Modeling Cyprus Stock Market

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Abstract: The intention of this research is to understand the behavior of the Cyprus Stock Market. Two time series are used as representatives: the FTSE/CySE 20 and the General index. Both return series are characterized by the presence of heavy tails and reject the Gaussian models. We use *a*-stable distributions to model the data. Although statistical tests accept the null hypothesis empirical findings of FTSE/CySE 20 show that return distribution takes the shape of a Gaussian distribution at 345 days and the tails appear to become less heavy for less frequent series. Self-similarity is also explored and Hurst exponent is $H \in (0.6, 0.65)$, showing persistent return time series.

Key–Words: a-stable distribution, Hurst exponent, heavy tails, infinite variance, Anderson-Darling, Kolmogorov-Smirnov criteria.

1 Introduction

The Cyprus Stock Exchange is the primary stock market in Cyprus and is considered to be a small emerging capital market with a very short history.

On March 29 1996, transactions start taking place. The main index for this market is the General Price index. Alternatively, FTSE/CySE 20 is another important index designed to provide a real-time measure of the Cyprus Stock Market on which index-linked derivatives can be traded. It was constructed with the cooperation of the Cyprus Stock Exchange, the Financial Times and the London Stock Exchange in November 2000.

The aim of this paper is to provide valuable information about possible models of the return series calculated by the FTSE/CySE 20 and the General index. In this sense, we add new knowledge to the scarce literature on the above regional market in an adequate and efficient way as FTSE/CySE 20 is studied from the entire spectrum of its operation and the General index from the year 2005 to 2011. In our knowledge there are no previous characterizations of distribution or trajectory properties of the indices for the Cyprus Stock Market.

In general, stock market modeling is an old issue and has been approached with Gaussian models (Brownian motion and Black-Schools option pricing formula) which cannot detect the problems of asymmetry, heavy tails, and persistence of shocks. Empirical studies (Mandelbrot [16, 18]) show that real log-returns are not Gaussian. Stable models are an alternative to this problem, being the most desirable because it takes advantage of the generalized central limit theorem. The characterization of being stable refers to stability under addition: the distribution of appropriately normalized sums of independent and identically distributed (i. i. d.) stable distributions is the same as the distribution of the summands. The key parameter of stable distributions is the stability index (which is invariant under convolution). Examples of stability analysis can be found in Rachev [23] and Belov et al. [4]. The latter analysis 26 international financial series focusing on the issues of stability and self-similarity.

The main drawback of stable distributions is the lack of closed forms of many probability densities functions. The recent existence of reliable computation packages to compute stable densities, distribution functions and quantities makes possible the use of stable models.

2 Tools and methods

Study of the two time series requires the following steps to be done:

- \circ The normality study.
- The study of *a*-stable distributions and the analysis of infinite variance.
- Self-similarity analysis.

Normality study

We study the normality by considering two ways: Statistical tests such as the Jarque-Bera test and graphical methods such as the QQ-plot and the comparison of empirical distribution to normal one.

Jarque-Bera test

2.1

The Jarque-Bera test is used when the data distribution is unknown and its parameters should be estimated. It is a two-sided goodness-of-fit test and the corresponding test statistic is

$$JB = \frac{n}{6} \left(\sigma^2 + \frac{k-3}{4} \right)$$

where n is the sample size, σ is the sample skewness and k is the sample kurtosis. For large sample sizes n the test statistic is chi-square distributed with two degrees of freedom.

QQ-plots

QQ-plot is a quantile-quantile plot of the sample data quantiles against the theoretical Gaussian quantiles. A normal distributed sample is almost approximated by a straight line.

2.2 Stable distributions

Stable distributions are a rich class of probability distributions with characteristics of skewness and heavy tails. Random variables with stable laws justify the generalized central limit theorem which states that stable distributions are the only asymptotic distributions for adequately scaled and centered sums of i. i. d. random variables [13]. Mittnik and Rachev [21] make another convincing argument for the accuracy of stable data generating the process: it is unlikely that we will discover the exact distribution from which observed data is generated. However because stable distributions have domains of attraction, it is likely that observed data will belong to the domain of attraction of a stable law and therefore appear to be generated from a stable distribution [9].

a-stable distributions need four parameters to be completely described and are denoted as $S(a, \beta, \gamma, \delta)$. The *a* parameter called is tail index and expresses the properties of distribution tails. In general $a \in (0, 2]$. The β parameter expresses the asymmetry of distribution with $\beta \in [-1, 1]$. The parameters γ , δ are correspondingly the scale with $\gamma > 0$ and location with $\delta \in \mathbb{R}$. In contrast to the probability density, which does not exist in closed form for the majority of the *a*-stable distributions, the characteristic function is always defined and given by the relation

$$\Phi(t) = E(e^{itx})$$

$$\Phi(t) = \begin{cases} e^{\left(-\gamma^a |t|^a \left(1 - i\beta sign(t) \tan\left(\frac{\pi a}{2}\right)\right) + i\delta t\right)} & \text{if } a \neq 1\\ e^{\left(-\gamma |t| \left(1 + i\beta sign(t) \frac{2}{\pi} \log|t|\right) + i\delta t\right)} & \text{if } a = 1 \end{cases}$$

Stable distributions form a closed family under temporal aggregation, in other words, if X_1, X_2, \dots, X_n is a sequence of i. i. d. random variables with characteristic exponent a then the distribution of any non-overlapping sum of X_1, X_2, \dots, X_n is also a member of the stable class with characteristic exponent a.

A stable random variable has the following property, which may be stated in two equivalent forms:

- 1. If X_1, X_2, \dots, X_n are independent random variables belonging to the $S(a, \beta, \gamma, \delta)$, then $\sum_{i=1}^n X_i$ will be distributed as $S(a, \beta, \gamma n^{\frac{1}{\alpha}}, n\delta)$.
- 2. If X_1, X_2, \dots, X_n are independent random variables belonging to the $S(a, \beta, \gamma, \delta)$, then

$$\sum_{i=1}^{n} X_i \stackrel{d}{=} \begin{cases} n^{1/a} X_i + \delta(n - n^{\frac{1}{a}}), & \text{if } a \neq 1 \\ n X_i + \frac{2}{\pi} \beta \gamma n \ln n, & \text{if } a = 1 \end{cases}$$

Examples of known distributions with closedform probability densities are the Gaussian $S(2, 0, \gamma, \delta)$ with tail index a = 2, the Cauchy $S(1, 0, \gamma, \delta)$ with a = 1 and Levy $S(1/2, 1, \gamma, \delta)$ with a = 1/2. Let X have distribution $S(\alpha, \beta, \gamma, 0)$ with a < 2. Then there exist two i. i. d. random variables Y_1 and Y_2 with the common distribution $S(\alpha, \beta, \gamma, 0)$ such that

$$X = \left(\frac{1+\beta}{2}\right)^{\frac{1}{\alpha}} Y_1 - \left(\frac{1-\beta}{2}\right)^{\frac{1}{\alpha}} Y_2, \text{ if } a \neq 1$$

Let X_1 and X_2 be independent random variables with $S(\alpha, \beta_i, \gamma_i, \delta_i)$, for i = 1, 2. Then $X_1 + X_2 \sim S(a, \beta, \gamma, \delta)$ with

$$\gamma = (\gamma_1^a + \gamma_2^a)^{1/a},$$
$$\beta = \frac{\beta_1 \gamma_1^a + \beta_2 \gamma_2^a}{\gamma_1^a + \gamma_2^a},$$
$$\delta = \delta_1 + \delta_2.$$

The *p*th moment is defined as $EX^p = \int_0^\infty P(|X|^p > y) dy$ for a random variable *X*. This exists and is finite if and only 0 ; otherwise, it does not exist. Specifically, for all <math>a < 2 (heavy tails) and $-1 < \beta < 1$, both tail probabilities and densities are asymptotically power laws. One consequence of heavy tails is that the non-existence of each moment. If the smallest tail index is greater

than 2, then the random variable has a finite variance; when the index belongs to the interval (1, 2], the random variable has a finite mean but an infinite variance; finally, when its value is less than 1, the random variable does not have a finite mean.

Therefore the stability of a given distribution is checked with the method of infinite variance. This method was proposed by Granger and Orr [10] and Rachev [23] and tests the convergence of the sample variance

$$S_n^2 = 1/n \sum_{i=1}^n (X_i - \mu_n)^2, 1 < n < N < \infty$$

as $n \to \infty$. If X_1, X_2, \dots, X_n are i. i. d. random variables with a distribution of finite variance then $S_n^2 \to c$, $1 < n < N < \infty$ almost surely for a constant such that $0 < c < \infty$.

A stochastic process $(X_t, t \ge 0)$ is stable if all its finite dimensional distributions are stable. Let $(X_t, t \ge 0)$ be a stochastic process, the process is *a*-stable if and only if every linear combination $\sum_{k=1}^{d} b_k X(t_k)$ is *a*-stable, where $d \ge 1$, t_1, t_2, \dots, t_d , b_1, b_2, \dots, b_d are real numbers. A stochastic process $(X_t, t \ge 0)$ is called the (standard) *a*-stable Levy motion if:

- 1. X(0) = 0 (almost surely)
- 2. $(X_t, t \ge 0)$ has independent increments
- 3. $X_t X_s \sim S\left(a, \beta, (t-s)^{\frac{1}{a}}, \delta\right)$, for any $0 \leq s < t < \infty$ and $0 < a \leq 2, -1 \leq \beta \leq 1$.

According to the Property 3, an *a*-stable Levy motion has stationary increments. As a = 2 we have the Brownian motion.

2.3 Self-similarity

The concept of self-similarity is based on the fact that there are processes (for example fractional Brownian motion) which exhibit the same behaviour at different scales on space or time, allowing us to know longtime behaviour from short time one. The reasons of its existence arise from either high variability (in situations where increments are independent) and heavytailed, as in stable Levy processes or strong dependence between increments (as in fractional Brownian motion). These two mechanisms for self-similarity have been called the Noah effect and the Joseph effect, respectively by Mandelbrot [5, 19], being coexistent (heavy tails and strong dependence between increments). Fractional stable processes [3, 24] provide such examples. The index a of stability and the exponent H of self-similarity satisfy the following relation a = 1/H.

The most common definition of self-similar processes is the following:

A continuous time process $(X_t, t \ge 0)$ is said to be self-similar if there exists H > 0 such that for any scaling factor c > 0, the processes $(X_{ct}, t \ge 0)$ and $(c^H X_t, t \ge 0)$ have the same law: $X_{ct} \stackrel{d}{=} c^H X_t$ for every t > 0, for every c > 0 and $0 \le H < 1$. The parameter H is called the self-similarity exponent of the process $X = (X_t, t \ge 0)$.

The value of the Hurst exponent provides a measure of the serial correlation and Self-similarity. Valid values range between 0 and 1. Specifically there are the following classes:

- 0 < H < 0.5, H characterizes a process called anti-persistent which tends to behave as follows: increase of its value is more probably followed by decrease and vice versa.
- *H* = 0.5 implies a random process (the Brownian Motion) in which increase to its value is almost the same likely to be followed by decrease.
- 0.5 < H < 1, H characterizes a process called persistent in which an increase of the value is more probably followed by increase and vice versa.

An example of the last class is the fractional Brownian motion $(X_t, t \ge 0)$ with covariance function given by the relation $cov(X_s, X_t) = \frac{1}{2} \left(t^{2H} + s^{2H} - (t-s)^{2H} \right)$, $s \le t$. Simulations of different trajectories can be carried out by using of Cholesky decomposition method. For H = 0.5 the covariance is $cov(X_s, X_t) = |t - s|$ indicating the Brownian Motion as mentioned before.



Figure 1: The relation between Self-similar, Gaussian and Levy stable process.

In this paper, the calculation of the Hurst exponent is made by using of the Selfis tool [25] according to Karagiannis et al. [12]. This package estimates the Hurst exponents with estimators which either employ time-domain methods or frequency-based methods. Time domain methods show a high degree of correlation between separated data points for persistent processes. In this case, we have the following estimators

- R/S methods, ([17, 18, 27]),
- Absolute Value method ([27, 28]),
- Variance of Residuals ([22, 28]) and
- Aggregate Variance method ([27, 28]).

In frequency domain the same processes show a significant level of power at frequencies close to zero. Estimators in the frequency domain are the

- Periodogram ([8, 27, 28])
- Whittle [11] and
- Abry-Veitch [1].

The existence of numerous estimators is firstly justified by the asymptotic nature of the Hurst exponent and secondly by the different properties of the timeseries which estimator looks at.

3 **Results of the Cyprus stock indexes**

The FTSE/CySE 20 series data and the General index are available for download in Cyprus Stock Exchange website [4]. FTSE/CySE 20 daily values covers the period 2/1/2001 to 30/11/2011, the sample size is N=2699 values. The General index includes data from 3/1/2005 to 30/12/2011, the sample size is N=1730 values. We pay more attention to the FTSE/CySE 20 data series due to the longest sample size. Both samples exclude the weekend and holiday periods. In order to study the properties of return data series we compute the continuously compounded return series r_t . For the period t the financial series X_t defines the continuously compounded return series r_t as follows:

$$r_t = \ln\left(\frac{X_t}{X_{t-1}}\right)$$

All computations were made with the MATLAB software.

3.1 Results from descriptive statistics and Jarque-Bera test

We begin the analysis by calculating the first four statistics from the sample: mean standard deviation, skewness and kurtosis (represented in Table 1). Table 2 shows the correlation between the two data series. The skewness and excess kurtosis can give a hint about how the empirical distribution and consequently the distribution of the generated process differ from the Gaussian one.

| Return series period | |
|------------------------|--|
| 2/1/2001 to 30/11/2011 | |
| -0,00088 | |
| C.1. 95% | |
| (-0,0019, 0,0000) | |
| 0,0228 | |
| C.1 95% | |
| (0,02222, 0,023439) | |
| 0,0947 S.E 0,04713 | |
| 3,9884 S.E 0,09422 | |
| 1783.6816 | |
| p-value 0,001* | |
| Return series period | |
| 3/1/2005 to 30/12/2011 | |
| -0.0007 | |
| C.1. 95% | |
| (-0, 00174, 0,0005) | |
| 0,02606 | |
| C.1 95% | |
| (0,025132, 0,02686) | |
| 0,084 S.E 0,059 | |
| 3,352 S.E 0,118 | |
| 926 0026 | |
| 820.9030 | |
| | |

Note: ' r at the significant level 0,05

| Table 2. | Correlation co | efficient for | the period |
|-----------------|-----------------------|---------------|------------|
| | 2005 to | 2011 | |

| Spearman's R | | |
|---------------|---------|---------|
| FTSE/CySE 20 | 1,000 | 0,304** |
| p-value | | 0,000 |
| General index | 0,304** | 1,00 |
| p-value | 0,00 | |

Note: **Correlation is significant at the 0,01 level (2-tailed).

Sample return series looks like to be positively skewed but these are statistically insignificant. The excess kurtosis which is the kurtosis minus three

describes a leptokurtic distribution when its value is positive. Because the values are positive and statistically significant, we can conclude that the distributions generated by the data probably have positive excess kurtosis and therefore fat tails. Distributions with fat tails of returns decay slower than those being Gaussian. This implies that return processes are not a random walk. Lastly the Jarque-Bera test with null hypothesis, the return series follows the Gaussian distribution with unknown mean and variance against the alternative: the return series does not follow a Gaussian distribution which suggests no normality for both return series.

Further evidence come from the QQ plot and the empirical density, shown in Figure 2, 3, 4, and 5. The QQ plots in Figure 2 and 3 show a deviation from the straight line for the FTSE/CySE 20 and the General index. The graphs reveal that both positive and negative shocks are responsible for the non-normality of the series. The empirical distributions in Figure 4 and Figure 5, (dashed line) show that the tails are much fatter and the central part of the empirical density is more peaked, than the Gaussian distribution. This picture is in accordance with the descriptive statistics in Table 1 and the results of Jarque-Bera test.

The non-parametric correlation coefficient Spearman R measures the statistical dependence between the two indices for the period 2005-2011 and has the statistically significant value of 0.304 at the 0.01 level as 0.00 < 0.01. Therefore we reject the null hypothesis of no correlation.



Figure 2: QQ plot of the return series FTSE/CySE 20 return series against the Normal.

Although the FTSE/CySE 20 return data series do not follow a Gaussian distribution, it is interesting to check if one can measure time L which the distribu-



Figure 3: QQ plot of the return series General index return series against the Normal.

tion is needed to reduce the kurtosis to 3 and to acquire a symmetric probability density shape. For this purpose, we consider the return series as a function of lag L

$$r_t(L) = L^{-1} \ln\left(\frac{X_{t+L}}{X_t}\right)$$
 for $L = 2, 3, 4, \cdots$.

The skewness and the kurtosis are considered to be

$$S(L) = \frac{E(r_t^3(L))}{(E(r_t^2(L)))^{3/2}}$$

and

$$K(L) = \frac{E(r_t^4(L))}{(E(r_t^2(L)))^2} - 3$$

We plot these functions against L and we conclude that the skewness and the Kurtosis reaches the value 0 after 345 days, Figure 6.

3.2 Results from the *a* - stable distribution

In order to see the suitability of both data series for a - stable distributions, we continue with the empirical analysis of moments as a function of n when $n \rightarrow \infty$, (see Figure 7 and Figure 8). The analysis of moments suggests that the variance looks to oscillate for the FTSE/CySE 20 and diverge for the General index. The graphs of the skewness are erratic and slightly fluctuate around zero. Kurtosis diverges.

The estimations of parameters for the stable distributions were made by the regression method proposed by Koutrouvelis [14]. The MATLAB software was downloaded from the site of free routines by Veillette [31]. The results are shown in Table 3.





| | FTSE/CySE 20 | General index |
|----------------|--------------|---------------|
| $\hat{\delta}$ | -0.00091 | -0.00099 |
| $\hat{\gamma}$ | 0.01172 | 0.01375 |
| \hat{eta} | -0.00465 | -0.05903 |
| â | 1.53987 | 1.56451 |
| Anderson- | 1.665 | 1.969 |
| Dalling | (0.142)* | (0.095)* |
| K-S test | 0.0345 | 0.754335 |
| | (0.079)* | (0.035571)* |

Table 3. Results of a-stable distribution andHypothesis tests

Note: *at the significant level 0.05.

We check the fitted distribution laws by applying two hypothesis tests: Anderson-Darling and Kolmogorov-Smirnov. Both tests are based on statistics which compare the empirical distribution function (EDF) of the sample with the hypothesized distribution F(x).

Anderson-Darling test has the feature that it is especially sensitive towards differences at the tails of distributions. Second, there exist evidences about the best capability to detect very small differences, even between large sample sizes.

General index return and FTSE/CySE 20 data series give Anderson-Darling statistic 1.24097 and 1.97183 respectively. Their corresponding approximate p-values are 0.25217 and 0.09516. Anderson-Darling results for the General index return series accept the law S(1.56451, -0.05903, 0.01375, -0.00099);

whereas FTSE/CySE 20 the law



Figure 5: Empirical distribution of the General index return series, dashed line: normal distribution: solid line.

S(1.53987, -0.00465, 0.01172, -0.00091). Evaluation of the fitness of stable laws by using the Kolmogorov-Smirnov test does not change the above results. This test is more sensitive towards differences at the central part of both distributions. Kolmogorov-Smirnov test in the case of FTSE/CySE 20 returns p - value = 0.079 at the significant level a = 0.05 accepting the null hypothesis. The General index returns p - value = 0.35571 allowing to accept the *a*-stable distribution.

Although both hypothesis tests accept the calculated a - stable distributions, empirical analysis of the FTSE/CySE 20 return data does not keep constant the index of stability at all levels of aggregation. The tail index was found a = 1.53987 for daily returns and a = 2 at the aggregated level of n = 345 days and the result is inconsistent with the theory of a- stable distributions.

Because of the relation H = 1/a, we expect a Hurst coefficient close to H = 1/1.53987 = 0.6494 for the FTSE/CySE 20 return data and H = 1/1.56451 = 0.6392 for the General index.

3.3 Self-similarity results

All Hurst exponent estimates were calculated with SELFIS tool (Table 4). The results show that the estimators of Hurst exponent calculated by Aggregate variance (97,03%), R/S (99,72%) and Variance of residuals (99,27%) method have correlation coefficient above 97% and values larger than 0.5 indicating significant dependence for the FTSE/CySE 20 index. Results from the relation H = 1/a agree with those



Figure 6: Skewness and Kurtosis as a function of L



Figure 7: Sample variance, skewness and Kurtosis for FTSE/CySE 20.

of the R/S method although estimations of the parameter H come from two different methods. The calculated results for the General index show the largest correlation coefficient from the same methods: Aggregate variance (95,85%), R/S (99,91%) and Variance of residuals (97,73%). In this case no estimator agrees with the Hurst exponent calculated through the stable model.

In both cases, Hurst exponent is greater than or equal to 0.6 indicating that the data generated process can be modeled by a persistent process perhaps from an ARIMA model with appropriate serial correlation or more accurately by the fractional Brownian motion.

Table 4. Estimators of the Hurst exponents



Figure 8: Sample variance, skewness and Kurtosis for General index returns.

| | FTSE/CySE | General |
|--------------|-----------------|------------------|
| | 20 | index |
| Ag. variance | H = 0.576 | H = 0.728 |
| method | Cor.coef.: | Cor.coef.: |
| | 97.03% | 95.85 % |
| R/S method | H = 0.648 | H = 0.606 |
| method | Cor.coef.: | Cor.coef.: |
| | 99.72% | 99.91 % |
| Absolute | H = 0.82 | H = 1.013 |
| moments | Cor.coef.: | Cor.coef.: |
| | 38.91% | 3.229 % |
| Periodogram | H = 0.555 | H = 0.482 |
| Variance of | H = 0.680 | H = 0.729 |
| res. method | Cor.coef.: | Cor.coef.: |
| | 99.27% | 97.73 % |
| | | |
| Abry-Veitch | H = 0.581 | H = 0.549 |
| method | 95% Con. Inter. | 95% Con. Inter. |
| | (0.544, 0.618) | (0.492, 0.606) |
| Whittle | H = 0.683 | H = 0.564 |
| estim. | 95% Con. Inter. | 95% Con. Inter. |
| | (0.533, 0.588)% | (0.525, 0.603) % |

4 Conclusion

The goal of this research is to analyze the behavior of the Cyprus Stock Market. The return data of both FTSE/CySE 20 and the General index reject the hypothesis of Gaussian distribution according to the Jarque-Bera test at the significant level 0.05. They show statistically significant kurtosis and fat tails. Distributions with fat tails decay slower than a Gaussian, which implies return processes are not a random walk.

We model the series with a-stable distributions based on their infinite variance and both Anderson-Darling and Kolmogorov hypothesis tests accept the null hypothesis. For the FTSE/CySE 20 daily return series we define the a-stable law S(1.53987, -0.00465, 0.01172, -0.00091) with tail index a = 1.53987 contrary to the fact that less frequent return series show evidence of reduced kurtosis (Figure 6) and consequently increase to tail index. Gencay et al. [7] have pointed out the phenomenon of low tail index for more frequent returns and higher for less frequent returns. Akgirav and Booth [2] consider the above feature inconsistent with stable models of returns and one explanation may be that financial returns are not independent and identically distributed. The practical importance of this contradiction has been reduced by Taleb [26]. Via this method the estimated value of H = is 0.6494 > 0.5. The calculation of the Hurst exponent using SELFIS tool and the R/S estimator agrees with the previous result.

The General index is characterized by a smaller sample size and both hypothesis tests give results of acceptance for the stable distribution S(1.56451, -0.05903, 0.01375, -0.00099) with tail index a = 1.56451. This method defines the value H = 0.6392 for the Hurst exponent. Hence, it does not agree with any estimator of the selfish tool. We accept as the best estimator that given by the R/S method with correlation coefficient: 99,91% and a value H = 0.606 > 0.5.

In summary, the Cyprus Stock Market analysis leads to non-Gaussian modeling. The *a*-stable models, although are acceptable from the hypothesis tests, do not keep constant the tail index through aggregated levels of time. At n = 345 days FTSE/CySE 20 is characterized by the tail index of 2. Therefore, both return data series are not independent. Self-similarity analysis concludes a Hurst exponent $H \in (0.6, 0.65)$ clearly greater than 0.5 showing a positive serial correlation. Our analysis shows appropriate ARIMA models or fractional Brownian motion with positive correlation.

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