Study on the dynamic model of a duopoly game with delay in insurance market

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Abstract: On the basis of domestic and foreign workers' study, this paper considers the dynamic model of a duopoly price game in insurance market. In the duopoly model, we theoretically analyze the existence and stability of the Nash equilibrium point of the dynamic system, when one player or both players make a delayed decision, then stability conditions are obtained. The numerical simulation results further confirmed the accuracy of the theory. We observe that consideration of the delayed decision cannot change the equilibrium point of the system, but the time to the situation of stabilize will be changed because of the influence of the delayed variable and other parameters. If we increase the value of delayed parameter, the system will take longer time to be stable. If we change the speed of price adjustment and the weight of price of different periods, the time to the situation of stabilize will be changed.

Key–Words: insurance market,duopoly,dynamic game,delayed decision

1 Introduction

With the development of China's insurance market, the number of the insurance company of domestic and foreign investment is gradually increasing. Although the monopoly of China's insurance market has declined, it still in the stage of oligopoly. A lot of references have studied the static game of oligopoly in insurance market, but the number of references to study the dynamic game is not enough.

Han Shufeng [1] uses game theory to analyze the price competition in insurance market, he shows that insurance company do not have to dominate the market by lower price, thus he proposes some corresponding pricing strategies. Yao Hongxing [2] applies Cournot model to bank competition, analyzes the stability of fixed points in this model and proposes the parameters' effect on the stability of this system. Pan Yurong [3] considers all the market participants have heterogeneous rationality, in the market two players use output decision of delayed bounded rationality and optimal response respectively, so the model of duopoly game based on heterogeneous rationality is obtained. Xu Feng [4] takes delayed decisions into a duopoly advertising game model, one player considers delayed decision and the other considers bounded rationality, then the author analyzes the effect of delayed decisions on this system. Ma Junhai [5] supposes all game players consider bounded rationality with delay. In the condition of different parameters, he investigates the stability, Hopf bifurcation and chaos of the system. It concludes that the delayed parameter and other parameter values will change the stability of system. E.Ahmed [6] investigates the Puu's economic dynamic model and shows delayed decisions will make the system stable. So enterprises that consider bounded rationality with time delay will reach the Nash equilibrium easier. H.N.Agiza [7] analyzes the dynamics of Bowley's model with bounded rationality, studies the existence and stability of the equilibria and the Bowley's model with delayed bounded rationality in monopoly. H.N.Agiza [8]investigates the dynamic Cournot duopoly game model with bounded rationality. Two players will adjust their outputs according to the current marginal profit. He also analyzes the complex dynamics of the system. Zhang Jixiang [9] studies the Bertrand model with bounded rationality and proposes that an increase or decrease of the speed of price adjustment may change the stability of the Nash equilibrium point. Song Yongli [10] studies a competitive Lotka-Volterra system with two delays, then analyzes the stability of the equilibrium point and the existence of Hopf bifurcations. Zhang Jinzhu [11] considers a two-species Lotka-Volterra competition system with dependent delays. Gao Qin [12] uses the theory of ecology to analyze the competition and cooperation between two container terminal companies. She improves the Lotka-Volterra model to constitute nonlinear models and investigates the stability and Hopf bifurcation of no-delay and delay models. Woo-Sik Son [13] investigates the dynamical model of a financial system, analyzes the local stability of the system with delays and the effect of delayed feedbacks on the financial model. Sanling Yuan [14] analyzes a predator-prey system with same feedback delays, Rui Xu [15] analyzes a predator-prey system with stage structure for the predator. Both of them investigate the stability of the equilibrium and the existence of Hopf bifurcation of the model. Rui Xu [16] also investigates a delayed predator-prey model with prey dispersal and analyzes the local stability of all nonnegative equilibria. Junhai Ma [17] studies a dynamic triopoly game model characterized by firms with different expectations. Junhai Ma [18] has considered a three-species symbiosis Lotka-Volterra model with discrete delays and investigated the local stability of positive equilibrium.

Most of investigations are on the discrete dynamical system and a few researches apply it to the insurance market. Here the study is based on the insurance market and a dynamic model of a duopoly game is constituted, to investigate situations that one player considers delayed decisions and both players make delayed decisions. According to the analysis of numerical simulations, the parameters' effects on the stability of this system are obtained.

2 Model

With the assumption that an insurance market is a duopoly market, the two insurance companies make different prices for their products. Let $p_i(t), i = 1,2$ represent the price of ith insurance company, $q_i, i = 1,2$ represent the quantity of ith insurance company. So the linear inverse demand functions of two insurance companies are as follows

$$\begin{cases} q_1 = a - b_1p_1 + d_1p_2 \\ q_2 = a - b_2p_2 + d_2p_1 \end{cases} \tag{1}$$

where $a, b_i, d_i > 0, i = 1,2$ and d_i represents the extent of ith insurance company's products substitute for the other's. If both of the two insurance companies have no fixed costs, let $c_i > 0, i = 1,2$ represent the marginal costs of ith insurance company, so the cost function with linear form is

$$C_i(q_i) = c_iq_i, i = 1,2 \tag{2}$$

Therefore, the profit function of the two insurance companies is given by

$$\pi_i(p_1,p_2) = p_iq_i - c_iq_i, i = 1,2 \tag{3}$$

according to the above analysis, we have

$$\begin{cases} \pi_1(p_1,p_2) = (p_1 - c_1)(a - b_1p_1 + d_1p_2) \\ \pi_2(p_1,p_2) = (p_2 - c_2)(a - b_2p_2 + d_2p_1) \end{cases} \tag{4}$$

then the marginal profits of the two insurance companies are

$$\begin{cases} \frac{\partial\pi_1(p_1,p_2)}{\partial p_1} = a - 2b_1p_1 + d_1p_2 + b_1c_1 \\ \frac{\partial\pi_2(p_1,p_2)}{\partial p_2} = a - 2b_2p_2 + d_2p_1 + b_2c_2 \end{cases} \tag{5}$$

As we know, the market information of all firms is incomplete. Some firms will take marginal profit into consideration in their price game in order to maximize their profits. If the marginal profit is positive, the firm will increase its price; otherwise, decrease its price. So the dynamic adjustment of the price is modeled as

$$\dot{p}_i(t) = \alpha_i(p_i)\frac{\partial\pi_i(p_1,p_2)}{\partial p_i}, i = 1,2 \tag{6}$$

where $\alpha_i(p_i)$ is a positive function which represents the extent of the price variation of ith insurance company according to its marginal profit. We assume $\alpha_i(p_i)$ has a linear form

$$\alpha_i(p_i) = v_ip_i, i = 1,2 \tag{7}$$

where v_i is positive which represents the speed of price adjustment of ith insurance company. According to (5), (6) and (7), we have the dynamic model of price game of the two insurance companies, the differential equation of this model is

$$\begin{cases} \dot{p}_1(t) = v_1p_1(a - 2b_1p_1 + d_1p_2 + b_1c_1) \\ \dot{p}_2(t) = v_2p_2(a - 2b_2p_2 + d_2p_1 + b_2c_2) \end{cases} \tag{8}$$

When the two insurance companies with bounded rationality need to determine their own price, they will consider not only the current marginal profit but also the marginal profit of τ time ago. Here we assume 1th insurance company considers delay first and the delay variable is τ. The other company makes price decisions with bounded rationality. So the dynamic model of price game becomes

$$\dot{p}_i(t) = v_ip_i\frac{\partial\pi_i(p_1^d,p_2)}{\partial p_i}, i = 1,2 \tag{9}$$

where

$$p_1^d = wp_1(t) + (1-w)p_1(t-\tau)$$

$0 < w < 1$ represents the weight of the current price, $1-w$ represents the weight of price of $t-\tau$ time.

Therefore, the final dynamic system of price game is given by

$$\begin{cases} \dot{p}_1(t) = & v_1p_1[a - 2b_1wp_1(t) \\ & -2b_1(1-w)p_1(t-\tau) \\ & +d_1p_2(t) + b_1c_1] \\ \dot{p}_2(t) = & v_2p_2[a - 2b_2p_2(t) + d_2wp_1(t) \\ & +d_2(1-w)p_1(t-\tau) + b_2c_2] \end{cases} \quad (10)$$

3 Equilibrium points and local stability

3.1 One player makes delayed decision

We know that system (10) and system (8) have same equilibrium points. And the equilibrium points of the dynamic system of the duopoly price game must be nonnegative because our model is an economic model. So the four fixed points of system (8) are

$$E_1(0,0), E_2(0, \tfrac{a+b_2c_2}{2b_2}),$$

$$E_3(\tfrac{a+b_1c_1}{2b_1}, 0), E_4(p_1^*, p_2^*).$$

where

$$\begin{cases} p_1^* = \frac{2ab_2+2b_1b_2c_1+ad_1+b_2c_2d_1}{4b_1b_2-d_1d_2} \\ p_2^* = \frac{2ab_1+2b_1b_2c_2+ad_2+b_1c_1d_2}{4b_1b_2-d_1d_2} \end{cases} \quad (11)$$

Obviously, E_1, E_2, E_3 are boundary equilibria, E_4 is the only Nash equilibrium and has economic meaning when $4b_1b_2 - d_1d_2 > 0$.

Let $u_1(t) = p_1(t) - p_1^*, u_2(t) = p_2(t) - p_2^*$,system (10) is written as

$$\begin{cases} \dot{u}_1(t) = & v_1(u_1(t)+p_1^*)[-2b_1wu_1(t) \\ & -2b_1(1-w)u_1(t-\tau) + d_1u_2(t)] \\ \dot{u}_2(t) = & v_2(u_2(t)+p_2^*)[-2b_2u_2(t) \\ & +d_2wu_1(t) + d_2(1-w)u_1(t-\tau)] \end{cases} \quad (12)$$

Now we can investigate the stability of point (0,0)instead of the stability of equilibrium point E_4.

The linearization of system (12) when $u = 0$ is

$$\begin{cases} \dot{u}_1(t) = & -2b_1wv_1p_1^*u_1(t) \\ & -2b_1(1-w)v_1p_1^*u_1(t-\tau) \\ & +d_1v_1p_1^*u_2(t) \\ \dot{u}_2(t) = & -2b_2v_2p_2^*u_2(t) + d_2wv_2p_2^*u_1(t) \\ & +d_2(1-w)v_2p_2^*u_1(t-\tau) \end{cases} \quad (13)$$

Thus, we can obtain the characteristic equation of the system (13)

$$\begin{vmatrix} \begin{matrix}(\lambda + 2b_1wv_1p_1^* \\ +2b_1(1-w)v_1p_1^*e^{-\lambda\tau})\end{matrix} & -d_1v_1p_1^* \\ \begin{matrix}(-d_2wv_2p_2^* \\ -d_2(1-w)v_2p_2^*e^{-\lambda\tau})\end{matrix} & \lambda + 2b_2v_2p_2^* \end{vmatrix} = 0 \quad (14)$$

Compute the determinant (14), we will have

$$\lambda^2 + l_1\lambda + l_2 + (l_3\lambda + l_4)e^{-\lambda\tau} = 0 \quad (15)$$

where

$$l_1 = 2b_1wv_1p_1^* + 2b_2v_2p_2^*,$$

$$l_2 = v_1v_2p_1^*p_2^*w(4b_1b_2 - d_1d_2),$$

$$l_3 = 2b_1(1-w)v_1p_1^*,$$

$$l_4 = v_1v_2p_1^*p_2^*(1-w)(4b_1b_2 - d_1d_2).$$

If $\lambda = i\omega(\omega > 0)$ is a root of equation (15), then

$$l_2 - \omega^2 + il_1\omega + il_3\omega\cos(\omega\tau) - il_4\sin(\omega\tau) + l_4\cos(\omega\tau) + l_3\omega\sin(\omega\tau) = 0$$

Separating the real and imaginary parts, we have

$$\begin{cases} \omega^2 - l_2 = l_4\cos(\omega\tau) + l_3\omega\sin(\omega\tau) \\ -l_1\omega = l_3\omega\cos(\omega\tau) - l_4\sin(\omega\tau) \end{cases} \quad (16)$$

We should square both sides of equations (16) and plus each other, then we have

$$\omega^4 + (l_1^2 - 2l_2 - l_3^2)\omega^2 + l_2^2 - l_4^2 = 0 \quad (17)$$

where

$$l_1^2 - 2l_2 - l_3^2 = 4(b_1v_1p_1^*)^2(2w-1) + 4(b_2v_2p_2^*)^2 + 2v_1v_2d_1d_2p_1^*p_2^*w,$$

$$l_2^2 - l_4^2 = (v_1v_2p_1^*p_2^*)^2(4b_1b_2 - d_1d_2)^2(2w-1)$$

We can see that if

$$\begin{array}{l} either l_1^2 - 2l_2 - l_3^2 < 0, and \\ \Delta = (l_1^2 - 2l_2 - l_3^2)^2 - 4(l_2^2 - l_4^2), \\ or l_2^2 - l_4^2 < 0 \end{array} \quad (18)$$

then $2w - 1 < 0$, thus $w < 0.5$ and the only positive root of the equation (17) is

$$\omega_0 = \sqrt{\frac{-(l_1^2 - 2l_2 - l_3^2) + \sqrt{\Delta}}{2}} \quad (19)$$

Therefore, the characteristic equation (15) has only a pair of purely imaginary roots $\pm i\omega_0$.

Substituting ω_0 into equations (16), we can determine τ_j

$$\tau_j = \frac{1}{\omega_0}\arccos[\frac{(l_4-l_1l_3)\omega_0^2-l_2l_4}{l_3^2\omega_0^2+l_4^2}] + \frac{2j\pi}{\omega_0}, \quad j = 0, 1, 2, \cdots \tag{20}$$

Taking the derivative with respect to ω in equation (15), we have

$$[\frac{d\lambda}{d\tau}]^{-1} = \frac{(2\lambda+l_1)e^{\lambda\tau}}{\lambda(l_3\lambda+l_4)} + \frac{l_3}{\lambda(l_3\lambda+l_4)} - \frac{\tau}{\lambda}$$

Together with (16), it leads to

$$\begin{aligned}
&Re[\frac{d\lambda}{d\tau}]^{-1}|_{\tau=\tau_j} \\
&= Re[\frac{(2\lambda+l_1)e^{\lambda\tau}}{\lambda(l_3\lambda+l_4)}]|_{\tau=\tau_j} + Re[\frac{l_3}{\lambda(l_3\lambda+l_4)}]|_{\tau=\tau_j} \\
&= Re[\frac{l_1\cos(\omega_0\tau)-2\omega_0\sin(\omega_0\tau)+i(2\omega_0 cos(\omega_0\tau)+l_1 sin(\omega_0\tau))}{-l_3\omega_0^2+il_4\omega_0}] + Re[\frac{l_3}{-l_3\omega_0^2+il_4\omega_0}] \\
&= \frac{1}{A}[-l_1\omega_0(l_3\omega_0\cos(\omega_0\tau) - l_4\sin(\omega_0\tau)) + 2\omega_0^2(l_4\cos(\omega_0\tau) + l_3\omega_0\sin(\omega_0\tau)) - l_3\omega_0^2] \\
&= \frac{\omega_0^2}{A}[2\omega_0^2 + l_1^2 - 2l_2 - l_3^2] \\
&= \frac{\omega_0^2}{A}\sqrt{\Delta}
\end{aligned}$$

where $A = l_3^2\omega_0^4 + l_4^2\omega_0^2$.

It is easy to see $A > 0$, so if $\Delta \neq 0$, then we have

$$Re[\frac{d\lambda}{d\tau}]^{-1}|_{\tau=\tau_j} = \frac{\omega_0^2}{A}\sqrt{\Delta} > 0$$

Thus, the Hopf bifurcation will occur at the Nash equilibrium E_4 of system (10).

When $\tau = 0$, the equation (15) will have this form

$$\lambda^2 + (l_1 + l_3)\lambda + l_2 + l_4 = 0 \tag{21}$$

It is easy to obtain $l_1, l_2, l_3, l_4 > 0$, so the two roots of the equation (20) have real parts and the equilibrium E_4 is local stable when $\tau = 0$. If the real parts of the roots of the characteristic equation (15) are zero, then we can see τ_0 is the minimum among $\tau_j (j = 0, 1, 2\cdots)$ to make it true. Therefore, the Nash equilibrium E_4 is locally asymptotically stable for $\tau \in [0, \tau_0)$, it is unstable for $\tau > \tau_0$.

3.2 Both players make delayed decision

We suppose that both insurance companies make delayed decision, but the values of their delayed parameter and the weight of price are different. Delayed parameters and weight of price are τ,τ' and w,w', respectively. The dynamic model of price game is

$$\dot{p}_i(t) = v_i p_i \frac{\partial\pi_i(p_1^d, p_2^d)}{\partial p_i}, i = 1, 2 \tag{22}$$

where

$$p_1^d = wp_1(t) + (1-w)p_1(t-\tau)$$

$$p_2^d = w'p_2(t) + (1-w')p_2(t-\tau')$$

w and w' represent the weight of the current price, $1-w$ and $1-w'$represent the weight of price of $t-\tau$ time.

Therefore, the differential equation model of price game is given by

$$\begin{cases} \dot{p}_1(t) = & v_1p_1[a - 2b_1wp_1(t) \\ & -2b_1(1-w)p_1(t-\tau) + d_1w'p_2(t) \\ & +d_1(1-w')p_2(t-\tau') + b_1c_1] \\ \dot{p}_2(t) = & v_2p_2[a - 2b_2w'p_2(t) \\ & -2b_2(1-w')p_2(t-\tau') + d_2wp_1(t) \\ & +d_2(1-w)p_1(t-\tau) + b_2c_2] \end{cases} \tag{23}$$

Maybe insurance companies cannot obtain the latest information on price when they are making decision, so the game is only based on the price of $t-\tau$ time. Suppose $w = w' = 0$, the system will be simplified as

$$\begin{cases} \dot{p}_1(t) = & v_1p_1[a - 2b_1p_1(t-\tau) \\ & +d_1p_2(t-\tau') + b_1c_1] \\ \dot{p}_2(t) = & v_2p_2[a - 2b_2p_2(t-\tau') \\ & +d_2p_1(t-\tau) + b_2c_2] \end{cases} \tag{24}$$

This system also has four boundary equilibria E_1, E_2, E_3 and E_4. It is obvious that E_4 is the only Nash equilibrium.

Let $u_1(t) = p_1(t) - p_1^*, u_2(t) = p_2(t) - p_2^*$, the system will become

$$\begin{cases} \dot{u}_1(t) = & v_1(u_1(t) + p_1^*)[-2b_1u_1(t-\tau) \\ & +d_1u_2(t-\tau')] \\ \dot{u}_2(t) = & v_2(u_2(t) + p_2^*)[-2b_2u_2(t-\tau') \\ & +d_2u_1(t-\tau)] \end{cases} \tag{25}$$

Now we can investigate the stability of point (0,0) instead of the stability of equilibrium point E_4.

The linearization of system (25) when $u = 0$ is

$$\begin{cases} \dot{u}_1(t) = & -2b_1v_1p_1^*u_1(t-\tau) \\ & +d_1v_1p_1^*u_2(t-\tau') \\ \dot{u}_2(t) = & -2b_2v_2p_2^*u_2(t-\tau') \\ & +d_2v_2p_2^*u_1(t-\tau) \end{cases} \tag{26}$$

Then we can obtain the characteristic equation of system (26)

$$\begin{vmatrix} \lambda + 2b_1v_1p_1^*e^{-\lambda\tau} & -d_1v_1p_1^*e^{-\lambda\tau'} \\ -d_2v_2p_2^*e^{-\lambda\tau'} & \lambda + 2b_2v_2p_2^*e^{-\lambda\tau} \end{vmatrix} = 0 \tag{27}$$

Compute the determinant , we have

$$\lambda^2 + 2b_1v_1p_1^* e^{-\lambda\tau}\lambda + 2b_2v_2p_2^* e^{-\lambda\tau'}\lambda + (4b_1b_2 - d_1d_2)v_1v_2p_1^*p_2^* e^{-\lambda(\tau+\tau')} = 0$$

Here we only investigate the situation of $\tau' = \tau$, then we have

$$\lambda^2 + m_1 e^{-\lambda\tau}\lambda + m_2 e^{-2\lambda\tau} = 0 \tag{28}$$

where

$$m_1 = 2(b_1v_1p_1^* + b_2v_2p_2^*)$$

$$m_2 = (4b_1b_2 - d_1d_2)v_1v_2p_1^*p_2^*$$

When $\tau = 0$, the equation (28) becomes

$$\lambda^2 + m_1\lambda + m_2 = 0 \tag{29}$$

This equation always has two roots with negative real part.

When $\tau > 0$, if $\lambda = i\omega(\omega > 0)$ is a root of equation (28), then

$$-\omega^2 + m_1\omega\cos(\omega\tau)i + m_1\omega\sin(\omega\tau) + m_2\cos(2\omega\tau) - m_2\sin(2\omega\tau)i = 0$$

Separating the real and imaginary parts, we have

$$\begin{cases} -\omega^2 + m_1\omega\cos(\omega\tau) + m_2\cos(2\omega\tau) = 0 \\ m_1\omega\cos(\omega\tau) - m_2\sin(2\omega\tau) = 0 \end{cases} \tag{30}$$

Then we have

$$\begin{cases} (m_2 - \omega^2)\cos(\omega\tau) = 0 \\ (m_2 + \omega^2)\sin(\omega\tau) = m_1\omega \end{cases} \tag{31}$$

If $\cos(\omega\tau) = 0$, we have $\sin(\omega\tau) = 1$, so $\omega^2 - m_1\omega + m_2 = 0$ and

$$\omega_{1,2} = \frac{m_1 \pm \sqrt{m_1^2 - 4m_2}}{2}$$

$$\tau_{1,j} = \frac{1}{\omega_{1,2}}\left(\frac{\pi}{2} + 2j\pi\right), j = 0, 1, 2, \cdots$$

If $m_2 - \omega^2 = 0$, we have $\omega_3 = \sqrt{m_2}$,

$$\tau_{2,j} = \frac{1}{\omega_3}\arccos\left[\pm\frac{\sqrt{4m_2 - m_1^2}}{2\sqrt{m_2}}\right] + \frac{2j\pi}{\omega_3},$$

$$j = 0, 1, 2, \cdots$$

The model in 3.2 is complex, here we only compute the values of the delayed parameter, we will deeply investigate this theory in the future.

4 Numerical simulations

4.1 One player makes delayed decision

Numerical simulations are carried to show the stability of system (10). First we assume that the two insurance companies don't consider the delay τ In order to investigate the local stability properties of the Nash equilibrium conveniently, here we use certain value data to simulate the dynamics of the system. The parameters are taken to be $a = 5, b_1 = 4.5, b_2 = 5, d_1 = 0.7, d_2 = 0.6, v_1 = 0.5, v_2 = 0.5$, the marginal costs of two insurance companies are $c_1 = 0.002, c_2 = 0.001$, and the initial prices of their products are $p_1(0) = 0.3, p_2(0) = 0.4$.

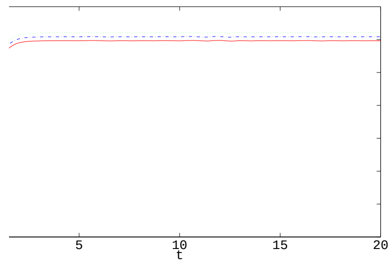

Figure 1: The orbits of p_1 with no delay,$d_1 = 0.7, 0.8, 0.9$

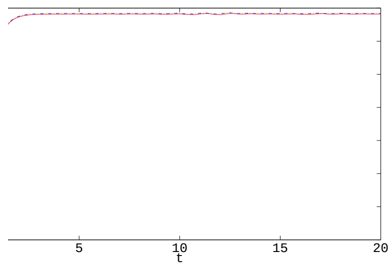

Figure 2: The orbits of p_2 with no delay,$d_1 = 0.7, 0.8, 0.9$

Figure 1 and 2 show that the orbits of p_1 and p_2 will approach to the equilibrium point $E_4(p_1^* = 0.5983, p_2^* = 0.5364)$. For $d_1 = 0.7$ the orbits are the red solid curves. We can easily observe the advantage of 1th insurance company. The cost and price of 1th insurance company are higher than the other's. The two players' profits are $\pi_1 = 1.5999, \pi_2 = 1.4332$.

From Fig.1 and 2, we can observe that if d_1 increasing, the Nash equilibrium point E_4 and two players' profits will be changed. For $d_1 = 0.8$, we can obtain $p_1^* = 0.6043, p_2^* = 0.5368, \pi_1 = 1.6323, \pi_2 = 1.4352$. For $d_1 = 0.9$, we can obtain $p_1^* = 0.6103, p_2^* = 0.5371, \pi_1 = 1.6650, \pi_2 = 1.4371$. It is obvious that as d_1 increases, the prices and profits will also increase. The parameter d_1 has greater impact on p_1 than p_2. We can conclude that the player of the duopoly game model should increase the extent of his own products substitute for the other's.Therefore, the products of 1th insurance company must contain some characteristics of the other's products. Then the parameter d_1 will increase and advantages will be obtained.

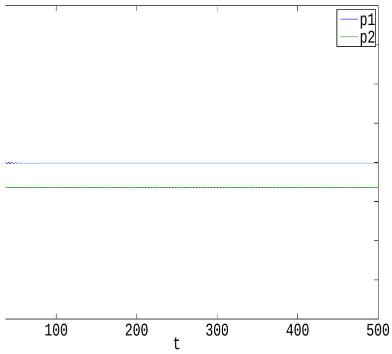

Figure 3: The orbits of p_1 and $p_2(\tau = 1.5)$

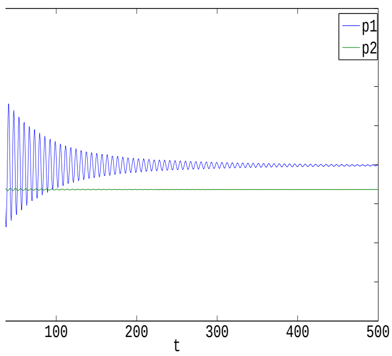

Figure 4: The orbits of p_1 and $p_2(\tau = 2.4994)$

Now we study the situation that 1th insurance company takes delayed decision into consideration. Suppose that $w = 0.45$ and the values of all parameters have not changed, together with (20), we can obtain $\tau_0 = 2.4994$. Fig.3– Fig.8 show the dynamics of system (10) for $\tau = 1.5, 2, 2.4, 2.4994, 2.6, 3$, respectively. If $\tau < \tau_0$, the Nash equilibrium point of system (10) is stable. As τ increases, it will take longer time to be stable. If $\tau = \tau_0$, Hopf bifurcation and period orbits are appeared. If $\tau > \tau_0$, the Nash equilibrium becomes unstable. As τ increases, the periods of solutions of system (10) will also increase.

When only one insurance company considers

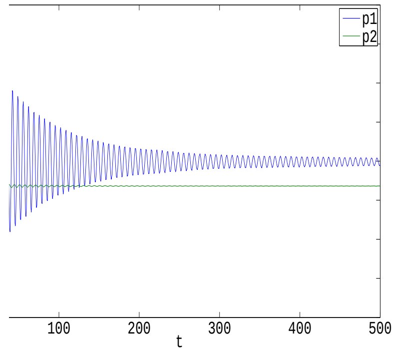

Figure 5: The orbits of p_1 and $p_2(\tau = 2.6)$

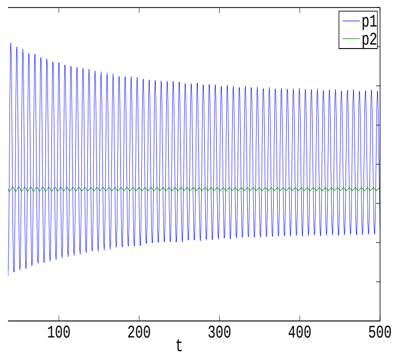

Figure 6: The orbits of p_1 and $p_2(\tau = 3)$

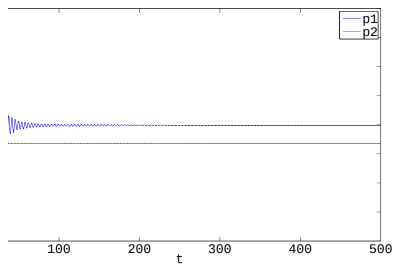

Figure 7: The orbits of p_1 and $p_2(\tau = 1.5, v_1 = 0.7)$

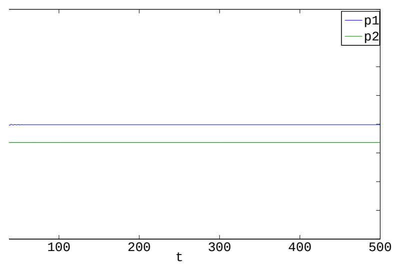

Figure 8: The orbits of p_1 and $p_2(\tau = 1.5, v_2 = 0.7)$

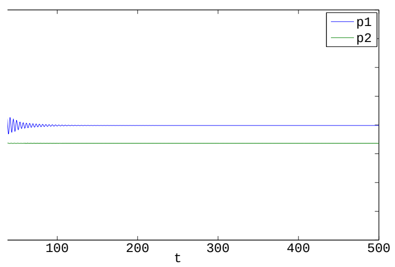

Figure 9: The orbits of p_1 and $p_2(\tau = 1.5, v_1 = v_2 = 0.7)$

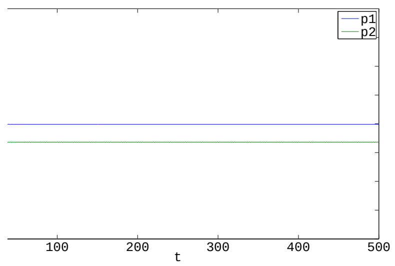

Figure 10: The orbits of p_1 and $p_2(\tau = 1.5, v_1 = v_2 = 0.4)$

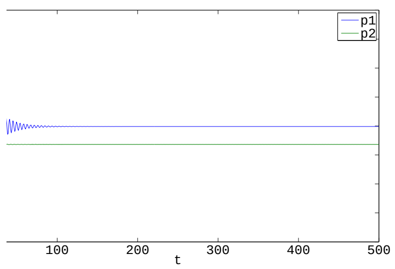

Figure 11: The orbits of p_1 and $p_2(\tau = 1.5, w = 0.4)$

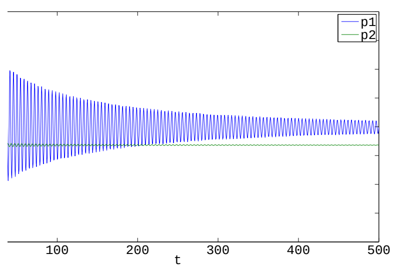

Figure 12: The orbits of p_1 and $p_2(\tau = 1.5, w = 0.36)$

delayed decisions, his own price and profit will be changed but it has almost no effect on the other company. As we know, the two players cannot control the insurance market if the period orbits occur. So we should decrease the value of parameter τ to make it stable quickly for $\tau < \tau_0$. When the insurance company need to consider the marginal profits of past time, it is easy to see that a appropriate value of parameter τ is very important to the whole insurance market.

From Fig.9-Fig.14, when we fix the value of τ , if we increase one player's speed of price adjustment or decrease the weight of the current price, the system will need longer time to be stable. Therefore, if insurance companies want to make delayed decisions, they must decrease the speed of adjustment or increase the weight of the current price. But the values of all the parameters must be appropriate.

4.2 Both players make delayed decision

Here we suppose both players have considered delayed decision.The parameters are taken to be $a = 5, b_1 = 4.5, b_2 = 5, d_1 = 0.7, d_2 = 0.6, v_1 = 0.2, v_2 = 0.2$, the marginal costs of two insurance companies are $c_1 = 0.002, c_2 = 0.001$, and the initial prices of their products are $p_1(0) = 0.3, p_2(0) = 0.4$. Maybe insurance companies cannot obtain the latest information on price when they are making decision, so the game is only based on the price of $t - \tau$ time. and we have $w = w' = 0$.Here the numerical simulations are based on the varying values of delayed parameter.

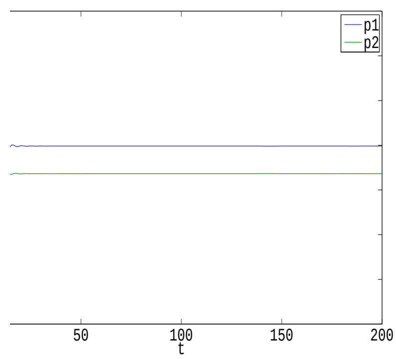

Figure 13: The orbits of p_1 and $p_2(\tau = \tau' = 1)$

From Fig.13- Fig.20, we can see that as the increase of the delayed parameter, the system will take longer time to the reach the equilibrium point. It will be unstable in the end. Therefore, the two insurance companies should get the latest price information as

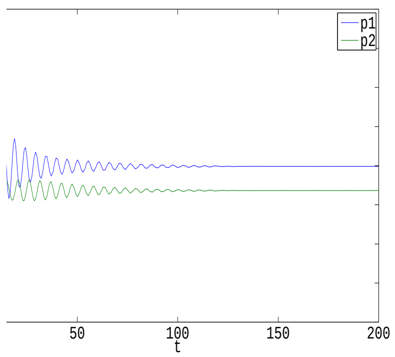

Figure 14: The orbits of p_1 and $p_2(\tau = \tau' = 1.3)$

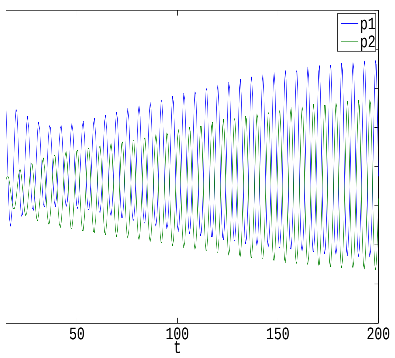

Figure 15: The orbits of p_1 and $p_2(\tau = \tau' = 1.4)$

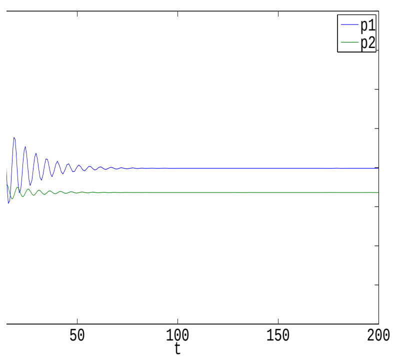

Figure 16: The orbits of p_1 and $p_2(\tau = 1.3, \tau' = 1)$

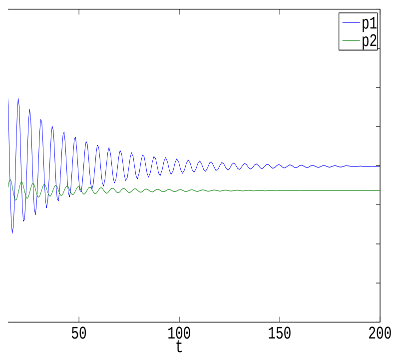

Figure 17: The orbits of p_1 and $p_2(\tau = 1.4, \tau' = 1)$

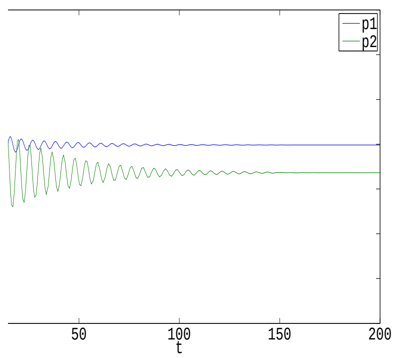

Figure 18: The orbits of p_1 and $p_2(\tau = 1, \tau' = 1.4)$

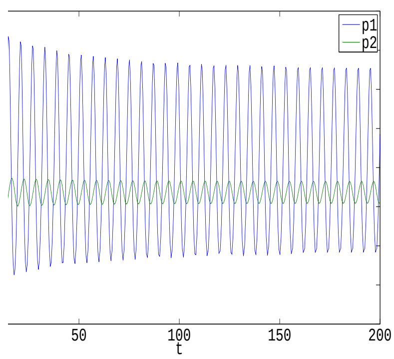

Figure 19: The orbits of p_1 and $p_2(\tau = 1.5, \tau' = 1)$

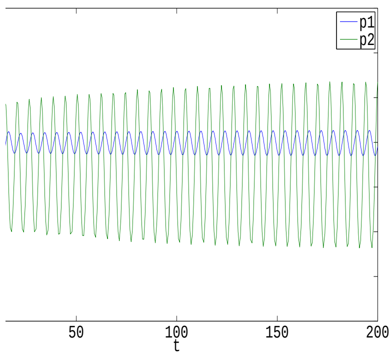

Figure 20: The orbits of p_1 and $p_2(\tau = 1, \tau' = 1.5)$

soon as possible and make a reasonable price decision. If the value of delayed parameter of one player is not appropriate, then the system will also be unstable. It will have a large negative impact on the insurance market and the insurance market will become chaotic. So better control the values of delayed parameter of both players is the most important.

5 Conclusion

This paper investigates the dynamic model of a duopoly game with delay in insurance market. Then we study three situations: neither of two players make delayed decision; one player makes delayed decision; both players make delayed decision. According to the analysis and numerical simulations of the differential equation model, we can conclude:

Delayed decisions cannot change Nash equilibrium point of the system. But they may change the stability of the system and have a greater impact on the player who considers delayed decisions.

As d_i increasing, the profits of two insurance company will also increase. And d_i has a greater impact on ith insurance company.

It is better to control the insurance market for the player with delayed decisions, if he increases the speed of price adjustment or decreases the weight of the current price.

The two insurance companies should get the latest price information as soon as possible and choose one reasonable value of delayed parameter. It is significant to the development of the whole insurance market.

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