Three dimensional visco-elastic flow with heat and mass transfer past a vertical porous plate in presence of variable suction

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Abstract: An analysis is carried out to study the unsteady three -dimensional flow of a visco-elastic fluid past a vertical porous plate in presence of variable suction .The effects of heat and mass transfer are taken into account .The fluid is characterized by Rivlin-Ericksen second - order fluid model .The analytical expressions for the velocity ,temperature and species concentration have been obtained .The non -dimensional shearing stress, rate of heat transfer in terms of Nusselt number and the rate of mass transfer in terms of Sherwood number at the plate have been demonstrated graphically in possible cases. It is found that the flow field is significantly affected by the visco-elastic parameter in combination with other flow parameters.

Key–Words: Visco-elastic, Porous, Rivlin-Ericksen tensors, Grashof number, Schmidt number, Nusselt number, Sherwood number.

1 Introduction

In recent years, progress has been considerably made in the study of heat and mass transfer in visco-elastic fluid flows because of its possible applications in diverse fields of science and technology such as Soil sciences, Astrophysics, Geophysics, Nuclear power-reactor etc. The effects of different arrangements and configurations of the suction holes and slits on the drag have been discussed by various scholars. Hydromagnetic effects on three dimensional flow past a porous plate has been discussed by Singh [1]. Singh [2] has also studied the three -dimensional viscous flow and heat transfer along a porous plate. Later, this idea has been extended by applying fluctuating flow and heat transfer along a plate with suction by Singh et al.[3]. Singh et al.[4] have also analyzed the three dimensional free convective flow and heat transfer along a porous plate. Guria et al. [5] have studied the hydromagnetic effect on three dimensional flow past a vertical porous plate. Three dimensional free convective Couette flow with transpiration cooling has been studied by Jain et al.[6]. Ahmed et al.[7] have investigated three dimensional free convective flow and mass transfer along a porous plate. The application of non-Newtonian fluid flow mechanism in modern technology and industries have attracted the researchers in a large scale. Authors like Vajravelu et al. [8], Hayat et al. [9], Rajagopal et al. [10], Soundalgekar et al.[11], Choudhury et al. [12-17] etc. have contributed their efforts in this line for analyzing the characteristics of visco-elastic fluid flows. Visco-elastic MHD free convective flow past a semi infinite porous plate with variable suction is a study which has many applications such as purification of crude oil in petroleum industries, polymer technology, arodynamic heating and accelaterators. Meteorologius can use this study to understand dynamics of meteorology and air pollution. In the light of above fact, this study will be useful to welfare of mankind. The objective of this paper is to analyze the visco-elastic effects of buoyancy forces and time dependent periodic suction on three-dimensional flow past a vertical porous plate. The visco-elastic fluid is characterized by Rivlin-Ericksen second-order fluid model. The constitutive equation for the second-order fluid is taken in the form

$$\sigma = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \qquad (1)$$

where σ is the stress tensor, A_n are the kinematic Rivlin-Ericksen tensors; μ_1 ,μ_2, μ_3 are the material coefficients describing the viscosity, elasticity and cross-viscosity respectively. From thermodynamic consideration, it is noticed that the material coefficients μ_1 andμ_3 are positive and μ_2 is negative [Coleman and Markovitz [16]]. The equation (1) was derived by Coleman and Noll [18] from that of simple fluids by assuming that stress is more sensitive to the recent deformation than to the deformation that occurred in the distant past. It is reported that solution of poly-isobutulene in Cetane at 30^0 C simulate a second-order fluid and the material constants for the solutions of various concentrations have been determined by Markovitz.

2 Problem Formulation

Consider the unsteady visco-elastic fluid along a semi-infinite vertical porous plate. Here, the $\bar{x}$-axis is chosen along the vertical plate, that is, the direction of the flow, $\bar{y}$-axis is perpendicular to the plate, and $\bar{z}$-axis is normal to the $\bar{x}\bar{y}$-plane. The plate is subjected to periodic suction velocity distribution of the form

$$\bar{v} = -V_0(1+\varepsilon\cos(\frac{\pi u_\infty \bar{z}}{\nu_1} - c\bar{t})) \quad (2)$$

where $\varepsilon(<< 1)$ is the amplitude of the suction velocity. Denoting velocity components $\bar{u}, \bar{v}, \bar{w}$ in the directions $\bar{x}, \bar{y}, \bar{z}$-axes respectively, the flow is governed by the following equations:

$$\frac{\partial \bar{v}}{\partial \bar{y}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \quad (3)$$

$$\begin{aligned}
&\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{u}}{\partial \bar{z}} \\
&= \nu_1\left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right) + \nu_2\left(\frac{\partial^3 \bar{u}}{\partial \bar{y}^2\partial \bar{t}} + \frac{\partial^3 \bar{u}}{\partial \bar{z}^2\partial \bar{t}}\right. \\
&+2\frac{\partial \bar{v}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} + \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{w}}{\partial \bar{y}^2}\frac{\partial \bar{u}}{\partial \bar{z}} \\
&+2\frac{\partial \bar{w}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{y}\partial \bar{z}} + \bar{w}\frac{\partial^3 \bar{u}}{\partial \bar{y}^2\partial \bar{z}} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2}\frac{\partial \bar{u}}{\partial \bar{y}} \\
&+2\frac{\partial \bar{v}}{\partial \bar{z}}\frac{\partial^2 \bar{u}}{\partial \bar{y}\partial \bar{z}} + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}\partial \bar{z}^2} + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\frac{\partial \bar{u}}{\partial \bar{z}} \\
&\left.+2\frac{\partial \bar{w}}{\partial \bar{z}}\frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + \bar{w}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3}\right) + \nu_3\left(\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\frac{\partial \bar{u}}{\partial \bar{y}}\right. \\
&+2\frac{\partial \bar{v}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + 2\frac{\partial^2 \bar{u}}{\partial \bar{y}\partial \bar{z}}\frac{\partial \bar{w}}{\partial \bar{y}} + \frac{\partial \bar{u}}{\partial \bar{z}}\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} \\
&+2\frac{\partial^2 \bar{u}}{\partial \bar{y}\partial \bar{z}}\frac{\partial \bar{v}}{\partial \bar{z}} + \frac{\partial \bar{u}}{\partial \bar{z}}\frac{\partial^2 \bar{w}}{\partial \bar{z}^2} + \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{z}^2} \\
&\left.+2\frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\frac{\partial \bar{w}}{\partial \bar{z}}\right) + g\beta_r\left(\bar{T} - T_\infty\right) \\
&+g\beta_m\left(\bar{C} - C_\infty\right)
\end{aligned} \quad (4)$$

$$\begin{aligned}
&\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{v}}{\partial \bar{z}} \\
&= -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{y}} + \nu_1\left(\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2}\right) \\
&+\nu_2\left(\frac{\partial^3 \bar{v}}{\partial \bar{y}^2\partial \bar{t}} + \frac{\partial^3 \bar{v}}{\partial \bar{z}^2\partial \bar{t}} + \bar{v}\frac{\partial^3 \bar{v}}{\partial \bar{y}^3} + \right. \\
&\bar{w}\frac{\partial^3 \bar{v}}{\partial \bar{y}^2\partial \bar{z}} + \bar{v}\frac{\partial^3 \bar{v}}{\partial \bar{z}^2\partial \bar{y}} + \bar{w}\frac{\partial^3 \bar{v}}{\partial \bar{z}^3} + \\
&2\frac{\partial \bar{w}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{y}} + 3\frac{\partial \bar{v}}{\partial \bar{z}}\frac{\partial^2 \bar{v}}{\partial \bar{y}\partial \bar{z}} + 2\frac{\partial \bar{u}}{\partial \bar{z}}\frac{\partial^2 \bar{u}}{\partial \bar{y}\partial \bar{z}} + \\
&13\frac{\partial \bar{v}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + 3\frac{\partial^2 \bar{w}}{\partial \bar{y}^2}\frac{\partial \bar{v}}{\partial \bar{z}} + 4\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \\
&\left.4\frac{\partial \bar{w}}{\partial \bar{y}}\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + 2\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + \frac{\partial \bar{v}}{\partial \bar{z}}\frac{\partial^2 \bar{v}}{\partial \bar{z}^2}\right) \\
&+\nu_3\left(2\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + 8\frac{\partial \bar{v}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + 2\frac{\partial \bar{w}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{y}\partial \bar{z}} + \right. \\
&2\frac{\partial \bar{v}}{\partial \bar{z}}\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + 2\frac{\partial \bar{v}}{\partial \bar{z}}\frac{\partial^2 \bar{v}}{\partial \bar{y}\partial \bar{z}} + 2\frac{\partial \bar{w}}{\partial \bar{y}}\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + \\
&\left.\frac{\partial^2 \bar{u}}{\partial \bar{y}\partial \bar{z}}\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right)
\end{aligned} \quad (5)$$

$$\begin{aligned}
&\frac{\partial \bar{w}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{w}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{w}}{\partial \bar{z}} \\
&= -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{z}} + \nu_1\left(\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\right) + \\
&\nu_2\left(\frac{\partial^3 \bar{w}}{\partial \bar{y}^2\partial \bar{t}} + \frac{\partial^3 \bar{w}}{\partial \bar{z}^2\partial \bar{t}} + \bar{w}\frac{\partial^3 \bar{v}}{\partial \bar{y}^3} + \right. \\
&\bar{v}\frac{\partial^3 \bar{v}}{\partial \bar{y}\partial \bar{z}^2} + \bar{w}\frac{\partial^3 \bar{w}}{\partial \bar{z}^3} + 2\frac{\partial \bar{v}}{\partial \bar{z}}\frac{\partial^2 \bar{w}}{\partial \bar{y}\partial \bar{z}} \\
&+2\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{y}\partial \bar{z}} + 3\frac{\partial \bar{w}}{\partial \bar{y}}\frac{\partial^2 \bar{w}}{\partial \bar{y}\partial \bar{z}} + 13\frac{\partial \bar{w}}{\partial \bar{z}}\frac{\partial^2 \bar{w}}{\partial \bar{z}^2} \\
&+3\frac{\partial^2 \bar{v}}{\partial \bar{z}^2}\frac{\partial \bar{w}}{\partial \bar{y}} + 4\frac{\partial \bar{u}}{\partial \bar{z}}\frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + \\
&\left.4\frac{\partial \bar{v}}{\partial \bar{z}}\frac{\partial^2 \bar{v}}{\partial \bar{z}^2} + 2\frac{\partial \bar{u}}{\partial \bar{z}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\partial \bar{w}}{\partial \bar{z}}\frac{\partial^2 \bar{w}}{\partial \bar{y}^2}\right) \\
&+\nu_3\left(2\frac{\partial \bar{u}}{\partial \bar{z}}\frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + 8\frac{\partial \bar{w}}{\partial \bar{z}}\frac{\partial^2 \bar{w}}{\partial \bar{z}^2} + 2\frac{\partial \bar{v}}{\partial \bar{z}}\frac{\partial^2 \bar{w}}{\partial \bar{y}\partial \bar{z}}\right. \\
&+2\frac{\partial \bar{w}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{z}^2} + 2\frac{\partial \bar{w}}{\partial \bar{y}}\frac{\partial^2 \bar{w}}{\partial \bar{y}\partial \bar{z}} + 2\frac{\partial \bar{v}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{z}^2} + \\
&\left.\frac{\partial^2 \bar{u}}{\partial \bar{y}\partial \bar{z}}\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{u}}{\partial \bar{z}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)
\end{aligned} \quad (6)$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{T}}{\partial \bar{z}} = \frac{k}{\rho c_p}\left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right) \quad (7)$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{C}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{C}}{\partial \bar{z}} = D\left(\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{C}}{\partial \bar{z}^2}\right) \quad (8)$$

where ρ is the density, $\bar{p}$ is the fluid pressure, g is the acceleration due to gravity , β_r is the coefficient of thermal expansion, β_m is the coefficient of mass expansion, k is the coefficient of heat conduction , C_p is the specific heat at constant pressure, $\bar{T}$ is the fluid temperature, $\vec{C}$ is the fluid concentration and $\nu_i = \frac{\mu_i}{\rho}$, where i=1,2,3 .

The boundary conditions are:

$$\begin{aligned}
&\bar{y} = 0 : \bar{u} = 0,\\
&\bar{v} = -V_0[1 + \varepsilon\cos(\frac{\pi u_\infty \bar{z}}{\nu_1} - ct)],\\
&\bar{w} = 0, \bar{T} = T_w, \bar{C} = C_w\\
\\
&\bar{y} \to \infty : \bar{u} = u_\infty,\\
&\bar{v} = V_0, \bar{w} = 0, \bar{p} = p_\infty,\\
&\bar{T} = T_\infty, \bar{C} = C_\infty
\end{aligned} \quad (9)$$

We now introduce the following dimensionless quantities:

$$\begin{aligned}
&y = \frac{u_\infty \bar{y}}{\nu_1},\ z = \frac{u_\infty \bar{z}}{\nu_1},\ t = c\bar{t},\ p = \frac{\bar{p}}{\rho u_\infty^2},\\
&u = \frac{\bar{u}}{u_\infty},\ v = \frac{\bar{v}}{u_\infty},\ w = \frac{\bar{w}}{u_\infty},\ \theta = \frac{\bar{T} - T_\infty}{T_w - T_\infty},\\
&C = \frac{\bar{C} - C_\infty}{C_w - C_\infty},\ Gm = \frac{g\beta_m(C_w - C_\infty)}{u_\infty^3},\\
&Gr = \frac{g\beta_r(T_w - T_\infty)}{u_\infty^3},\ \omega = \frac{c\nu_1}{u_\infty^2},\\
&P_r = \frac{\rho\nu_1 C_p}{k},\ S_c = \frac{\nu_1}{D}
\end{aligned} \quad (10)$$

where Gm is the Grashof number for mass transfer, Gr is the Grashof number for heat transfer, ω is the frequency parameter, Pr is the Prandtl number, Sc is the Schmidt number, T_w is the temperature at the plate, T_∞ is the temperature outside the boundary layer, C_w is the concentration at the plate, C_∞ is the concentration outside the boundary layer.
Substituting (10) into the equations (2)-(8) we obtain the following dimensionless equations:

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (11)$$

$$\begin{aligned}
&\omega\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\\
&= \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + \alpha_1\Bigg(\omega\frac{\partial^3 u}{\partial y^2\partial t} + \omega\frac{\partial^3 u}{\partial z^2\partial t}\\
&+2\frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y^2}\frac{\partial u}{\partial z}\\
&+2\frac{\partial w}{\partial y}\frac{\partial^2 u}{\partial y\partial z} + w\frac{\partial^3 u}{\partial y^2\partial z} + \frac{\partial^2 v}{\partial z^2}\frac{\partial u}{\partial y} + 2\frac{\partial v}{\partial z}\frac{\partial^2 u}{\partial y\partial z}\\
&+v\frac{\partial^3 u}{\partial y\partial z^2} + \frac{\partial^2 w}{\partial z^2}\frac{\partial u}{\partial z} + 2\frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial z^2} + w\frac{\partial^3 u}{\partial z^3}\Bigg)\\
&+\beta_1\Bigg(\frac{\partial^2 v}{\partial y^2}\frac{\partial u}{\partial y} + 2\frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial y^2} + 2\frac{\partial^2 u}{\partial y\partial z}\frac{\partial w}{\partial y} +\\
&\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial y^2} + 2\frac{\partial^2 u}{\partial y\partial z}\frac{\partial v}{\partial z} + \frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial z^2} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial z^2}\\
&+2\frac{\partial^2 u}{\partial z^2}\frac{\partial w}{\partial z}\Bigg) + Gr\theta + GmC
\end{aligned} \quad (12)$$

$$\begin{aligned}
&\omega\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\\
&= -\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} + \alpha_1\Bigg(\omega\frac{\partial^3 v}{\partial y^2\partial t} +\\
&\omega\frac{\partial^3 v}{\partial z^2\partial t} + v\frac{\partial^3 v}{\partial y^3} + w\frac{\partial^3 v}{\partial y^2\partial z} + v\frac{\partial^3 v}{\partial y\partial z^2}\\
&+w\frac{\partial^3 v}{\partial z^3} + 2\frac{\partial w}{\partial y}\frac{\partial^2 v}{\partial y\partial z} + 3\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial y\partial z}\\
&+2\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial y\partial z} + 13\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2} + 3\frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial y^2}\\
&+4\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} + 4\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2} + 2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial z^2}\\
&+\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial z^2}\Bigg) + \beta_1\Bigg(2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} + 8\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2} +\\
&2\frac{\partial w}{\partial y}\frac{\partial^2 v}{\partial y\partial z} + 2\frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial y^2} + 2\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial y\partial z} +\\
&2\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 u}{\partial y\partial z}\frac{\partial u}{\partial z} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial z^2}\Bigg)
\end{aligned} \quad (13)$$

$$\begin{aligned}
&\omega\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\\
&= -\frac{\partial p}{\partial y} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} + \alpha_1\Bigg(\omega\frac{\partial^3 w}{\partial y^2\partial t} +\\
&\omega\frac{\partial^3 w}{\partial z^2\partial t} + w\frac{\partial^3 v}{\partial y^3} + v\frac{\partial^3 v}{\partial z^2\partial y} + w\frac{\partial^3 w}{\partial z^3}
\end{aligned}$$

$$+v\frac{\partial^3 w}{\partial y^3}+2\frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial z\partial y}+3\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial z\partial y}$$
$$+2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial z\partial y}+13\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial z^2}+3\frac{\partial^2 v}{\partial z^2}\frac{\partial w}{\partial y}$$
$$+4\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2}+4\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial z}+2\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial y^2}$$
$$+\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial y^2}\Bigg)+\beta_1\Bigg(2\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2}+8\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial z^2}$$
$$+2\frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial z\partial y}+2\frac{\partial^2 v}{\partial z^2}\frac{\partial w}{\partial y}+2\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial z\partial y}$$
$$+2\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial z^2}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial z\partial y}+\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial y^2}\Bigg) \quad (14)$$

$$\omega\frac{\partial \theta}{\partial t}+v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{1}{Pr}\left(\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right) \quad (15)$$

$$\omega\frac{\partial C}{\partial t}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=\frac{1}{Sc}\left(\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right) \quad (16)$$

where $\alpha_1=\frac{\nu_2 u_\infty^2}{\nu_1}$, $\beta_1=\frac{\nu_3 u_\infty^2}{\nu_1}$ are visco-elastic parameters.

The modified boundary conditions are

$$\begin{aligned}&y=0:\ u=0, w=0,\ \theta=1,\ C=1\\ &v=-S[1+\varepsilon\cos(\pi z-t)],\\ &y\to\infty:\ u=1\ , w=0,\ \theta=0,\ C=0,\\ &v=-S, \quad (17)\end{aligned}$$

where $S=\frac{V_0}{u_\infty}$ is the suction parameter.

3 Method of Solution

We assume the solutions of (11) to (16) to be of the forms

$$u(x,y)=u_0(y)+\varepsilon u_1(y,z,t)+o(\varepsilon^2) \quad (18)$$

$$v(x,y)=v_0(y)+\varepsilon v_1(y,z,t)+o(\varepsilon^2) \quad (19)$$

$$w(x,y)=w_0(y)+\varepsilon w_1(y,z,t)+o(\varepsilon^2) \quad (20)$$

$$p(x,y)=p_0(y)+\varepsilon p_1(y,z,t)+o(\varepsilon^2) \quad (21)$$

$$\theta(x,y)=\theta_0(y)+\varepsilon \theta_1(y,z,t)+o(\varepsilon^2) \quad (22)$$

$$C(x,y)=C_0(y)+\varepsilon C_1(y,z,t)+o(\varepsilon^2) \quad (23)$$

Substituting these in the equations (11) to (16) and equating the coefficients of like powers of ε and neglecting ε^2 and higher powers, we get the following differential equations:

Zeroth-Order equations

$$v_0'=0 \quad (24)$$

$$\alpha_1 v_0 u_0'''+u_0''-v_0u_0'+Gr\theta_0+GmC_0=0 \quad (25)$$

$$p_0'-(4\alpha_1+2\beta_1)u_0'u_0''=0 \quad (26)$$

$$\theta_0''-v_0Pr\theta_0'=0 \quad (27)$$

$$C_0''-v_0ScC_0'=0 \quad (28)$$

The relevant boundary conditions are:

$$\begin{aligned}y=0&:u_0=0, v_0=-S, \theta_0=0, C_0=0\\ y\to\infty&:u_0=1, v_0=-S, \theta_0=0, C_0=0 \quad (29)\end{aligned}$$

The solutions of (24), (27) and (28) under the boundary condition (29) are given by

$$v_0(y)=-S, \theta_0(y)=e^{-SyPr}, C_0(y)=e^{-SySc} \quad (30)$$

To solve (25), we use multiparameter perturbation scheme following Nowinski and Ismail (1965) as $\alpha_1 << 1$ for small rate of shear. Then we consider

$$u_0(y)=u_{00}(y)+\alpha_1u_{01}(y)+o(\alpha_1^2) \quad (31)$$

Substituting (31) in (25) and comparing the like terms we obtain the following ordinary differential equations:

$$u_{00}''-v_0u_{00}'+Gr\theta_0+GmC_0=0 \quad (32)$$

$$v_0u_{00}'''+u_{01}''-v_0u_{01}'=0 \quad (33)$$

subject to boundary conditions:

$$\begin{aligned}y=0&:u_{00}=0, v_{00}=-S, \theta_0=1,\\ &C_0=1, u_{01}=0, v_{01}=0;\\ y\to\infty&:u_{00}=1, v_{00}=-S, \theta_0=0\\ &C_0=0, u_{01}=0, v_{01}=0. \quad (34)\end{aligned}$$

Solving (32) and (33) under the boundary conditions (34) we obtain

$$u_0(y) = 1 - e^{-Sy} + \frac{Gr}{S^2Pr(Pr-1)}(e^{-Sy} - e^{-SPry}) + \frac{Gm}{S^2Sc(Sc-1)}(e^{-Sy} - e^{-SScy}) + \alpha_1\Bigg(\frac{SGr}{Pr(Pr-1)}ye^{-Sy} + \frac{SGm}{Sc(Sc-1)}ye^{-Sy} - \frac{PrGr}{(Pr-1)^2}e^{-Sy} - \frac{GmSc}{(Sc-1)^2}e^{-Sy} - S^3ye^{-Sy} + \frac{GrPr}{(Pr-1)^2}e^{-SPry} + \frac{ScGm}{(Sc-1)^2}e^{-SScy}\Bigg) \quad (35)$$

for $Pr \neq 1$,$Sc \neq 1$.

First-order equations:

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \quad (36)$$

$$\omega\frac{\partial u_1}{\partial t} + v_1\frac{\partial u_0}{\partial y} - S\frac{\partial u_1}{\partial y} = \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} + Gr\theta_1 + GmC_1 + \alpha_1\Bigg(\omega\frac{\partial^3 u_1}{\partial y^2\partial t} + \omega\frac{\partial^3 u_1}{\partial z^2\partial t} - S\frac{\partial^3 u_1}{\partial y^3} + v_1\frac{\partial^3 u_0}{\partial y^3} - S\frac{\partial^3 u_1}{\partial z^2\partial y} - 2\frac{\partial v_1}{\partial y}\frac{\partial^2 u_0}{\partial y^2} - \frac{\partial u_0}{\partial y}\frac{\partial^2 v_1}{\partial y^2} - \frac{\partial^2 v_1}{\partial z^2}\frac{\partial u_0}{\partial y}\Bigg) \quad (37)$$

$$\omega\frac{\partial v_1}{\partial t} - S\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2} + \alpha_1\Bigg(\omega\frac{\partial^3 v_1}{\partial y^2\partial t} + \omega\frac{\partial^3 v_1}{\partial z^2\partial t} + -S\frac{\partial^3 v_1}{\partial y\partial z^2} - S\frac{\partial^3 v_1}{\partial y^3}\Bigg) \quad (38)$$

$$\omega\frac{\partial w_1}{\partial t} - S\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2} + \alpha_1\Bigg(\omega\frac{\partial^3 w_1}{\partial y^2\partial t} + \omega\frac{\partial^3 w_1}{\partial z^2\partial t} - S\frac{\partial^3 w_1}{\partial y\partial z^2} - S\frac{\partial^3 w_1}{\partial y^3}\Bigg) \quad (39)$$

$$\omega\frac{\partial \theta_1}{\partial t} + v_1\frac{\partial \theta_0}{\partial y} - S\frac{\partial \theta_1}{\partial y} = \frac{1}{Pr}\Bigg(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\Bigg) \quad (40)$$

$$\omega\frac{\partial C_1}{\partial t} + v_1\frac{\partial C_0}{\partial y} - S\frac{\partial C_1}{\partial y} = \frac{1}{Sc}\Bigg(\frac{\partial^2 C_1}{\partial y^2} + \frac{\partial^2 C_1}{\partial z^2}\Bigg) \quad (41)$$

where $\beta_1 = -2\alpha_1$ (with the assumption $p_0' = 0$ in the equation (26)) The relevant boundary conditions are:

$$\begin{aligned} y = 0 : u_1 = 0, v1 = -S\cos(\pi z - t), \\ w_1 = 0, \theta_1 = 0, C_1 = 0 \\ y \to \infty : u_1 = 0, v_1 = 0, w_1 = 0 \\ \theta_1 = 0, C_1 = 0 \end{aligned} \quad (42)$$

We assume the velocity components:

$$\begin{aligned} u_1(y,z,t) &= u_{11}(y)e^{i(\pi z - t)} \\ v_1(y,z,t) &= v_{11}(y)e^{i(\pi z - t)} \\ w_1(y,z,t) &= \frac{i}{\pi}v_{11}'(y)e^{i(\pi z - t)} \\ p_1(y,z,t) &= p_{11}(y)e^{i(\pi z - t)} \\ \theta_1(y,z,t) &= \theta_{11}(y)e^{i(\pi z - t)} \\ C_1(y,z,t) &= C_{11}(y)e^{i(\pi z - t)} \end{aligned} \quad (43)$$

Substituting (43) in (36) to (41), we get following differential equations:

$$v_{11}'' + Sv_{11}' - (\pi^2 - i\omega)v_{11} = p_{11}' + \alpha_1\Bigg(i\omega v_{11}'' - i\omega v_{11}\pi^2 - S\pi^2 v_{11}' + Sv_{11}'''\Bigg) \quad (44)$$

$$v_{11}''' + Sv_{11}'' - (\pi^2 - i\omega)v_{11}' = \pi^2 p_{11} + \alpha_1\Bigg(i\omega {v_{11}}''' - i\omega {v_{11}}'\pi^2 - S\pi^2 v_{11}'' + Sv^{IV}\Bigg) \quad (45)$$

$$\theta_{11}'' + SPr\theta_{11}' - (\pi^2 - iPr\omega)\theta_{11} = Prv_{11}\theta_0' \quad (46)$$

$$C_{11}'' + SSc\theta_{11}' - (\pi^2 - iSc\omega)C_{11} = Scv_{11}C_0' \quad (47)$$

$$u''_{11} + Su'_{11} - (\pi^2 - i\omega)u_{11} = v_{11}u'_0 - Gr\theta_{11} - GmC_{11} + \alpha_1\Bigg(i\omega u_{11}'' - i\omega\pi^2 u_{11} + Su_{11}''' - v_{11}u'''_0 - S\pi^2 u_{11}' + 2v_{11}'u_0'' + u'_0 v_{11}'' - \pi^2 v_{11}u_0'\Bigg) \quad (48)$$

subject to boundary conditions:

$$y = 0 : u_{11} = 0, v_{11} = -S, \theta_{11} = 0, C_{11} = 0;$$
$$y \to \infty : u_{11} = 0, v_{11} = 0, \theta_{11} = 0, C_{11} = 0. \quad (49)$$

On eliminating p'_{11} from equations (44) and (45), we get

$$v_{11}^{IV} + Sv_{11}''' - (\pi^2 - i\omega)v''_{11} - \pi^2 v_{11}'' - \pi^2 Sv'_{11} + \pi^2(\pi^2 - i\omega)v_{11} = \alpha_1\Bigg(i\omega v_{11}{}^{IV} - 2i\omega v''_{11}\pi^2 - 2S\pi^2 v_{11}''' + Sv_{11}{}^V + Sv'_{11}\pi^4\Bigg) \quad (50)$$

Again, we consider

$$u_{11}(y) = u_{110}(y) + \alpha_1 u_{111}(y) + o(\alpha_1^2) \quad (51)$$

$$v_{11}(y) = v_{110}(y) + \alpha_1 v_{111}(y) + o(\alpha_1^2) \quad (52)$$

$$\theta_{11}(y) = \theta_{110}(y) + \alpha_1\theta_{111}(y) + o(\alpha_1^2) \quad (53)$$

$$C_{11}(y) = C_{110}(y) + \alpha_1 C_{111}(y) + o(\alpha_1^2) \quad (54)$$

Substituting these into (48), (49) and (50) and comparing the like terms we get the following differential equations:

$$u''_{110} + Su_{110}' - (\pi^2 - i\omega)u_{110} = v_{110}u'_0 - Gr\theta_{110} - GmC_{110} \quad (55)$$

$$u''_{111} + Su'_{111} - (\pi^2 - i\omega)u_{111} = v_{111}u'_0 - Gr\theta_{111} - GmC_{111} + i\omega u_{110}'' - i\omega\pi^2 u_{110} + Su_{110}''' - v_{110}u'''_0 - S\pi^2 u_{110}' + 2v_{110}'u_0'' + u'_0 v''_{110} - \pi^2 v_{110}u'_0 \quad (56)$$

$$v_{110}^{IV} + Sv_{110}''' - (\pi^2 - i\omega)v''_{110} - \pi^2 v''_{110} - \pi^2 Sv'_{110} + \pi^2(\pi^2 - i\omega)v_{110} = 0 \quad (57)$$

$$v_{111}^{IV} + Sv_{111}''' - (\pi^2 - i\omega)v''_{111} - \pi^2 v_{111}'' - \pi^2 Sv'_{111} + \pi^2(\pi^2 - i\omega)v_{111} = i\omega v_{110}^{IV} - 2i\omega\pi^2 v''_{110} - 2S\pi^2 v_{110}''' + Sv_{110}{}^V + S\pi^4 v_{110}' \quad (58)$$

The corresponding boundary conditions are:

$$y = 0 : u_{110} = u_{111} = 0, v_{110} = -S, v_{111} = 0, v'_{110} = v_{111} = 0, \theta_{110} = \theta_{111} = 0, C_{110} = C_{111} = 0$$
$$y \to \infty : u_{110} = u_{111} = 0, v_{110} = v_{111} = 0, v'_{110} = v'_{111} = 0, \theta_{110} = \theta_{111} = 0, C_{110} = C_{111} = 0 \quad (59)$$

The solutions of the equations (46) to (48), (51) to (58) with relevant boundary conditions are:

$$v_{110} = A_1 e^{-\pi y} - A_2 e^{-r_1 y} \quad (60)$$

$$v_{111} = A_3 e^{-\pi y} + A_4 e^{-r_1 y} + A_5 y e^{-\pi y} + A_6 y e^{-r_1 y} \quad (61)$$

$$\theta_{110} = A_7 e^{-r_2 y} + A_8 e^{-(r_1 + SPr)y} + A_9 e^{-(\pi + SPr)y} \quad (62)$$

$$\theta_{111} = A_{10}e^{-r_2 y} + A_{11}e^{-(r_1 + SPr)y} + A_{12}e^{-(\pi + SPr)y} + A_{13}ye^{-(\pi + SPr)y} + A_{14}ye^{-(r1 + SPr)y} \quad (63)$$

$$C_{110} = A_{15}e^{-r_3 y} + A_{16}e^{-(r_1 + SSc)y} + A_{17}e^{-(\pi + SSc)y} \quad (64)$$

$$C_{111} = A_{18}e^{-r_3 y} + A_{19}e^{-(r_1 + SSc)y} + A_{20}e^{-(\pi + SSc)y} + A_{21}ye^{-(\pi + SSc)y} - A_{22}ye^{-(r1 + SSc)y} \quad (65)$$

$$u_{110} = A_{23}e^{-r_1 y} + A_{24}e^{-(\pi + S)y} + A_{25}e^{-(r_1 + SPr)y} + A_{26}e^{-(\pi + SPr)y} - A_{27}e^{-r_2 y} + A_{28}e^{-(\pi + S)y} - A_{29}ye^{-(r_1 + S)y} + A_{30}e^{-(\pi + SSc)y} + A_{31}e^{-(r_1 + SSc)y} - A_{32}e^{-r_3 y} + A_{33}e^{-(r_1 + S)y} \quad (66)$$

$$
\begin{aligned}
u_{111} \;=\;& A_{34}e^{-r_3y}+A_{35}e^{-(\pi+S)y}\\
&+A_{36}e^{-(r_1+SPr)y}+A_{37}ye^{(\pi+SPr)y}\\
&+A_{38}e^{-(r_1+S)y}+A_{39}ye^{-(r_1+SPr)y}\\
&+A_{40}ye^{-(r_1+S)y}-A_{41}y^2e^{-(\pi+S)y}\\
&+A_{42}e^{-r_2y}+A_{43}e^{-r_1y}\\
&+A_{44}ye^{-(\pi+S)y}+A_{45}e^{-(\pi+SSc)y}\\
&+A_{46}e^{-(r_1+SSc)y}+A_{47}ye^{-(\pi+SSc)y}\\
&+A_{48}e^{-(\pi+SPr)y}+A_{49}ye^{-r_1y}\\
&+A_{50}y^2e^{-(r_1+S)y}+A_{51}ye^{-(r_1+SSc)y}
\end{aligned}\tag{67}
$$

$$
\begin{aligned}
u_{11} \;=\;& A_{52}e^{-r_1y}+A_{53}e^{-(\pi+S)y}\\
&+A_{54}e^{(\pi+SPr)y}+A_{55}e^{-(r_1+SPr)y}\\
&+A_{56}e^{-r_2y}+A_{57}ye^{-(\pi+S)y}\\
&+A_{58}e^{-(r_1+S)y}+A_{59}ye^{-(r_1+S)y}\\
&+A_{60}e^{-(\pi+SSc)y}+A_{61}ye^{-(r_1+SSc)y}\\
&+A_{62}e^{-r_3y}+A_{63}ye^{(\pi+SPr)y}\\
&+A_{64}ye^{-(\pi+SSc)y}+A_{65}ye^{-(r_1+SPr)y}\\
&+A_{66}ye^{-(r_1+SSc)y}-A_{67}ye^{-r_1y}\\
&+A_{68}y^2e^{-(r_1+S)y}-A_{69}y^2e^{-(\pi+S)y}
\end{aligned}\tag{68}
$$

$$
\begin{aligned}
v_{11} \;=\;& A_{70}e^{-\pi y}+A_{71}e^{-r_1y}+A_{72}ye^{-\pi y}\\
&+A_{73}ye^{-r_1y}
\end{aligned}\tag{69}
$$

$$
\begin{aligned}
\theta_{11} \;=\;& A_{74}e^{-r_2y}+A_{75}e^{-(r_1+SPr)y}\\
&+A_{76}e^{-(\pi+SPr)y}+A_{77}ye^{-(\pi+SPr)y}\\
&+A_{78}ye^{-(r_1+SPr)y}
\end{aligned}\tag{70}
$$

$$
\begin{aligned}
C_{11} \;=\;& A_{79}e^{-r_3y}+A_{80}ye^{-(r_1+SSc)y}\\
&+A_{81}e^{-(\pi+SSc)y}+A_{82}ye^{-(\pi+SSc)y}\\
&+A_{83}ye^{-(r_1+SSc)y}
\end{aligned}\tag{71}
$$

Thus the respective velocity, temperature and concentration are given by

$$
\begin{aligned}
u =\;& 1-e^{-Sy}+\frac{Gr}{S^2Pr(Pr-1)}(e^{-Sy}-e^{-SPry})\\
&+\frac{Gm}{S^2Sc(Sc-1)}(e^{-Sy}-e^{-SScy})\\
&+\alpha_1\Bigg(\frac{SGr}{Pr(Pr-1)}ye^{-Sy}-\frac{PrGr}{(Pr-1)^2}e^{-Sy}\\
&-\frac{GmSc}{(Sc-1)^2}e^{-Sy}-S^3ye^{-Sy}+\frac{SGm}{Sc(Sc-1)}ye^{-Sy}\\
&+\frac{GrPr}{(Pr-1)^2}e^{-SPry}+\frac{GmSc}{(Sc-1)^2}e^{-SySc}\Bigg)\\
&+\varepsilon\Bigg(A_{52}e^{-r_1y}+A_{53}e^{-(\pi+S)y}\\
&+A_{54}e^{(\pi+SPr)y}+A_{55}e^{-(r_1+SPr)y}\\
&+A_{56}e^{-r_2y}+A_{57}ye^{-(\pi+S)y}\\
&+A_{58}e^{-(r_1+S)y}+A_{59}ye^{-(r_1+S)y}\\
&+A_{60}e^{-(\pi+SSc)y}+A_{61}e^{-(r_1+SSc)y}\\
&+A_{62}e^{-r_3y}+A_{63}ye^{(\pi+SPr)y}\\
&+A_{64}ye^{-(\pi+SSc)y}+A_{65}ye^{-(r_1+SPr)y}\\
&+A_{66}ye^{-(r_1+SSc)y}-A_{67}ye^{-r_1y}\\
&+A_{68}y^2e^{-(r_1+S)y}-A_{69}y^2e^{-(\pi+S)y}\Bigg)e^{i(\pi z-t)}
\end{aligned}\tag{72}
$$

$$
\begin{aligned}
v \;=\;& -S+\varepsilon\Bigg(A_{70}e^{-\pi y}+A_{72}ye^{-\pi y}\\
&+A_{71}e^{-r_1y}+A_{73}ye^{-r_1y}\Bigg)e^{i(\pi z-t)}
\end{aligned}\tag{73}
$$

$$
\begin{aligned}
\theta \;=\;& e^{-SPry}+\varepsilon\Bigg(A_{74}e^{-r_2y}+A_{75}e^{-(r_1+SPr)y}\\
&+A_{76}e^{-(\pi+SPr)y}+A_{77}ye^{-(\pi+SPr)y}\\
&+A_{78}ye^{-(r_1+SPr)y}\Bigg)e^{i(\pi z-t)}
\end{aligned}\tag{74}
$$

$$
\begin{aligned}
C \;=\;& e^{-SScy}+\varepsilon\Bigg(A_{79}e^{-r_3y}+A_{80}ye^{-(r_1+SSc)y}\\
&+A_{81}e^{-(\pi+SSc)y}+A_{82}ye^{-(\pi+SSc)y}\\
&+A_{83}ye^{-(r_1+SSc)y}\Bigg)e^{i(\pi z-t)}
\end{aligned}\tag{75}
$$

$$
\begin{aligned}
w \;=\;& \frac{i\varepsilon}{\pi}\Bigg(A_{84}e^{-\pi y}+A_{85}e^{-r_1y}+A_{86}ye^{-\pi y}\\
&+A_{87}ye^{-r_1y}\Bigg)e^{i(\pi z-t)}
\end{aligned}\tag{76}
$$

The non-dimensional shearing stress at the plate ($y = 0$) is given by

$$
\sigma=\left[\frac{\partial u}{\partial y}\right]_{y=0}+\alpha_1\Bigg[\omega\frac{\partial^2u}{\partial y\partial t}-3\frac{\partial u}{\partial y}\frac{\partial v}{\partial y}+
v\frac{\partial^2u}{\partial y^2}-\frac{\partial w}{\partial y}\frac{\partial u}{\partial z}+w\frac{\partial^2u}{\partial y\partial z}-2\frac{\partial v}{\partial z}\frac{\partial u}{\partial z}\Bigg]_{y=0}
$$

$$= A_{88} + A_{89} + \alpha_1 \Bigg(\omega A_{90} - 3(A_{88} + A_{89})A_{91}$$
$$+ v(A_{92} + A_{93}) - A_{94}A_{95} + wA_{96} - 2A_{97}A_{98} \Bigg) \quad (77)$$

Again, the non-dimensional heat flux coefficient at the plate in terms of Nusselt number Nu is given by

$$Nu = \left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$
$$= \quad -SPr + \varepsilon \Bigg(A_{99} + A_{100} + A_{77} + A_{78} \Bigg) e^{i(\pi z - t)} \quad (78)$$

The mass transfer coefficient in terms of Sherwood number Sh is given by

$$Sh = \left(\frac{\partial C}{\partial y}\right)_{y=0} = -SSc + \varepsilon \Bigg(A_{101} + A_{102}$$
$$+ A_{103} + A_{82} + A_{83} \Bigg) e^{i(\pi z - t)} \quad (79)$$

4 Result and Discussion

In this analysis, we discuss the unsteady three-dimensional visco-elastic flow with heat and mass transfer in presence of variable suction. The visco-elastic effect is exhibited through the non-dimensional parameter α_1. The corresponding results for Newtonian fluid are obtained by setting $\alpha_1 = 0$. The non-zero values of $\alpha_1 = 0$ characterize the visco-elastic fluid flow phenomenon. The real part of the solutions is implied throughout the discussion. To get physical insight into the problem, the fluid velocity and the shearing stress at the plate have been illustrated graphically by assigning some specific values to the parameters involved in the problem and the effects of the visco-elastic parameter on the governing flow have been discussed in detail. The values of the parameters $\varepsilon = 0.3$, z=0.2, $\omega = 10$, t=0.2 are kept fixed throughout the discussion.

The figures 1 to 6 demonstrate the fluid velocity u against y for different values of Prandtl number Pr, Grashof number for heat transfer Gr, Grashof number for mass transfer Gm, Suction parameter S , Schmidt number Sc and visco-elastic parameter $|\alpha_1|$. In all the cases, the fluid velocity accelerates near the plate but shows uniformly away from the plate in both Newtonian and non-Newtonian cases. The variation of the

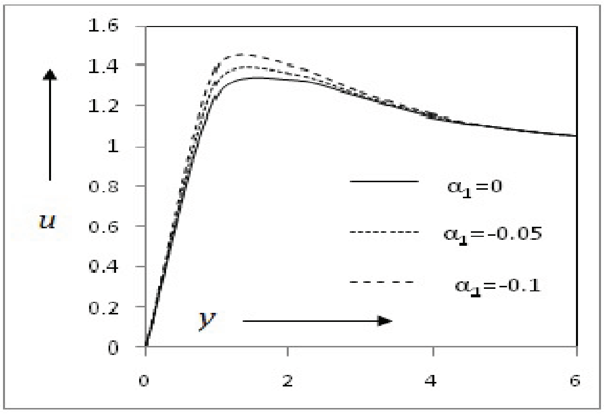

Figure 1: Fluid velocity u against y for $Pr = 2, S = 1, Sc = 0.6, Gm = 0.5, Gr = 2$

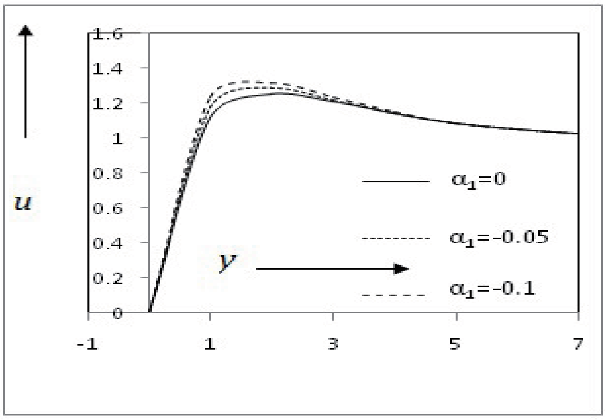

Figure 2: Fluid velocity u against y for $Pr = 3, S = 1, Sc = 0.6, Gm = 0.5, Gr = 2$

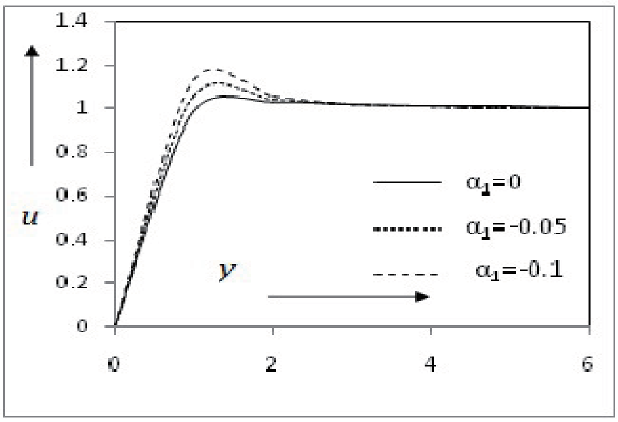

Figure 3: Fluid velocity u against y for$Pr = 2, S = 2, Sc = 0.6, Gm = 0.5, Gr = 2$

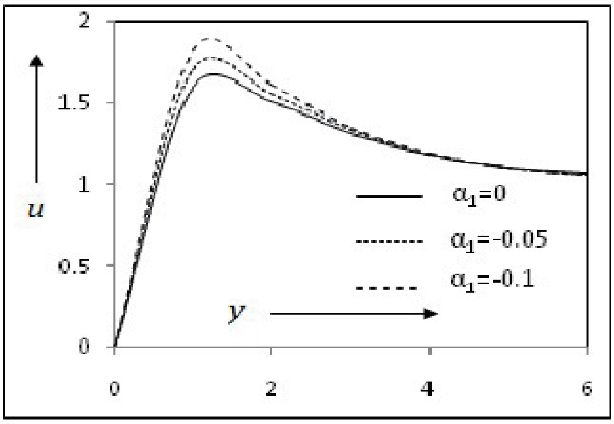

Figure 4: Fluid velocity u against y for $Pr = 2, S = 1, Sc = 0.6, Gm = 0.5, Gr = 5$

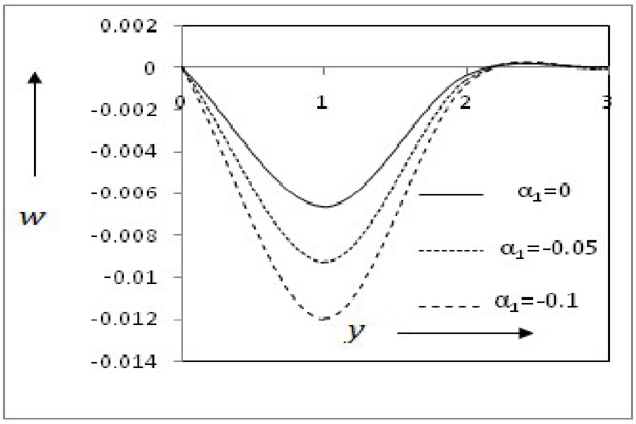

Figure 7: Cross velocity w against y for$Pr = 2, S = 1, Sc = 0.6, Gm = 0.5.Gr = 2$

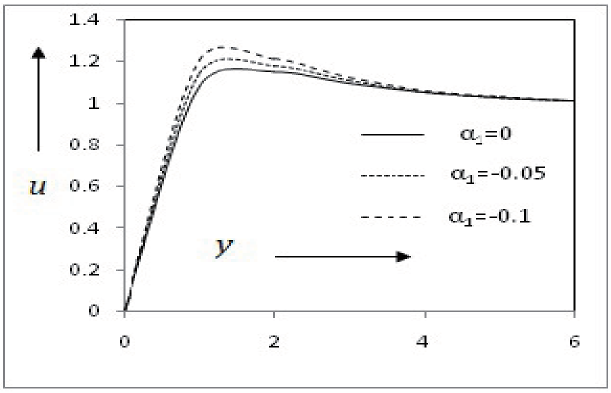

Figure 5: Fluid velocity u against y for $Pr = 2, S = 1, Sc = 0.9, Gm = 0.5, Gr = 2$

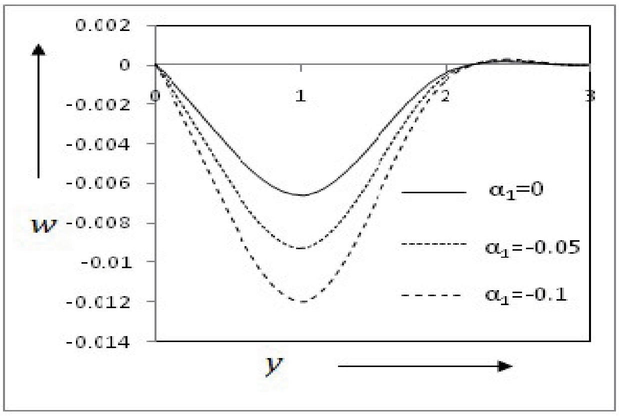

Figure 8: Cross velocity$w$ against y for $Pr = 3, S = 1, Sc = 0.6, Gm = 0.5, Gr = 2$

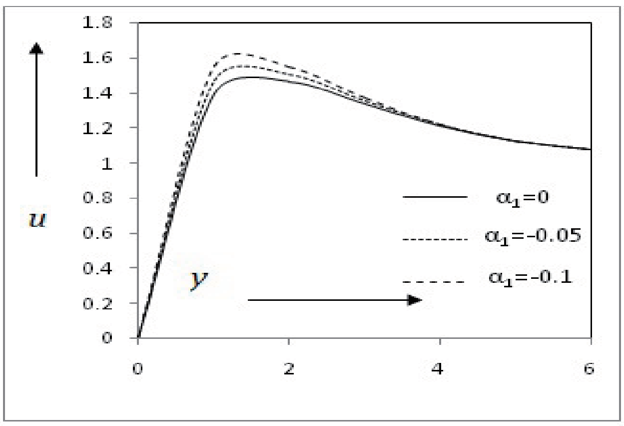

Figure 6: Fluid velocity u against y for$Pr = 2, S = 1, Sc = 0.6, Gm = 0.7, Gr = 2$

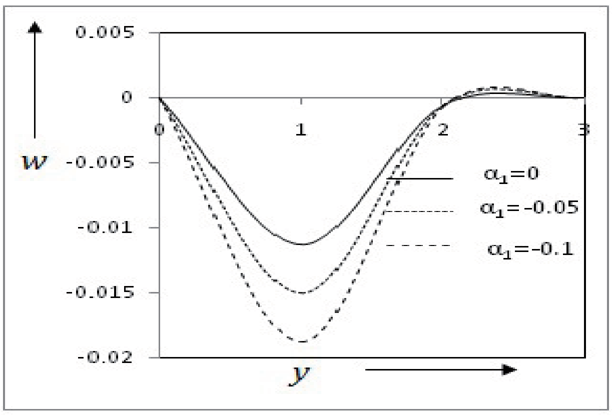

Figure 9: Cross velocity w against y for $Pr = 2, S = 2, Sc = 0.6, Gm = 0.5, Gr = 2$

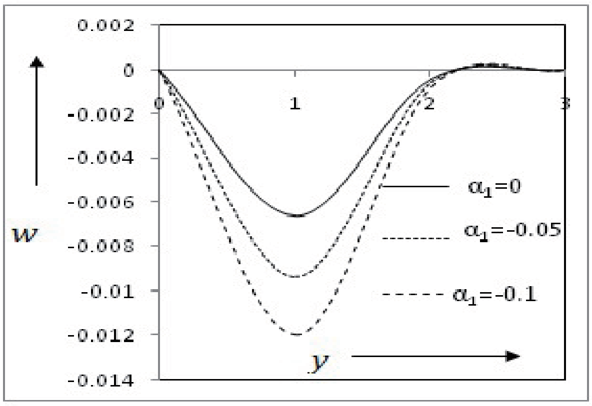

Figure 10: Cross velocity w against y for $Pr = 2, S = 1, Sc = 0.6, Gm = 0.5, Gr = 5$

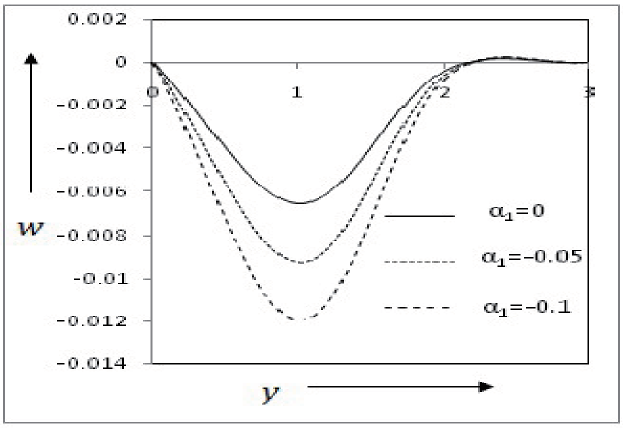

Figure 11: Cross velocity w against y for $Pr = 2, S = 1, Sc = 0.9, Gm = 0.5, Gr = 2$

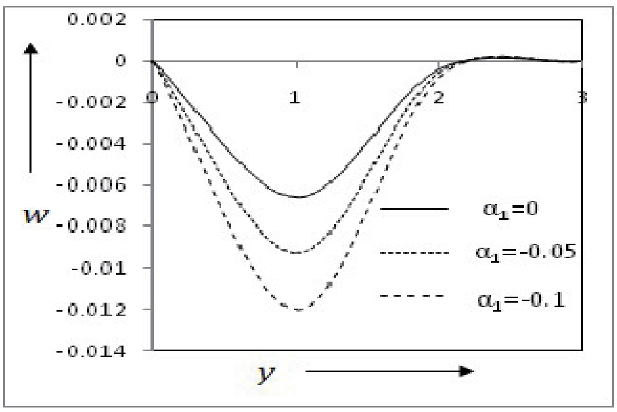

Figure 12: Cross velocity w against y for $Pr = 2, S = 1, Sc = 0.6, Gm = 0.7, Gr = 2$

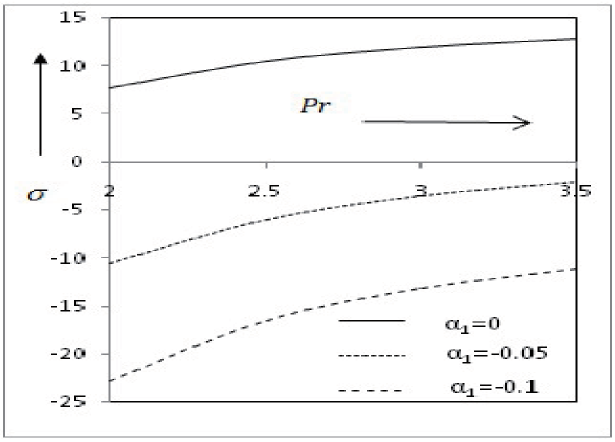

Figure 13: Shearing stress against Prandtl number Pr for $S = 1, Sc = 0.6, Gm = 0.5, Gr = 2$

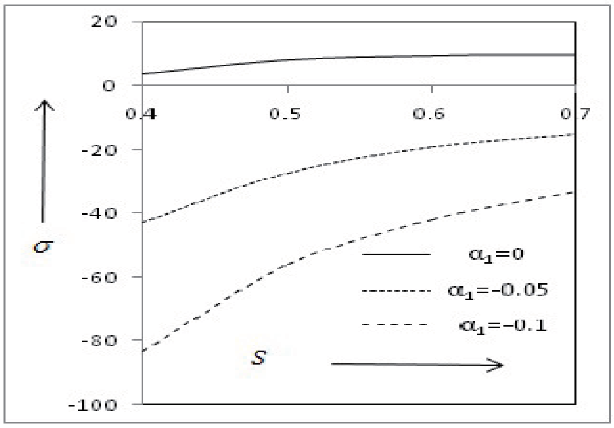

Figure 14: Shearing stress against suction parameter S for $Pr = 2, Sc = 0.6, Gm = 0.5, Gr = 2$

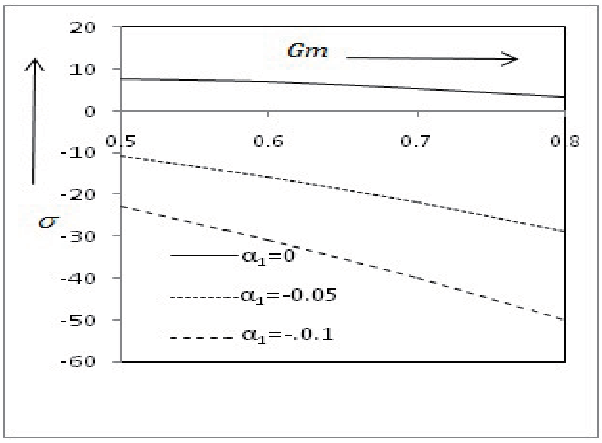

Figure 15: Shearing stress against Grashof number for mass transfer with $Pr = 2, Sc = 0.6, S = 1, Gr = 2$

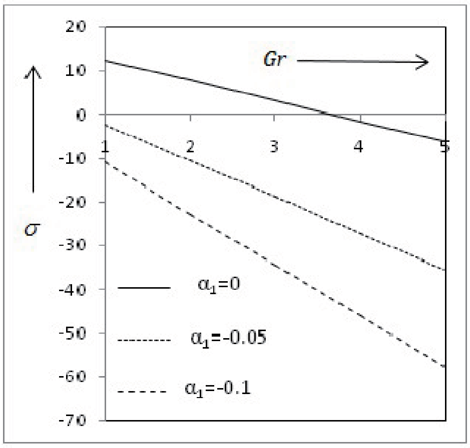

Figure 16: Shearing stress against Grashof number for heat transfer with $Pr = 2, Sc = 0.6, S = 1, Gm = 0.5$

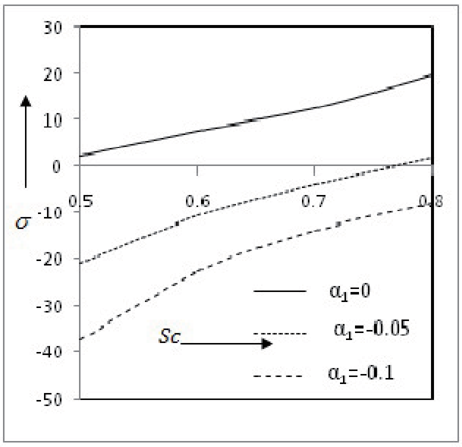

Figure 17: Shearing stress against Schmidt number for $Pr = 2, Gr = 2, S = 1, Gm = 0.5$

physical parameters like Pr, Gr, Gm, S, and Sc do not alter the pattern of the profiles. But in all the cases, the growth of absolute value of the visco-elasticity enhances the fluid velocity in comparison with Newtonian fluid flow phenomenon.
The cross velocity w against y has been depicted in the figures 7 to 12. In all the figures, it is noticed that the cross velocity w in both Newtonian and non-Newtonian cases first diminishes and then boost up to considerable amount. The rising values of visco-elastic parameter$|\alpha_1|$ show a decelerating trend in speed.
Figures 13 to 17 illustrate the variations of shearing stress against the Prandtl number Pr, Suction parameter S, Grashof number for mass transfer Gm, Grashof number for heat transfer Gr, and the Schmidt number Sc respectively with fixed values of other physical parameters viz. $Pr = 2$, $S = 1$, $Sc = .6$, $Gm = 0.5$ and $Gr = 2$. With the enhancement of Gr and Gm the shearing stress shows a diminishing trend in both Newtonian and non-Newtonian fluid flows but depict rising trends with the variation of Pr, S and Sc. In all the cases, the rising values of $|\alpha_1|$diminish the shearing stress in the fluid flow region in comparison with simple Newtonian fluid.
From the expressions (77) and (79), it can be observed that the rate of heat transfer and rate of mass transfer are not noticeably affected by the visco-elastic parameter.

5 Conclusion

The investigation leads to the following conclusions:

1. The velocity field is significantly affected at every point of the fluid flow region by the visco-elastic parameter in combination of other flow parameters.

2. The shearing stress exhibits an accelering trend in both Newtonian and non-Newtonian cases with the increase of Prandtl number, suction parameter and Schmidt number but reverse trend is observed with the increase of Grashof number for mass transfer and Grashof number for heat transfer. In all the cases, the absolute value of visco-elastic parameter diminishes the shearing stress in comparison with Newtonian fluid.

3. The rate of heat transfer in the form of Nusselt number and the rate of mass transfer in the form of Sherwood number are not significantly affected by the visco-elastic parameter.

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