Algorithms for Approximations in MGRSM Based on Maximal Compatible Granules

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Abstract—This paper emphasizes studying on the properties of approximations in rough set and multi-granulation rough set models based on maximal compatible classes as primitive ones in which any two objects are mutually compatible, obtains several theorem results, proposes and designs the upper and lower approximation computation algorithms in multi-granulation rough set model. It verifies the correctness of algorithms by examples and experiments.

Keywords- incomplete information system; rough set model; maximal compatible class; algorithm; multi-granulation

1 Introduction

Rough set theory (RST for short) , put forward Pawlak in 1982 ([1]), is an efficient mathematics tool for complete information system processing and for intelligent system processing characterized by uncertainty, vague, uncertainty ([2],[3]). It is now widely applied in many research fields such as decision making, pattern recognition, knowledge discovery, and so on. Incomplete information systems (IIS for short), usually studied by constructing non-indiscernibility relation, instead of indiscernibility relation, such as tolerance relation suggested by M. Kryszkiewicz([4]), similarity relation put forward by J.Stefanowski([5]), limited tolerant relation proposed by W.Guoying([6]), and so on ([7],[8]) to deal with due to the existence of null or missing attribute values. In this way, the rough set model is extended and gotten more and more wide and better investigated. These days, researchers further suggest multi-granulation rough set models from the view point of granules and different perspectives. Combining rough set model with granule and multi-granulation to study has been become a hot topic in the related academic regions.

Concerning granule view with maximal compatible classes as primitive granules based on tolerance relation to promote the model to handle IIS([9]), the present paper proceeds with some new work about the definitions of optimistic and pessimistic lower and upper approximations in single attribute subset through this new granule view. It introduces the related computation methods of finding approximation into multi-granulation rough set model (MGRSM) in both optimistic and pessimistic cases. It also discusses the properties of them and the relationships between single and multiple granulation models and between the optimistic and pessimistic cases. The main task of it is to analyze and design algorithms for solving lower and upper approximations in multi-granulation rough set model([10],[11]). It brings even a new expect to produce a new approach for transacting multi-granulation RST problems in IIS and people may us them to acquire determinative and possible decision rules or knowledge from massive information system conveniently and efficiently in the future. Through proofs, examples and experiments, it verifies that the knowledge acquisition approach is validated. So the work done here is of great importance.

2 Definitions

An IIS is a quadruple $S=(U,AT,V,f)$ According to the definitions in ([4]). $TOL(A)=\{(x,y)\in U\times U: \forall a\in A, f_a(x)=f_a(y)\vee f_a(x)=*\vee f_a(y)=*\}$ is called the tolerance relation derived by $A\subset AT$.

Definition 1. Let $A\subset AT$. $C(A)$ is defined as

$$C(A)=\{X\subseteq U:\max\{X^2\subseteq TOL(A)\}\}$$

where max means that the operation is operator $\subseteq$ on a series of sets. $C(A)$ also forms a cover or a knowledge expression system on U .

Definition 2. Let $x\in U$, $A\subseteq AT$. The compatible class(es) containing x is defined as

$$C_A(x)=\max\{X:x\in X, X^2\subseteq TOL(A)\}$$

where max also means $\subseteq$. Because the compatible class(es) containing x may be not unique for some $x\in U$, $C_A(x)$ may be a set of compatible classes .

It can be easily proved that $C(A)=\cup_{x\in U}C_A(x)$.

Definition 3. The upper and lower approximations for $X\subseteq U$ in knowledge expression system $C(A)$ are respectively defined as follows:

$\underline{A}^o(X)=\{x\in U:\exists C\in C(A)(x\in C\wedge C\subseteq X)\}$;

$\overline{A}^o(X)=\{x\in U:\exists C\in C(A)(x\in C\wedge(C\cap X\neq\varnothing))\}$.

The optimistic approximation precision for $X\subseteq U$ in $C(A)$ is $|\underline{A}^o(X)|/|\overline{A}^o(X)|$.

Definition 4. The pessimist upper and lower approximations for $X\subseteq U$ in knowledge expression system $C(A)$ are respectively defined as follows:

$\underline{A}^p(X)=\{x\in U:\forall C\in C(A)(x\in C\rightarrow(C\subseteq X))\}$;

$\overline{A}^p(X)=\{x\in U:\forall C\in C(A)(x\in C\rightarrow(C\cap X\neq\varnothing))\}$.

The pessimistic approximation precision for $X\subseteq U$ in $C(A)$ is $|\underline{A}^p(X)|/|\overline{A}^p(X)|$.

In the literature([10],[11]), Qian et al. proposed a multi-granulation rough set model, which includes optimistic multi-granulation rough set and pessimistic multi-granulation rough set([12]). Combined with our above compatible granules, we can introduce them into multi-granulation rough set using knowledge expression system $C(A)$, we can also obtain related new research results.

Definition 5. Let $A_1, A_2, ..., A_m \subseteq AT$ be m attribute subsets. for $\forall X \subseteq U, M = \{1,2,...,m\}$, the optimistic multi-granulation lower and upper approximations of X with respect to $A_1, A_2, ..., A_m$ are respectively defined as:

$$\underline{\sum_{i=1}^{m} A_i}^{o}(X) = \{x \in U : \exists i \in M(\exists C \in C(A_i)(x \in C \wedge (C \subseteq X)))\};$$

$$\overline{\sum_{i=1}^{m} A_i}^{o}(X) = \{x \in U : \exists i \in M(\exists C \in C(A_i)(x \in C \wedge (C \cap X \neq \varnothing)))\}.$$

The optimistic multi-granulation boundary region of X is

$$Bn^{o}_{\sum_{i=1}^{m} A_i}(X) = \overline{\sum_{i=1}^{m} A_i}^{o}(X) - \underline{\sum_{i=1}^{m} A_i}^{o}(X).$$

The optimistic approximation precision for $X \subseteq U$ in multi-granulation model with respect to $C(A)$ is $|\underline{\sum_{i=1}^{m} A_i}^{o}(X)| / |\overline{\sum_{i=1}^{m} A_i}^{o}(X)|$.

Definition 6. The pessimistic multi-granulation lower and upper approximations are respectively defined as:

$$\underline{\sum_{i=1}^{m} A_i}^{p}(X) = \{x \in U : \forall i \in M(\forall C \in C(A_i)(x \in C \to C \subseteq X))\};$$

$$\overline{\sum_{i=1}^{m} A_i}^{p}(X) = \{x \in U : \forall i \in M(\forall C \in C(A_i)(x \in C \to C \cap X \neq \varnothing))\}.$$

The pessimistic multi-granulation boundary region of X is

$$Bn^{p}_{\sum_{i=1}^{m} A_i}(X) = \overline{\sum_{i=1}^{m} A_i}^{p}(X) - \underline{\sum_{i=1}^{m} A_i}^{p}(X).$$

The pessimistic approximation precision for $X \subseteq U$ in multi-granulation model with respect to $C(A)$ is $|\underline{\sum_{i=1}^{m} A_i}^{p}(X)| / |\overline{\sum_{i=1}^{m} A_i}^{p}(X)|$.

3 Properties and relationships

Theorem 1. (i) $\underline{A}^{o}(X) = \cup_{C \in C(A), C \subseteq X} C$;

(ii) $\overline{A}^{o}(X) = \cup_{C \in C(A), C \cap X \neq \varnothing} C$.

PROOF. (i) $y \in \underline{A}^{o}(X) = \{x \in U : \exists C \in C(A)(x \in C \wedge C \subseteq X)\} \Rightarrow \exists C \in C(A)(y \in C \wedge C \subseteq X)$ $\Rightarrow y \in \cup_{C \in C(A), C \subseteq X} C$, so $\underline{A}^{o}(X) \subseteq \cup_{C \in C(A), C \subseteq X} C$.

Conversely, For any $y \in \cup_{C \in C(A), C \subseteq X} C$ $\Rightarrow \exists C \in C(A)(C \subseteq X)$ such that $y \in C$.

So $y \in \{x \in U : \exists C \in C(A)(x \in C \wedge C \subseteq X)\} = \underline{A}^{o}(X)$. That is $\cup_{C \in C(A), C \subseteq X} C \subseteq \underline{A}^{o}(X)$.

Thus $\underline{A}^{o}(X) = \cup_{C \in C(A), C \subseteq X} C$.

The result of this theorem implies that the definition of $\underline{A}^{o}(X)$ can be equivalently defined by $\cup_{C \in C(A), C \subseteq X} C$.

(ii) $y \in \overline{A}^{o}(X) = \{x \in U : \exists C \in C(A)(x \in C \wedge C \cap X \neq \varnothing)\}$
$\Rightarrow \exists C \in C(A)(y \in C \wedge C \cap X \neq \varnothing)$
$\Rightarrow y \in \cup_{C \in C(A), C \cap X \neq \varnothing} C$.

So $\overline{A}^{o}(X) \subseteq \cup_{C \in C(A), C \cap X \neq \varnothing} C$.

Conversely, $y \in \cup_{C \in C(A), C \cap X \neq \varnothing} C$
$\Rightarrow \exists C \in C(A)(C \cap X \neq \varnothing)$
such that $y \in C$.

So $y \in \{x \in U : \exists C \in C(A)(x \in C \wedge C \cap X \neq \varnothing)\} = \underline{A}^{o}(X)$. That is $\cup_{C \in C(A), C \cap X \neq \varnothing} C \subseteq \overline{A}^{o}(X)$. Thus, $\overline{A}^{o}(X) = \cup_{C \in C(A), C \cap X \neq \varnothing} C$.

The result of this theorem implies that the definition of $\overline{A}^{o}(X)$ can be equivalently defined by $\cup_{C \in C(A), C \cap X \neq \varnothing} C$.

Theorem 2. (i) $\sim \underline{A}^{o}(\sim X) = \overline{A}^{p}(X)$;

(ii) $\sim \underline{A}^{o}(\sim X) \subseteq \overline{A}^{o}(X)$.

PROOF. (i) $y \in \sim \underline{A}^{o}(\sim X) \Leftrightarrow y \notin \underline{A}^{o}(\sim X)$
$\Leftrightarrow y \notin \{x \in U : \exists C \in C(A)(x \in C \wedge C \subseteq \sim X)\}$.
$\neg \exists C \in C(A)(y \in C \wedge C \subseteq \sim X)$
$\Leftrightarrow \forall C \in C(A)(\neg(y \in C) \vee C \not\subseteq \sim X)$
$\Leftrightarrow \forall C \in C(A)(\neg(y \in C) \vee (C \cap X \neq \varnothing))$
$\Leftrightarrow \forall C \in C(A)(y \in C \to C \cap X \neq \varnothing)$. So $\sim \underline{A}^{o}(\sim X)$ $= \{x \in U : \forall C \in C(A)(x \in C \to C \cap X \neq \varnothing)\}$ $= \overline{A}^{p}(X)$.

(ii) $y \in \sim \underline{A}^{o}(\sim X) \Leftrightarrow y \notin \cup_{C \in C(A), C \subseteq \sim X} C$
$\Leftrightarrow \forall C \in C(A), C \subseteq \sim X, y \notin C$
$\Rightarrow y \in \cup_{C \in C(A), C \cap X \neq \varnothing} C = \overline{A}^{o}(X)$. So
$\sim \underline{A}^{o}(\sim X) \subseteq \overline{A}^{o}(X)$.

Form (ii) of Theorem 3, we can immediately obtain that $\sim \overline{A}^{o}(\sim X) \subseteq \underline{A}^{o}(X)$.

Theorem 3. $\sim \underline{A}^{p}(\sim X) \subseteq \overline{A}^{p}(X)$.

PROOF. $y\in \sim \underline{A}^p(\sim X) \Leftrightarrow y\notin \underline{A}^p(\sim X)$
$\Leftrightarrow y\notin \{x\in U: \forall C\in C(A)(x\in C\to (C\subseteq \sim X))\}$.
Because $\neg\forall C\in C(A)(y\in C\to (C\subseteq \sim X))$
$\Leftrightarrow \exists C\in C(A)\neg(\neg(y\in C)\vee C\subseteq \sim X))$
$\Leftrightarrow \exists C\in C(A)\neg(\neg(y\in C)\vee C\cap X=\emptyset))$
$\Leftrightarrow \forall C\in C(A)(y\in C\wedge C\cap X\neq\emptyset)$
$\Rightarrow \forall C\in C(A)(y\in C\to C\cap X\neq\emptyset)$, we have
$\sim \underline{A}^p(\sim X)\subseteq \overline{A}^p(X)$.

Theorem 4. $\sim \overline{A}^p(\sim X)=\underline{A}^o(X)$.

PROOF. $y\in \sim \overline{A}^p(\sim X)\Leftrightarrow y\notin \overline{A}^p(\sim X)$
$\Leftrightarrow y\notin \{x\in U: \forall C\in C(A)(x\in C\to (C\cap \sim X\neq\emptyset))\}$.
Because $\neg\forall C\in C(A)(x\in C\to (C\cap\sim X\neq\emptyset))$
$\Leftrightarrow \neg\forall C\in C(A)(\neg(x\in C)\vee (C\cap \sim X\neq\emptyset))$
$\Leftrightarrow \exists C\in C(A)((x\in C)\wedge (C\cap \sim X=\emptyset))$
$\Leftrightarrow \exists C\in C(A)((x\in C)\wedge (C\subseteq X))$.
So $\sim \overline{A}^p(\sim X)=\underline{A}^o(X)$.

The proof of this theorem can also be obtained immediately from the (i) of Theorem 3.

Theorem 5. (i) $\underline{\sum_{i=1}^m A_i}^o(X)=\cup_{i=1}^m \underline{A_i}^o(X)$;

(ii) $\overline{\sum_{i=1}^m A_i}^o(X)=\cup_{i=1}^m \overline{A_i}^o(X)$.

PROOF. (i) $y\in \underline{\sum_{i=1}^m A_i}^o(X)=\{x\in U: \exists i\in M$
$(\exists C\in C(A_i)\ (x\in C\wedge (C\subseteq X)))\}$
$\Rightarrow \exists i\in M(\exists C\in C(A_i)(y\in C\wedge (C\subseteq X)))$
$\Rightarrow y\in \underline{A_i}^o(X)\Rightarrow y\in \cup \underline{A_i}^o(X)$.

Conversely, for any $y\in \cup_{i=1}^m \underline{A_i}^o(X)$
$\Rightarrow \exists i\in M$, $y\in \underline{A_i}^o(X)$
$\Rightarrow \exists C\in C(A_i)(y\in C\wedge (C\subseteq X))$
$\Rightarrow y\in \underline{\sum_{i=1}^m A_i}^o(X)$.

Therefore, $\underline{\sum_{i=1}^m A_i}^o(X)=\cup_{i=1}^m \underline{A_i}^o(X)$.

(ii) $y\in \overline{\sum_{i=1}^m A_i}^o(X)=$
$\{x\in U: \exists i\in M(\exists C\in C(A_i)(x\in C\wedge (C\cap X\neq\emptyset)))\}$
$\Rightarrow \exists i\in M(\exists C\in C(A_i)(y\in C\wedge (C\cap X\neq\emptyset)))$
$\Rightarrow y\in \overline{A_i}^o(X)\Rightarrow y\in \cup_{i=1}^m \overline{A_i}^o(X)$.

Conversely, For any $y\in \cup_{i=1}^m \overline{A_i}^o(X)$
$\Rightarrow \exists i\in M(\ y\in \overline{A_i}^o(X))$
$\Leftrightarrow \exists i\in M(\ \exists C\in C(A_i)(x\in C\wedge (C\cap X\neq\emptyset)))$
$\Rightarrow y\in \overline{\sum_{i=1}^m A_i}^o(X)$.

Therefore, $\overline{\sum_{i=1}^m A_i}^o(X)=\cup_{i=1}^m \overline{A_i}^o(X)$.

Theorem 6. (i) $\sim \underline{\sum_{i=1}^m A_i}^o(\sim X)=\overline{\sum_{i=1}^m A_i}^p(X)$;

(ii) $\sim \underline{\sum_{i=1}^m A_i}^p(\sim X)=\overline{\sum_{i=1}^m A_i}^o(X)$

PROOF. (1) $y\in \sim \underline{\sum_{i=1}^m A_i}^o(\sim X)$
$\Leftrightarrow y\notin \{x\in U: \exists i\in M(\exists C\in C(A_i)(x\in C\wedge (C\subseteq \sim X)))\}$
$\Leftrightarrow y\notin \{x\in U: \exists i\in M(\exists C\in C(A_i)(x\in C$
$\wedge (C\cap X=\emptyset)))\}$.

Because $\neg\exists i\in M(\exists C\in C(A_i)(y\in C\wedge (C\cap X=\emptyset)))$
$\Leftrightarrow \forall i\in M(\forall C\in C(A_i)(y\in C\to C\cap X\neq\emptyset))$,
therefore, $\sim \underline{\sum_{i=1}^m A_i}^o(\sim X)=$
$\{x\in U: \forall i\in M(\forall C\in C(A_i)(x\in C\to C\cap X\neq\emptyset))\}$
$=\overline{\sum_{i=1}^m A_i}^p(X)$.

(ii) $y\in \sim \underline{\sum_{i=1}^m A_i}^p(\sim X)$
$\Leftrightarrow y\notin \{x\in U: \forall i\in M(\forall C\in C(A_i)(x\in C\to C\subseteq \sim X))\}$.
Because $\neg\forall i\in M(\forall C\in C(A_i)(x\in C\to C\subseteq \sim X))$
$\Leftrightarrow \exists i\in M(\exists C\in C(A_i)(\neg(\neg(x\in C)\vee (C\subseteq \sim X))))$
$\Leftrightarrow \exists i\in M(\exists C\in C(A_i)((x\in C)\wedge (C\not\subseteq \sim X)))$
$\Leftrightarrow \exists i\in M(\exists C\in C(A_i)(x\in C\wedge C\cap X\neq\emptyset))$.
Therefore,
$\sim \underline{\sum_{i=1}^m A_i}^p(\sim X)=\{x\in U: \exists i\in M(\exists C\in C(A_i)$
$(x\in C\wedge C\cap X\neq\emptyset))\}=\overline{\sum_{i=1}^m A_i}^o(X)$.

Lemma. (i) $\overline{\sum_{i=1}^m A_i}^p(X)=\sim \underline{\sum_{i=1}^m A_i}^o(\sim X)$;

(ii) $\underline{\sum_{i=1}^m A_i}^p(X)=\sim \overline{\sum_{i=1}^m A_i}^o(\sim X)$.

The two equations in this lemma can be easily proved through Theorem 7.

Theorem 7. Let $S=(U,AT,V,f)$ be an incomplete information system and $A_i \subseteq AT(i=1,2,\ldots,m)$ be m attribute subsets. Then for any $X \subseteq U$, we have

(i) $\underline{\sum_{i=1}^{m} A_i}^{p}(X) = \cap_{i=1}^{m} \underline{A_i}^{p}(X)$;

(ii) $\overline{\sum_{i=1}^{m} A_i}^{p}(X) = \cap_{i=1}^{m} \overline{A_i}^{p}(X)$.

PROOF. (i) Because $\underline{\sum_{i=1}^{m} A_i}^{p}(X) = \{x \in U : \forall i \in M$

$(\forall C \in C(A_i)(x \in C \to C \subseteq X))\}$

$\Leftrightarrow \{x \in U : \forall C \in C(A_i)(x \in C \to C \subseteq X)(i=1,2,\ldots,m)\}$

$\Leftrightarrow \cap_{i=1}^{m} \{x \in U : \forall C \in C(A_i)(x \in C \to C \subseteq X)\}$

$\Leftrightarrow \cap_{i=1}^{m} \underline{A_i}^{p}(X)$.

therefore, this theorem is held.

(ii) The proof of it is similar to (i).

Theorem 8. Let $S=(U,AT,V,f)$ be an incomplete information system and $A_i \subseteq AT(i=1,2,\ldots,m)$ be m attribute subsets. Then for any $X \subseteq U$, we have

(i) $\underline{\sum_{i=1}^{m} A_i}^{o}(X) \subseteq X \subseteq \overline{\sum_{i=1}^{m} A_i}^{o}(X)$;

(ii) $\underline{\sum_{i=1}^{m} A_i}^{o}(\varnothing) = \overline{\sum_{i=1}^{m} A_i}^{o}(\varnothing) = \varnothing$,

$\underline{\sum_{i=1}^{m} A_i}^{o}(U) = \overline{\sum_{i=1}^{m} A_i}^{o}(U) = U$;

(iii) $\underline{\sum_{i=1}^{m} A_i}^{o}(\underline{\sum_{i=1}^{m} A_i}^{o}(X)) = \underline{\sum_{i=1}^{m} A_i}^{o}(X)$,

$\overline{\sum_{i=1}^{m} A_i}^{o}(\overline{\sum_{i=1}^{m} A_i}^{o}(X)) = \overline{\sum_{i=1}^{m} A_i}^{o}(X)$;

(iv) $\underline{\sum_{i=1}^{m} A_i}^{o}(\sim X) = \sim \overline{\sum_{i=1}^{m} A_i}^{p}(X)$,

$\overline{\sum_{i=1}^{m} A_i}^{o}(\sim X) = \sim \underline{\sum_{i=1}^{m} A_i}^{p}(X)$.

Theorem 9. Let $S=(U,AT,V,f)$ be an incomplete information system and $A_i \subseteq AT(i=1,2,\ldots,m)$ be m attribute subsets. Then for any $X \subseteq U$, we have

(i) $\underline{\sum_{i=1}^{m} A_i}^{p}(X) \subseteq X \subseteq \overline{\sum_{i=1}^{m} A_i}^{p}(X)$;

(ii) $\underline{\sum_{i=1}^{m} A_i}^{p}(\varnothing) = \overline{\sum_{i=1}^{m} A_i}^{p}(\varnothing) = \varnothing$,

$\underline{\sum_{i=1}^{m} A_i}^{p}(U) = \overline{\sum_{i=1}^{m} A_i}^{p}(U) = U$;

(iii) $\underline{\sum_{i=1}^{m} A_i}^{p}(\underline{\sum_{i=1}^{m} A_i}^{p}(X)) = \underline{\sum_{i=1}^{m} A_i}^{p}(X)$,

$\overline{\sum_{i=1}^{m} A_i}^{p}(\overline{\sum_{i=1}^{m} A_i}^{p}(X)) = \overline{\sum_{i=1}^{m} A_i}^{p}(X)$;

(iv) $\underline{\sum_{i=1}^{m} A_i}^{p}(\sim X) = \sim \overline{\sum_{i=1}^{m} A_i}^{o}(X)$,

$\overline{\sum_{i=1}^{m} A_i}^{p}(\sim X) = \sim \underline{\sum_{i=1}^{m} A_i}^{o}(X)$.

Example 1. An incomplete information system is shown in Table 1, where *Price*, *Mileage*, *Size*, *Max-Speed* are conditional attributes, d is a decision attribute. For convenience, we use P, M, S, X to represent *Price*, *Mileage*, *Size*, *Max-Speed* in short in Table 1. Let $A=AT=\{P,M,S,X\}$. We obtain: $C(A)=\{\{1\},\{2,6\},\{3\},\{4,5\},\{5,6\}\}$. $C_A(1)=\{\{1\}\}$, $C_A(2)=\{\{2,6\}\}$, $C_A(3)=\{\{3\}\}$, $C_A(4)=\{\{4,5\}\}$, $C_A(5)=\{\{4,5\},\{5,6\}\}$, $C_A(6)=\{\{2,6\},\{5,6\}\}$. Let $X=d_{good}=\{1,2,4,6\}$. We get:

$\underline{A}^{o}(X) = \{1,2,6\}$, $\overline{A}^{o}(X) = \{1,2,4,5,6\}$, $\underline{A}^{p}(X) = \{x \in U : \forall C \in C(A)(x \in C \to (C \subseteq X))\} = \{1,2\}$,

$\overline{A}^{p}(X) = \{x \in U : \forall C \in C(A)(x \in C \to (C \cap X \neq \varnothing))\} = \{1,2,4,5,6\}$

The optimistic approximation precision for $X \subseteq U$ in $C(A)$ is $|\underline{A}^{o}(X)| / |\overline{A}^{o}(X)|$=3/5=0.5, The pessimistic approximation precision for $X \subseteq U$ in $C(A)$ is $|\underline{A}^{p}(X)| / |\overline{A}^{p}(X)|$=2/5.

Table 1. An IIS about cars

Car	P	M	S	X	d
1	high	low	full	low	good
2	low	*	full	low	good
3	*	*	compact	low	poor
4	high	*	full	high	good
5	*	*	full	high	excellent
6	low	high	full	*	good

Example 2. Still use the incomplete information system shown in Table 1. Let $A_1=\{P,M\}$, $A_2=\{S,X\}$, $A_3=\{M,X\}$.

Then $C(A_1)=\{\{1,3,4,5\},\{2,3,5,6\}\}$, $C(A_2)=\{\{1,2,6\},\{3\},\{4,5,6\}\}$, $C(A_3)=\{\{1,2,3\},\{2,3,6\},\{4,5,6\}\}$.

$\underline{\sum_{i=1}^{m} A_i}^{o}(X) = \cup \underline{A_i}^{o}(X)$=$\{1,2,6\}$,

$\overline{\sum_{i=1}^{m} A_i}^{o}(X) = \cup_{i=1}^{m} \overline{A_i}^{o}(X)$=$\{1,2,3,4,5,6\}$.

$\underline{\sum_{i=1}^{m} A_i}^{p}(X) = \{x \in U : \forall i \in M(\forall C \in C(A_i)(x \in C$

$\to C \subseteq X))\} = \cap_{i=1}^{m} \underline{A_i}^{p}(X) = \varnothing$,

$\overline{\sum_{i=1}^{m} A_i}^{p}(X) = \{x \in U : \forall i \in M(\forall C \in C(A_i)(x \in C$

$\rightarrow C \cap X \neq \varnothing))\} = \cap_{i=1}^{m} \overline{A_i}^{p}(X)$ ={1,2,4,5,6}.

The optimistic approximation precision for $X \subseteq U$ in multi-granulation model with respect to $C(A)$ is $|\underline{\sum_{i=1}^{m} A_i}^{o}(X)| / |\overline{\sum_{i=1}^{m} A_i}^{o}(X)|$=3/6. The pessimistic approximation precision for $X \subseteq U$ in multi-granulation model with respect to $C(A)$ is

$|\underline{\sum_{i=1}^{m} A_i}^{p}(X)| / |\overline{\sum_{i=1}^{m} A_i}^{p}(X)|$=0.

4 Algorithms for approximations in multi-granulation model

Using maximal compatible class $C(A_i)$ in $A_i\,(M = \{1,2,...,m\})$, the algorithm is referred in ([13]) as basic granules, we can not hardly design related algorithms to compute lower and upper approximations of a given subset in optimistic multi-granulation and pessimistic multi-granulation rough set models proposed in the present paper.

Let $U = \{x_i \mid i = 1,2,...,n\}$. We use $M_s = (m_{ij}^{(s)})_{n\times n}(s = 1,2,...,m)$, where $m_{ij}^{(s)}$ equals 1, if $(x_i, x_j) \in T(A_i)$, 0 otherwise, as adjacent matrix for attribute subset A_i and a 2-dimensional binary matrix $P_{l\times n}^{(s)}$, where $P^{(s)}(v, j) = 1$ means that x_j belongs to the v-th maximal compatible class, 0 means not, v=1,2,…,l, to store all maximal compatible classes, where $l <= (n*n)/2$, but l may be greater than n in some cases. So we set l be an enough big positive integer. Suppose there are totally k maximal compatible classes. After finishing computation, they are stored in the first k rows of $P_{l\times n}^{(s)}$, where $k \le l$.

Let $P^{(1)}, P^{(2)},..., P^{(m)}$, which are obtained according to algorithm A respectively, be respectively the maximal compatible class matrices of $A_1, A_2,..., A_m \subseteq AT$. We can firstly design the algorithm to find the optimistic multi-granulation lower approximations of $X \subseteq U$.

Algorithm 1: Finding the optimistic multi-granulation lower approximation of X, i.e. $\underline{\sum_{i=1}^{m} A_i}^{o}(X)$ $= \{x \in U : \exists i \in M(\exists C \in C(A_i)(x \in C \wedge (C \subseteq X)))\}$.

Input: $P^{(i)}$ (i=1,2,…,m): m matrices; Y: a coded $1\times n$ matrix, representing X, $Y[i]$=1 means $x_i \in X$, 0 means not.

Initialization: $T = [0,0,...,0]$, an $1\times n$ matrix;

Description:
```
for ( int i=0; i<m; i++ )
    for (j=0; j<k; j++)
    {   int tag=1;
        for ( int u=0; u<n; u++ )
          if (P(i)[j][u]==1&& Y[u]==1)
              continue;
          else{ tag=0; break; }
        if (tag==1)   T = P(i)[j] ∨ T ;
    }
```

Output: T, T is the lower approximations of X

The time complexity of it is $O(mnk)$.

Now we design the algorithm to find the optimistic multi-granulation upper approximation of $X \subseteq U$.

Algorithm 2: Finding the optimistic multi-granulation upper approximation of X, i.e. $\overline{\sum_{i=1}^{m} A_i}^{o}(X) =$ $\{x \in U : \exists i \in M(\exists C \in C(A_i)(x \in C \wedge (C \cap X \neq \varnothing)))\}$.

Input: $P^{(i)}$ (i=1,2,…,m): m matrices; Y: a coded $1\times n$ matrix, representing X, $Y[i]$=1 means $x_i \in X$, 0 means not.

Initialization: $T = [0,0,...,0]$, an $1\times n$ matrix;

Description:
```
for ( int i=0; i<m; i++ )
    for (j=0; j<k; j++)
    {   int tag=0;
        for ( int u=0; u<n; u++ )
          if (P(i)[j][u]==1&& Y[u]==1)
            { tag=1;
              break ;
            }
        if (tag==1)  T = P(i)[j] ∨ T ;
    }
```

Output: T, T is the upper approximations of X

The time complexity of it is $O(mnk)$.

We then design the algorithm to find the pessimistic multi-granulation lower approximation of $X \subseteq U$.

Algorithm 3: Finding the pessimistic multi-granulation lower approximations of X, i.e. $\underline{\sum_{i=1}^{m} A_i}^{p}(X) = \{x \in U : \forall i \in M(\forall C \in C(A_i)(x \in C$ $\rightarrow C \subseteq X))\}$. But from the (ii) of the lemma in the above, we can calculate $\underline{\sum_{i=1}^{m} A_i}^{p}(X)$ by

$\underline{\sum_{i=1}^{m} A_i}^{p}(X) = \sim \overline{\sum_{i=1}^{m} A_i}^{o}(\sim X)$ using **Algorithm 2**.

Input: $P^{(i)}$ (i=1,2,…,m): m matrices; Y: a coded $1\times n$ matrix, representing X, $Y[i]$=1 means $x_i \in X$, 0 means not.

Initialization: $T = [0,0,...,0]$, an $1\times n$ matrix;

Description:

a) Let Y be the reverse code of Y, that is, Y=~Y;

b) Getting the optimistic multi-granulation upper approximation T of Y by calling **Algorithm 2**;

d) T=~T;

Output: T, T is the pessimistic multi-granulation lower approximation of X.

The time complexity of it is $O(mnk)$.

Now we finally design the algorithm to find the pessimistic multi-granulation upper approximations of $X \subseteq U$.

Algorithm 4: Finding the pessimistic multi-granulation upper approximation of X , i.e.

$\overline{\sum_{i=1}^{m} A_i}^{p}(X) = \{x \in U : \forall i \in M(\forall C \in C(A_i)(x \in C \rightarrow C \cap X \neq \varnothing))\}$. But from the (i) of the lemma in the above, we can calculate $\overline{\sum_{i=1}^{m} A_i}^{p}(X)$ by

$\overline{\sum_{i=1}^{m} A_i}^{p}(X) = \sim \underline{\sum_{i=1}^{m} A_i}^{o}(\sim X)$ using **Algorithm 1**.

Input: P_i (i=1,2,…,m): m matrices; Y: a coded $1 \times n$ matrix, representing X,Y[i]=1 means xi∈X,0,means not.

Initialization: $T = [0, 0, ..., 0]$, an $1 \times n$ matrix;

Description:

a) Let Y be the reverse of Y , that is, Y=~Y;

b) Getting the optimistic multi-granulation lower approximation T of Y by calling **Algorithm 1**;

d) T=~T;

Output: T , T is the pessimistic multi-granulation upper approximation of X .

The time complexity of it is $O(mnk)$.

5 Conclusions

Using maximal compatible classes as primitive granules ([9]), this paper defines $C(A)$ as a knowledge representing system and the optimistic and pessimistic lower and upper approximations based on $C(A)$. It extends single granulation rough set model to multi-granulation model. It studies properties of the two kinds of approximations in single granulation rough set model and multi-granulation model, and discusses the relationships of the approximations in both models. Using the relationships of optimistic and pessimistic lower and upper approximation in multi-granulation model and through binary vectors and matrices, it designs algorithms to solve upper and lower approximations at some advantages. The correctness of the algorithms is verified by experiments through programming and execution on computers on several data sets. It provides a new forming granule view to solve multi-granulation problems in multi-granulation rough set model in dealing with incomplete information systems. This novel granular approach leads to enriching study methods confronting with multi-granulation rough set models. Our next work will be the rule generations through the approximations in multi-granulation model with maximal compatible classes as primitive granules.

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