Axiological Impact Analysis of Legal Regulations and Policies. A Seminal Case-Study from Land Law for a Mathematical Theory

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Abstract: This paper offers a seminal case-study of an elementary legal micro-system composed of estate property right and right to privacy. We intend to decompose the structure of such legal positions, identifying their most basic factors and logical relations and adopting a mathematical model for the quantitative analysis of law that we have recently introduced, still to be validated by the scientific community. We will describe the conflicts among identical legal values (property right vs. property right) and different legal values (property right vs. privacy right), originating by a claim to open a window next to the land boundary. Comparing two possible regulatory options for this pattern of conflict, we will be ultimately able to unveil the truly legal nature of each option (*status personarum* policy *vs.* axiological balancing policy), as well as the axiological (in)-efficiency of every devised solution. This method might lead the way to more extensive implementations of mathematical analysis in the realm of axiological impact deriving from legal regulations and policies.

Key–Words: Law, Mathematics, Value, Number, Conflict, Property, Privacy, Impact, Regulation, Policy

1 Introduction

We have recently construed and introduced a mathematical model for the quantitative analysis of law, as a first result of an ongoing research project, through a series of published monographs [1, 2, 3], articles and papers [4, 5, 6, 7, 8], while some other contributions on the topics are still under either peer review for important scientific international conferences [9, 10, 11] or editorial revision [12]. Our intent focuses on the formulation of a system of axioms, rules and procedures allowing the operator to manage effectively legal values, regulations and policies, in a fairly objective and transparent way. By doing so, if we succeeded in building up a neutral model to measure legal entities, we might think of codifying a new *lingua franca* for the dialogue among legal cultures and traditions: this is why we have been carrying out our project under the auspice of Meda*lics-Research Center for Mediterranean Relations*, whose mission is that of fostering knowledge flows in interdisciplinary and intercultural relations.

2 Objective of this Paper

In the present paper we want to feature a first complete case-study of quantitative legal assets management, in order to try and demonstrate that our model, by now, is ready to be tested, and eventually validated and massively implemented, or instead ameliorated by means of other scholars contribution, what we are eagerly looking for.

Through the step-by-step illustration of our case-study, we will be able to highlight any operation required for the full implementation of our model. The reader will receive then complete information about how the model works, how it can perform and what kind of outcome it can deliver.

3 Description of our Case-Study

In two previous works of ours [4,5], we have illustrated how our model can translate a legal rule expressed in a string of text (art. 3 of the *Universal Declaration on Human Rights* approved by the *General Assembly* of the *United Nations Organization*, enshrining the right to human life) in a logical device, ready for quantitative and mathematical analysis, allegedly capable of conveying axiological legal information.

Now we want to move our explanatory purposes one step forward. To this end, we will not just describe, classify and measure one legal *situation* (a legal *organism* of second order, according to our

nomenclature already introduced in a previous article [6]), but a *micro-system* composed of more legal *organisms*, conflicting among them. What we would like to maintain in the next paragraphs of the present paper, is that our model can be used to effectively manage such patterns of legal axiological conflicts: starting from a descriptive phase, the model will ultimately deliver normative prescriptions and suggestions about the nature of legal policies and the axiological balance of each option of regulation. Finally, comparing the balance of the options between them, and each of them with the axiological *equilibrium* governing *status quo ante* endowments of legal entitlements, we will draw ax axiological quantitative analysis of the *impact* of *regulations* and *policies* in terms of legal values and axiological potentials.

Next we will explicit the terms of our proposed *case-study*, describing the *micro-system* we have thought of and the *options* that we will be measuring and comparing. Our choices have been guided by the intent of *maximum* accessibility for a not necessarily legally-specialized scholar and with no bias in favor of any specific legal tradition. The usage of latin expressions would like to be neutral, considering that historically the first complete regulation of *jura luminum* (the rights of the landlords who want to open windows next to their boundary [21]) was enacted by ancient *Roman Law*, not yet into force, and that these roots are practically universally spread even nowadays. Our model should be as much interdisciplinary and of cross-border applicability as possible.

3.1 Rules of Law for our Case-Study

We assume that the legal *micro-system*, within which we will be performing our *case-study*, is composed of some general and specific legal *rules*, governing it.

Through the step-by-step illustration of our case-study, we will be able to highlight any operation required for the full implementation of our model. The reader will receive then complete information about how the model works, how it can perform and what kind of outcome it can deliver.

3.1.1 General Framework of our Hypothesized Legal Micro-System

We will then assume that the legal system is composed of a series of legal rules giving value to facts and events of reality [13]. Whether such rules are put by a legislator or a judge seems to be not material for the proper functioning of our model. The crucial point is that the operator has to be able to logically identify the *factors* of the *situation* featured by the rule, and the logical *relations* among them [14].

We will assume that *titulary* of a legal *position* supposes legal *capacity*, which in turn supposes legal *subjectivity*. This three factors describe the *subjective cluster* [12] of the legal *organisms* at issue.

As to the *objective cluster* [12], a legal *object* is a *thing* capable of attracting human *interest*.

Furthermore, we will not be comparing each right in itself as a whole axiological unit (for ex. *property* and *privacy*), but instead the single *facultas agendi* entangled within its legal content (*jus ne prospectui officiatur; jus excludendi omnes alios*). We will suppose that a legal *facultas* is a single behavior covered by a legal *entitlement*: it is the object of a *claim* vested by a legal *situation*. We will be comparing *facultates* rather than *jura*, what allows moreover a fully concrete and contextualized shaping of the axiological *terms* of the legal *conflict*.

3.1.2 The Legal Claims (*facultates*) at Stake: *jus ne prospectui officiatur; jus excludendi alienam inspectionem*

The *claims*, giving birth to the *conflicts* under scrutiny, are basically two:

- A land owner claims to open a window (*jus prospectum tollendi*) next to the boundary with his neighbor, without the latter being allowed to interfere with such behavior (*jus ne prospectui officiatur*). The same *claim* is symmetrically moved by the neighbor.

- Any individual claims to be allowed to exclude other peoples interference on his sphere of private life (right to *privacy: jus excludendi omnes alios*) and specifically from other peoples attempt to observe (*inspicere*) what happens within his property (*jus excludendi alienam inspectionem*).

  Within our micro-system we assume that:

- the right to *privacy* belongs to any human being;

- a window (*prospectus*) is a opening in wall, characterized by the facts that it allows the owner to look at the neighborhood (*inspectio*) and to put his body so as to have a fairly easy view (*prospectio*). An opening lacking of *inspectio* and/or *prospectio* is not a *prospectus*, but a mere *lumen: jus luminum* (i.e. the claim to open a *lumen*) is not at issue in our case-study. We observe that *prospectio* logically supposes *inspectio*, but not *viceversa*.

3.1.3 The Patterns of Conflict: *Robinsonian* Conflict *vs. Axiological* Conflict

We will analyze the two following legal *conflicts*:

- *Jus tollendi* vs. *Jus tollendi*: this *conflict* entails two identical legal *situations*, symmetrically opposed to

each other. The *conflict* arises in *abstract* terms, for the two *claims* are ontologically incompatible. It is not possible to open two *prospectus* just on the same limit of the two lands, face-to-face. This incompatibility between two symmetrical *claims* is an example of *Robinsons Recht* (RR) *conflict* (according to our notion of RR, already explained in some of our previous works [8, 10, 11]). It is as though the only two people on an island conflicted for the appropriation of the one coconut. We have already stated our view on the point [8, 10, 11]: the only effective *policy* measure to solve a truly RR legal *conflict* is a *status personarum* policy: we will deal with such assumption later in this paper.

- *Jus tollendi* vs. *Jus excludendi*: this legal *conflict* opposes two different legal *positions*, hence two different legal *values*. *jus tollendi* pertains to land *property* right (*jus in rem*); *jus excludendi* pertains to *privacy* right (*jus personalis*). We have already stated, in some previous contributions of ours [8, 10, 11], that a legal *axiological conflict* can be solved either on *status personarum* basis, or on *axiological balancing* basis.

3.2   The *Options* within our Case-Study.

The scope of our *case-study* will be confined to what is strictly necessary to depict all the operations required for the functioning of our *model*. We will discuss the interactions between two *facultates* (as shown in 3.1 above) and we will compare two alternative legal *policies*, respectively concretized by two alternative legal *regulations*, for their resolution.

3.2.1   Options of Legal Policy: *Status Personarum* Policy vs. *Axiological Balancing* Policy

A measure of *status personarum* policy discriminates against some subjective position irrespective of its concrete situation [8, 10]. It is not the *structure* of the legal *position* what makes the (axiological) difference, but instead an attribute qualifying the (dis-)favored subject as *status quo* owner (*incumbent*) or *challenger*. Such kind of policy manifests a *conservative* orientation if it favors the incumbent, or a *progressist* orientation if it favors the challenger.

The *levers* capable of capturing the measure on *status* policy are the parameters (X, Y, W and Z) describing the *proportions* (*realis* and *personalis*) which substantiate the elementary axiological mechanism of axiological creation of a legal *entitlement* [7, 8, 9]. Within this complex system of legal *proportions*, the *status* position of the *incumbent* is measured by the segments X and Z, whilst the *status* position of the *challenger* is measured by the segments Y and W. In

a neutral system, X=Y=1 and Z=W=0. If *Height* (*Hn*) and *Strength* (*Sn*) are the axiological dimensions of legal values [15] measuring the axiological contribution (*Wn*) of the *n*-units of legal *Mass* (i.e. the *factors* of the legal *structure*), instead the *status* parameters (*X, Y, W* and *Z*) measure the axiological impact of the subjective positions to the *equilibrium* legal entity. *Status personae* is the systematic measure of legal *subjectivity*. Our model assumes that the *subjectivity* is a terminal for both legal *interests* and legal *attributions* [13]: it does not constitute a *unit* of legal *mass*, hence it has not got neither *Height* nor *Strength*. Instead, the *subject* assumes the *positions* of:

- Center of human *interest*: it expresses the interest of the *subject* towards a given *object*, the object of a *claim* (i.e. the ultra-peripheral factor of the string). This *interest* is what generates gravitation (*momentum* [1, 2, 8]) of the legal *mass* towards the two (subjective) terminals of the *string*. [1, 2, 6, 7, 8]. More specifically:

  ○ Parameter *X* measures the *Challenger* in its *position* of center of interest;

  ○ Parameter *Y* measures the *Incumbent* in its *position* of center of interest;

- Center of legal *attribution*: the *subject* receives in this *position* the axiological impulse [1, 2, 6, 7, 8. The axiological *momentum* of the *mass*, triggered by the counterpart (in its *position* of center of *interest*), is conveyed to the other *subject*/terminal as addressee of the legal *attribution*. More specifically:

  ○ Parameter Z measures the *Challenger* as center of legal *attribution*;

  ○ Parameter W measures the *Incumbent* as center of legal *attribution*.

We observe here that the above said on *status personarum* parameters appears satisfactory in a regime of *individual* feedback [8, 9, 10, 11], whereas it is up to the *incumbent* to directly face the *challengers claim*, opposing its *status quo ante entitlement* in a pattern of immediate axiological *conflict*. Instead, in a regime of *social* feedback [8, 9, 10, 11], the system sets up an *impersonal* agent to emit a *medium*-intensity axiological feedback to vest a legal *entitlement* on the *challenger claim*; only in a subsequent phase the *incumbent*, dissatisfied with the *social* feedback, will have to face the already vested *challenger entitlement*, opposing his own *status quo ante entitlement*, thus leading the way to an axiological *conflict*. We thus reinterpret the classical notions of statuses and contract [18, 19].

Moreover, we note that the distinction between *individual* and *social* feedback regimes is not material in RR *conflicts*, by definition: this is why we have already written that RR embodies a sort of *reductio*

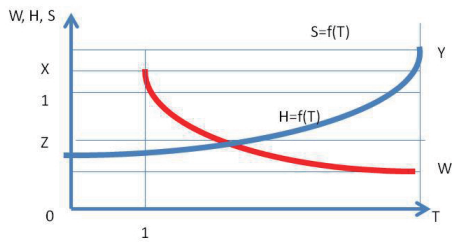*ad unum* for both axiological feedback *regimes* [11]. This is shown in Fig.1 below.

Figure 1: *Status personarum* Parameters

Instead a *policy* of axiological *balancing* is based upon the comparison among axiological *potentials* associated to legal *mass*. It is a situational way of conflict *adjudication*, in the sense that it depends upon the *situation*, hence upon the allocation of *mass* units within the *strings* (positions, situations, rules [6, 8]). However, we have already explained [6, 8] that an axiological balancing *policy* can even be non-situational, because it entails legal *organisms* with no time allocation: this is the case for the legal *organisms* of *third order* (the legal *principles*), which can be represented as informational vectors with two *dimensions* (axiological *Height*; axiological *Strength*), and with no logical time allocation [6, 8]. This is depicted in next Fig.2.
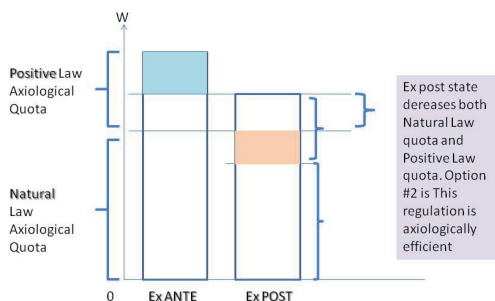
Figure 2: Axiological Balancing Policy

The *levers* which capture axiological *balancing* policies are basically the two *parameters* which determine the *slope* of the axiological elementary *curves* of *Height* [$Hn=f(Tn)$] and *Strength* [$Sn=f(Tn)$]. More specifically:
- Parameter α determines the slope of Hn curve;
- Parameter β determines the slope of Sn curve.

Another *endogenous* variable, capable of expressing a choice of axiological balancing policy, is the quota of *discretionary* potential associated to *Gap-Mass* [1, 4, 5]. This is a fraction of the axiological *potential* deriving from a legal *structure* that is at disposal of the *policy-maker* (*Positive Law*) for the decision of *favor* or *disfavor* concerning a certain legal *situation*.

3.2.2 Options of Legal Regulation: *Preemption vs. Distance* from the Boundary

We will analyze and compare two possible *regulatory options*:

- Regulatory Option #1 (*Preemption*): only the landlord who builds first next to the boundary, has the right to open the window; furthermore he is allowed to exercise *inspectio* and *prospectio* on his neighbors land and he is entitled of excluding his neighbor from exercising in future the right to open a window face-to-face.

- Regulatory Option #2 (*Distance*): each of the two landlords can open a window next to the boundary; however each of them has to respect a distance of (at least) three meters from the boundary. If this distance is respected, the neighbor cannot exclude other *inspectio* and *prospectio*.

4 Preliminary insights on the Model

Is it possible and practically useful the idea of identifying some *mathematical laws* underlying the process of *creation* of any legal *value* in any legal *system*?

Yes, in our humble opinion it is.

In order to measure legal *values* with *numbers*, it appears necessary to describe the logical *structure* of a legal *entity*, ordering its basic *ingredients* by using *Logics*. Each *concept* is the content of a *factor* of the legal *structure* [1, 8]; *factors* are linked by logical *relations* of:

- *presupposition/consequentiality* (from one factor placed at Time T to the other *factor* put at Time T+1);

- *attribute*: from an *attribute-factor* placed at Time T to the principal-*factor* even placed at time T.

The *origin* of logical *Time* (situated at T=0) is the legal (claiming) *subject*, in its *position* of center of any human *interest* not satisfied by *status quo ante* allocation. The (challenging or claiming) *subject* moves a *claim* towards an *object* and the legal *system* requires a certain logically orderly *sequence/string* of *factors* to be accomplished, in order to vest this *claim* with a legal *entitlement*, capable of resisting once conflicting with other subject *entitlements* [1, 8, 16], giving rise to an axiological *adjudication* process [1, 8, 9, 11]. This is illustrated in following Fig.3.
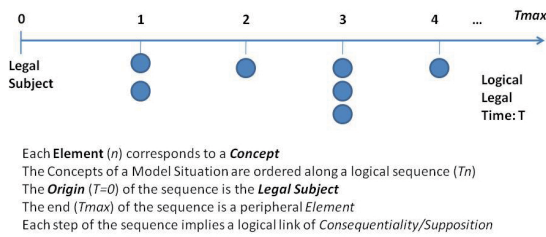
Each **Element** (*n*) corresponds to a **Concept**
The Concepts of a Model Situation are ordered along a logical sequence (*Tn*)
The **Origin** (*T=0*) of the sequence is the **Legal Subject**
The end (*Tmax*) of the sequence is a peripheral *Element*
Each step of the sequence implies a logical link of *Consequentiality/Supposition*

Figure 3: Orderly Sequence of Elements

We have already proposed [1,2,3,4] two basic *equations* describing the two *dimensions* of legal values [15], i.e. *Height* and *Strength*. We have more recently modified their formulation [8], keeping unvaried the main features of their trends. Referring to our previous works for more details on the topics [1,4,8], we explicit the two basic equations for axiological *Height* "*Hn*"(1) and *Strength* "*Sn*"(2), considering a *neutral* policy as to parameters α and β (i.e. $\alpha=\beta=1$), as shown in the final passage of (2) and in (3).

$$H_n = e^{-\infty(T_n-1)} \qquad (1)$$

$$S_n = e^{-\beta(T_{max}-T_n)} = e^{-\Delta T_n} \qquad (2)$$

$$H_n = e^{-\infty(T_n-1)} = e^{1-T_n} \qquad (3)$$

Legenda:
α, β: parameters for the slope of the axiological curves;
n: units of Legal Mass;
T: legal logical Time;
T_n: the level of T associated with each n-unit of Mass;
H_n: axiological Height associated with n;
S_n: axiological Strength associated with n;
T_{max}: peripheral level of T for a given String;
$\Delta T_n = (T_{max}T_n)$
e: *Neperus* mathematical constant.

The legal *subject* is the center of any human *interest* towards the *objects* existing in (material and/or spiritual) reality [13]. The legal *system* shapes and admits certain *paths* to gain an *entitlement* over each eligible *object of claim*. The path-shaping is a matter of legal *Logos* [14].The intensity of the *claim* decreases the more this path is (logically) long. Fig.4 below depicts this trend of the axiological curve for a *Claim*.
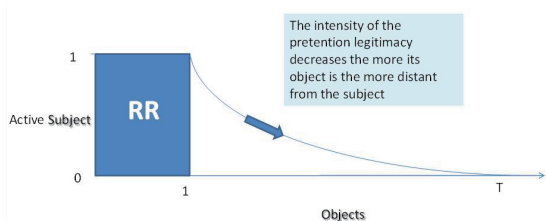

Figure 4: Pretention curve: *realis proportio*

As the legal *phenomenon* is inter-individual [17], the *claim* has to be acknowledged by another *subject*: the latter can be the *status quo* owner (*individual* feedback) or an impersonal agent set up by the system to make the process of creation of legal values more efficient (*social* feedback) [8, 9]. Fig 5 shows the trend of *acknowledgement* elementary axiological curve [7]:
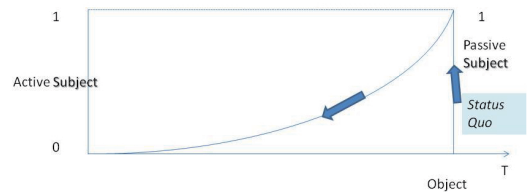

Figure 5: Acknowledgement curve: *personalis proportio*

Putting together a *claim* and its *acknowledgment*, a legal *entitlement* is vested by the legal *system* [7, 8]: a legal *value* is associated with such *entitlement*, determining its capacity to *conflict* with other entitlements, within a *due process of axiology* for the *adjudication* of the *object of conflict* [1, 7, 8]. We have addressed this basic union of *claim* and *acknowledgment* as legal *position*, i.e. a legal *relation* of *first order* [6, 7, 8], as shown in Fig.6 below.
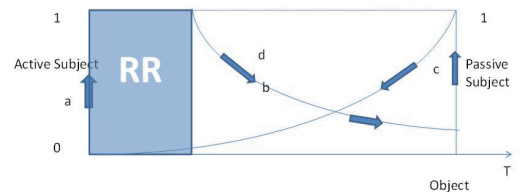

Figure 6: Legal relation of I Order: legal Position

In a regime of *social* feedback, each legal *entitlement* is axiologically bi-dimensional [11], and an axiological *conflict* can be represented as follow in Fig.7.
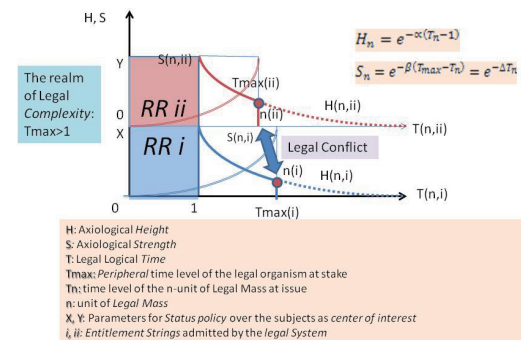

H: Axiological *Height*
S: Axiological *Strength*
T: Legal Logical *Time*
Tmax: *Peripheral* time level of the legal organism at stake
Tn: time level of the n-unit of Legal Mass at issue
n: unit of *Legal Mass*
X, Y: Parameters for *Status policy* over the subjects as *center of interest*
i, ii: *Entitlement Strings* admitted by the *legal System*

Figure 7: Legal Complexity, Robinsons Recht (**RR**) **and** Social **Feedback**

5 Problem Solution

We will solve our problem using the following operative *protocol* for each *Claim* and each Legal *Rule* at stake:

- Description of the *Fact* of the Rule;

- Distinguishing of Principal and Ancillary *Paths* within the Fact;

- *Ordering* of the factors of the Fact, with the emergence of Gap-Mass.

- Description of the *Effect* of the Rule;

- Distinguishing of Principal and Ancillary *Paths* within the Effect;

- *Ordering* of the factors of the Effect, with the Emergence of Gap-Mass;

- *Synthesis* of the *Rule*, assembling Fact and Effect via a Legal *Causality* Link;

- *Ordering* of the Factors of the Legal Rule;

- *Calculation* of H_n, S_n and W_n for the Legal *Rule*.

After having classified and measured all the legal *Rules* at stake, we will compare the axiological *impact* of each *regulatory* option, comparing the *status quo ante* and the *ex post* situation once the *regulatory option* is applied. We will then be able to inquire deeper on the axiological *impact* of each option, the truly nature of each *policy option* and to compare them so as to assess their *equalitarian* and *efficiency* axiological *quality*.

5.1 Calculation of the Axiological Potentials for the Claims at Issue

We begin by representing the *claim* named as *Jus prospectum tollendi*, which is an expression of landlords estate property right (*jure dominii*). Its logical framework is shown below in Fig.8.
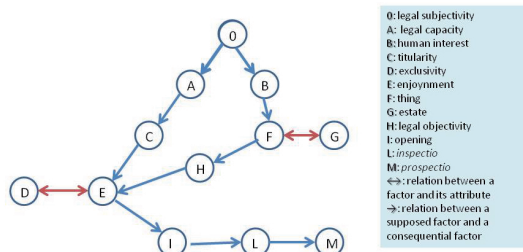
Figure 8: *Claim* to open a window *jure dominii*

We have already defined the *Principal Path* as a legal *position* leading from the *claiming subject* to the *object of claim*, characterized by the greatest level of *complexity* within the *organism* at stake [1, 4, 5, 6,

8]. We thus reinterpret the concept of complexity in a segment of social sciences, the legal one: the concept of *social complexity* is currently under revision [22]. The other *positions/paths* are less complex than the principal one. This distinction is material because it allows to objectively establish the emergence of *Gap-Mass* whenever a *factor* is geminated within the various *paths* of the *organism* and it would assume different Time levels in the several paths. For more details on *Principal/Ancillary* paths and on *Conceptual/Gap Mass*, we must refer here our above cited previous contributions. Here it is just necessary to refer that *Gap-Mass* is a *mass* unit geminated inside the paths of an *organism* at different *time* levels, thus losing its effective association with conceptual informational content, though keeping its informational capacity (for ex. as to its axiological *History*): it is the matrix of indetermination, uncertainty, and subsequently of discretionary legal *power*.

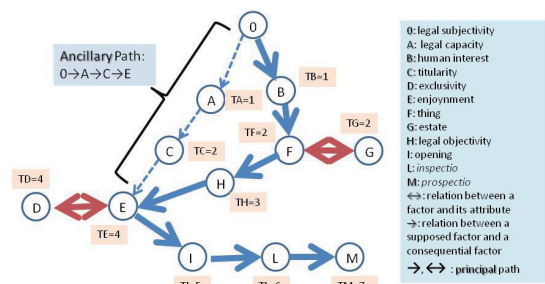Fig.9 below shows the logical *paths* within the legal *situation* at issue.

Figure 9: Paths in Fig.8 *Situation*

After having identified the *paths* within the *organism* at stake, we join them at an *organic* level of first order [6], building the aggregate legal *situation* under scrutiny: we then highlight the *Gap-mass* originated by this operation in next Fig.10.
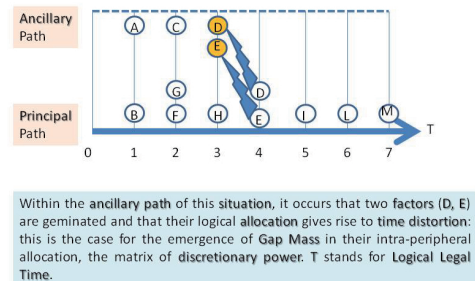

Figure 10: Order of Fig.8 *Situation*

Now, we can apply the two basic axiological *equations* of H_n and S_n, in order to measure the axiological contributions of each *factor*, and finally the

aggregate axiological *potentials* for the *situation* involved (W_n), discriminating the quotas of *Natural* Law (associated with *Conceptual* Mass: *Hnat*, *Snat* and *Wnat*) and the quotas of Positive Law (associated with *Gap* Mass: *Hpos*, *Spos* and *Wpos*). We thus reinterpret the well established and disputed notions of *Natural Law*

$$H_{Nat} = H_A + H_B + H_C + H_F + $$
$$+ H_G + H_H + H_E + H_D + H_I + $$
$$+ H_L + H_M = 1 + 1 + 3e^{-1} + e^{-2} + \quad (4)$$
$$+ 2e^{-3} + e^{-4} + e^{-5} + e^{-6} = 3.36606$$

$$H_{Pos} = H_{D*} + H_{E*} = 2e^{-2} = 0.73756 \quad (5)$$

$$S_{Nat} = S_A + S_B + S_C + S_F + S_G + $$
$$+ S_H + S_E + S_D + S_I + S_L + S_M = $$
$$2e^{-6} + 3e^{-5} + e^{-4} + 2e^{-3} + \quad (6)$$
$$+ e^{-2} + e^{-1} + 1 = 1.64626$$

$$S_{Pos} = S_{D*} + S_{E*} = 2e^{-4} = 0.03662 \quad (7)$$

$$W_{Nat} = H_{Nat} + S_{Nat} = $$
$$3.36606 + 1.64626 = 5.01232 \quad (8)$$

$$W_{Pos} = H_{Pos} + S_{Pos} = $$
$$0.73756 + 0.03662 = 0.77418 \quad (9)$$

Then we represent the *claim* named as *Jus excludendi alienam inspectionem*, which is an expression of the *right to privacy*.

As before, we begin by describing the logical *structure* of this legal *entity* in following Fig.11.
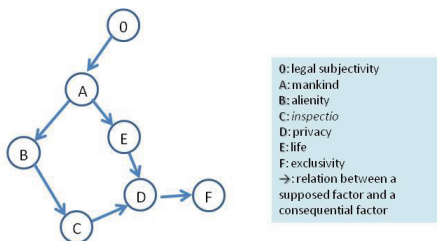

Figure 11: *Claim* to privacy right against the opening of a window

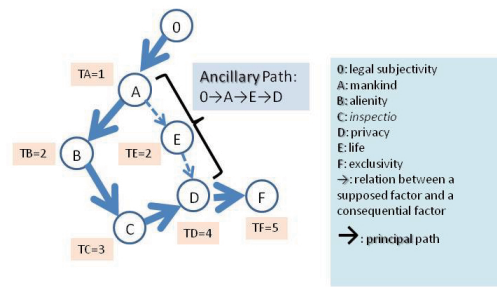Then we distinguish the *paths* within this entity in Fig.12 below.

Figure 12: Paths in Fig.11 *Situation*

Next we put the *paths* together, organically representing the whole *situation* at issue, underscoring the *Gap-Mass* created by doing so, in following Fig.13.
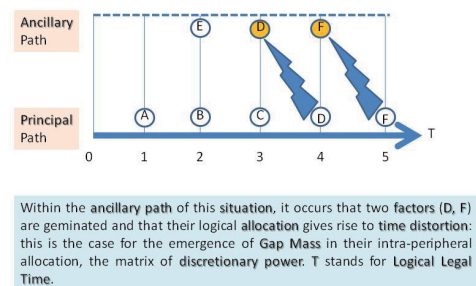

Figure 13: Order of Fig.11 *Situation*

Now, we can apply the two basic axiological *equations* of H_n and S_n, in order to measure the axiological contributions of each *factor*, and finally the aggregate axiological *potentials* for the *situation* involved, discriminating the quotas of *Natural Law* (associated with *Conceptual Mass*) and the quotas of *Positive Law* (associated with *Gap-Mass*).

$$H_{Nat} = H_A + H_B + H_E + H_C + $$
$$+ H_D + H_F = 1 + 2e^{-1} + e^{-2} + \quad (10)$$
$$+ e^{-3} + e^{-4} = 1.93918$$

$$H_{Pos} = H_{D*} + H_{E*} = e^{-2} + e^{-3} = 0.18511 \quad (11)$$

$$S_{Nat} = S_A + S_B + S_E + S_C + S_D + S_L = $$
$$e^{-4} + 2e^{-3} + e^{-2} + e^{-1} + 1 = 1.62108 \quad (12)$$

$$S_{Pos} = S_{D*} + S_{E*} = e^{-2} + e^{-1} = 0.50321 \quad (13)$$

$$W_{Nat} = H_{Nat} + S_{Nat} = $$
$$1.93918 + 1.62108 = 3.56026 \quad (14)$$

$$W_{Pos} = H_{Pos} + S_{Pos} = $$
$$0.18511 + 0.50321 = 0.68832 \quad (15)$$

5.2 Calculation of the Axiological Potentials for the Regulatory Options at Stake

Next we will calculate axiological quotas for the two *regulatory options* under consideration.

5.2.1   Calculation for Option #1 (*Preemption*)

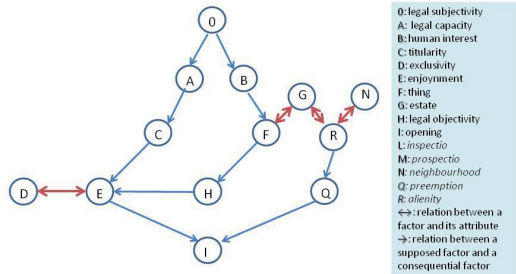The *Fact* in Option #1 is below represented in Fig.14.

Figure 14: Option 1 (*Preemption*). Fact

Within this logical *structure*, the following *paths* can be identified, as shown in Fig.15.
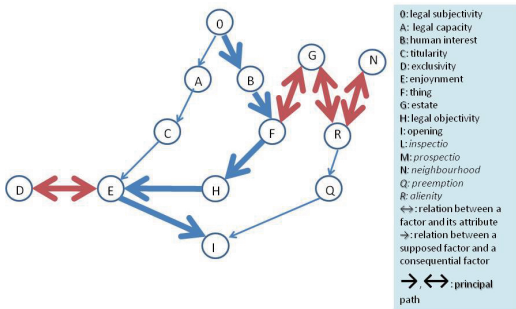

Figure 15: Option 1 (*Preemption*). Fact:Paths

This allows for the *ordering* of all the *factors* of the legal *entity* at stake, with clear identification of *Gap-Mass*, as depicted in Fig.16.
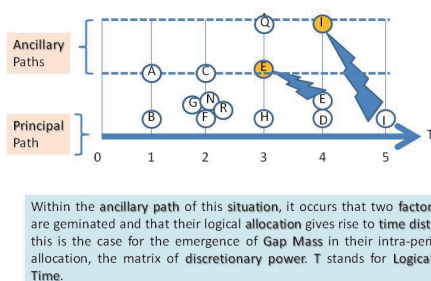

Figure 16: Option 1 (*Preemption*). Fact:Order

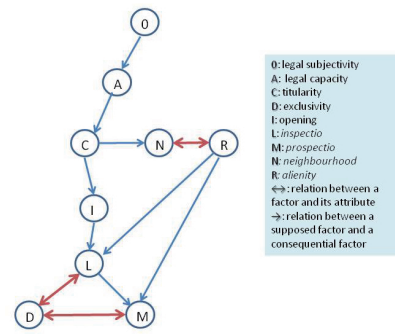The *Effect* in Option #1 is below represented in Fig.17.

Figure 17: Option 1 (*Preemption*). Effect

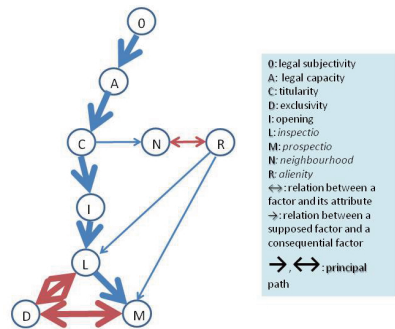Within this logical *structure*, the following *paths* can be identified, as featured in Fig.18.

Figure 18: Option 1 (*Preemption*). Effect: Paths

This allows for the *ordering* of all the *factors* of the legal *entity* at stake, with clear identification of *Gap-Mass*, as represented in Fig.19.
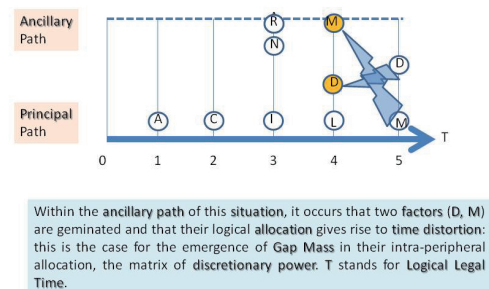

Figure 19: Option 1 (*Preemption*). Effect: Order

Then we put together the above depicted factual and effectual *situations*, to build up a legal *organism* of second order [6, 8] encompassing our regulatory Option #1.

Firstly, we describe the overall *structure* of the *rule* thus assembled, in Fig.20 below.
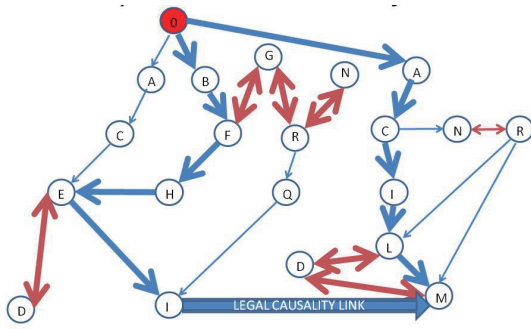
Figure 20: Option 1 (*Preemption*). *Synthesis* of the *Rule of Law*

Secondly, we order all the legal *mass* within this *organism*, identifying all the emerging *Gap-Mass*, as illustrated in Fig.21.

After the **synthesis** of the legal **rule**, some more units of **Gap Mass** emerge. A same **concept** can be geminated in more than one **Mass** unit only in different **Time levels**. For any given **Time level**, each **concept** can be contained in no more than one **Unit of Mass**.
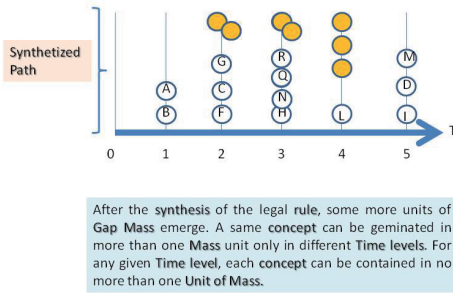
Figure 21: Option 1 (*Preemption*). Order

Finally, we apply the elementary axiological *functions* (H_n and S_n) to calculate the *potential* ($W_n = H_n + S_n$) generated by this *rule*.

$$H_{Nat} = H_A + H_B + H_C + H_F + H_G +$$
$$+ H_L + H_M + H_D = 1 + 1 + 3e^{-1} + \qquad (16)$$
$$+ 4e^{-2} + e^{-3} + 3e^{-4} = 3.74967$$

$$H_{Pos} = 2e^{-1} + 2e^{-2} + 3e^{-3} = 1.15576 \qquad (17)$$

$$S_{Nat} = S_A + S_B + S_C + S_F + S_G +$$
$$+ S_L + S_M + S_D = 2e^{-4} + 3e^{-3} + \qquad (18)$$
$$+ 4e^{-2} + e^{-1} + 3 = 3.60797$$

$$S_{Pos} = 2e^{-3} + 2e^{-2} + 3e^{-1} = 1.47386 \qquad (19)$$

$$W_{Nat} = H_{Nat} + S_{Nat} =$$
$$3.74967 + 3.60797 = 7.35764 \qquad (20)$$

$$W_{Pos} = H_{Pos} + S_{Pos} =$$
$$1.15576 + 1.47386 = 2.62962 \qquad (21)$$

5.2.2 Calculation for Option #2 (Distance)

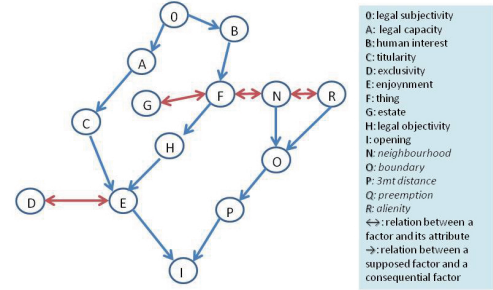The *Fact* in Option #2 is below represented in Fig.22.

Figure 22: Option 2 (*Distance*). Fact

Within this logical *structure*, the following *paths* can be identified, as shown in Fig.23.
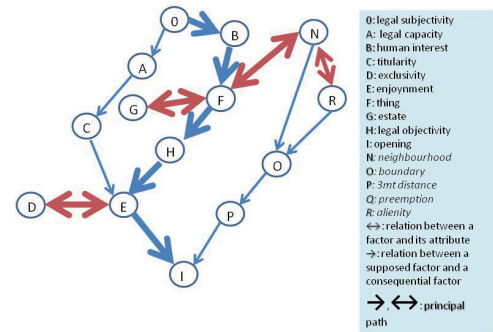

Figure 23: Option 2 (*Distance*). Fact: Paths

This allows for the ordering of all the factors of the legal entity at stake, with clear identification of Gap Mass.
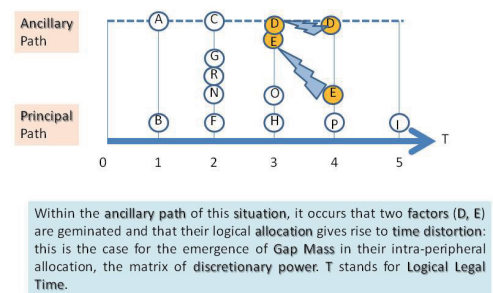

Within the **ancillary path** of this **situation**, it occurs that two **factors** (D, E) are geminated and that their logical **allocation** gives rise to **time distortion**: this is the case for the emergence of **Gap Mass** in their intra-peripheral allocation, the matrix of **discretionary power**. T stands for **Logical Legal Time**.

Figure 24: Option 2 (*Distance*). Fact: Order

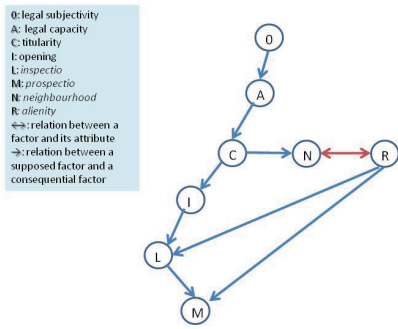The *Effect* in Option #1 is below represented in Fig.25.

Figure 25: Option 2 (*Distance*). Effect

Within this logical *structure*, the following *paths* can be identified as depicted in Fig.26.
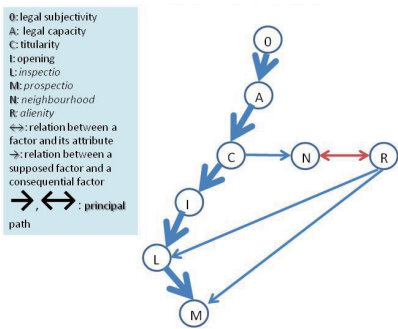

Figure 26: Option 2 (*Distance*). Effect: Paths

This allows for the ordering of all the *factors* of the legal *entity* at stake, with clear identification of *Gap-Mass*, as shown in Fig.27 below.

Within the **ancillary path** of this situation, it occurs that one factor (D, E) isgeminated and that its logical **allocation** gives rise to **time distortion**: this is the case for the emergence of **Gap Mass** in their intra-peripheral allocation, the matrix of **discretionary power**. T stands for **Logical Legal Time**.
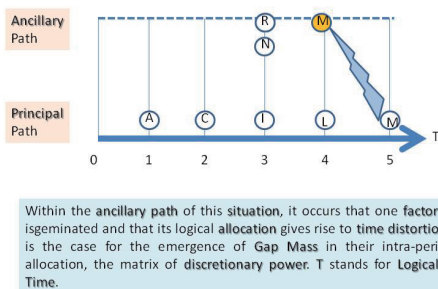
Figure 27: Option 2 (*Distance*). Effect: Order

Then we put together the above depicted factual and effectual *situations*, to build up a legal *organism* of second order [6, 8] encompassing our regulatory Option #1.

Firstly, we describe the overall *structure* of the *rule* thus assembled, what is illustrated in Fig.28.
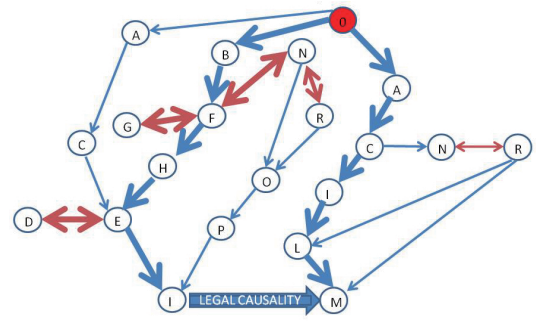

Figure 28: Option 2 (*Distance*). Rule of Law: Synthesis

Secondly, we order all the legal *mass* within this *organism*, identifying all the emerging *Gap-Mass* in Fig.29.
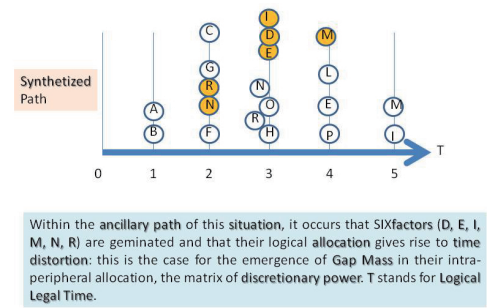

Within the **ancillary path** of this situation, it occurs that SIXfactors (D, E, I, M, N, R) are geminated and that their logical **allocation** gives rise to **time distortion**: this is the case for the emergence of **Gap Mass** in their intra-peripheral allocation, the matrix of **discretionary power**. T stands for **Logical Legal Time**.

Figure 29: Option 2 (*Distance*). Rule of Law: Order

Finally, we apply the elementary axiological *functions* (H_n and S_n) to calculate the *potential* ($W_n = H_n + S_n$) generated by this rule.

$$H_{Nat} = H_A + H_B + H_C + H_F + H_H+$$
$$+H_N + H_O + H_R + H_L + H_E + H_P+$$
$$+H_I + H_M = 1 + 1 + 2e^{-1}+$$
$$+4e^{-2} + 3e^{-3} + 2e^{-4} = 3.51462 \tag{22}$$

$$H_{Pos} = 2e^{-1} + 3e^{-2} + e^{-3} = 1.19153 \tag{23}$$

$$S_{Nat} = S_A + S_B + S_C + S_F + S_H+$$
$$+S_N + S_O + S_R + S_L + S_E + S_P+$$
$$+S_I + S_M = 2e^{-4} + 2e^{-3}+$$
$$+4e^{-2} + 3e^{-1} + 2 = 4.14902 \tag{24}$$

$$S_{Pos} = 2e^{-3} + 3e^{-2} + e^{-1} = 0.87343 \tag{25}$$

$$W_{Nat} = H_{Nat} + S_{Nat} =$$
$$3.51462 + 4.14902 = 7.66364 \tag{26}$$

$$W_{Pos} = H_{Pos} + S_{Pos} =$$
$$1.19153 + 0.87343 = 2.06496 \tag{27}$$

5.3 Calculation of the Axiological *Impact* for the Policy *Options* under Scrutiny

At this point, we will calculate the axiological *impact* of each regulatory *option*, comparing axiological *status quo ante* balance with *ex post* axiological balance.

5.3.1 Option #1 (*Preemption*) axiological Impact

In *status quo ante* situation, the axiological *potential* associated with the *claim* to open a window gives rise to a RR *adjudication*: this axiological area is equally shared (in a *socialist* regime) or divided (in a *liberal* regime) by the two neighbors (named neighbor I and neighbor II). Moreover, each of the neighbor enjoys of the *claim* to *privacy*.

Therefore, the axiological aggregate *status quo ante* balance is the following:

$$W_{Tot} = W_I + W_{II} =$$
$$(W_{NAT}^I + W_{POS}^I) + (W_{NAT}^{II} + W_{POS}^{II}) =$$
$$(5.01232 + 0.77418) + 2(3.56026 + 0.68832) \tag{28}$$

$$W_{Tot}^{NAT} = W_I^{NAT} + W_{II}^{NAT} =$$
$$5.01232 + 2(3.56026) = 12.13284 \tag{29}$$

$$W_{Tot}^{POS} = W_I^{POS} + W_{II}^{POS} =$$
$$0.77418 + 2(0.68832) = 2.15082 \tag{30}$$

After Option #1 (*Preemption*) has been applied, the *situation* of the two neighbors is really different. If neighbor I is the one who builds the wall in preemption, his legal sphere will contain *Jus ne prospectui officiatur* and *Jus excludendi alienam inspectionem*. Instead, neighbor II will have lost both *Jura*. The *ex post* Option I *aggregate* axiological asset will coincide with Neighbor I axiological asset.

$$W_{Tot} = W_I = (W_{NAT}^I + W_{POS}^I) =$$
$$(7.35764 + 2.62962) + (3.56026 + 0.68832) =$$
$$10.9179 + 3.31794 \tag{31}$$

$$W_{TOT}^{NAT} = W_I^{NAT} =$$
$$7.35764 + 3.56026 = 10.9179 \tag{32}$$

$$W_{TOT}^{POS} = W_I^{POS} =$$
$$2.62962 + 0.68832 = 3.31794 \tag{33}$$

To sum up, in axiological *aggregate* terms, the transition from *ex ante* Option #1 to *ex post* Option #1 is characterized by a decrease of *Natural* Law *potential* (from 12.13284 to 10.9179) and by an increase in *Positive* Law *potential* (from 2.15082 to 3.31794).

5.3.2 Option #2 (*Distance*) axiological Impact

The *status quo ante* situation is the same as the one depicted in previous 5.3.1 for ex ante Option #1 *status quo*.

In *ex post* Option #2 *status*, each neighbor enjoys *Jus prospectus*, whilst they have no more *Jus excludendi*. The *situation* of the two neighbors is symmetrical both *ex ante* and *ex post*.

$$W_{TOT}^{POS} = W_I^{POS} + W_{II}^{POS} =$$
$$2(2.06496) = 4.12992 \tag{34}$$

$$W_{TOT}^{NAT} = W_I^{NAT} + W_{II}^{NAT} =$$
$$2(7.66364) = 15.32728 \tag{35}$$

To sum up, in axiological *aggregate* terms, the transition from *ex ante* Option #2 to *ex post* Option #2 is characterized by an increase both in *Natural* Law potential (from 12.13284 to 15.32728) and in *Positive* Law potential (from 2.15082 to 4.12992).

5.4 Cluster *Functional* Analysis for the Devised Regulatory Options.

We have recently introduced in our model a segment for cluster analysis of legal structures [12], in order to investigate *functional* multi-level framework of legal *entities*. The basic idea of this *segment* is that each legal *organism* has a proper *overall function* [6, 8] but even some of its *components* (the *clusters* of *factors*) own a *specific function*, within the whole *organism* at issue.

We have identified in a previous paper four *clusters*, which seem to be the most typical of very legal *entity*:

- A cluster for the qualification of legal *subject*;

- A cluster for the qualification of legal *object*;

- A cluster for the *rise* of a legal *conflict*;

- A cluster for the *resolution* of a legal *conflict*.

In the two following Fig.30 and Fig.31, we will signal the *clusters* for the *rise* and the *resolution* of the legal *conflict* within the *organisms* representing the two *regulatory options* already analyzed (option #1: *preemption*; option #2: *distance*).
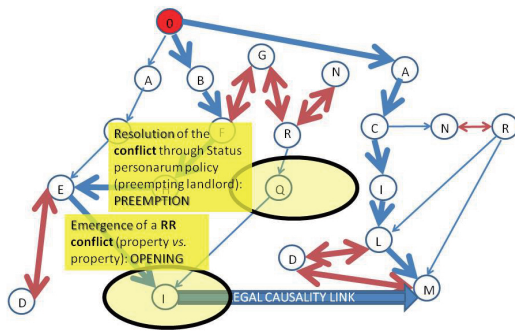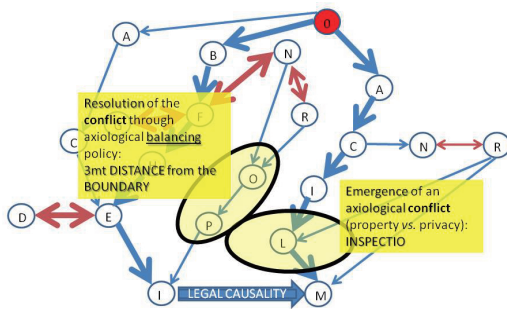
Figure 30: Cluster analysis: Option 1

Figure 31: Cluster analysis: Option 2

6 Conclusions

The end of the implementation of our *model* consists of useful and practical suggestions about how to build effective *rules*, in an *axiological* sense, and eventually to objectively *challenge* ineffective *rules*, by means of a series of fairly transparent tests of *reasonableness*, a notion with an established tradition in legal doctrine [20].

Such implementation might interact with models founded upon either systems theory [24] or economic utility functions [25] and growth [26, 27, 28, 29].

Given the above results of our case-study, we will draw our final conclusions on the *policy* and *regulatory options* under scrutiny.

6.1 Unveiling the Truly Legal Nature of Regulations

Regulatory Option #1 is a measure of *status personarum* policy, non-situational in itself. It solves a RR *conflict* between two symmetrical *claims* (*jura prospectus*) deriving from the same legal *position* (*land property right*). It discriminates the *status* of the first builder against the *status* of the second builder. It does not convey any *information* about axiological *balancing* of the legal *positions* at stake.

Instead, regulatory option #2 is a measure expressing a truly axiological *balancing* policy among

different legal *positions* (*jus prospectus; jus excludendi*).

6.2 Axiological *Balance* of the Regulations at Stake

After having identified the truly nature of the *regulatory* options under scrutiny, we will assess their effectiveness under the lenses of two axiological *criteria*: axiological *equality*; axiological *efficiency*. We have outlined the fundamental *trade-off* among the two axiological *criteria* thereof [1, 8].

6.2.1 Axiological *Balance* of the Regulations at Stake

Under an equalitarian point of view, regulatory option #1 seems to be unreasonable, for it enshrines a *status personarum* policy determining the unilateral *adjudication* of the entire axiological *potential* at stake in favor of the one neighbor who builds the wall first. Perhaps such a choice might be motivated by the public interest in fostering urban edification on the territory, but in any case it appears too much unbalanced discriminating excessively in a non-situational way the *status* of the preempting neighbor from the other counterparty.

Under a strictly efficiency axiological point of view, regulatory option #1 disrupts *Natural* Law potential in comparison with *status quo ante*. This decrease might only be compensated by the charge of *Positive* Law quotas in ex ante situation. We hope that in some future contribution we will able to investigate the *limit* of lawfully *feasibility* of such an operation of axiological *compensation*. By now, it appears somehow odd the attempt to justify a decrease in *Natural* (intangible) law potential by a mere movement over *Positive* (discretionary) law quotas of legal *value*.

Fig.32 below shows the axiological *impact* cipher deriving from *regulatory* option #1 (*preemption*).
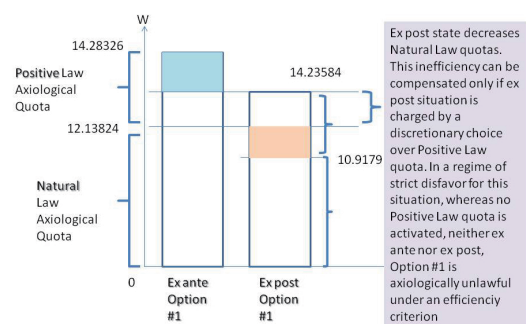

Figure 32: Axiological Impact (Option # 1)

6.2.2 Distance Regulation: axiological equality and efficiency

Under an axiological *equalitarian* viewpoint, *regulatory* option #2 is certainly *lawful* and *effective*, for both *ex ante* and *ex post* situations establish a fifty/fifty *adjudication* of the axiological assets between the two neighbors involved by our case-study.

Moreover, as far as axiological *efficiency* is concerned, *regulatory* option #2 is *efficient*, for its *ex post* situation increments both *natural* law and *positive* law quotas of *value*.

Following Fig.33 shows the axiological *impact* deriving from *regulatory* option #2.
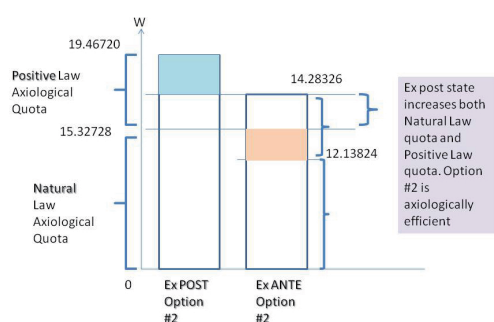

Figure 33: Axiological Impact (Option # 2)

It seems to depend upon *constitutional* arrangements into force within the legal *system* at issue, whether the *axiological* judgment of *effectiveness* and *lawfulness* of the *regulatory* options under scrutiny must be construed:

- In a *narrow* sense, as the comparison between *ex ante* and *ex post* states for a regulatory option. The option is not unlawful if it does not hinder axiological *equality* and *efficiency*. The simple respect of both axiological *criteria* for a single option makes it viable. This kind of axiological test might be suitable for *constitutional* rules, *statutory* laws and by-laws;

- In a *large* sense, as a compared test among the options at stake, not being sufficient that the devised option be better than *status quo ante*, instead being required that the devised option be the best one, under both axiological criteria, out of all the options under consideration. This *species* of axiological test would seem to match with *judge-made* law and *administrative* acts.

7 Acknowledgement

We hope that the scientific community will test the viability of our model, and eventually validate it or suggest effective ways of improving it, in order to ultimately discuss the setting up of a newly conceived discipline, "*Law&Mathematics"(Mathematical analysis of law)*.

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