij}(t+kT)| \\ = |v_{ij}(t+kT)-(-v_{ij}(t+kT+T)| \\ \le K\xi_{ij}e^{-\lambda(t+kT)} = K\xi_{ij}e^{-\lambda t}\left(\frac{1}{e^{\lambda T}}\right)^k \\ \text{for all } t+kT>0, ij\in\Omega. \qquad (18)$$

Thus, for any natural number $q$, we have

$$(-1)^{q+1}v_{ij}(t+(q+1)T) \\ = v_{ij}(t)+\sum_{k=0}^{q}[(-1)^{k+1}v_{ij}(t+(k+1)T) \\ -(-1)^k v_{ij}(t+kT)]. \qquad (19)$$

In view of (18) and (19), we know that $\{(-1)^q v(t+qT)\}$ uniformly converges to a continuous function $x^*(t) = (x^*_{11}(t), x^*_{12}(t), \ldots, x^*_{mn}(t))^T$ on any compact set of $R^{m+n}$.

Now we show that $x^*(t)$ is $T$-anti-periodic solution of (1). Firstly, $x^*(t)$ is $T$-anti-periodic, since

$$\begin{aligned} x^*(t+T) &= \lim_{q\to\infty}(-1)^q v(t+T+qT) \\ &= -\lim_{(q+1)\to\infty}(-1)^{q+1}v(t+(q+1)T) \\ &= -x^*(t). \qquad (20)\end{aligned}$$

In the sequel, we prove that $x^*(t)$ is a solution of (1). Because of the continuity of the right-hand side of (1), (17) implies that $\{((-1)^{q+1}v_{ij}(t+(q+1)T))'\}$ uniformly converges to a continuous function on any compact subset of $R$. Thus, letting $q\to\infty$, we can easily obtain

$$\dot{x}^*_{ij}(t) = -a_{ij}(t)x^*_{ij}(t-\tau_{ij}(t)) \\ -\sum_{C_{kl}\in N_r(i,j)} C^{kl}_{ij}(t)\int_0^\infty K_{ij}(u)f(x^*_{kl}(t-u))du \\ \times x^*_{ij}(t)+L_{ij}(t), \qquad (21)$$

where $ij \in \Omega$. Therefore, $x^*(t)$ is a solution of (1). Finally, by applying Lemma 3, it is easy to check that $x^*(t)$ is globally exponentially stable. This completes the proof. □

# 4 An example

In this section, we give an example to illustrate our main results obtained in previous sections. Consider the shunting inhibitory cellular neural network with delays

$$\dot{x}_{ij}(t) = -a_{ij}(t)x_{ij}(t-\tau_{ij}(t)) \\ -\sum_{C_{kl}\in N_r(i,j)} C^{kl}_{ij}(t)\int_0^\infty K_{ij}(u)f(x_{kl}(t-u))du \\ \times x_{ij}(t)+L_{ij}(t), \qquad (22)$$

where

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1+|\cos t| & 2+|\cos t| & 3+|\cos t| \\ 1+|\cos t| & 2+|\cos t| & 3+|\cos t| \\ 1+|\cos t| & 2+|\cos t| & 3+|\cos t| \end{bmatrix},$$

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & c_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 0.01|\sin t| & 0.01|\sin t| & 0.01|\sin t| \\ 0.01|\sin t| & 0.01|\sin t| & 0.01|\sin t| \\ 0.01|\sin t| & 0.01|\sin t| & 0.01|\sin t| \end{bmatrix},$$

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} = \begin{bmatrix} 0.05\sin t & 0.05\sin t & 0.05\sin t \\ 0.05\sin t & 0.05\sin t & 0.05\sin t \\ 0.05\sin t & 0.05\sin t & 0.05\sin t \end{bmatrix}.$$

Set $r = 1$, $f(x) = \frac{1}{20}|\sin x|$, $\tau_{ij}(t) = 0.02\sin 2t$, $K_{ij}(u) = e^{-u}\cos u$. Then

$$\begin{bmatrix} \sum_{C_{kl}\in N_1(1,1)}|C^{kl}_{11}| & \sum_{C_{kl}\in N_1(1,2)}|C^{kl}_{12}| & \sum_{C_{kl}\in N_1(1,3)}|C^{kl}_{13}| \\ \sum_{C_{kl}\in N_1(2,1)}|C^{kl}_{21}| & \sum_{C_{kl}\in N_1(2,2)}|C^{kl}_{22}| & \sum_{C_{kl}\in N_1(2,3)}|C^{kl}_{23}| \\ \sum_{C_{kl}\in N_1(3,1)}|C^{kl}_{31}| & \sum_{C_{kl}\in N_1(3,2)}|C^{kl}_{32}| & \sum_{C_{kl}\in N_1(3,3)}|C^{kl}_{33}| \end{bmatrix}$$

$$= \begin{bmatrix} 0.03|\sin t| & 0.06|\sin t| & 0.04|\sin t| \\ 0.06|\sin t| & 0.09|\sin t| & 0.04|\sin t| \\ 0.06|\sin t| & 0.04|\sin t| & 0.04|\sin t| \end{bmatrix}$$

and $L^f = 0.05$, $M = 0.05$, $L^+_{ij} = 0.05$.

Let $\eta = 1.9$, $\xi_{ij} = 3$, $\gamma = 2$. Then

$$
\begin{aligned}
&-a_{ij}(t)\gamma + |a_{ij}(t)|\tau_{ij}(t)\Bigg[a_{ij}^{+}\gamma + \sum_{C_{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl} \\
&\times \int_0^{\infty} |K_{ij}(u)|du(L^f\gamma + |f(0)|)\gamma + L_{ij}^{+}\Bigg] \\
&+ \sum_{C_{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl}(t) \int_0^{\infty} |K_{ij}(u)|du(L^f\gamma \\
&+|f(0)|)\gamma + L_{ij}^{+} \\
&< -1\times 2 + (4\times 0.02)[4\times 2 + 0.09\times 0.05 \\
&\times 2 + 0.05] + 0.09\times 0.05\times 2 + 0.05 \\
&= -1.29628 < 0
\end{aligned}
$$

and

$$
\begin{aligned}
&-a_{ij}(t)\xi_{ij} + a_{ij}(t)\xi_{ij}\tau_{ij}^{+}\Bigg[a_{ij}^{+}\xi_{ij} + \gamma \sum_{C_{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl} \\
&\times \int_0^{\infty} |K_{ij}(u)|L\xi_{kl}du + \sum_{C_{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl} \\
&\times \int_0^{\infty} |K_{ij}(u)|M\xi_{ij}du\Bigg] + \gamma \sum_{C_{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl} \\
&\times \int_0^{\infty} |K_{ij}(u)|L\xi_{kl}du + \sum_{C_{kl}\in N_r(i,j)} \bar{C}_{ij}^{kl} \\
&\times \int_0^{\infty} |K_{ij}(u)|M\xi_{ij}du \\
&< -1\times 3 + (4\times 0.02)[4\times 3 + 2\times 0.09 \\
&\times 3\times 0.05 + 0.09\times 0.05\times 3] + 2\times 0.09 \\
&\times 0.05\times 3 + 0.09\times 0.05\times 3 \\
&= -1.99818 < -1.9 < 0,
\end{aligned}
$$

which implies that system (22) satisfies all the conditions in Theorem 5. Thus, (22) has exactly one $\pi$-anti-periodic solution which is globally exponentially stable. The fact is verified by the numerical simulations in Figures 1-4.

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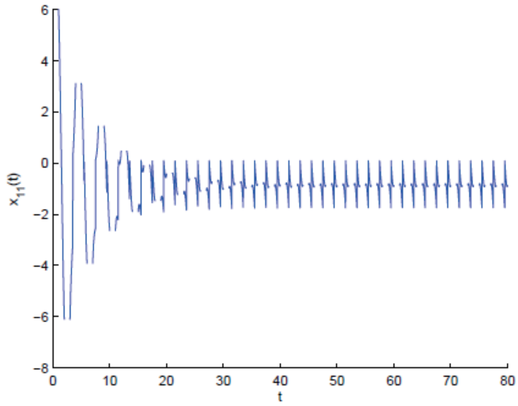


Figure 1: The time response of state variable $x_{11}(t)$ of system (22)

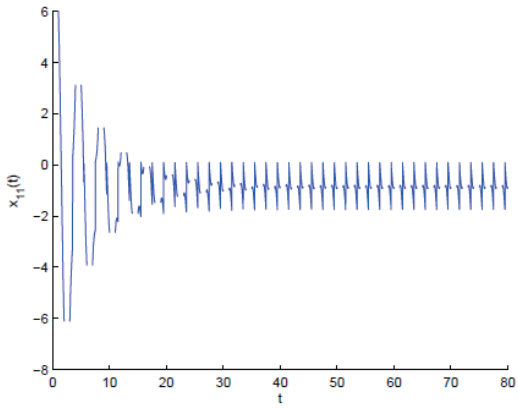


Figure 2: The time response of state variable $x_{12}(t)$ of system (22)

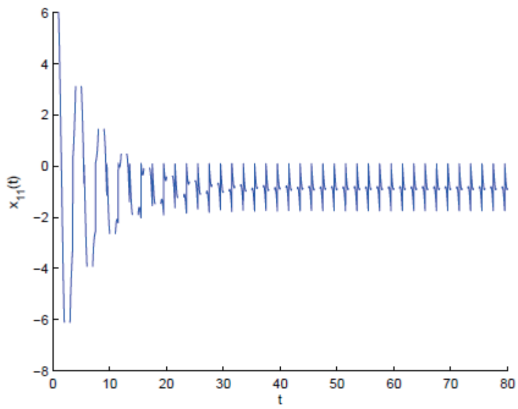


Figure 3: The time response of state variable $x_{21}(t)$ of system (22)

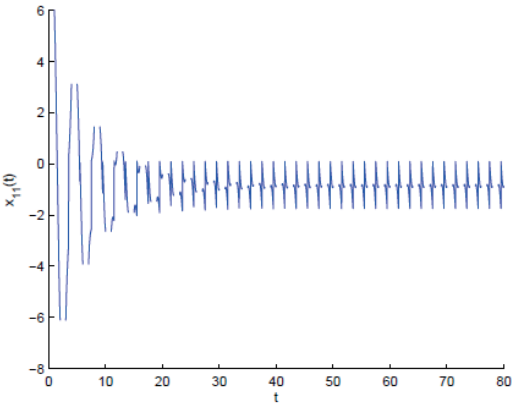


Figure 4: The time response of state variable $x_{22}(t)$ of system (22)


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