Traveling Wave Solutions For Two Nonlinear Lattice Equations By An Extended Riccati Sub-equation Method

Chuanbao Wen
School of Science
Shandong University of Technology
Zhangzhou Road 12, Zibo, 255049
China
wcb2001171@126.com

Abstract: In this paper, we apply an extended Riccati sub-equation method to establish new exact solutions for two nonlinear lattice equations. As a result, new traveling wave solutions including hyperbolic function solutions, trigonometric function solutions and rational function solutions are obtained, and some of them are generalizations of some known results in the literature obtained by the (G'/G)-expansion method.

Key–Words: Nonlinear lattice equations; Riccati sub-equation method; Exact solutions; Traveling wave solutions; Differential-difference equations; Nonlinear evolution equations
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1 Introduction

Nonlinear lattice equations can find their applications in many aspects of mathematical physics such as condensed matter physics, biophysics, atomic chains, molecular crystals and quantum physics and so on. Since the work of Fermi, Pasta and Ulam in the 1960s [1], nonlinear lattice equations have been the focus of many nonlinear studies, and much attention have been paid to the research of the theory of nonlinear lattice equations during the last decades (for example, see [2-10] and the references therein). Among these research works, the investigation of exact solutions of nonlinear lattice equations plays an important role in the study of nonlinear physical phenomena. As we all know, it is hard to generalize one method for nonlinear differential equations to solve nonlinear lattice equations due to the difficulty to search for iterative relations from indices n to $n \pm 1$. Recently, the extensions of some effective methods have been presented and applied for solving some nonlinear lattice equations successfully in the literature. For example, these methods include the exp-function method [11], the exponential function rational expansion method [12-13], the Jacobi elliptic function method [14-15], Hirota's bilinear method [16], the extended simplest equation method [17], the tanh function method [18] and so on. In [19-21], Zayed et al. established abundant exact solutions for some nonlinear partial differential equations using the (G'/G)-expansion method. But we notice that the obtained results by Zayed are not related to nonlinear lattice equations. In fact, relatively few results on the application of the (G'/G)-expansion method to nonlinear lattice equations have been obtained so far in the literature [22-25].

In this paper, the Riccati sub-equation method is extended for solving nonlinear lattice equations, which can be regarded as a generalization of the (G'/G)-expansion method, and in which the iterative relations from indices n to $n\pm1$ are established. We are concerned of two lattice equations: the two-component Volterra lattice equations [23]

$$\begin{cases} \dot{u}_n(t) = u_n(v_n - v_{n-1}), \\ \dot{v}_n(t) = v_n(u_{n+1} - u_n), \end{cases} \tag{1}$$

and the following lattice equation [24]

$$\dot{u}_n(t) = (\alpha + \beta u_n + \gamma u_n^2)(u_{n-1} - u_{n+1}), \tag{2}$$

where $u_n = u_n(t),\ v_n = v_n(t),\ n \in \mathbb{Z}$.

When $\alpha = 1$, Eq. (2) becomes the known Hybrid lattice equation [23, 26]. When $\beta = 0,\ \gamma = 1$, Eq. (2) becomes the known m-KdV lattice equation [23]. When $\alpha = \beta = 0,\ \gamma = -1$, Eq. (2) becomes the modified Volterra lattice equation [27].

We organize this paper as follows. In Section 2, we give the description of the extended Riccati sub-equation method. Then in Section 3 we apply the method to solve Eqs. (1) and (2). Comparisons between the present method and the known (G'/G)-expansion method are also made. Some Conclusions are presented at the end of the paper.

2 Description of the extended Riccati sub-equation method

The main steps of the extended Riccati sub-equation method for solving nonlinear lattice equations are summarized as follows:

Step 1. Consider a system of M polynomial nonlinear lattice equations in the form

$$P(u_{n+p_1}(x),...,u_{n+p_k}(x),...,u'_{n+p_1}(x),...,$$
$$u'_{n+p_k}(x),...,u^{(r)}_{n+p_1}(x),...,u^{(r)}_{n+p_k}(x))=0, \quad (3)$$

where the dependent variable u has M components u_i, the continuous variable x has N components x_j, the discrete variable n has Q components n_i, the k shift vectors $p_s \in \mathbb{Z}^{\mathbb{Q}}$ has Q components p_{sj}, and $u^{(r)}(x)$ denotes the collection of mixed derivative terms of order r.

Step 2. Using a wave transformation

$$u_{n+p_s}(x)=U_{n+p_s}(\xi_n),\ \xi_n=\sum_{i=1}^{Q} d_i n_i+\sum_{j=1}^{N} c_j x_j+\zeta,$$

where $d_i,\ c_j,\ \zeta$ are all constants, we can rewrite Eq. (3) as the following nonlinear form:

$$P(U_{n+p_1}(\xi_n),...,U_{n+p_k}(\xi_n),...,U'_{n+p_1}(\xi_n),...,$$
$$U'_{n+p_k}(\xi_n),...,U^{(r)}_{n+p_1}(\xi_n),...,U^{(r)}_{n+p_k}(\xi_n))=0. \quad (4)$$

Step 3. Suppose the solutions of Eq. (4) can be denoted by

$$U_n(\xi_n)=\sum_{i=0}^{l} a_i\phi^i(\xi_n), \quad (5)$$

where a_i are constants to be determined later, l is a positive integer that can be determined by balancing the highest order linear term with the nonlinear terms in Eq. (4), $\phi(\xi_n)$ satisfies the known Riccati equation:

$$\phi'(\xi_n)=\sigma+\phi^2(\xi_n). \quad (6)$$

Step 4. We present some special solutions $\phi_1,...,\phi_6$ for Eq. (6):

When $\sigma<0$,

$$\begin{cases}\phi_1(\xi_n)=-\sqrt{-\sigma}\tanh(\sqrt{-\sigma}\xi_n+c_0),\\ \phi_2(\xi_n)=-\sqrt{-\sigma}\coth(\sqrt{-\sigma}\xi_n+c_0),\\ \phi_{1,2}(\xi_{n+p_s})=\dfrac{\phi_{1,2}(\xi_n)-\sqrt{-\sigma}\tanh(\sqrt{-\sigma}\sum_{i=1}^{Q}d_ip_{si})}{1-\frac{\phi_{1,2}(\xi_n)}{\sqrt{-\sigma}}\tanh(\sqrt{-\sigma}\sum_{i=1}^{Q}d_ip_{si})},\end{cases} \quad (7)$$

where c_0 is an arbitrary constant.

When $\sigma>0$,

$$\begin{cases}\phi_3(\xi_n)=\sqrt{\sigma}\tan(\sqrt{\sigma}\xi_n+c_0),\\ \phi_4(\xi_n)=-\sqrt{\sigma}\cot(\sqrt{\sigma}\xi_n+c_0),\\ \phi_{3,4}(\xi_{n+p_s})=\dfrac{\phi_{3,4}(\xi_n)+\sqrt{\sigma}\tan(\sqrt{\sigma}\sum_{i=1}^{Q}d_ip_{si})}{1-\frac{\phi_{3,4}(\xi_n)}{\sqrt{\sigma}}\tan(\sqrt{\sigma}\sum_{i=1}^{Q}d_ip_{si})},\end{cases} \quad (8)$$

and

$$\begin{cases}\phi_5(\xi_n)=\sqrt{\sigma}[\tan(2\sqrt{\sigma}\xi_n+c_0)+|\sec(2\sqrt{\sigma}\xi_n+c_0)|],\\ \phi_5(\xi_{n+p_s})=\dfrac{\phi_5^{(1)}(\xi_n)+\sqrt{\sigma}\tan(2\sqrt{\sigma}\sum_{i=1}^{Q}d_ip_{si})}{1-\frac{\phi_5^{(1)}(\xi_n)}{\sqrt{\sigma}}\tan(2\sqrt{\sigma}\sum_{i=1}^{Q}d_ip_{si})}\\ \qquad+\dfrac{\phi_5^{(2)}(\xi_n)\sec(2\sqrt{\sigma}\sum_{i=1}^{Q}d_ip_{si})}{1-\frac{\phi_5^{(1)}(\xi_n)}{\sqrt{\sigma}}\tan(2\sqrt{\sigma}\sum_{i=1}^{Q}d_ip_{si})},\end{cases} \quad (9)$$

where $\phi_5^{(1)}(\xi_n)=\sqrt{\sigma}\tan(2\sqrt{\sigma}\xi_n+c_0)$, $\phi_5^{(2)}(\xi_n)=\sqrt{\sigma}|\sec(2\sqrt{\sigma}\xi_n+c_0)|$, and c_0 is an arbitrary constant.

When $\sigma=0$,

$$\begin{cases}\phi_6(\xi_n)=-\frac{1}{\xi_n+c_0},\\ \phi_6(\xi_{n+p_s})=\dfrac{\phi_6(\xi_n)}{1-\phi_6(\xi_n)\sum_{i=1}^{Q}d_ip_{si}},\end{cases} \quad (10)$$

where c_0 is an arbitrary constant.

Step 5. Substituting (5) into Eq. (4), by use of Eqs. (6)-(10), the left hand side of Eq. (4) can be converted into a polynomial in $\phi(\xi_n)$. Equating each coefficient of $\phi^i(\xi_n)$ to zero, yields a set of algebraic equations. Solving these equations, we can obtain the values of $a_i, d_i,\ c_j$.

Step 6. Substituting the values of a_i into (5), and combining with the various solutions of Eq. (6), we can obtain a variety of exact solutions for Eq. (3).

3 Application of the extended Riccati sub-equation method

In this section, we will apply the extended Riccati sub-equation method described in Section 2 to two nonlinear lattice equations. First we consider the two-component Volterra lattice equations denoted by Eqs. (1).

Using a wave transformation

$$u_n = U_n(\xi_n),\ v_n = V_n(\xi_n),\ \xi_n = d_1 n + c_1 t + \zeta, \tag{11}$$

where $d_1,\ c_1,\ \zeta$ are all constants, the system (1) can be rewritten as the following form:

$$\begin{cases} c_1 U_n' = U_n(V_n - V_{n-1}), \\ c_1 V_n' = V_n(U_{n+1} - U_n), \end{cases} \tag{12}$$

Suppose the solutions for (12) can be denoted by

$$U_n(\xi_n) = \sum_{i=0}^{l_1} a_i \phi^i(\xi_n), \tag{13}$$

$$V_n(\xi_n) = \sum_{i=0}^{l_2} b_i \phi^i(\xi_n), \tag{14}$$

where $\phi(\xi_n)$ satisfies Eq. (6). Balancing the order of U_n' and $U_n V_n$ in Eq. (13), the order of V_n' and $V_n U_n$ in Eq. (14), we obtain $l_1 = l_2 = 1$. So we have

$$U_n(\xi_n) = a_0 + a_1\phi(\xi_n). \tag{15}$$

$$V_n(\xi_n) = b_0 + b_1\phi(\xi_n). \tag{16}$$

We will proceed to solve Eqs. (12) in several cases.

Case 1: If $\sigma < 0$, and assume (6) and (7) hold, then substituting (15), (16), (6) and (7) into Eqs. (12), collecting the coefficients of $\phi_{1,2}^i(\xi_n)$ and equating them to zero, we obtain a series of algebra equations:

$$-c_1 a_1 \tanh(\sqrt{-\sigma} d_1) + \tanh(\sqrt{-\sigma} d_1) b_1 a_1 = 0,$$

$$-c_1 a_1 \sqrt{-\sigma} + \tanh(\sqrt{-\sigma} d_1) b_1 a_0 = 0,$$

$$-c_1 a_1 \sigma \tanh(\sqrt{-\sigma} d_1) + \tanh(\sqrt{-\sigma} d_1) b_1 a_1 \sigma = 0,$$

$$c_1 a_1 (-\sigma)^{\frac{3}{2}} + \tanh(\sqrt{-\sigma} d_1) b_1 a_0 \sigma = 0,$$

$$c_1 b_1 \tanh(\sqrt{-\sigma} d_1) + \tanh(\sqrt{-\sigma} d_1) b_1 a_1 = 0,$$

$$-c_1 b_1 \sqrt{-\sigma} + a_1 \tanh(\sqrt{-\sigma} d_1) b_0 = 0,$$

$$c_1 b_1 \sigma \tanh(\sqrt{-\sigma} d_1) + \tanh(\sqrt{-\sigma} d_1) b_1 a_1 \sigma = 0,$$

$$c_1 b_1 (-\sigma)^{\frac{3}{2}} + a_1 \tanh(\sqrt{-\sigma} d_1) b_0 \sigma = 0.$$

Solving these equations, yields

$$a_1 = -c_1, a_0 = -\frac{c_1\sqrt{-\sigma}}{\tanh(\sqrt{-\sigma} d_1)},\ b_1 = c_1,$$

$$b_0 = -\frac{c_1\sqrt{-\sigma}}{\tanh(\sqrt{-\sigma} d_1)},\ d_1 = d_1,\ c_1 = c_1,$$

or

$$a_1 = \frac{a_0 \tanh(\sqrt{-\sigma} d_1)}{\sqrt{-\sigma}},\ a_0 = a_0,$$

$$b_1 = -\frac{a_0 \tanh(\sqrt{-\sigma} d_1)}{\sqrt{-\sigma}}, b_0 = a_0,$$

$$c_1 = -\frac{a_0 \tanh(\sqrt{-\sigma} d_1)}{\sqrt{-\sigma}},\ d_1 = d_1.$$

So we obtain the following four groups of solitary wave solutions:

$$\begin{cases} u_n(t) = c_1\sqrt{-\sigma}\tanh[\sqrt{-\sigma}(d_1 n + c_1 t + \zeta) + c_0] \\ \qquad -\frac{c_1\sqrt{-\sigma}}{\tanh(\sqrt{-\sigma} d_1)}, \\ v_n(t) = -c_1\sqrt{-\sigma}\tanh[\sqrt{-\sigma}(d_1 n + c_1 t + \zeta) + c_0] \\ \qquad -\frac{c_1\sqrt{-\sigma}}{\tanh(\sqrt{-\sigma} d_1)}, \end{cases} \tag{17}$$

$$\begin{cases} u_n(t) = c_1\sqrt{-\sigma}\coth[\sqrt{-\sigma}(d_1 n + c_1 t + \zeta) + c_0] \\ \qquad -\frac{c_1\sqrt{-\sigma}}{\tanh(\sqrt{-\sigma} d_1)}, \\ v_n(t) = -c_1\sqrt{-\sigma}\coth[\sqrt{-\sigma}(d_1 n + c_1 t + \zeta) + c_0] \\ \qquad -\frac{c_1\sqrt{-\sigma}}{\tanh(\sqrt{-\sigma} d_1)}, \end{cases} \tag{18}$$

$$\begin{cases} u_n(t) = -a_0\tanh(\sqrt{-\sigma} d_1)\tanh[\sqrt{-\sigma}(d_1 n \\ \qquad -\frac{a_0\tanh(\sqrt{-\sigma} d_1)}{\sqrt{-\sigma}} t + \zeta) + c_0] + a_0, \\ v_n(t) = a_0\tanh(\sqrt{-\sigma} d_1)\tanh[\sqrt{-\sigma}(d_1 n \\ \qquad -\frac{a_0\tanh(\sqrt{-\sigma} d_1)}{\sqrt{-\sigma}} t + \zeta) + c_0] + a_0, \end{cases} \tag{19}$$

$$\begin{cases} u_n(t) = -a_0\tanh(\sqrt{-\sigma} d_1)\coth[\sqrt{-\sigma}(d_1 n \\ \qquad -\frac{a_0\tanh(\sqrt{-\sigma} d_1)}{\sqrt{-\sigma}} t + \zeta) + c_0] + a_0, \\ v_n(t) = a_0\tanh(\sqrt{-\sigma} d_1)\coth[\sqrt{-\sigma}(d_1 n \\ \qquad -\frac{a_0\tanh(\sqrt{-\sigma} d_1)}{\sqrt{-\sigma}} t + \zeta) + c_0] + a_0, \end{cases} \tag{20}$$

where $d_1,\ c_0,\ a_0$ are arbitrary constants.

Case 2: If $\sigma > 0$, and assume (6) and (8) hold, then substituting (15), (16), (6) and (8) into (12), collecting the coefficients of $\phi_{3,4}^i(\xi_n)$ and equating them to zero, we obtain a series of algebra equations:

$$c_1 a_1 \tan(\sqrt{\sigma} d_1) - \tan(\sqrt{\sigma} d_1) b_1 a_1 = 0,$$

$$c_1a_1\sqrt{\sigma}-\tan(\sqrt{\sigma}d_1)b_1a_0=0,$$
$$c_1a_1\sigma\tan(\sqrt{\sigma}d_1)-\tan(\sqrt{\sigma}d_1)b_1a_1\sigma=0,$$
$$c_1a_1\sigma^{\frac{3}{2}}-\tan(\sqrt{\sigma}d_1)b_1a_0\sigma=0,$$
$$-c_1b_1\tan(\sqrt{\sigma}d_1)-\tan(\sqrt{\sigma}d_1)b_1a_1=0,$$
$$c_1b_1\sqrt{\sigma}-a_1\tan(\sqrt{\sigma}d_1)b_0=0,$$
$$-c_1b_1\sigma\tan(\sqrt{\sigma}d_1)-\tan(\sqrt{\sigma}d_1)b_1a_1\sigma=0,$$
$$c_1b_1\sigma^{\frac{3}{2}}-a_1\tan(\sqrt{\sigma}d_1)b_0\sigma=0.$$

Solving these equations, yields

$$a_1=-c_1, a_0=-\frac{c_1\sqrt{\sigma}}{\tan(\sqrt{\sigma}d_1)},\ b_1=c_1,$$

$$b_0=-\frac{c_1\sqrt{\sigma}}{\tan(\sqrt{\sigma}d_1)},\ d_1=d_1,\ c_1=c_1,$$

or

$$a_1=\frac{a_0\tan(\sqrt{\sigma}d_1)}{\sqrt{\sigma}},\ a_0=a_0,\ b_1=-\frac{a_0\tan(\sqrt{\sigma}d_1)}{\sqrt{\sigma}},$$

$$b_0=a_0,\ c_1=-\frac{a_0\tan(\sqrt{\sigma}d_1)}{\sqrt{\sigma}},\ d_1=d_1.$$

So we obtain the following solitary wave solutions:

$$\begin{cases} u_n(t)=-c_1\sqrt{\sigma}\tan[\sqrt{\sigma}(d_1n+c_1t+\zeta)+c_0] \\ \qquad -\frac{c_1\sqrt{\sigma}}{\tan(\sqrt{\sigma}d_1)}, \\ v_n(t)=c_1\sqrt{\sigma}\tan[\sqrt{\sigma}(d_1n+c_1t+\zeta)+c_0] \\ \qquad -\frac{c_1\sqrt{\sigma}}{\tan(\sqrt{\sigma}d_1)}, \end{cases} \tag{21}$$

$$\begin{cases} u_n(t)=c_1\sqrt{\sigma}\cot[\sqrt{\sigma}(d_1n+c_1t+\zeta)+c_0] \\ \qquad -\frac{c_1\sqrt{\sigma}}{\tan(\sqrt{\sigma}d_1)}, \\ v_n(t)=-c_1\sqrt{\sigma}\cot[\sqrt{\sigma}(d_1n+c_1t+\zeta)+c_0] \\ \qquad -\frac{c_1\sqrt{\sigma}}{\tan(\sqrt{\sigma}d_1)}, \end{cases} \tag{22}$$

where $d_1,\ c_1,\ a_0$ are arbitrary constants, and

$$\begin{cases} u_n(t)=a_0\tan(\sqrt{\sigma}d_1)\tan[\sqrt{\sigma}(d_1n \\ \qquad -\frac{a_0\tan(\sqrt{\sigma}d_1)}{\sqrt{\sigma}}t+\zeta)+c_0]+a_0, \\ v_n(t)=-a_0\tan(\sqrt{\sigma}d_1)\tan[\sqrt{\sigma}(d_1n \\ \qquad -\frac{a_0\tan(\sqrt{\sigma}d_1)}{\sqrt{\sigma}}t+\zeta)+c_0]+a_0, \end{cases} \tag{23}$$

$$\begin{cases} u_n(t)=-a_0\tan(\sqrt{\sigma}d_1)\cot[\sqrt{\sigma}(d_1n \\ \qquad -\frac{a_0\tan(\sqrt{\sigma}d_1)}{\sqrt{\sigma}}t+\zeta)+c_0]+a_0, \\ v_n(t)=a_0\tan(\sqrt{\sigma}d_1)\cot[\sqrt{\sigma}(d_1n \\ \qquad -\frac{a_0\tan(\sqrt{\sigma}d_1)}{\sqrt{\sigma}}t+\zeta)+c_0]+a_0, \end{cases} \tag{24}$$

where $d_1,\ c_0,\ a_0$ are arbitrary constants.

Case 3: If $\sigma>0$, and assume (6) and (9) hold, then substituting (15), (16), (6) and (9) into (12), using $[\phi_5^{(2)}(\xi_n)]^2=\sigma+[\phi_5^{(1)}(\xi_n)]^2$, collecting the coefficients of $[\phi_5^{(1)}(\xi_n)]^i[\phi_5^{(2)}(\xi_n)]^j$ and equating them to zero, we obtain a series of algebra equations:

$$-2b_1a_1\sin(2\sqrt{\sigma}d_1)+2c_1a_1\sin(2\sqrt{\sigma}d_1)=0,$$
$$-b_1a_1\sqrt{\sigma}\cos(2\sqrt{\sigma}d_1)-b_1a_0\sin(2\sqrt{\sigma}d_1)$$
$$+2c_1a_1\sqrt{\sigma}\cos(2\sqrt{\sigma}d_1)+b_1a_1\sqrt{\sigma}=0,$$
$$-2b_1a_1\sin(2\sqrt{\sigma}d_1)+2c_1a_1\sin(2\sqrt{\sigma}d_1)=0,$$
$$(c_1a_1\sin(2\sqrt{\sigma}d_1)-b_1a_1\sin(2\sqrt{\sigma}d_1))\sigma$$
$$-b_1a_1\sigma\sin(2\sqrt{\sigma}d_1)+c_1a_1\sigma\sin(2\sqrt{\sigma}d_1)=0,$$
$$-b_1a_1\sqrt{\sigma}\cos(2\sqrt{\sigma}d_1)-b_1a_0\sin(2\sqrt{\sigma}d_1)$$
$$+2c_1a_1\sqrt{\sigma}\cos(2\sqrt{\sigma}d_1)+b_1a_1\sqrt{\sigma}=0,$$
$$b_1a_0\sqrt{\sigma}-b_1a_1\sigma\sin(2\sqrt{\sigma}d_1)-b_1a_0\sqrt{\sigma}\cos(2\sqrt{\sigma}d_1)=0,$$
$$c_1a_1\sigma^{\frac{3}{2}}cos(2\sqrt{\sigma}d_1)-b_1a_0\sigma\sin(2\sqrt{\sigma}d_1)+(-b_1a_1\sqrt{\sigma}$$
$$\cos(2\sqrt{\sigma}d_1)+c_1a_1\sqrt{\sigma}\cos(2\sqrt{\sigma}d_1)+b_1a_1\sqrt{\sigma})\sigma=0,$$
$$-2c_1b_1\sin(2\sqrt{\sigma}d_1)-2b_1a_1\sin(2\sqrt{\sigma}d_1)=0,$$
$$-a_1b_0\sin(2\sqrt{\sigma}d_1)-b_1a_1\sqrt{\sigma}+b_1a_1\sqrt{\sigma}\cos(2\sqrt{\sigma}d_1)$$
$$+2c_1b_1\sqrt{\sigma}\cos(2\sqrt{\sigma}d_1)=0,$$
$$-2c_1b_1\sin(2\sqrt{\sigma}d_1)-2b_1a_1\sin(2\sqrt{\sigma}d_1)=0,$$
$$(-c_1b_1\sin(2\sqrt{\sigma}d_1)-b_1a_1\sin(2\sqrt{\sigma}d_1))\sigma$$
$$-c_1b_1\sigma\sin(2\sqrt{\sigma}d_1)-b_1a_1\sigma\sin(2\sqrt{\sigma}d_1)=0,$$
$$-a_1b_0\sin(2\sqrt{\sigma}d_1)-b_1a_1\sqrt{\sigma}+b_1a_1\sqrt{\sigma}\cos(2\sqrt{\sigma}d_1)$$
$$+2c_1b_1\sqrt{\sigma}\cos(2\sqrt{\sigma}d_1)=0,$$
$$-b_1a_1\sigma\sin(2\sqrt{\sigma}d_1)+a_1b_0\sqrt{\sigma}\cos(2\sqrt{\sigma}d_1)-a_1b_0\sqrt{\sigma}=0,$$

$$c_1b_1\sigma^{\frac{3}{2}}\cos(2\sqrt{\sigma}d_1)-a_1b_0\sigma\sin(2\sqrt{\sigma}d_1)+(b_1a_1\sqrt{\sigma}$$
$$\cos(2\sqrt{\sigma}d_1)+c_1b_1\sqrt{\sigma}\cos(2\sqrt{\sigma}d_1)-b_1a_1\sqrt{\sigma})\sigma=0.$$

Solving these equations, we get that

$$a_1=-c_1,\ a_0=0,\ b_1=c_1,\ b_0=0,$$

$$d_1=\frac{\pi}{2\sqrt{\sigma}},\ c_1=c_1,$$

or

$$a_1=-c_1,\ a_0=b_0,\ b_1=c_1,\ b_0=b_0,$$

$$d_1=\frac{1}{2\sqrt{\sigma}}\arcsin(-\frac{2c_1b_0\sqrt{\sigma}}{b_0^2+c_1^2\sigma}),\ c_1=c_1.$$

So we obtain the following trigonometric function solutions:

$$\begin{cases} u_n(t) = -c_1\sqrt{\sigma}\{\tan[2\sqrt{\sigma}(\frac{\pi}{2\sqrt{\sigma}}n + c_1t + \zeta) + c_0] \\ \qquad +|\sec[2\sqrt{\sigma}(\frac{\pi}{2\sqrt{\sigma}}n + c_1t + \zeta) + c_0]|\}, \\ v_n(t) = c_1\sqrt{\sigma}\{\tan[2\sqrt{\sigma}(\frac{\pi}{2\sqrt{\sigma}}n + c_1t + \zeta) + c_0] \\ \qquad +|\sec[2\sqrt{\sigma}(\frac{\pi}{2\sqrt{\sigma}}n + c_1t + \zeta) + c_0]|\}, \end{cases} \tag{25}$$

where $c_1,\ c_0$ are an arbitrary constants, and

$$\begin{cases} u_n(t) = -c_1\sqrt{\sigma}\{\tan[2\sqrt{\sigma}(\frac{1}{2\sqrt{\sigma}}\arcsin(-\frac{2c_1b_0\sqrt{\sigma}}{b_0^2+c_1^2\sigma})n \\ \quad +c_1t + \zeta) + c_0] \\ \quad +|\sec[2\sqrt{\sigma}(\frac{1}{2\sqrt{\sigma}}\arcsin(-\frac{2c_1b_0\sqrt{\sigma}}{b_0^2+c_1^2\sigma})n \\ \quad +c_1t + \zeta) + c_0]|\} + b_0, \\ v_n(t) = c_1\sqrt{\sigma}\{\tan[2\sqrt{\sigma}(\frac{1}{2\sqrt{\sigma}}\arcsin(-\frac{2c_1b_0\sqrt{\sigma}}{b_0^2+c_1^2\sigma})n \\ \quad +c_1t + \zeta) + c_0] \\ \quad +|\sec[2\sqrt{\sigma}(\frac{1}{2\sqrt{\sigma}}\arcsin(-\frac{2c_1b_0\sqrt{\sigma}}{b_0^2+c_1^2\sigma})n \\ \quad +c_1t + \zeta) + c_0]|\} + b_0, \end{cases} \tag{26}$$

where $c_1,\ b_0,\ c_0$ are an arbitrary constants.

Case 4: If $\sigma = 0$, and assume (6) and (10) hold, then substituting (15), (16), (6) and (10) into (12), collecting the coefficients of $\phi_6^i(\xi_n)$ and equating them to zero, we obtain a series of algebra equations:

$$c_1a_1d_1 - d_1b_1a_1 = 0,$$

$$c_1a_1 - d_1b_1a_0 = 0,$$

$$c_1b_1d_1 + d_1b_1a_1 = 0,$$

$$-c_1b_1 + a_1d_1b_0 = 0.$$

Solving these equations, yields

$$a_1 = d_1b_0,\ a_0 = b_0,\ b_1 = -d_1b_0,$$

$$b_0 = b_0,\ d_1 = d_1,\ c_1 = -d_1b_0.$$

Then we obtain the following rational solutions:

$$\begin{cases} u_n(t) = \frac{-d_1b_0}{d_1n - d_1b_0t + \zeta + c_0} + b_0, \\ v_n(t) = \frac{d_1b_0}{d_1n - d_1b_0t + \zeta + c_0} + b_0, \end{cases} \tag{27}$$

where $d_1,\ b_0,\ c_0$ are an arbitrary constants.

Remark 1 In [23, Eqs. (46), (47), (51), (52)], Ayhan and Bekir presented some exact solutions for the two-component Volterra lattice equations by the (G'/G)-expansion method. We note that our results (17), (18) are generalizations of [23, Eqs. (46), (47)], while (21), (22) are generalizations of [23, Eqs. (51), (52)]. In fact, if we let

$$c_0 = \arctan(\frac{C_2}{C_1}),\ \sigma = \frac{4\mu - \lambda^2}{4}$$

or

$$c_0 = arcoth(\frac{C_1}{C_2}),\ \sigma = \frac{4\mu - \lambda^2}{4},$$

then our results (17), (18) reduce to [23, Eq. (46), (47)]. If we let

$$c_0 = \arctan(-\frac{C_2}{C_1}),\ \sigma = \frac{4\mu - \lambda^2}{4}$$

or

$$c_0 = arcot(-\frac{C_1}{C_2}),\ \sigma = \frac{4\mu - \lambda^2}{4},$$

then our results (21), (22) reduce to [23, Eq. (51), (52)].

Remark 2 The established results by (25)-(27) are new exact solutions for the two-component Volterra lattice equations so far to our best knowledge.

Next we will apply the extended Riccati subequation method to the lattice equation denoted by Eq. (2). Using a wave transformation

$$u_n = U_n(\xi_n),\ \xi_n = d_1n + c_1t + \zeta, \tag{28}$$

where $d_1,\ c_1,\ \zeta$ are all constants, Eq. (2) can be rewritten as the following form:

$$c_1U_n' - (\alpha + \beta U_n + \gamma U_n^2)(U_{n-1} - U_{n+1}) = 0. \tag{29}$$

Suppose the solutions of Eq. (29) can be denoted by

$$U_n(\xi_n) = \sum_{i=0}^{l} a_i\phi^i(\xi_n), \tag{30}$$

where $\phi(\xi_n)$ satisfies Eq. (6). Balancing the order of U_n' and U_n^2 in Eq. (29) we obtain $l + 1 = 2l$, and then $l = 1$. So we have

$$U_n(\xi_n) = a_0 + a_1\phi(\xi_n). \tag{31}$$

Then similar as the previous process, we will proceed to solve Eq. (29) in several cases.

Case 1: If $\sigma < 0$, and assume (6) and (7) hold, then substituting (31), (6) and (7) into Eq. (29), collecting the coefficients of $\phi_{1,2}^i(\xi_n)$ and equating them to zero, we obtain a series of algebra equations:

$$c_1\tanh(\sqrt{-\sigma}d_1)^2-2\sqrt{-\sigma}\gamma a_1^2\tanh(\sqrt{-\sigma}d_1)=0,$$

$$-2\sqrt{-\sigma}\beta a_1\tanh(\sqrt{-\sigma}d_1)$$
$$-4\sqrt{-\sigma}\gamma a_1a_0\tanh(\sqrt{-\sigma}d_1)=0,$$

$$c_1\sigma\tanh(\sqrt{-\sigma}d_1)^2+c_1\sigma-2\sqrt{-\sigma}\alpha\tanh(\sqrt{-\sigma}d_1)$$
$$-2\sqrt{-\sigma}\beta a_0\tanh(\sqrt{-\sigma}d_1)+2\gamma a_1^2(-\sigma)^{\frac{3}{2}}\tanh(\sqrt{-\sigma}d_1)$$
$$-2\sqrt{-\sigma}\gamma a_0^2\tanh(\sqrt{-\sigma}d_1)=0,$$

$$2\beta a_1(-\sigma)^{\frac{3}{2}}\tanh(\sqrt{-\sigma}d_1)$$
$$+4\gamma a_1a_0(-\sigma)^{\frac{3}{2}}\tanh(\sqrt{-\sigma}d_1)=0,$$

$$\sigma(-2\gamma a_0^2\sqrt{-\sigma}\sinh(\sqrt{-\sigma}d_1)-2\beta a_0\sqrt{-\sigma}\sinh(\sqrt{-\sigma}d_1)$$
$$+c_1\sigma\cosh(\sqrt{-\sigma}d_1)-2\alpha\sqrt{-\sigma}\sinh(\sqrt{-\sigma}d_1))=0.$$

Solving these equations, yields

$$\begin{aligned}&a_1=\pm\sqrt{\tfrac{4\alpha\gamma-\beta^2}{\sigma}\tfrac{\tanh(\sqrt{-\sigma}d_1)}{2\gamma}},\\&a_0=-\tfrac{\beta}{2\gamma},\\&d_1=d_1,\ c_1=\tfrac{\beta^2-4\alpha\gamma}{2\sqrt{-\sigma}\gamma}\tanh(\sqrt{-\sigma}d_1),\\&\beta^2-4\alpha\gamma>0.\end{aligned}$$

So we obtain the following solitary wave solutions:

$$\begin{aligned}&u_n(t)=\pm\sqrt{\beta^2-4\alpha\gamma}\tfrac{\tanh(\sqrt{-\sigma}d_1)}{2\gamma}\times\\&\tanh[\sqrt{-\sigma}(d_1n+\tfrac{\beta^2-4\alpha\gamma}{2\sqrt{-\sigma}\gamma}\tanh(\sqrt{-\sigma}d_1)t+\zeta)+c_0]\\&-\tfrac{\beta}{2\gamma},\end{aligned}\tag{32}$$

and

$$\begin{aligned}&u_n(t)=\pm\sqrt{\beta^2-4\alpha\gamma}\tfrac{\tanh(\sqrt{-\sigma}d_1)}{2\gamma}\times\\&\coth[\sqrt{-\sigma}(d_1n+\tfrac{\beta^2-4\alpha\gamma}{2\sqrt{-\sigma}\gamma}\tanh(\sqrt{-\sigma}d_1)t+\zeta)+c_0]\\&-\tfrac{\beta}{2\gamma},\end{aligned}\tag{33}$$

where d_1, c_0 are arbitrary constants.

Case 2: If $\sigma>0$, and assume (6) and (8) hold, then substituting (31), (6) and (8) into Eq. (29), collecting the coefficients of $\phi_{3,4}^i(\xi_n)$ and equating them to zero, we obtain a series of algebra equations:

$$c_1\tan(\sqrt{\sigma}d_1)^2-2\sqrt{\sigma}tan(\sqrt{\sigma}d_1)\gamma a_1^2=0,$$

$$-2\sqrt{\sigma}\tan(\sqrt{\sigma}d_1)\beta a_1-4\sqrt{\sigma}\tan(\sqrt{\sigma}d_1)\gamma a_1a_0=0,$$

$$c_1\sigma\tan(\sqrt{\sigma}d_1)^2-c_1\sigma-2\sqrt{\sigma}\tan(\sqrt{\sigma}d_1)\alpha$$
$$-2\sqrt{\sigma}tan(\sqrt{\sigma}d_1)\beta a_0-2\sigma^{\frac{3}{2}}tan(\sqrt{\sigma}d_1)\gamma a_1^2$$
$$-2\sqrt{\sigma}\tan(\sqrt{\sigma}d_1)\gamma a_0^2=0,$$

$$-2\sigma^{\frac{3}{2}}\tan(\sqrt{\sigma}d_1)\beta a_1-4\sigma^{\frac{3}{2}}tan(\sqrt{\sigma}d_1)\gamma a_1a_0=0,$$

$$(2\sqrt{\sigma}\sin(\sqrt{\sigma}d_1)\beta a_0+2\sqrt{\sigma}\sin(\sqrt{\sigma}d_1)\gamma a_0^2$$
$$+2\sqrt{\sigma}\sin(\sqrt{\sigma}d_1)\alpha+c_1\sigma\cos(\sqrt{\sigma}d_1))\sigma=0.$$

Solving these equations, yields

$$\begin{aligned}&a_1=\pm\sqrt{\tfrac{\beta^2-4\alpha\gamma}{\sigma}\tfrac{\tan(\sqrt{\sigma}d_1)}{2\gamma}},\\&a_0=-\tfrac{\beta}{2\gamma},\\&d_1=d_1,\\&c_1=\tfrac{\beta^2-4\alpha\gamma}{2\sqrt{\sigma}\gamma}\tan(\sqrt{\sigma}d_1),\\&\beta^2-4\alpha\gamma>0.\end{aligned}$$

Then we have the following trigonometric function solutions:

$$\begin{aligned}&u_n(t)=\pm\sqrt{\beta^2-4\alpha\gamma}\tfrac{\tanh(\sqrt{\sigma}d_1)}{2\gamma}\times\\&\tan[\sqrt{\sigma}(d_1n+\tfrac{\beta^2-4\alpha\gamma}{2\sqrt{\sigma}\gamma}\tanh(\sqrt{\sigma}d_1)t+\zeta)+c_0]\\&-\tfrac{\beta}{2\gamma},\end{aligned}\tag{34}$$

and

$$\begin{aligned}&u_n(t)=\pm\sqrt{\beta^2-4\alpha\gamma}\tfrac{\tanh(\sqrt{\sigma}d_1)}{2\gamma}\times\\&\cot[\sqrt{\sigma}(d_1n+\tfrac{\beta^2-4\alpha\gamma}{2\sqrt{\sigma}\gamma}\tanh(\sqrt{\sigma}d_1)t+\zeta)+c_0]\\&-\tfrac{\beta}{2\gamma},\end{aligned}\tag{35}$$

where $d_1,\ c_0$ are arbitrary constants.

Case 3 If $\sigma>0$, and assume (6) and (9) hold, then substituting (31), (6) and (9) into Eq. (29), using $[\phi_5^{(2)}(\xi_n)]^2=\sigma+[\phi_5^{(1)}(\xi_n)]^2$, collecting the coefficients of $[\phi_5^{(1)}(\xi_n)]^i[\phi_5^{(2)}(\xi_n)]^j$ and equating them to zero, we obtain a series of algebra equations:

$$2c_1\cos(2\sqrt{\sigma}d_1)^2-2c_1+4\sqrt{\sigma}sin(2\sqrt{\sigma}d_1)$$
$$\gamma a_1^2\cos(2\sqrt{\sigma}d_1)+4\sqrt{\sigma}\sin(2\sqrt{\sigma}d_1)\gamma a_1^2=0,$$

$$4\sqrt{\sigma}sin(2\sqrt{\sigma}d_1)\gamma a_1a_0\cos(2\sqrt{\sigma}d_1)$$
$$+4\sqrt{\sigma}\sin(2\sqrt{\sigma}d_1)\gamma a_1a_0+2\sqrt{\sigma}\sin(2\sqrt{\sigma}d_1)\beta a_1$$
$$+2\sqrt{\sigma}sin(2\sqrt{\sigma}d_1)\beta a_1\cos(2\sqrt{\sigma}d_1)=0,$$

$$2c_1\cos(2\sqrt{\sigma}d_1)^2-2c_1$$
$$+4\sqrt{\sigma}\sin(2\sqrt{\sigma}d_1)\gamma a_1^2\cos(2\sqrt{\sigma}d_1)$$
$$+4\sqrt{\sigma}\sin(2\sqrt{\sigma}d_1)\gamma a_1^2=0,$$

$$3c_1\sigma\cos(2\sqrt{\sigma}d_1)^2+2\sqrt{\sigma}\sin(2\sqrt{\sigma}d_1)\beta a_0\cos(2\sqrt{\sigma}d_1)$$
$$-c_1\sigma+2\sqrt{\sigma}\sin(2\sqrt{\sigma}d_1)\gamma a_0^2\cos(2\sqrt{\sigma}d_1$$
$$+2\sqrt{\sigma}\sin(2\sqrt{\sigma}d_1)\alpha\cos(2\sqrt{\sigma}d_1)$$
$$+4\sigma^{\frac{3}{2}}\sin(2\sqrt{\sigma}d_1)\gamma a_1^2\cos(2\sqrt{\sigma}d_1)$$
$$+(4\sqrt{\sigma}\sin(2\sqrt{\sigma}d_1)\gamma a_1^2-c_1+c_1\cos(2\sqrt{\sigma}d_1)^2$$
$$+2\sqrt{\sigma}\sin(2\sqrt{\sigma}d_1)\gamma a_1^2\cos(2\sqrt{\sigma}d_1))\sigma=0,$$

$$4\sqrt{\sigma}\sin(2\sqrt{\sigma}d_1)\gamma a_1a_0\cos(2\sqrt{\sigma}d_1)$$
$$+4\sqrt{\sigma}\sin(2\sqrt{\sigma}d_1)\gamma a_1a_0$$
$$+2\sqrt{\sigma}\sin(2\sqrt{\sigma}d_1)\beta a_1$$
$$+2\sqrt{\sigma}sin(2\sqrt{\sigma}d_1)\beta a_1\cos(2\sqrt{\sigma}d_1)=0.$$

$$4\sigma^{\frac{3}{2}} sin(2\sqrt{\sigma}d_1)\gamma a_1 a_0 \cos(2\sqrt{\sigma}d_1)$$
$$+(2\sqrt{\sigma}\sin(2\sqrt{\sigma}d_1)\beta a_1 + 4\sqrt{\sigma}\sin(2\sqrt{\sigma}d_1)\gamma a_1 a_0)\sigma$$
$$+2\sigma^{\frac{3}{2}}\sin(2\sqrt{\sigma}d_1)\beta a_1\cos(2\sqrt{\sigma}d_1) = 0,$$

$$2\sigma^{\frac{3}{2}}\sin(2\sqrt{\sigma}d_1)\beta a_1\cos(2\sqrt{\sigma}d_1)$$
$$+4\sigma^{\frac{3}{2}}\sin(2\sqrt{\sigma}d_1)\gamma a_1 a_0\cos(2\sqrt{\sigma}d_1) = 0,$$

$$2\sigma^{\frac{3}{2}}\sin(2\sqrt{\sigma}d_1)\gamma a_0^2 cos(2\sqrt{\sigma}d_1) + c_1\sigma^2\cos(2\sqrt{\sigma}d_1)^2$$
$$+2\sigma^{\frac{3}{2}}\sin(2\sqrt{\sigma}d_1)\beta a_0\cos(2\sqrt{\sigma}d_1)$$
$$+2\sigma^{\frac{3}{2}}\sin(2\sqrt{\sigma}d_1)\alpha\cos(2\sqrt{\sigma}d_1)$$
$$+(2\sigma^{\frac{3}{2}}\sin(2\sqrt{\sigma}d_1)\gamma a_1^2\cos(2\sqrt{\sigma}d_1)$$
$$+c_1\sigma\cos(2\sqrt{\sigma}d_1)^2)\sigma = 0.$$

Solving these equations, yields

$$\begin{aligned}&a_1 = a_1,\ a_0 = -\tfrac{\beta}{2\gamma},\\ &d_1 = \pm\tfrac{\pi}{4\sqrt{\sigma}},\\ &c_1 = \pm 2\sqrt{\sigma}\gamma a_1^2,\\ &\beta^2 - 4\alpha\gamma > 0,\end{aligned}$$

or

$$\begin{aligned}&a_1 = \sqrt{\tfrac{\beta^2-4\alpha\gamma}{\sigma}}\tfrac{1}{2\gamma},\\ &a_0 = -\tfrac{\beta}{2\gamma},\\ &d_1 = \pm\tfrac{\pi}{4\sqrt{\sigma}},\\ &c_1 = \pm\tfrac{\beta^2-4\alpha\gamma}{2\sqrt{\sigma}\gamma}, \beta^2 - 4\alpha\gamma > 0,\end{aligned}$$

or

$$\begin{aligned}&a_1 = -\sqrt{\tfrac{\beta^2-4\alpha\gamma}{\sigma}}\tfrac{1}{2\gamma},\\ &a_0 = -\tfrac{\beta}{2\gamma},\\ &d_1 = \pm\tfrac{\pi}{4\sqrt{\sigma}},\\ &c_1 = \pm\tfrac{\beta^2-4\alpha\gamma}{2\sqrt{\sigma}\gamma},\ \beta^2 - 4\alpha\gamma > 0,\end{aligned}$$

or

$$\begin{aligned}&a_1 = a_1,\\ &a_0 = -\tfrac{\beta}{2\gamma},\\ &c_1 = \pm a_1\sqrt{\beta^2 - 4\alpha\gamma},\\ &d_1 = \tfrac{1}{2\sqrt{\sigma}}\arccos(\tfrac{-\beta^2+4\alpha\gamma+4\sigma\gamma^2a_1^2}{-4\sigma\gamma^2a_1^2-\beta^2+4\alpha\gamma}),\\ &\beta^2 - 4\alpha\gamma > 0.\end{aligned}$$

So we obtain the following four groups of trigonometric function solutions:

$$\begin{aligned}&u_n(t) = a_1\{\\ &\tan[2\sqrt{\sigma}(\pm\tfrac{\pi}{4\sqrt{\sigma}}n \pm 2\sqrt{\sigma}\gamma a_1^2 t + \zeta) + c_0]\\ &+|\sec[2\sqrt{\sigma}(\pm\tfrac{\pi}{4\sqrt{\sigma}}n \pm 2\sqrt{\sigma}\gamma a_1^2 t + \zeta) + c_0]|\} - \tfrac{\beta}{2\gamma},\end{aligned} \tag{36}$$

$$\begin{aligned}&u_n(t) = \tfrac{\sqrt{\beta^2-4\alpha\gamma}}{2\gamma}\{\\ &\tan[2\sqrt{\sigma}(\pm\tfrac{\pi}{4\sqrt{\sigma}}n \pm \tfrac{\beta^2-4\alpha\gamma}{2\sqrt{\sigma}\gamma}t + \zeta) + c_0]\\ &+|\sec[2\sqrt{\sigma}(\pm\tfrac{\pi}{4\sqrt{\sigma}}n \pm \tfrac{\beta^2-4\alpha\gamma}{2\sqrt{\sigma}\gamma}t + \zeta) + c_0]|\} - \tfrac{\beta}{2\gamma},\end{aligned} \tag{37}$$

$$\begin{aligned}&u_n(t) = -\tfrac{\sqrt{\beta^2-4\alpha\gamma}}{2\gamma}\{\\ &\tan[2\sqrt{\sigma}(\pm\tfrac{\pi}{4\sqrt{\sigma}}n \pm \tfrac{\beta^2-4\alpha\gamma}{2\sqrt{\sigma}\gamma}t + \zeta) + c_0]\\ &+|\sec[2\sqrt{\sigma}(\pm\tfrac{\pi}{4\sqrt{\sigma}}n \pm \tfrac{\beta^2-4\alpha\gamma}{2\sqrt{\sigma}\gamma}t + \zeta) + c_0]|\} - \tfrac{\beta}{2\gamma},\end{aligned} \tag{38}$$

$$\begin{aligned}&u_n(t) = a_1\{\\ &\tan[2\sqrt{\sigma}(\tfrac{1}{2\sqrt{\sigma}}\arccos(\tfrac{-\beta^2+4\alpha\gamma+4\sigma\gamma^2a_1^2}{-4\sigma\gamma^2a_1^2-\beta^2+4\alpha\gamma})n\\ &\pm a_1\sqrt{\beta^2-4\alpha\gamma}) + c_0]\\ &+|\sec[2\sqrt{\sigma}(\tfrac{1}{2\sqrt{\sigma}}\arccos(\tfrac{-\beta^2+4\alpha\gamma+4\sigma\gamma^2a_1^2}{-4\sigma\gamma^2a_1^2-\beta^2+4\alpha\gamma})n\\ &\pm a_1\sqrt{\beta^2-4\alpha\gamma}t + \zeta) + c_0]|\} - \tfrac{\beta}{2\gamma},\end{aligned} \tag{39}$$

where $a_1,\ c_0$ are arbitrary constants, and $a_1 \neq 0$.

Case 4 If $\sigma = 0$, and assume (6) and (10) hold, then substituting (31), (6) and (10) into Eq. (29), collecting the coefficients of $\phi^i(\xi_n)$ and equating them to zero, we obtain a series of algebra equations:

$$c_1 d_1^2 - 2d_1\gamma a_1^2 = 0,$$

$$-4d_1\gamma a_1 a_0 - 2d_1\beta a_1 = 0,$$

$$-c_1 - 2d_1\alpha - 2d_1\gamma a_0^2 - 2d_1\beta a_0 = 0.$$

Solving these equations, yields

$$a_1 = \pm\frac{\sqrt{\beta^2 - 4\alpha\gamma}d_1}{2\gamma},\ a_0 = -\frac{\beta}{2\gamma},\ d_1 = d_1,$$

$$c_1 = \frac{(\beta^2 - 4\alpha\gamma)d_1}{2\gamma},\quad \beta^2 - 4\alpha\gamma > 0.$$

Then we obtain the following rational solution:

$$u_n(t) = \pm\tfrac{\sqrt{\beta^2-4\alpha\gamma}d_1}{2\gamma}\times (\tfrac{1}{d_1 n + \frac{(\beta^2-4\alpha\gamma)d_1}{2\gamma}t + \zeta + c_0}) - \tfrac{\beta}{2\gamma}, \tag{40}$$

where$d_1,\ c_0$ are arbitrary constants.

Remark 3 . In [24, Eqs. (34), (37)], Zhang et al. presented some exact solutions for Eq. (2) by the (G'/G)-expansion method. We note that our results (32), (34) are generalizations of Zhang's results. In fact, if we let

$$c_0 = arctan(\frac{C_2}{C_1}),\ \sigma = \frac{4\mu - \lambda^2}{4}$$

or

$$c_0 = arcoth(\frac{C_1}{C_2}),\ \sigma = \frac{4\mu - \lambda^2}{4},$$

then our result (32) reduces to [24, Eq. (34)]. If we let

$$c_0 = arctan(-\frac{C_2}{C_1}),\ \sigma = \frac{4\mu - \lambda^2}{4}$$

or

$$c_0 = arccot(-\frac{C_1}{C_2}),\ \sigma = \frac{4\mu - \lambda^2}{4},$$

then our result (34) reduces to [24, Eq. (37)].

Remark 4 To our best knowledge, The established results by (36-40) are new exact solutions for the two-component Volterra lattice equations, and have not been reported by other authors.

Remark 5 From the analysis above, we notice that more general exact solutions for the two lattice equations mentioned above are obtained by the proposed extended Riccati sub-equation method than by the (G'/G)-expansion method. In fact, in the (G'/G)-expansion method, the solutions $U_n(\xi_n)$ is denoted by a polynomial in $(G'(\xi_n)/G(\xi_n))$, and G satisfies

$$G'' + \lambda G' + \mu G = 0, \tag{41}$$

where λ, μ are constants. If we let in Eq. (41) $(G'(\xi_n)/G(\xi_n)) = -\phi(\xi_n) - \frac{\lambda}{2}$, $\frac{4\mu-\lambda^2}{4} = \sigma$, then Eq. (41) reduces to $\phi'(\xi_n) = \sigma + \phi^2(\xi_n)$, which is the Riccati equation (6). So $(G'(\xi_n)/G(\xi_n))$ can be generalized by $\phi(\xi_n)$.

Remark 6 All of the solutions presented in this paper have been checked with Maple 11 by putting them back into the original equations.

4 Conclusions

We have proposed an extended Riccati sub-ODE method for solving nonlinear lattice equations, and applied it to find exact solutions of two nonlinear lattice equations. As a result, some generalized exact solutions and solitary wave solutions for them have been successfully found. We have also compared this method with the known (G'/G)-expansion method. Comparison results show that more exact solutions are obtained by the proposed method than by the (G'/G)-expansion method, which is to some extent in accordance with the analysis in [28].

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