Multiple periodic solutions for a general class of delayed cooperative systems on time scales

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Abstract: In this paper, we consider a general class of delayed nonautonomous logistic Lotka-Volterra type multi-species cooperative system with harvesting terms on time scales. The model invovles the intraspecific cooperative terms defined by functions which depend on population densities. An existence theorem of at least 2^n periodic solutions is established by using the coincidence degree theory. An example is given to illustrate the effectiveness of our result.

Key–Words: Time scales; Periodic solutions; Delayed cooperative system; Coincidence degree; Harvesting term.

1 Introduction

In the past decades, the applications of functional differential equations in ecosystem have developed rapidly. Various mathematical models with delays have been proposed in study of population dynamics (for example, see[1-15]). Owing to its theoretical and practical significance, Lotka-Volterra system have been studied extensively [16-33].

It is well known that the focus in theoretical models of population and community dynamics must be not only on how populations dependent on their own population densities or the population densities of other organisms, but also on how population change in response to the physical environment. To consider periodic environment factors (e.g, seasonal effects of weather, food supplies, mating habits, etc), it is reasonable to study Lotka-Volterra systems with periodic coefficients.

A very basic and important problem in the study of a population growth model with a periodic environment is the global existence and stability of a positive periodic solution, which plays a similar role as a globally stable equilibrium does in an autonomous model. Recently, there have been many nice works on the existence problem of positive periodic solutions of Lotka-Volterra systems with periodic coefficients [9-14,16-23]. Since the exploitation of biological resources and the harvest of population species are commonly practiced in fishery, forestry and wildlife management, the study of population dynamics with harvesting is an important subject in mathematical bioeconomics, which is related to the optimal management of renewable resources (see [34-36]). This motivates us to consider the following nonautonomous delay n species cooperative system with harvesting terms:

$$\begin{aligned} \dot{x_i}(t) &= x_i(t)\cdot \\ &\left(a_i(t) - \frac{f_i(t, x_i(t-\tau_{ii}(t)))}{K_i} x_i(t-\tau_{ii}(t)) \right. \\ &\left. + \sum_{j=1, j\neq i}^{n} c_{ij}(t) x_j(t-\tau_{ij}(t))\right) - h_i(t), \end{aligned} \tag{1}$$

where $x_i(t)(i = 1, 2, \ldots, n)$ stands for the ith species population density. $a_i(t) > 0, f_i(t, x_i) > 0, c_{ij}(t) > 0$ and $h_i(t) > 0$ are the natural reproduction rate, the intraspecific competition, the interspecific cooperation and the harvesting term, respectively, $K_i > 0$ is the carrying capacity which is a constant, $\tau_{ij}(t) \geq 0$ is the time-lag in the process of cooperation between ith and jth species, $a_i(t), c_{ij}(t), \tau_{ij}(t)$ and $h_i(t)(i, j = 1, 2, \ldots, n)$ are all continuous ω-periodic functions and $f_i(t, x) \in C(\mathbb{R}^2, \mathbb{R}^+)$ and $f_i(t, x) = f_i(t+\omega, x)$, for all $t \in \mathbb{R}$ and $x \in \mathbb{R}$.

If the estimates of the population size and all coefficients in (1) are made at equally spaced time intervals, then we can incorporate this aspect in (1) and obtain the following discrete analogue of system (1):

$$\begin{aligned} &\tfrac{x_i(k+1)}{x_i(k)} = \\ &\exp\left\{a_i(k) - \tfrac{f_i(k, x_i(k-\tau_{ii}(k)))}{K_i} x_i(k-\tau_{ii}(k)) \right. \\ &\left. + \sum_{j=1, j\neq i}^{n} c_{ij}(k) x_j(k-\tau_{ij}(k)) - \tfrac{h_i(k)}{x_i(k)}\right\}, \end{aligned} \tag{2}$$

where for $i,\ j = 1, 2, \dots, n$, $a_i : \mathbb{Z} \to \mathbb{R}^+, c_{ij} : \mathbb{Z} \to \mathbb{R}^+, \tau_{ij} : \mathbb{Z} \to \mathbb{Z}^+, h_i : \mathbb{Z} \to \mathbb{R}^+, f_i : \mathbb{Z} \times \mathbb{R} \to \mathbb{R}^+$ are all ω-periodic, that is, $a_i(k+\omega) = a_i(k)$, $c_{ij}(k+\omega) = c_{ij}(k), \tau_{ij}(k+\omega) = \tau_{ij}(k), h_i(k+\omega) = h_i(k),\ f_i(k+\omega, x) = f_i(k, x)$, for any $\mathbb{Z}$ (the set of all integers), ω is a fixed positive integer. However, dynamics in each equally spaced time interval may vary continuously. So, it may be more realistic to assume that the population dynamics involves the hybrid discrete-continuous processes. For example, Gamarra and Solé pointed out that such hybrid processes appear in the population dynamics of certain species that feature nonoverlapping generations: the change in population from one generation to the next is discrete and so is modeled by a difference equation, while within-generation dynamics vary continuously (due to mortality rates, resource consumption, predation, interaction, etc.) and thus are described by a differential equation [37]. The theory of calculus on time scales (see [38-39] and references cited therein) was initiated by Hilger in his Ph.D. thesis in 1988 [40] in order to unify continuous and discrete analysis, and it has become an effective approach to the study of mathematical models involving the hybrid discrete-continuous processes. This motivates us to unify systems (1) and (2) to the system on time scales $\mathbb{T}$ as follows:

$$\begin{aligned} u_i^{\Delta}(t) &= a_i(t) - \frac{f_i(t, e^{u_i(t-\tau_{ii}(t))})}{K_i} e^{u_i(t-\tau_{ii}(t))} \\ &\quad + \sum_{j=1, j\neq i}^{n} c_{ij}(t) e^{u_j(t-\tau_{ij}(t))} - h_i(t) e^{-u_i(t)}, \end{aligned} \tag{3}$$

where $a_i(t) > 0, c_{ij}(t) > 0, \tau_{ij}(t) \geq 0$ and $h_i(t) > 0 (i, j = 1, 2, \dots, n)$ are all rd-continuous ω-periodic functions and $f_i(t, x) \in C_{rd}(\mathbb{T} \times \mathbb{R}, \mathbb{R}^+)$ and $f_i(t, u) = f_i(t + \omega, u)$, for all $t \in \mathbb{T}$ and $u \in \mathbb{R}$.

In (3), let $x_i(t) = e^{u_i(t)}, i = 1, 2, \dots, n$. If $\mathbb{T} = \mathbb{R}$ (the set of all real numbers), then (3) reduces to (1). If $\mathbb{T} = \mathbb{Z}$ (the set of all integers), then (3) reduces to (2).

To the best of our knowledge, no work has been done for the existence of periodic solutions of system (3) yet. Our main purpose of this paper is to establish the existence of at least 2^n positive periodic solutions for system (3) by using Mawhin's continuation theorem of coincidence degree theory [41]. For the work concerning the multiple existence of periodic solutions of periodic population models which was done using coincidence degree theory, we refer to [42-52].

The remain of this paper is organized as follows. In Section 2, some notations and basic theorem or lemmas on time scales are given. In Section 3, the main results of the existence of multiple periodic solutions of system (3) is obtained. In Section 4, one example is given to illustrate the effectiveness of our results. Finally, some brief conclusions are presented in Section 5.

2 Preliminaries on Time Scales

In this section, we briefly recall some basic definitions and lemmas on time scales which are used in what follows. For more details, one can see [38-40].

Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+$ are defined, respectively, by $\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}$, $\rho(t) = \sup\{s \in \mathbb{T} : s < t\}$ and $\mu(t) = \sigma(t) - t$.

A point $t \in \mathbb{T}$ is called left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T} \setminus \{m\}$; otherwise $\mathbb{T}^k = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m, then $\mathbb{T}_k = \mathbb{T} \setminus \{m\}$; otherwise $\mathbb{T}^k = \mathbb{T}$.

Let $\omega > 0$. Throughout this paper, the time scale T is assumed to be ω-periodic, that is, $t \in \mathbb{T}$ implies $t + \omega \in \mathbb{T}$. In particular, the time scale $\mathbb{T}$ under consideration is unbounded above and below.

Definition 1. *A function $f : \mathbb{T} \to \mathbb{R}$ is called $regulated$ provided its right-side limits exist (finite) at all right-side points in $\mathbb{T}$ and its left-side limits exist (finite) at all left-side points in $\mathbb{T}$.*

Definition 2. *A function $f : \mathbb{T} \to \mathbb{R}$ is called rd-continuous provided it is continuous at right-dense point in $\mathbb{T}$ and its left-side limits exist (finite) at left-dense points in $\mathbb{T}$. The set of rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C_{rd} = C_{rd}(\mathbb{T}) = C_{rd}(\mathbb{T}, \mathbb{R})$.*

Definition 3. *Assume $f : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^k$. Then we define $f^{\Delta}(t)$ to be to be the number (if it exists) with the property that given any $\varepsilon > 0$ there exists a neighborhood U of t (i.e., $U = (t - \delta, t + \delta) \cap \mathbb{T}$ for some $\delta > 0$) such that*

$$|[f(\sigma(t)) - f(s)] - f^{\Delta}(t)[\sigma(t) - s]| < \varepsilon|\sigma(t) - s|$$

for all $s \in U$. we call $f^{\Delta}(t)$ the delta (or Hilger) derivative of f at t. The set of functions $f : \mathbb{T} \to \mathbb{R}$ that are differentiable and whose derivative is $rd-$continuous is denoted by $C^1_{rd} = C^1_{rd}(\mathbb{T}) = C^1_{rd}(\mathbb{T}, \mathbb{R})$.

If f is continuous, then f is rd-continuous. If f is rd-continuous, the f is regulated. If f is delta differentiable at t, then f is continuous at t.

Lemma 4. *Let f be regulated, then there exists a function F which is delta differentiable with region of differentiation D such that*

$$F^{\Delta}(t) = f(t) \text{ for all } t \in D.$$

Definition 5. *Assume $f : \mathbb{T} \to \mathbb{R}$ is a regulated function. Any function F as in Lemma 2.1 is called a Δ-antiderivative of f. We define the indefinite integral of a regulated function f by*

$$\int f(t)\Delta t = F(t) + C,$$

Where C is an arbitrary constant and F is a Δ-antiderivative of f. We define the Cauchy integral by

$$\int_a^b f(s)\Delta s = F(b) - F(a) \text{ for all } a, b \in \mathbb{T}.$$

A function $F : \mathbb{T} \to \mathbb{R}$ is called an antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided

$$F^{\Delta}(t) = f(t) \text{ for all } t \in \mathbb{T}^k.$$

Lemma 6. *If $a, b \in \mathbb{T}$, $\alpha, \beta \in \mathbb{R}$ and $f, g \in C(\mathbb{T}, \mathbb{R})$, then*

(i) $\int_a^b [\alpha f(t) + \beta g(t)]\Delta t = \alpha \int_a^b f(t)\Delta t + \beta \int_a^b g(t)\Delta t$;

(ii) *if $f(t) \geq 0$ for all $a \leq t < b$, then $\int_a^b f(t)\Delta t \geq 0$;*

(iii) *if $|f(t)| \leq g(t)$ on $[a, b) := \{t \in \mathbb{T} : a \leq t < b\}$, then $|\int_a^b f(t)\Delta t| \leq \int_a^b g(t)\Delta t$.*

Lemma 7. [53] *Assume that $\{f_n\}_{n\in N}$ is a function on J such that*

(i) *$\{f_n\}_{n\in N}$ is uniformly bounded on J,*

(ii) *$\{f_n^{\Delta}\}_{n\in N}$ is uniformly bounded on J.*

Then there is a subsequence of $\{f_n\}_{n\in N}$ which converges uniformly on J.

3 Existence of at least 2^n periodic solutions

In this section, by using Mawhin's continuation theorem, we shall show the existence of multiple periodic solutions of (3). To do so, we need to make some preparations.

Let X and Z be real normed vector spaces. Let $L : \operatorname{Dom} L \subset X \to Z$ be a linear mapping and $N : X \times [0, 1] \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if dim Ker L = codim Im $L < \infty$ and Im L is closed in Z. If L is a Fredholm mapping of index zero, then there exists continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $\operatorname{Im} P = \operatorname{Ker} L$ and $\operatorname{Ker} Q = \operatorname{Im} L = \operatorname{Im}(I - Q)$, and $X = \operatorname{Ker} L \bigoplus \operatorname{Ker} P$, $Z = \operatorname{Im} L \bigoplus \operatorname{Im} Q$. It follows that $L|_{\operatorname{Dom} L \cap \operatorname{Ker} P} : (I - P)X \to \operatorname{Im} L$ is invertible and its inverse is denoted by K_P. If Ω is a bounded open subset of X, the mapping N is called L-compact on $\bar{\Omega} \times [0, 1]$, if $QN(\bar{\Omega} \times [0, 1])$ is bounded and $K_P(I - Q)N : \bar{\Omega} \times [0, 1] \to X$ is compact. Because Im Q is isomorphic to Ker L, there exists an isomorphism $J : \operatorname{Im} Q \to \operatorname{Ker} L$.

The Mawhin's continuous theorem [41, p.40] is given as follows:

Lemma 8. [41] *Let L be a Fredholm mapping of index zero and let N be L-compact on $\bar{\Omega} \times [0, 1]$. Assume*

(a) *for each $\lambda \in (0, 1)$, every solution x of $Lx = \lambda N(x, \lambda)$ is such that $x \notin \partial\Omega \cap \operatorname{Dom} L$;*

(b) *$QN(x, 0)x \neq 0$ for each $x \in \partial\Omega \cap \operatorname{Ker} L$;*

(c) $\deg(JQN(x, 0), \Omega \cap \operatorname{Ker} L, 0) \neq 0$.

Then $Lx = N(x, 1)$ has at least one solution in $\overline{\Omega} \cap \operatorname{Dom} L$.

For the sake of convenience, we denote by

$$\kappa = \min\{[0, +\infty) \cap \mathbb{T}\}, \quad I_\omega = [\kappa, \kappa + \omega] \cap \mathbb{T},$$

$$g^M = \sup_{t \in I_\omega} g(t), g^l = \inf_{t \in I_\omega} g(t),$$

$$\bar{g} = \frac{1}{\omega}\int_{I_\omega} g(s)\Delta s = \frac{1}{\omega}\int_\kappa^{\kappa+\omega} g(s)\Delta s,$$

where $g \in C_{rd}(\mathbb{T})$ is an ω-periodic real function, that is, $g(t + \omega) = g(t)$ for all $t \in \mathbb{T}$.

$$l_i^{\pm} = \frac{a_i^l K_i \pm \sqrt{(a_i^l K_i)^2 - 4b_i^M h_i^M K_i}}{2b_i^M},$$

$$\begin{aligned} L_i^{\pm} &= \pm\frac{\sqrt{\Big(K_i\big(a_i^M + \sum_{j=1,j\neq i}^n c_{ij}^M K_j\big)\Big)^2 - 4b_i^l h_i^l K_i}}{2b_i^l} \\ &\quad + \frac{K_i\big(a_i^M + \sum\limits_{j=1,j\neq i}^n c_{ij}^M K_j\big)}{2b_i^l}, \quad i = 1, 2, \ldots, n. \end{aligned}$$

Throughout this paper, we need the following assumptions:

(H_1) $f_i(t,x) \in C_{rd}(\mathbb{T}\times\mathbb{R},\mathbb{R}^+)$ is bounded and

$$\inf_{x\in\mathbb{R}}\min_{t\in I_\omega} f_i(t,x) = b_i^l$$

and

$$\sup_{x\in\mathbb{R}}\max_{t\in I_\omega} f_i(t,x) = b_i^M,$$

for all $i = 1,2,\ldots,n$.

(H_2) $a_i^l > 2\sqrt{\frac{b_i^M h_i^M}{K_i}}, i = 1,2,\ldots,n.$

Lemma 9. *Assume that* (H_1) *and* (H_2) *hold, then we have the following inequalities:*

$$L_i^- < l_i^- < l_i^+ < L_i^+, i = 1,2,\ldots,n.$$

Proof: In fact,

$$L_i^- < \frac{2h_i^M}{a_i^l K_i \pm \sqrt{(a_i^l K_i)^2 - 4b_i^M h_i^M K_i}} = l_i^- < l_i^+,$$

$$L_i^+ > \frac{a_i^l K_i + \sqrt{(a_i^l K_i)^2 - 4b_i^M h_i^M K_i}}{2b_i^M} = l_i^+.$$

□

Theorem 10. *Assume that* (H_1) *and* (H_2) *hold. Then system* (3) *has at least* 2^n *positive* ω*-periodic solutions.*

Proof: In order to apply Lemma 8 to (3), we first define

$$X = Z = \Big\{u = (u_1,\ldots,u_n)^T \in C_{rd}(\mathbb{T},R^n) : u_i(t+\omega) = u_i(t)\Big\}$$

and

$$\|u\| = \sum_{i=1}^n \max_{t\in I_\omega}|u_i(t)|, \quad u\in X \textit{ or } Z.$$

Equipped with the above norm $\|\cdot\|$, X and Z are Banach spaces. Let for all $u\in X$

$$N(u,\lambda) = \begin{pmatrix} N_1(u,\lambda)\\ \vdots \\ N_i(u,\lambda) \\ \vdots \\ N_n(u,\lambda)\end{pmatrix}_{n\times 1}, \quad Lu = \begin{pmatrix} u_1^\Delta \\ \vdots \\ u_i^\Delta \\ \vdots \\ u_n^\Delta\end{pmatrix}_{n\times 1},$$

$$Pu = \begin{pmatrix} \frac{1}{\omega}\int_\kappa^{\kappa+\omega} u_1(t)\Delta t \\ \vdots \\ \frac{1}{\omega}\int_\kappa^{\kappa+\omega} u_i(t)\Delta t \\ \vdots \\ \frac{1}{\omega}\int_\kappa^{\kappa+\omega} u_n(t)\Delta t\end{pmatrix}_{n\times 1}, \quad u\in X,$$

$$Qz = \begin{pmatrix} \frac{1}{\omega}\int_\kappa^{\kappa+\omega} z_1(t)\Delta t \\ \vdots \\ \frac{1}{\omega}\int_\kappa^{\kappa+\omega} z_i(t)\Delta t \\ \vdots \\ \frac{1}{\omega}\int_\kappa^{\kappa+\omega} z_n(t)\Delta t\end{pmatrix}_{n\times 1}, \quad z\in Z,$$

where

$$\begin{aligned} N_i(u,\lambda) &= a_i(t) - \frac{f_i(t, e^{u_i(t-\tau_{ii}(t))})}{K_i} e^{u_i(t-\tau_{ii}(t))} \\ &\quad +\lambda\sum_{j=1,j\neq i}^n c_{ij}(t)e^{u_j(t-\tau_{ij}(t))} \\ &\quad -h_i(t)e^{-u_i(t)}. \end{aligned}$$

Thus it follows that $\mathrm{Ker}L = \{u\in X : (u_1,u_2,\ldots,u_n)^T = (h_1,h_2,\ldots,h_n)^T \in \mathbb{R}^n, \forall t\in\mathbb{T}\} = \mathbb{R}^n$, $\mathrm{Im}L = \{z\in Z : \int_\kappa^{\kappa+\omega} z(t)\Delta t = 0\}$ is closed in Z, $\dim \mathrm{Ker}\ L = n = \mathrm{codim\ Im}\ L$, and P,Q are continuous projectors such that

$$\mathrm{Im}\ P = \mathrm{Ker}\ L, \ \ \mathrm{Ker}\ Q = \mathrm{Im}\ L = \mathrm{Im}\ (I-Q).$$

Hence, L is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L) K_P : $\mathrm{Im}\ L \to \mathrm{Ker}\ P\bigcap \mathrm{Dom}\ L$ is given by

$$K_P(z) = \int_\kappa^t z(s)\Delta s - \frac{1}{\omega}\int_\kappa^{\kappa+\omega}\int_\kappa^\theta z(s)\Delta s\Delta\theta.$$

Then

$$QN(u,\lambda) = \begin{pmatrix} \frac{1}{\omega}\int_\kappa^{\kappa+\omega} F_1(s,\lambda)\Delta s \\ \vdots \\ \frac{1}{\omega}\int_\kappa^{\kappa+\omega} F_i(s,\lambda)\Delta s \\ \vdots \\ \frac{1}{\omega}\int_\kappa^{\kappa+\omega} F_n(s,\lambda)\Delta s\end{pmatrix}_{n\times 1}$$

and

$$
\begin{aligned}
& K_p(I-Q)N(u,\lambda) \\
= & \begin{pmatrix}
\int_\kappa^t F_1(s,\lambda)\Delta s - \frac{1}{\omega}\int_\kappa^{\kappa+\omega}\int_\kappa^\theta F_1(s,\lambda)\Delta s\Delta\theta \\
+(\frac{1}{2}-\frac{t}{\omega})\int_\kappa^{\kappa+\omega} F_1(s,\lambda)\Delta s \\
\vdots \\
\int_\kappa^t F_i(s,\lambda)\Delta s - \frac{1}{\omega}\int_\kappa^{\kappa+\omega}\int_\kappa^\theta F_i(s,\lambda)\Delta s\Delta\theta \\
+(\frac{1}{2}-\frac{t}{\omega})\int_\kappa^{\kappa+\omega} F_i(s,\lambda)\Delta s \\
\vdots \\
\int_\kappa^t F_n(s,\lambda)\Delta s - \frac{1}{\omega}\int_\kappa^{\kappa+\omega}\int_\kappa^\theta F_n(s,\lambda)\Delta s\Delta\theta \\
+(\frac{1}{2}-\frac{t}{\omega})\int_\kappa^{\kappa+\omega} F_n(s,\lambda)\Delta s
\end{pmatrix}
\end{aligned}
$$

where

$$
\begin{aligned}
& F_i(s,\lambda) \\
= & a_i(s) - \frac{f_i(s,u_i(s,e^{u_i(s-\tau_{ii}(s))}))}{K_i}e^{u_i(s-\tau_{ii}(s))} \\
& +\lambda\sum_{j=1,j\neq i}^{n} c_{ij}(s)e^{u_j(s-\tau_{ij}(s))} - h_i(s)e^{-u_i(s)}.
\end{aligned}
$$

Obviously, QN and $K_P(I-Q)N$ are continuous. By Lemma 7, it is not difficult to show that $\overline{K_P(I-Q)N(\overline{\Omega})}$ is compact for any open bounded set $\Omega \subset X$. Moreover, $QN(\overline{\Omega})$ is clearly bounded. Thus, N is L-compact on $\overline{\Omega}$ with any open bounded set $\Omega \subset X$.

In order to use Lemma 8, we have to find at least 2^n appropriate open bounded subsets in X. Considering the operator equation $Lu = \lambda N(u,\lambda)$, $\lambda \in (0,1)$, we have

$$
\begin{aligned}
u_i^\Delta(t) = & \lambda\bigg(a_i(t) - \frac{f_i(t,e^{u_i(t-\tau_{ii}(t))})}{K_i}e^{u_i(t-\tau_{ii}(t))} \\
& +\lambda\sum_{j=1,j\neq i}^{n} c_{ij}(t)e^{u_j(t-\tau_{ij}(t))} \\
& -h_i(t)e^{-u_i(t)}\bigg),
\end{aligned} \tag{4}
$$

where $i = 1,\dots,n$. Assume that $u \in X$ is an ω-periodic solution of system (4) for some $\lambda \in (0,1)$. Since $u(t) \in X$, there exist $\xi_i, \eta_i \in I_\omega$, such that

$$u_i(\xi_i) = \max_{t\in I_\omega} u_i(t), \quad u_i(\eta_i) = \min_{t\in I_\omega} u_i(t).$$

According to the definition of Δ-differential calculus, we have

$$u_i^\Delta(\xi_i) \le 0, \quad u_i^\Delta(\eta_i) \ge 0, \quad i = 1,2,\dots,n. \tag{5}$$

By (4) and (5), one has

$$
\begin{aligned}
a_i^l &< a_i(\xi_i) + \lambda\sum_{j=1,j\neq i}^{n} c_{ij}(\xi_i)e^{u_j(\xi_i-\tau_{ij}(\xi_i))} \\
&\le \frac{f_i(\xi_i,e^{u_i(\xi_i-\tau_{ii}(\xi_i))})}{K_i}e^{u_i(\xi_i-\tau_{ii}(\xi_i))} \\
&\quad +h_i(\xi_i)e^{-u_i(\xi_i)} \\
&\le \frac{b_i^M}{K_i}e^{u_i(\xi_i)} + h_i^M e^{-u_i(\xi_i)},
\end{aligned}
$$

namely

$$b_i^M e^{2u_i(\xi_i)} - a_i^l K_i e^{u_i(\xi_i)} + h_i^M K_i > 0,$$

which imply that

$$u_i(\xi_i) > \ln l_i^+ \quad \text{or} \quad u_i(\xi_i) < \ln l_i^-, \tag{6}$$

and

$$
\begin{aligned}
& \frac{b_i^l}{K_i}e^{u_i(\eta_i)} + h_i^l e^{-u_i(\eta_i)} \\
< & \frac{f_i(\eta_i,e^{u_i(\eta_i-\tau_{ii}(\eta_i))})}{K_i}e^{u_i(\eta_i-\tau_{ii}(\eta_i))} \\
& +h_i(\eta_i)e^{-u_i(\eta_i)} \\
\le & a_i(\eta_i) + \lambda\sum_{j=1,j\neq i}^{n} c_{ij}(\eta_i)e^{u_j(\eta_i-\tau_{ij}(\eta_i))} \\
\le & a_i^M + \sum_{j=1,j\neq i}^{n} c_{ij}^M K_j,
\end{aligned}
$$

namely

$$\frac{b_i^l}{K_i}e^{2u_i(\eta_i)} - \big(a_i^M + \sum_{j=1,j\neq i}^{n} c_{ij}^M K_j\big)e^{u_i(\eta_i)} + h_i^l > 0,$$

which imply that

$$\ln L_i^- < u_i(\eta_i) < \ln L_i^+, \quad i = 1,2,\dots,n. \tag{7}$$

By the assumptions (H_1), (H_2) and Lemma 9, we have

$$\ln L_i^- < \ln l_i^- < \ln l_i^+ < \ln L_i^+, i = 1,2,\dots,n. \tag{8}$$

From (6), (7) and (8), we obtain, for all $t \in \mathbb{T}$,

$$\ln L_i^- < u_i(t) < \ln l_i^- \text{ or } \ln l_i^+ < u_i(t) < \ln L_i^+. \tag{9}$$

For convenience, we denote

$$G_i = \big(\ln L_i^-, \ln l_i^-\big), \; H_i = \big(\ln l_i^+, \ln L_i^+\big).$$

Clearly, $l_i^{\pm}$ and $L_i^{+}, i = 1,2,\ldots,n$ are independent of λ. For each $i = 1,2,\ldots,n$, we choose an interval between two intervals G_i and H_i and denote it as Δ_i, then define the set

$$\big\{u = (u_1,\ldots,u_n)^T \in X : u_i(t) \in \Delta_i, t \in R\big\}.$$

Obviously, the number of the above sets is 2^n. We denote these sets as $\Omega_k, k = 1,2,\ldots,2^n$. $\Omega_k, k = 1,2,\ldots,2^n$ are bounded open subsets of X, $\Omega_i \cap \Omega_j = \phi, i \neq j$. Thus $\Omega_k (k = 1,2,\ldots,2^n)$ satisfies the requirement (a) in Lemma 8.

Now we show that (b) of Lemma 8 holds, i.e., we prove when $u \in \partial\Omega_k \cap \operatorname{Ker} L = \partial\Omega_k \cap \mathbb{R}^n, QN(u,0) \neq (0,0,\ldots,0)^T, k = 1,2,\ldots,2^n$. If it is not true, then when $u \in \partial\Omega_k \cap \operatorname{Ker} L = \partial\Omega_k \cap \mathbb{R}^n, k = 1,2,\ldots,2^n$, constant vector $u = (u_1,u_2,\ldots,u_n)^T$ with $u \in \partial\Omega_k, k = 1,2,\ldots,2^n$, satisfies

$$\begin{aligned} 0 &= \int_\kappa^{\kappa+\omega} a_i(t)\,\Delta t - \int_\kappa^{\kappa+\omega} \frac{f_i(t,e^{u_i})}{K_i} e^{u_i}\,\Delta t \\ &\quad - \int_\kappa^{\kappa+\omega} h_i(t)e^{-u_i}\,\Delta t. \end{aligned} \tag{10}$$

By (10), we have

$$a_i^M + \sum_{j=1,j\neq i}^n c_{ij}^M K_j > a_i^M \geq \frac{b_i^l}{K_i} e^{u_i} + h_i^l e^{-u_i},$$

and

$$a_i^l < \frac{b_i^M}{K_i} e^{u_i} + h_i^M e^{-u_i}, \ \ i = 1,2,\ldots,n.$$

Similarly, we get

$$\ln L_i^- < u_i < \ln l_i^- \quad \text{or} \quad \ln l_i^+ < u_i(t_i) < \ln L_i^+. \tag{11}$$

(11) gives that u belongs to one of $\Omega_k \cap \mathbb{R}^n, k = 1,2,\ldots,2^n$. This contradicts the fact that $u \in \partial\Omega_k \cap \mathbb{R}^n, k = 1,2,\ldots,2^n$. Thus condition (b) in Lemma 8 is satisfied.

Finally, in order to show that (c) in Lemma 8 holds, we only prove that for $u \in \partial\Omega_k \cap \operatorname{Ker} L = \partial\Omega_k \cap \mathbb{R}^n, k = 1,2,\ldots,2^n$, then it holds that $\deg\{JQN(u,0), \Omega_k \cap \operatorname{Ker} L, (0,0,\ldots,0)^T\} \neq 0$. To this end, we define the mapping $\phi : \operatorname{Dom} L \times [0,1] \to X$ by

$$\phi(u,\mu) = \mu QN(u,0) + (1-\mu)G(u),$$

here $\mu \in [0,1]$ is a parameter and $G(u)$ is defined by

$$G(u) = \begin{pmatrix} \int_\kappa^{\kappa+\omega} \big(a_1(s) - \frac{b_1^M}{K_1} e^{u_1(s)} \\ -h_1(s)e^{u_1(s)}\big)\,\Delta s \\ \vdots \\ \int_\kappa^{\kappa+\omega} \big(a_n(s) - \frac{b_n^M}{K_n} e^{u_n(s)} \\ -h_n(s)e^{u_n(s)}\big)\,\Delta s \end{pmatrix}_{n\times 1}.$$

We show that for $u \in \partial\Omega_k \cap \operatorname{Ker} L = \partial\Omega_k \cap \mathbb{R}^n, k = 1,2,\ldots,2^n, \mu \in [0,1]$, then it holds that $\phi(u,\mu) \neq (0,0,\ldots,0)^T$. Otherwise, parameter μ and constant vector $u = (u_1,u_2,\ldots,u_n)^T \in \mathbb{R}^n$ satisfy $\phi(u,\mu) = (0,0\ldots,0)^T$, i.e.,

$$\begin{aligned} 0 &= \int_\kappa^{\kappa+\omega} [\mu\big(a_i(s) - \frac{f_i(s,e^{u_i})}{K_i} e^{u_i} - h_i(s)e^{-u_i}\big) \\ &\quad +(1-\mu)\big(a_i(s) - \frac{b_i^M}{K_i} e^{u_i} \\ &\quad -h_i(s)e^{-u_i}\big)]\Delta s, \end{aligned} \tag{12}$$

where $i = 1,2,\ldots,n$. From (12), we obtain

$$\begin{aligned} &\omega[a_i^l - \frac{b_i^M}{K_i} e^{u_i} - h_i^M e^{-u_i}] \\ \leq\ & \int_\kappa^{\kappa+\omega} [a_i(s) - \frac{b_i^M}{K_i} e^{u_i} - h_i(s)e^{-u_i}]\Delta s \\ =\ & -\mu \int_\kappa^{\kappa+\omega} [b_i^M - \frac{f_i(s,e^{u_i}))}{K_i} e^{u_i}]\Delta s < 0 \end{aligned}$$

and

$$\begin{aligned} 0 &= \int_\kappa^{\kappa+\omega} [a_i(s) - \frac{b_i^M}{K_i} e^{u_i} - h_i(s)e^{-u_i} \\ &\quad +\mu(\frac{b_i^M}{K_i} - \frac{f_i(s,e^{u_i})}{K_i})e^{u_i}]\Delta s \\ &< \int_\kappa^{\kappa+\omega} [a_i(s) - \frac{f_i(s,e^{u_i})}{K_i} e^{u_i} - h_i(s)e^{-u_i}]\Delta s \\ &< \omega[a_i^M - \frac{b_i^l}{K_i} e^{u_i} - h_i^l e^{-u_i}] \\ &< \omega[a_i^M + \sum_{j=1,i\neq j}^n c_{ij}^M K_j - \frac{b_i^l}{K_i} e^{u_i} - h_i^l e^{-u_i}], \end{aligned}$$

which imply that

$$\ln L_i^- < u_i < \ln l_i^- \quad \text{or} \quad \ln l_i^+ < u_i < \ln L_i^+, \tag{13}$$

where $i = 1,2,\ldots,n$. (13) gives that u belongs to one of $\Omega_k \cap \mathbb{R}^n, k = 1,2,\ldots,2^n$. This contradicts the fact that $u \in \partial\Omega_k \cap \mathbb{R}^n, k = 1,2,\ldots,2^n$. This proves $\phi(u,\mu) \neq (0,0,\ldots,0)^T$ holds. Note that the system of algebraic equations:

$$a_i(\bar{t}_i) - \frac{b_i^M}{K_i} e^{x_i} - h_i(\bar{t}_i)e^{-x_i} = 0, \ i = 1,2,\ldots,n$$

has 2^n distinct solutions since (H_1) and (H_2) hold, $(x_1^*, x_2^*, \ldots, x_n^*) = (\ln\hat{x}_1, \ln\hat{x}_2, \ldots, \ln\hat{x}_n)$, where $x_i^{\pm} = \frac{a_iK_i(\bar{t}_i) \pm \sqrt{(a_i(\bar{t}_i)K_i)^2 - 4b_i^M h_i(\bar{t}_i)K_i}}{2b_i^M}, \hat{x}_i = x_i^-$ or $\hat{x}_i = x_i^+, i = 1,2,\ldots,n$. Similar to the proof of Lemma 9, it is easy to verify that

$$\ln L_i^- < \ln x_i^- < \ln l_i^- < \ln l_i^+ < \ln x_i^+ < \ln L_i^+.$$

Therefore, $(x_1^*, x_2^*, \ldots, x_n^*)$ uniquely belongs to the corresponding Ω_k. Since $\operatorname{Ker} L = \operatorname{Im} Q$, we can take $J = I$. A direct computation gives, for $k = 1, 2, \ldots, 2^n$,

$$\begin{aligned} & \deg\left\{JQN(u,0), \Omega_k \cap \operatorname{Ker} L, (0,0,\ldots,0)^T\right\} \\ = \; & \deg\left\{\phi(u,1), \Omega_k \cap \operatorname{Ker} L, (0,0,\ldots,0)^T\right\} \\ = \; & \deg\left\{\phi(u,0), \Omega_k \cap \operatorname{Ker} L, (0,0,\ldots,0)^T\right\} \\ = \; & \operatorname{sign}\left[\prod_{i=1}^{n}\left(-\frac{b_i^M}{K_i}x_i^* + \frac{h_i(\bar{t}_i)}{x_i^*}\right)\right]. \end{aligned}$$

Since $a_i(\bar{t}_i) - \frac{b_i^M}{K_i}x_i^* - \frac{h_i(\bar{t}_i)}{x_i^*} = 0, \; i = 1, 2, \ldots, n$, then

$$\begin{aligned} & \deg\left\{JQN(u,0), \Omega_k \cap \operatorname{Ker} L, (0,0,\ldots,0)^T\right\} \\ = \; & \operatorname{sign}\left[\prod_{i=1}^{n}\left(a_i(\bar{t}_i)K_i - 2b_i^M x_i^*\right)\right] = \pm 1, \end{aligned}$$

where $k = 1, 2, \ldots, 2^n$. So far, we have prove that $\Omega_k (k = 1, 2, \ldots, 2^n)$ satisfies all the assumptions in Lemma 8. Hence, system (3) has at least 2^n different ω-periodic solutions. This completes the proof of Theorem 10. □

4 An example

Consider the following two species delayed logistic cooperative system with harvesting terms on time scales:

$$\begin{cases} u_1^{\Delta}(t) = a_1(t) - \frac{f_1(t, e^{u_1(t-\tau_{11}(t))})}{K_1} \\ \qquad \cdot e^{u_1(t-\tau_{11}(t))} + c_{12}(t)e^{u_2(t-\tau_{12}(t))} \\ \qquad - h_1(t)e^{-u_1(t)}, \\ u_2^{\Delta}(t) = a_2(t) - \frac{f_2(t, e^{u_2(t-\tau_{22}(t))})}{K_2} \\ \qquad \cdot e^{u_2(t-\tau_{22}(t))} + c_{21}(t)e^{u_1(t-\tau_{21}(t))} \\ \qquad - h_2(t)e^{-u_2(t)}, \end{cases} \tag{14}$$

where $\tau_{ij}(t) \geq 0 (i, j = 1, 2)$ are 4-periodic functions and

$$a_1(t) = 3 + \sin(0.5\pi t),$$

$$f_1(t, x) = \frac{4 + \sin(0.5\pi t) + \cos x}{10},$$

$$h_1(t) = \frac{9 + \cos(0.5\pi t)}{20},$$

$$a_2(t) = 3 + \cos(0.5\pi t),$$

$$f_2(t, y) = \frac{4 + \cos(0.5\pi t) + \sin y}{10},$$

$$h_2(t) = \frac{2 + \cos(0.5\pi t)}{5},$$

$$c_{12}(t) = c_{21}(t) = 1, \; K_1 = K_2 = 1.$$

If $\mathbb{T} = \mathbb{R}$, then (14) reduces to the following system:

$$\begin{cases} \dot{u}_1(t) = a_1(t) - \frac{f_1(t, e^{u_1(t-\tau_{11}(t))})}{K_1} \\ \qquad \cdot e^{u_1(t-\tau_{11}(t))} + c_{12}(t)e^{u_2(t-\tau_{12}(t))} \\ \qquad - h_1(t)e^{-u_1(t)}, \\ \dot{u}_2(t) = a_2(t) - \frac{f_2(t, e^{u_2(t-\tau_{22}(t))})}{K_2} \\ \qquad \cdot e^{u_2(t-\tau_{22}(t))} + c_{21}(t)e^{u_1(t-\tau_{21}(t))} \\ \qquad - h_2(t)e^{-u_2(t)}, \end{cases} \tag{15}$$

Let $x_i(t) = e^{u_i(t)} (i = 1, 2)$, then (15) can be changed into the following classical continuous system:

$$\begin{cases} \frac{\dot{x}_1(t)}{x_1(t)} = a_1(t) - \frac{f_1(t, x_1(t-\tau_{11}(t)))}{K_1}x_1(t - \tau_{11}(t)) \\ \qquad + c_{12}(t)x_2(t - \tau_{12}(t)) - \frac{h_1(t)}{x_1(t)}, \\ \frac{\dot{x}_2(t)}{x_2(t)} = a_2(t) - \frac{f_2(t, x_2(t-\tau_{22}(t)))}{K_2}x_2(t - \tau_{22}(t)) \\ \qquad + c_{21}(t)x_1(t - \tau_{21}(t)) - \frac{h_2(t)}{x_2(t)}. \end{cases}$$

A direct computation gives that

$$a_1^l = 2, \; a_1^M = 4, \; b_1^l = \inf_{x\in\mathbb{R}} \min_{t\in I_{2\pi}} f_1(t, x) = \frac{2}{10},$$

$$b_1^M = \sup_{x\in\mathbb{R}} \max_{t\in I_{2\pi}} f_1(t, x) = \frac{5}{10}, \; h_1^M = \frac{10}{20}, \; h_1^l = \frac{2}{5},$$

$$a_2^l = 2, \; a_2^M = 4, \; b_2^l = \inf_{x\in\mathbb{R}} \min_{t\in I_{2\pi}} f_2(t, x) = \frac{2}{10},$$

$$b_2^M = \sup_{x\in\mathbb{R}} \max_{t\in I_{2\pi}} f_2(t, x) = \frac{6}{10}, \; h_2^M = \frac{3}{5}, \; h_2^l = \frac{1}{5},$$

$$a_1^l = 2 > 1 = 2\sqrt{b_1^M h_1^M}, \;\; l_1^{\pm} = 2 \pm \sqrt{3},$$

$$a_2^l = 2 > \frac{6}{5} = 2\sqrt{b_2^M h_2^M}, \;\; l_2^{\pm} = \frac{5 \pm 4}{3},$$

$$L_1^{\pm} = \frac{25 \pm \sqrt{617}}{2}, \;\; L_2^{\pm} = \frac{25 \pm \sqrt{621}}{2},$$

$$G_1 = \left(\ln\frac{25 - \sqrt{617}}{2}, \;\; \ln(2 - \sqrt{3})\right),$$

$$H_1 = \left(\ln(2 + \sqrt{3}), \;\; \ln\frac{25 + \sqrt{617}}{2}\right),$$

$$G_2 = \left(\ln\frac{25 - \sqrt{621}}{2}, \;\; \ln\frac{1}{3}\right),$$

$$H_2 = \left(\ln 3, \;\; \ln\frac{25 + \sqrt{621}}{2}\right),$$

$$\Omega_1 = \left\{(u_1(t), \; u_2(t))^T : u_1(t) \in G_1, \; u_2(t) \in G_2\right\},$$

$$\Omega_2 = \left\{(u_1(t), \; u_2(t))^T : u_1(t) \in G_1, \; u_2(t) \in H_2\right\},$$

$$\Omega_3 = \{(u_1(t),\ u_2(t))^T : u_1(t) \in H_1,\ u_2(t) \in G_2\},$$
$$\Omega_4 = \{(u_1(t),\ u_2(t))^T : u_1(t) \in H_1,\ u_2(t) \in H_2\}.$$

Therefore, all conditions of Theorem 10 are satisfied. By Theorem 10, system (14) has at least four 4-periodic solutions.

5 Conclusions

Population dynamics is an important subject in various fields of mathematical biology. One of the famous models for dynamics of population is the Lotka-Volterra system. In this paper, we consider the Lotka-Volterra cooperative system which is different from the other Lotka-Voltera system because it not only involves intraspecies competition but also touches on interspecific cooperation. Owing to the delay and the periodic environment factors (e.g, seasonal effects of weather, food supplies, mating habits, etc), We investigated the existence of the multiple positive periodic solutions for the system (3). Some Sufficient criteria have been obtained. In addition, the system (3) unify the continuous system (1) and the discrete system (2). Our applying the techniques and methods are an effective approaches to the study of mathematical models involving the hybrid discrete-continuous processes.

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References:

[1] H. I. Freedman and Y. Kuang, Stability switches in linear scalar neutral delay equations, *Funkcial. Ekvac.*, vol.34, 1991, pp. 187–209.

[2] H. I. Freedman and J. Wu, Periodic solutions of single–species models with periodic delay, *SIAM J. Math. Anal.*, vol.23, 1992, pp. 689–701.

[3] K. Gopalsamy, Global asymptotic stability in a periodic Lotka–Volterra system, *J. Aust. Math. Soc. Ser. B*, vol.27, 1985, pp. 66–72.

[4] K. Gopalsamy, *Stability and Oscillations in Delay Differential Equations of Population Dynamics,* Kluwer Academic Publishers, Boston 1992.

[5] K. Gopalsamy X. He and L. Wen, On a periodic neutral logistic equation, *Glasg. Math. J.*, vol.33, 1991, pp. 281–286.

[6] Y. Kuang, *Delay Differential Equations with Applications in Population Dynamics,* Academic Press, New York 1993.

[7] Y. K. Li, Periodic solutions of a periodic delay predator–prey system, *Proc. Amer. Math. Soc.*, vol.127, 1999, pp. 1331–1335.

[8] K. Gopalsamy and B.G. Zhang, On a neutral delay logistic equation, *Dyn. Stab. Syst.*, vol.2, 1998, pp. 183–195.

[9] Y. K. Li, Periodic solutions for delay Lotka–Volterra competition systems, *J. Math. Anal. Appl.*, vol.246, 2000, pp. 230–244.

[10] Y. K. Li and Y. Kuang, Periodic solutions of periodic delay Lotka–Volterra equations and systems, *J. Math. Anal. Appl.*, vol.255, 2001, pp. 260–280.

[11] Y. K. Li and Y. Kuang, Periodic solutions in periodic delayed Gause–type predator–prey systems, *Proc. Dynam. Syst. Appl.*, vol.3, 2001, pp. 375–382.

[12] Y. K. Li and Y, Kuang, Periodic solutions in periodic state–dependent delay equations and population models, *Proc. Amer. Math. Soc.*, vol.130, 2002, pp. 135–153.

[13] Z. Jin and Z. E. Ma, Periodic solutions for delay differential equations model of plankton allelopathy, *Comput. Math. Appl.*, vol.44, 2002, pp. 491–500.

[14] M. Fan and K. Wang, Global existence of positive periodic solutions of periodic predator–prey system with infinite delay, *J. Math. Anal. Appl.*, vol.262, 2001, pp. 1–11.

[15] Z. E. Ma and W. D. Wang, Asymptotic behavior of predator-prey system with time dependent coefficients, *Appl. Anal.*, vol.34, 1989, pp. 79–90.

[16] P. Yang and R. Xu, Global attractivity of the periodic Lotka–Volterra system, *J. Math. Anal. Appl.*, vol.233, 1999, pp.221–232.

[17] Y. K. Li, Positive periodic solutions of periodic neutral Lotka-Volterra system with distributed delays, *Chaos, Solitons* & *Fractals*, vol.37, 2008, pp. 288–298.

[18] M. Fan, K. Wang and D. Jiang, Existence and global attractivity of positive periodic solutions of periodic n–species Lotka–Volterra competition systems with several deviating arguments, *Math. Biosci.*, vol.160, 1999, pp. 47–61.

[19] Y. K. Li, Positive periodic solutions of periodic neutral Lotka-Volterra system with state dependent delays, *J. Math. Anal. Appl.*, vol.330, 2007, pp. 1347–1362.

[20] B. Tang and Y. Kuang, Existence, uniqueness and as asymptotic stability of periodic solutions of periodic functional differential systems, *Tohoku Math. J.*, vol.49, 1997, pp. 217–239.

[21] Y. K. Li and L. F. Zhu, Existence of periodic solutions of discrete Lotka–Volterra systems with delays, *Bull. Inst. Math. Acad. Sin.*, vol.33, 2005, pp. 369–380.

[22] Z. Yang and J. Cao, Positive periodic solutions of neutral Lotka–Voltrra system with periodic delays, *Appl. Math. Comput.*, vol.149, 2004, pp. 661–687.

[23] H. Zhao and N. Ding, Existence and global attractivity of positive periodic solution for competition–predator system with variable delays, *Chaos, Solitons & Fractals* 29, 2006, pp. 162–170.

[24] J. Zhen, M. Han and G. Li, The persistence in a Lotka-Volterra competition systems with impulsive, *Chaos, Solitons & Fractals*, vol.24, 2005, pp. 1105–1117.

[25] J. Zhen, Z. Ma and M. Han, The existence of periodic solutions of the n–species Lotka–Volterra competition systems with impulsive, *Chaos, Solitons & Fractals*, vol.22, 2004, pp. 181–188.

[26] Y. Song, M. Han and Y. Peng, Stability and Hopf bifurcations in a competitive Lotka–Volterra system with two delays, *Chaos, Solitons & Fractals*, vol.22, 2004, pp. 1139–1148.

[27] J. R. Yan and J. R. Liu, Periodicity and stability for a Lotka–Volterra type competition system with feedback controls and deviating arguments, *Nonlinear Analysis: Theory, Methods & Applications* 74, 2011, pp. 2916–2928.

[28] J. D. Zhao, X. Guo, Z. X. Han and Z. Z. Chen, Average conditions for competitive exclusion in a nonautonomous two dimensional Lotka–Volterra system, *Mathematical and Computer Modelling*, vol.57, 2013, pp. 1131–1138.

[29] Răzvan M. Tudoran and Anania GÎrban, On a Hamiltonian version of a three–dimensional Lotka-Volterra system, *Nonlinear Analysis: Real World Applications*, vol.13, 2012, pp. 2304–2312.

[30] Z. Y. Hou, On permanence of Lotka–Volterra systems with delays and variable intrinsic growth rates, *Nonlinear Analysis: Real World Applications*, vol.14, 2013, pp. 960–975.

[31] C. L. Shi, Z. Li and F.D. Chen, Extinction in a nonautonomous Lotka–Volterra competitive system with infinite delay and feedback controls, *Nonlinear Analysis: Real World Applications*, vol.13, 2012, pp. 2214–2226.

[32] J. X. Li and J. R. Yan, Partial permanence and extinction in an N-species nonautonomous Lotka–Volterra competitive system, *Computers & Mathematics with Applications*, vol.55, 2008, pp. 76–88.

[33] Z. X. Yu and R. Yuan, Traveling waves for a Lotka–Volterra competition system with diffusion, *Mathematical and Computer Modelling*, vol.53, 2011, pp. 1035–1043.

[34] C. W. Clark, *Mathematical Bioeconomics: The Optimal Management of Renewable Resources,* Wiley, New York 1990.

[35] L. J. Trowtman, *Variational Calculus and Optimal Control*, Springer, New York 1996.

[36] A. W. Leung, Optimal harvesting-coefficient control of steady–state prey–predator diffusive Volterra-Lotka system, *Appl. Math. Optim.*, vol.31, 1995, pp. 219–241.

[37] J. G. P. Gamarra and R. V. Solé, Complex discrete dynamics from simple continuous population models, *Bulletin of Mathematical Biology*, vol.64, 2002, pp. 611–620.

[38] M. Bohner and A. Peterson, *Dynamic Equations on Time Scales: An Introduction with Applications,* Birkhäuser, Boston, Mass, USA 2001.

[39] M. Bohner and A. Peterson, *Advances in Dynamic Equations on Time Scales,* Birkhäuser, Boston, Mass, USA 2003.

[40] S. Hilger, Analysis on measure chains–a unified approach to continuous and discrete calculus, Results in Mathematics, *Resultate der Mathematik*, vol.18, 1990, pp.18–56.

[41] R. Gaines and J. Mawhin, *Coincidence Degree and Nonlinear Differetial Equitions,* Springer Verlag, Berlin 1977.

[42] Y. Chen, Multiple periodic solutions of delayed predator–prey systems with type IV functional responses, *Nonlinear Anal. RWA.*, vol.5, 2004, pp. 45–53.

[43] Q. Wang, B. Dai and Y. Chen, Multiple periodic solutions of an impulsive predator–prey model with Holling-type IV functional response, *Math. Comput. Modelling*, vol.49, 2009, pp. 1829–1836.

[44] D. Hu and Z. Zhang, Four positive periodic solutions to a Lotka–Volterra cooperative system with harvesting terms, *Nonlinear Anal. RWA.* 11, 2010, pp. 1115–1121.

[45] Z. Zhang and T. S. Tian, Multiple positive periodic solutions for a generalized predator–prey system with exploited terms, *Nonlinear Anal. RWA.*, vol.9, 2008, pp. 26–39.

[46] K. Zhao and Y. Ye, Four positive periodic solutions to a periodic Lotka–Volterra predatory–prey system with harvesting terms, *Nonlinear Anal. RWA.*, vol.11, 2010, pp. 2448–2455.

[47] K. Zhao and Y. Li, Four positive periodic solutions to two species parasitical system with harvesting terms, *Computers and Mathematics with Applications*, vol.59, 2010, pp. 2703–2710.

[48] Z. Li, K. Zhao and Y. Li, Multiple positive periodic solutions for a non-autonomous stage–structured predatory–prey system with harvesting terms, *Commun. Nonlinear Sci. Numer. Simulat.*, vol.15, 2010, pp. 2140–2148.

[49] Y. Li, K. Zhao and Y. Ye, Multiple positive periodic solutions of n species delay competition systems with harvesting terms, *Nonlinear Anal. RWA.*, vol.12, 2011, pp. 1013–1022.

[50] H. Fang and Y. F. Xiao, Existence of multiple periodic solutions for delay Lotka–Volterra competition patch systems with harvesting, *Applied Mathematical Modelling*, vol.33, 2009, pp.1086–1096.

[51] J. F. Wang, X. N. Zhou and L. H. Huang, Hopf bifurcation and multiple periodic solutions in Lotka–Volterra systems with symmetries, *Nonlinear Analysis: Real World Applications*, vol.14, 2013, pp. 1817–1828.

[52] Y. Li and Y. Ye, Multiple positive almost periodic solutions to an impulsive non–autonomous Lotka–Volterra predator–prey system with harvesting terms, *Communications in Nonlinear Science and Numerical Simulation*, 2013, dx.doi.org/10.1016/j.cnsns.2013.03.014.

[53] Y. Xing, M. Han and G. Zheng, Initial value problem for first–order integro–differential equation of Volterra type on time scales, *Nonlinear Analysis: Theory, Methods & Applications* vol.60, 2005, pp. 429–442.