

MS Excel Spreadsheet as a Tool for Mathematical Modeling and Computer Simulation of Simple Permutation Brainteaser

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Abstract: - Mathematical modeling of real phenomena and processes is understood as one of the ways of scientific research. The computer simulation supports and develops mathematical modeling in form of executing and visualization of given model. The computer simulation can be created not only in specialized programming languages designed specifically for the requirements of simulations, but also in standard programming languages e.g. in Visual Basic for Application in MS Excel Spreadsheet. The modeling and computer simulation supports the concept of the system and the system approach that is broadly used not only in the natural sciences and in technical disciplines, but also begins to be applied in other scientific disciplines such as sociology, psychology and other social sciences. Regarding the fact, the education has to reflect and support the application of scientific principles, the principles of system approach, modeling and computer simulation are applied even to pedagogical science and education. The paper describes both the principles of implementation system approach, modeling and computer simulation to teaching of informatics as well as the case study of student's final project – design and programming of serious game in form of computer simulation of a simple permutation brainteaser in programming language Visual Basic for Application in MS Excel Spreadsheet. The case study represents an example of computer simulation game that was designed and created by prospective teacher of informatics within their final project.

Key-Words: - Modeling, Computer Simulation, Informatics, Education, Case Study, Permutation Brainteaser.

1 Introduction

The education should reflect current trends of scientific knowledge. One of the approaches that are widely implemented to scientific work are principles of system approach together with scientific modeling and computer simulation. Moreover, computer simulation is now understood as “the third way” of the science [1]. System approach, modeling and simulation are implemented not only in the natural sciences and technical disciplines, but also in economy, humanities, sociology and other social sciences. Many universities have principles of system approach, modeling and simulation already implemented in their study programs. These principles should be successively applied in lower stages of the school education, in secondary schools. Implementation of systemic approach, modeling and simulation can support development of key competencies, particularly development of key

competency for solving the problems, discovering and creativity [2, 3]. Development of this key competency by means of implementation of systemic approach requires from the teacher well knowledge of these principles, the teacher should be able to apply modeling and simulation in educational process.

Strategy of the implementation of system approach, modeling and simulation to education is learned at study program Teaching of Informatics accredited by Department of Informatics, Faculty of Science, University of Hradec Kralove. The paper describes in the case studies of possible way of this learning strategy.

2 The System Theory, Modeling and Simulation in Education

2.1 Implementation of the System Approach to Education

The concept of system approach can be understood as the significant approach not only in science and technical disciplines, but also in humanities social sciences, psychology etc. The principles of the systemic approach is based on identification of the system that is intended to study. The system can be understood not only real object – reality, but also real phenomenon, processes and problems that has to be solved – identified. To identify the system means specify [4]:

- the elements of the system, their properties and quantities;
- interactions between the elements (how the elements interact between each other);
- interactions between system itself and its surroundings (how the system interact with other elements that are outside of the system);
- behavior of the system in time (whether the system is time dependent or independent).

The specification (identification) of the system enables determination of significant parts of the reality that are intended to study. The most important during identification of the system is also determination of the approximation. The approximation means the neglect of such elements and interactions of the system, that do not affect the behavior of study system or that affect the system in insignificant rate [5]. The approximation enables the study of the reality – we are not able to study the real system in whole complexity (to take into account all interactions between all elements of the reality). The approximation of the reality should be specified focusing on the part reality that would be investigated – e.g. the reality can be studied focusing on physical behavior, economy behavior, social behavior etc.

Education has to reflect the application of scientific principles and therefore the principles of system approach should be applied to pedagogical science and education. The system approach in education can be applied in all stages of the study – detailed information can be found e.g. in [6, 7].

The purpose of the school education is not only gain the broad facts and data, but also provide pupils and students with systematic and well-structured knowledge, teach them to incorporate the information into a meaningful context of everyday practice and motivate them to reach new knowledge and developed their skills [2]. The pupils and

students should be able to solve the everyday life problems. The secondary learning has to develop student's thinking in all areas of the learning. These ideas are also supported by educational frameworks of education. E.g. the Educational Framework for Secondary Schools of the Czech Republic [3] apart from others defines competency for problem solving and discovering. Development of this competency can be provided implementation of systemic approach to learning process at secondary schools. The pupils should be able at their age level to identify the studied system that helps them to develop their ability to discover and solve the problem – it can be understood as thinking in systemic approach.

To ensure the proper implementation of the systemic approach to education, teachers of the secondary schools have to be familiar with this approach as well as they have to be able this approach properly use. There are a lot of universities educating the prospective teacher, that have already implemented in their study programs lectures and seminars focusing on implementation of systemic approach to learning process.

2.2 Modeling and Simulation in Education

The system theory is in scientific research closely related to the theory of modeling and simulation.

The main task of the scientific modeling is investigation of the content (elements), structure (interactions) and behavior (time dependency) of the real system representing a part of the modeled reality.

The scientific modeling is based on approximation of the reality. The real systems cannot be modeled in whole complexity. The model is only simplification of studied reality. On the other hand scientific modeling due to simplifications enables study of the reality [8].

The result of the process of modeling is conceptual model of real system. The conceptual model can be expressed in different representations. The most commonly used are model expressed in form of [5]:

- Mathematical equation – mathematical model. This type of model is used for expression of basic (particularly physical) properties of the studied system;
- Process charts – process model. This type of model describes the real processes and is the most represented by the flowcharts that described the algorithm of the modeled process.

The computer simulation together with the scientific modeling is currently understood as the “third way” of scientific research which is associated with development of computer technology [1]. Scientific

simulation supports and completes process of modeling. Simulation enables representation of the modeled real system or real process and its behavior in real time by means of computer. The advantage of simulation is that enables numerical solution of irresolvable mathematical models.

Information and communication technology (ICT) are irreplaceable in current education. The implementation of the method of modeling and computer simulation in education supports pupils and students discover and understand science's phenomenon and processes, it helps them understand technical equipment, it develops algorithmic as well as programming thinking and it develops computational, logical and mathematical thinking. Application of principles modeling and simulation develops student's intuition. Finally yet importantly the method modeling and simulation helps to increase of ICT competency of pupils and students – see e.g. [2, 9, 10]. Computer simulation together with visualization (in form of charts, 2D and 3D models, animated diagrams etc.) enable better understanding of the basic features of the real systems and develop their discovering. It is also essential that the teaching by means of simulation is much cheaper and faster than the teaching carried by real experiment. In some cases providing the real experiment cannot be feasible.

Modeling and simulation can be and should be applied in all subjects of secondary school curricula thus it helps to complex development of children's personality.

The authors presents in this paper the case study of implementation of principles of system theory, modeling and computer simulation to learning of prospective teacher of informatics within the study specialization Teaching of Informatics in their compulsory subject Modeling and Simulation. The paper illustrates problem analysis (system identification), creation of conceptual model and simulation program of solution and visualization of simple permutation brainteaser.

3 Problem Solution - Case Study

3.1 Solution of Permutation Brainteaser in Programming Language Visual Basic for Application in MS Excel Spreadsheet

The students within the lectures of subject Modeling and simulation in study program Teaching of informatics are acquainted with the system theory, scientific modeling and simulation as well as with application of these principles in learning at secondary schools. The seminar are focused to

development of skills and knowledge in identification of real system and creation conceptual model and computer simulation program.

Within the subject, the student has to create and pass the final project. The main requirement for students is to create computer simulation program of selected problem, phenomenon or process.

In the following text there is in the form of case study demonstrated example of final project created by students in the last academic year. The presented project represents the simulation of the permutation brainteaser programmed in Visual Basic for Application (VBA) implemented to MS Excel Spreadsheet. Similar case studies can be found e.g. in [11, 12].

3.2 Problem analysis

Visual Basic for Application of MS Excel Spreadsheet can be understood as “appropriate tool” for solution of the permutation puzzle. Although the puzzle can be solved intuitively (the permutation puzzle is very simple), the finding of automatic solution is not entirely elementary. Conversely, for a number of undergraduate students can be intuitive solution problem. Finally, those who are able to solve the puzzle intuitively are not sure that their solution is optimal (e.g. in terms of the number of steps used).

Before the algorithm as well as program of the solution of permutation puzzle will be described, it will be proved that mentioned puzzle is indeed permutation puzzle whose arbitrary mixing is permutations of basic positions and various possible moves also represent certain permutations and their chaining can be modeled by combining of the permutations.

Denote puzzle segments according to Figure 1 by numbers 1, 2, 3, 4, 5, 6, 7 and 8. Then we the following four elementary strokes can be performed by counterclockwise rotation of the circle segments:

1. Rotation of the left circle by angle 90° counterclockwise: the basic permutation (1, 2, 3, 4, 5, 6, 7, 8) is permuted to (4, 1, 2, 3, 5, 6, 7, 8). Let name of the move is “**A**”.
2. Rotation of the right circle by angle 90° counterclockwise: the basic permutation (1, 2, 3, 4, 5, 6, 7, 8) is permuted to (1, 2, 3, 4, 8, 5, 6, 7). Let name of the move is “**B**”.
3. Movement the upper half of both circles so that the upper right semicircle is exactly above the lower left semicircle followed by rotation thus created circle by angle 90° counterclockwise: the original permutation (1, 2, 3, 4, 5, 6, 7, 8) we is permuted to (1, 2, 6, 3, 4, 5, 7, 8). Finally, the

upper half of both circles are returned to the starting position. Let name of the move is “C”.

- Movement the upper half of both circles so that the upper left semicircle is exactly above the lower right semicircle followed by rotation thus created circle by angle 90 ° counterclockwise: the original permutation (1, 2, 6, 3, 4, 5, 7, 8) we is permuted to (8, 1, 3, 4, 5, 6, 2, 7). Finally, the upper half of both circles are returned to the starting position. Let name of the move is “D”.

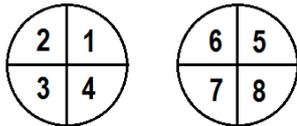


Fig. 1. Numbering of the segments of permutation brainteaser

Note that in each of these elementary strokes A, B, C, or D the positions of four segments of the puzzle are changed at once. This begs the question: How these elementary strokes can be combined (how the permutation can be composes), so the positions only two segments will be changed? If only the position of two segments is changed, the two segments are mutually replace, without changing the position of any of the remaining six segments.

The principle of the composition of the permutation A, C, B and D (movement of the puzzle) is shown on the Figure 2. The example represents composition of the four permutations, each leaving four elements (puzzle segments) of eight in its place. The resulting permutation changes the position of each of eight elements (each puzzle segment puzzle) – each element has different position with comparison of the original position.

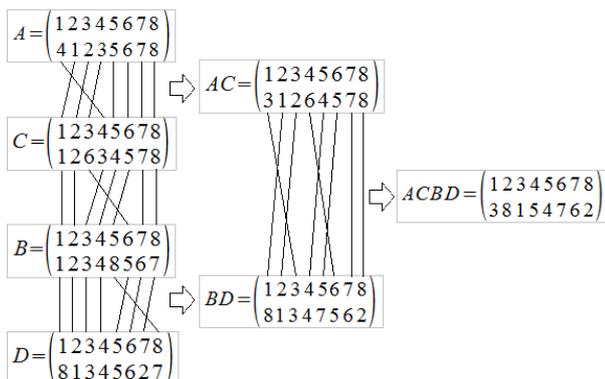


Fig. 2. Composition brainteaser movements in form of composition of the permutation

The task can be extended as follows: Does exist the sequence of the elementary moves {A, B, C, D} so

that just two elements are in different positions than their original positions are and other six elements return to their original positions. Intuitive finding of such sequences of movements is difficult, so the solution can be created in any simulation software or programming language.

3.3 Design of simulation program

One of the appropriate tool for finding and printing the required sequences is Microsoft Excel spreadsheet together with programming language Visual Basic for Application – see e.g. [5]

First, the function Set() will be created. The function defines the initial state of the puzzle (the variable H of type array contains the actual permutation of the segments of the puzzle):

```
Function Set() As Variant
    Dim H As Variant
    H = Array(0, 1, 2, 3, 4, 5, 6, 7, 8)
    Set = H
End Function
```

Another function Move (H, i) realizes the various elementary moves of the set {A, B, C, D}. The specific motion is defined by input of the second parameter i. The value of i = 1 represents move A, i = 2 represents the move B, i = 3 represents move of C and finally i = 4 represents move of D. Inside the functions are permuted only the elements that change their order within given permutation. The first argument represents the state of the puzzle before making the turn. The state of the puzzle after the turn is represented by the return value of the function (Move = H).

```
Function Move(H As Variant, i)
    As Variant
    If i = 1 Then
        p = H(4)
        H(4) = H(3)
        H(3) = H(2)
        H(2) = H(1)
    End If
    If i = 2 Then
        p = H(8)
        H(8) = H(7)
        H(7) = H(6)
        H(6) = H(5)
        H(5) = p
    End If
    If i = 3 Then
        p = H(6)
        H(6) = H(5)
        H(5) = H(4)
```

```

    H(4) = H(3)
    H(3) = p
End If
If i = 4 Then
    p = H(8)
    H(8) = H(7)
    H(7) = H(2)
    H(2) = H(1)
    H(1) = p
End If
Move = H
End Function

```

The last procedure Calculation () searches and writes (to Excel Activesheet) the sequence of elementary strokes that have changed just two values of the elements, while the six element remains in original place. The procedure consist of nine nested cycles, because it was experimentally discovered that the shortest sequence satisfying the requirements consists nine elementary movements of the set {A, B, C, D}.

```

Sub Calculation()
    Dim H As Variant
    H = Array(0, 1, 2, 3, 4, 5, 6, 7, 8)
    p = 0
    For i1 = 1 To 4
    For i2 = 1 To 4
    For i3 = 1 To 4
    For i4 = 1 To 4
    For i5 = 0 To 4
    For i6 = 0 To 4
    For i7 = 0 To 4
    For i8 = 0 To 4
    For i9 = 0 To 4
        H = Set()
        H = Move(H, i1)
        H = Move(H, i2)
        H = Move(H, i3)
        H = Move(H, i4)
        H = Move(H, i5)
        H = Move(H, i6)
        H = Move(H, i7)
        H = Move(H, i8)
        H = Move(H, i9)
        n = 8
        For j = 1 To 8
            k = H(j)
            If k = j Then
                n = n - 1
            End If
        Next j
        If n = 2 Then
            p = p + 1

```

```

s = Chr(64+i1) + Chr(64+i2)
s = s + Chr(64+i3) + Chr(64+i4)
s = s + Chr(64+i5) + Chr(64+i6)
s = s + Chr(64+i7) + Chr(64+i8)
s = s + Chr(64+i9)
v = Chr(48+H(1)) + " "
v = v + Chr(48+H(2)) + " "
v = v + Chr(48+H(3)) + " "
v = v + Chr(48+H(4)) + " "
v = v + Chr(48+H(5)) + " "
v = v + Chr(48+H(6)) + " "
v = v + Chr(48+H(7)) + " "
v = v + Chr(48+H(8))
Cells(p, 1) = s
Cells(p, 2) = v
End If
Next i9
Next i8
Next i7
Next i6
Next i5
Next i4
Next i3
Next i2
Next i1
End Sub

```

This procedure can be run as macro in activesheet of Excel workbook. The macro will fill the first two columns of the excel table and lists a total of 96 different sequences and their corresponding resulting permutations. By inspection the listed sequence it is evident, that a quarter of the sequences represents three consecutive repetitive sequence of turns X, Y and Z, which can be labeled as XYZXYZXYZ, or short $(XYZ)^3$. For such periodically repeated sequence (three repetitions of the same ordered strokes) there are three other sequences without any specified regularity. Thus, the number of strokes is reduced to quarter, therefore from 96 to 24 sequences. The 24 sequences are shown in Table 1.

The strokes A and B or C and D are immediately carried in some sequences. The order A and B (C and D) doesn't affect each other, i.e. it does not depend on execution of order of strokes, $AB = BA$ and $CD = DC$. This allows exclude always one of the sequences leading to the same result – such redundant sequences are indicated in Table 1 by gray fill of the cells. The number of the sequence is reduced to 16. Generally, however, the commutative law cannot apply to the connection of the permutations.

Table 1. Sequences of brainteaser solution

<i>Element's exchange</i>		<i>Final permutation</i>	<i>Sequences</i>	
Adjacent by sides	1,4	42315678	DACDACDAC	(DAC) ³
	2,3	13245678	CADCADCAD	(CAD) ³
	5,8	12348675	CBDCBDCBD	(CBD) ³
	6,7	12345768	DBCDBCDBC	(DBC) ³
Adjacent by vertex	1,3	32145678	CDACDACDA	(CDA) ³
			DCADCADCA	(DCA) ³
	2,4	14325678	ACDACDACD	(ACD) ³
			ADCADCADC	(ADC) ³
	5,7	12347658	CDBCDBCDB	(CDB) ³
			DCBDCBDCB	(DCB) ³
6,8	12345876	BCDBCBCDB	(BCD) ³	
In different circles, after move, adjacent by sides	1,8	82345671	ADBADBADB	(ADB) ³
	2,7	17345628	BDABDABDA	(BDA) ³
	3,6	12645378	ACBACBACB	(ACB) ³
	4,5	12354678	BCABCABCA	(BCA) ³
In different circles, after move, adjacent by vertex	1,7	72345618	ABDABDABD	(ABD) ³
			BADBADBAD	(BAD) ³
	2,8	18345672	DABDABDAB	(DAB) ³
			DBADBADBA	(DBA) ³
	3,5	12543678	ABCABCABC	(ABC) ³
			BACBACBAC	(BAC) ³
4,6	12365478	CABCABCAB	(CAB) ³	
			CBACBACBA	(CBA) ³

Sequences shown in Table 1, solve 16 of the possible 28 two-element combinations of eight elements. There are total 28 combinations.

The remaining of given 12 combinations can be easily obtained by rotation of the given segment to suitable place, followed by performing some of the exchanges from Table 1 and finally returning the replaced segment back to the correct position. The proposals for solutions of these exchanges are shown in Table 2.

Table 2. Sequences of brainteaser solution – finalization

<i>Element's exchange</i>		<i>Final permutation</i>	<i>Sequences</i>
Adjacent by sides	1,2	21345678	A ³ (DAC) ³ A
	3,4	12435678	A ³ (CAD) ³ A
	5,6	12346578	B ³ (CBD) ³ B

In different circles, after move, adjacent by sides	7,8	12345687	B ³ (DBC) ³ B
	1,6	62345178	B ² (ADB) ³ B ²
	2,5	15342678	B ² (BDA) ³ B ²
	3,8	12845673	B ² (ACB) ³ B ²
In different circles, in the same quadrants,	4,7	12375648	B ² (BCA) ³ B ²
	1,5	52341648	B ² (ABD) ³ B ²
	2,6	16345278	B ² (DAB) ³ B ²
	3,7	12745638	B ² (ABC) ³ B ²
4,8	12385674	B ² (CAB) ³ B ²	

The finding the sequence of commands solving the change of any two segments of a permutation puzzle show both, that any possible state of mixing of the segments can be solved as well as the algorithm of the solution was designed. For practical use it is not necessary to know all the sequences listed in Tables 1 and 2, but only four sequences can be selected, one from each group in Table 1, i.e. sequences for:

- replacement of elements from one circle adjacent by sides;
- replacement of elements from one circle adjacent by vertex;
- replacement of elements from different circles, that after appropriate shift adjoin by side;
- replacement of elements from different circles, that after appropriate shift adjoin by vertex.

For the last two groups one option can be selected. Computer simulation written in form of program in Visual Basic for Applications helps to find a set of sequences for easy solution of the permutation puzzle. The solution is not optimal in terms of the number of steps used. Finding the shortest solution excludes the scope of this paper.

4 Conclusion

The paper describes one of the possible way, how to provide the students of study program Teaching of informatics by the strategy of implementation of system approach, modeling and computer simulation to secondary school education. The case study present student's final projects in subject of Modeling and simulation. The paper presents the mathematical analysis of the permutation teaser. The system is identified with respect the mathematical solution of the teaser. The model in form of computational algorithm is designed by

process chart and process table. The model is validated by repeated simulation program execution. The simulation of the solution was programmed in Visual Basic for Application in MS Excel that easily enables transformation of the computational algorithm to simulation. The mathematic analysis of the permutation as well as simulation program are suitable for development of logical thinking of students. The simulation can be used as supporting tool in secondary stage of education.

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