# Research on The Prediction of Stock Market Based on Chaos and SVM

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*Abstract:* -The stock market is a very complex system, so it is necessary to use the support vector machine (SVM) algorithm with small sample learning characteristics. The stock market is also a chaotic system, whose financial time series data has chaotic characteristics of random, noise and strong nonlinear. However, the support vector machine for a given time series is usually not considered its chaotic characteristics, so that the regression results are easy to be disturbed and the accuracy is decreased. This paper fully takes into account the chaos characteristics of the stock market, and combines phase space reconstruction theory and support vector machine (SVM). The paper uses C-C algorithm to find the best time delay and the minimum embedding dimension as the input nodes of SVM and establish the chaps-SVM regression model of stock market. The experiment results with SSE (Shanghai Stock Exchange) Composite Index show that, the model can improve the prediction ability of the system with high precision accuracy and good results. This indicates that it is very promising of using the phase space reconstruction theory for economic prediction and it provides an effective method for the prediction of financial time series.

*Key-Words:* -Support vector machine (SVM); Chaos systems; Phase space reconstruction; Prediction

## 1 Introduction

The stock market is a complex giant system because of its complexity of the interactions of the internal factors and difficulty in handling multifarious external factors. The stock market is difficult to understand and describe. It is also inconsistent with efficient market theory (EMH), showing the characteristics of complex nonlinear and time-varying [1]. It is difficult to describe and predict its variation accurately. Therefore, the effective study of stock prediction method has the extremely important theory significance and application value. With the development of the stock market and the deepening of understanding of the laws on stock market, experts and scholars put forward various kinds of stock market prediction methods, which mainly are traditional prediction methods of expert forecast method, moving average method, linear regression, exponential smoothing, trend extrapolation method, seasonal variation method, artificial intelligence method, etc [2,3,4,5]. However, traditional forecasting methods have a large subjective arbitrariness in index determination; It also need to know all parameters in advance and how to correct the parameters under different circumstances [6].

The stock market is a very complex nonlinear system, whose statistical characteristics of price volatility are non-stationary. If using the traditional neural network (multi - layer perceptron) to predict the trend of the stock market, the network structure must be more complex due to the complexity of the problem. This potentially needs more samples, otherwise easily leads to over-learning. But the more samples, the more violent are the changes of statistical characteristics. In turn, it will result in learning difficulties of the network. Therefore, using the algorithm of support vector machine with small sample learning characteristics is very necessary [7].

Support vector machine (SVM) is a novel machine learning method based on statistical theory proposed by Vapnik et al in the mid 90's in last century. Because it uses the structural risk minimization principle, it can solve practical problems like small sample, non-linear, high dimension and local minimum point [8]. Presently, the time series prediction by SVM has reached very high accuracy, better than the neural network. But for the selection of support vector machine input node number has not a strict theory basis [9], what's more, a specific series's chaos characteristics are not considered in general. So we have to solve the problem that what is the input node number of a chaotic time series prediction by SVM.

The nonlinear prediction method based on the phase space reconstruction theory [10, 11] is a new prediction method developed in the past ten years. The method is mainly used in experiments of water conservancy, engineering field at present. Because of the complexity of the financial system and its own requirements of the phase space construction theory, there are few cases of the method using in economic prediction. Paper [12] used the method for financial outlier detection and got a good effect. Paper [13] combined the method with the neural network and presented an adaptive RBF neural network algorithm.

In this paper, we firstly used the singular value decomposition method (SVD) to reduce the noise of the stock market, according to the chaos characteristics of random, big noise, strong nonlinear in generating process of financial time series data. Then we used the C-C algorithm to determine the best time delay and the minimum embedding dimension. Then establish a chaos-SVM regression model by combing with the SVM method. By using the model to predict SSE Composite index, the results show that this method can significantly improve the prediction accuracy.

The remainder of this paper is organized as follows. Section 2 is introduced how to reconstruct the stock market with two subsection briefly describing the principle of singular value decomposition and phase space construction. Section 3 shows the support vector machine theory. The prediction model formed by phase space construction and support vector machine is introduced in section 4. Experimental results and discussion follows in section 5. Finally we report the conclusion in section 6.

## 2 Reconstruction of the Stock Market Series

Multiple factors influence the stock market, such as policy and economy. It is a very complex nonlinear system, and its data sequence contains noise inevitably. In order to improve the prediction accuracy and reduce the influence of noise effectively, we need to reduce the noise before the analysis of sequence, extracting the main trend as much as possible.

### 2.1 Singular Value Decomposition Denoising

Singular value exists in nonlinear natural processes, which can be used to describe the fractal or multifractal [14].

For a discrete signal $X=\{x_1, x_2, \cdots x_L\}$ polluted by noises constructs a m×n （m≤n） Hankel matrix as follows:

$$A_{m\times n}=\begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ x_2 & x_3 & \cdots & x_{n+1} \\ \vdots & \vdots & \vdots & \vdots \\ x_m & x_{m+1} & \cdots & x_L \end{bmatrix} \quad (1)$$

Where A is a Hankel matrix; m is the embedding dimension satisfying m+n-1=L.

Using the SVD on the Hankel matrix we can obtain a new matrix as follows:

$$A=U\sum V^T \quad (2)$$

Where U is m×m orthogonal matrix; V is n×n orthogonal matrix; Σis m×n matrix. The main diagonal elements of matrix are the singular value ranging from small to large.

Matrix A is Hankel matrix consisted of signal polluted by noise, it can express the signal subspace and the noise subspace without noise pollution:

$$A=\overline{A}+N=\left[U_r U_0\right]\begin{pmatrix}\Sigma_r & 0 \\ 0 & \Sigma_0\end{pmatrix}\begin{bmatrix}V_r^T \\ V_0^T\end{bmatrix} \quad (3)$$

Where $\overline{A}$ is the signal subspace without noise pollution; $N$ is the noise subspace. Denoising the original signal is converted into finding the optimal approximation of $\overline{A}$. The better approximation level, the more obvious denoising effect [15].

Retain the first k effective singular value of the diagonal matrix, and set the other singular value to zero. Get reconstructed matrix by the inverse process of SVD. Generally, the reconstructed matrix is no longer the Hankel matrix form. In order to get the signal after denoising, we use the following equation to average the elements against the angle in reconstructed matrix:

$$\overline{x}_i=\frac{1}{s-l+1}\sum_{j=1}^{s}\overline{A_{i-j+1},j} \quad (4)$$

Where $l=\max(1,i-m+1), s=\min(n,i)$.

$\overline{X}=\left\{\overline{x_1},\overline{x_2},\cdots,\overline{x_L}\right\}$is the discrete signal after denoising.

## 2.2 Phase Space Reconstruction Theory

In the chaos theory, an important idea is to reconstruct phase space [16]. The changes of any system component in high dimensional phase space are not isolated, There is a correlation between them and other components. Therefore, if we have a seemingly one-dimensional time series, in fact it has contained high dimensional information.

Phase space reconstruction is the first step in the analysis of time series prediction. The basic method is to choose the appropriate embed sequence through the one-dimensional time series and then calculated fractal dimension in high dimensional space and search for attractors in high dimensional space (or the singular attractors). Minimum dimension of the phase space can be obtained according to the shape of the attractors. It can construct inherent multidimensional phase space of the system. Thus we can establish a more actual model consistent with the system operation, according to the other features of the system.

According to the Takens theorem [17], for the observed time series{xi}, i=1, 2, 3,⋯, n, the choice of embedding dimension m can get another set of column vectors $y_i=\left\{x_i,x_{i+\tau},x_{i+2\tau},\cdots x_{i+(m-1)\tau}\right\}$, i=1, 2,⋯. τ is the delay time. So the m dimensional state space composed of the observation sequence and time sequence is the phase space after the reconstruction, which is a diffeomorphism with the primitive state space. Therefore, the key to reconstructing the phase space is to determine the optimal delay time and the minimum embedding dimension m of nonlinear time series [18]. The appropriate embedding dimension can save inherent deterministic nature of the power system. If m is too small, it's not enough to show the detailed structure of complex behavior. If m is too large, it will make the calculation work greatly complicate, and the noise will not be ignored. τ also has a great effect on the reconstructed attractor. If τ is too small, it cannot fully expand attractors, then the redundant error will increase; If τ is too large, the irrelevant error of one-dimensional original time series converted into a multidimensional space will increase, so the reconstructed attractor will become very complicated.

There are many methods to calculate the minimum embedding dimension and optimal delay time. At present, experts and scholars usually adopt C-C algorithm proposed by Kim. H. S et al [19].

For the time series $\{x_t, t = 1, 2, \cdots, n\}$, set the space point $\hat{x}_j = (x_j, x_{j+\tau}, \cdots, x_{j+(m-1)\tau})$ after the phase space reconstruction, and the correlation integral of embedded time series is proposed as follows:

$$C(m, N, r, \tau) = \frac{2}{m(m-1)} \sum_{1 \le i \le j \le m}^{2} \theta(r - d(r_i, r_j)) \quad (5)$$

Where $\theta(x) = \begin{cases} 1, x \ge 0 \\ 0, x < 0 \end{cases}$; m is the embedding dimension; N is the number of sequences; τ is time delay; $d(r_i, r_j)$ is the distance between point $r_i$ and $r_j$; Let $M = N - (m-1)\tau$ represent the number of the embedding point in the m dimensional phase space.

$C(m, N, r, \tau)$ divide the time sequence into several disjoint sub sequence. For a specific value, calculate the following equation:

$$S(m, N, r, t) = \frac{1}{t} \sum_{s=1}^{t} [C(m, \frac{N}{t}, r, t) - C(1, \frac{N}{t}, r, t)] \quad (6)$$

When N is large, $S(m, r, t)$ reflect the correlation of time series; We can choose the first zero of $S(m, r, t)$ as the best time delay τ. Let $\Delta S(m, r, t) = \max\{S(m, r_j, t)\} - \min\{S(m, r_j, t)\}$, then the best time delay is the minimum of $\Delta S(m, r, t)$. The best embedding dimension m can be calculated by $\tau_w = (m-1)\tau$.

Embedding the time series of stock market into reconstructed phase space $R_m$, then choosing the embedding time delay τ and embedding dimension m, then $\hat{x}_j = (x_j, x_{j+\tau}, \cdots, x_{j+(m-1)\tau})$.

Structuring mapping $f : R^m \to R$, using the reconstructed state vector to predict the observable value of the stock market sequences, we can get: $x(n+1) = f(\hat{x}_j(n))$.

## 3 The Principle of Support Vector Machine

Support vector machine (SVM) is a new kind of nonlinear regression forecast method, which is based on VC dimension theory and structural risk minimization principle. The input vectors are mapped to a high dimensional feature space by a nonlinear transformation, and construct the optimal decision function. The kernel function is used to replace the dot product operation in the high dimensional feature space. Obtain the global optimal solution by finite sample training [20].

This paper studies a nonlinear time series, so we use the theory of nonlinear regression support vector machine. Set $(x_i, y_i), i = 1, 2, \cdots$ as a training sample, $n$ has ε similarity, i.e. $|y_i - f(x_i)| \le \varepsilon, i = 1, 2, \cdots, n$, the estimating function $f(x)$ can be determined as follows: Firstly, mapping the input vector to a high dimensional feature space by a nonlinear transformation. Then constructing the optimal decision function in this space by using the structural risk minimization principle, and kernel function of the original space is used to replace dot product operation in the high dimensional feature space.

SVM estimation function as follows:

$$f(x) = w^T \varphi(X) + b \qquad (7)$$

Based on the statistical learning theory, the estimation function is transformed into the following optimization problems:

$$\min \frac{1}{2} w^T w + C\sum_{i=1}^{n}(\xi_i + \hat{\xi}_i)$$
$$s.t \begin{cases} wx_i + b_i - y_i \le \varepsilon + \xi_i \\ -wx_i - b_i + y_i \le \varepsilon + \hat{\xi}_i \\ \xi_i, \hat{\xi}_i \ge 0 \end{cases} \qquad (8)$$

Where C is the penalty factor; $\xi_i, \hat{\xi}_i$ are relaxation factors; b is offset; i=1,⋯, n. ε is the loss function which can ignore the regression error in the range ε . The expression is as follows:

$$L_\varepsilon(y, f(x)) = \begin{cases} 0, & if\,|y - f(x)| \le \varepsilon \\ |y - f(x)|, & otherwise \end{cases} \qquad (9)$$

The dual theory is used to solve quadratic programming in Eq.(8). It uses the following dual formula as the constraint expression:

$$\min \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(a_i^* - a_i)(a_j^* - a_j) < \varphi(X_i)$$
$$\varphi(X_j) > \sum_{i=1}^{n}(a_i - a_i^*)\varepsilon - \sum_{i=1}^{n}(a_i - a_i^*)y_i \qquad (10)$$
$$s.t \begin{cases} 0 \le a_i, a_i^* \le C \\ \sum_{i=1}^{n}\left(a_i - a_i^*\right) = 0 \end{cases}$$

Define $K(x_i, x_j) = (\Phi(x_i) \cdot \Phi(x_j)) = \Phi^T(x_j)\Phi(x_i)$ as the kernel function. Commonly used kernel function in SVM regression are the linear kernel function, the polynomial kernel function, the radial basis function (RBF) and the Sigmoid kernel function.

According to the Karush-Kuhn-Tucker theorem, $a_i, a_i^*, b$ can be obtained, and finally we can obtain the following SVM regression function:

$$f(x) = \sum_{i=1}^{n}(a_i - a_i^*)K(x_i, x) + b \qquad (11)$$

# 4 Chaos-SVM Prediction Model

In summary, establish the prediction model for the time series $x_t = \sum_{k=1}^{M} x_t^k, t = 1, 2, \cdots N$ can be divided into the following steps based on the chaos and SVM theory:

(1)Use SVD method to reduce noise of the original time series in order to effectively extract the main trend components of the original sequence, and obtain new sequence $\tilde{x}_t = \sum_{k=1}^{n} x_t^k, n < M, t = 1, 2, \cdots, N$;

(2)Reconstruct the phase space of new sequences by using C-C algorithm to obtain the best time delay τ and embedding dimension m. The points in reconstructed space are $\hat{x}_j = \left(x_j, x_{j+\tau}, \cdots, x_{j+(m-1)\tau}\right)$;

(3)Use the embedding dimension m as the number of input nodes of SVM, and establish training samples:

$$X = \begin{bmatrix} x_1 & x_2 & \cdots & x_m \\ x_2 & x_3 & \cdots & x_{m+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n-m} & x_{n-m+1} & \cdots & x_{n-1} \end{bmatrix}, Y = \begin{bmatrix} x_{m+1} \\ x_{m+2} \\ \vdots \\ x_n \end{bmatrix}$$

According to the SVM regression function in Eq. (11), we can get:

$$x_{n+1} = \sum_{i=1}^{n-m} (a_i - a_i^*) K(\vec{x}_i, \vec{x}_{n-m+1}) + b .$$

Where $\vec{x}_{n-m+1} = \{x_{n-m+1}, x_{n-m+2}, \cdots, x_n\}$, put it into the training sample $\vec{x}_{n-m+2}$ as $\vec{x}_{n-m+2} = \{x_{n-m+2}, x_{n-m+3}, \cdots, x_n, x_{n+1}\}$.

So the second step prediction is

$$x_{n+2} = \sum_{i=1}^{n-k} (a_i - a_i^*) K(\vec{x}_i, \vec{x}_{n-m+2}) + b .$$

Generally, the predicted value of the (n+s) data in sample is $x_{n+s} = \sum_{i=1}^{n-m} (a_i - a_i^*) K(\vec{x}_i, \vec{x}_{n-m+s}) + b$.

Where $\vec{x}_{n-m+s} = \{x_{n-m+s}, \cdots, x_{n+1}, \cdots, x_{n+s-1}\}$.

The model is shown in Fig. 1：

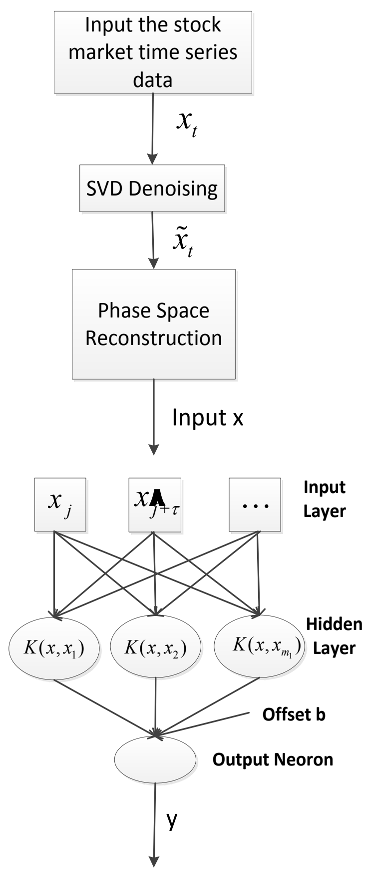


Fig. 1 Chaos-SVM Prediction Model

# 5 Tests and Analysis

This paper selected the opening price of 420 trading days of the SSE Composite Index from 2012.3.23 to 2013.12.18. The data is a 420×6 matrix with double type, each row represents each days' data, 6 columns represent the opening index, the highest index value, the lowest value of index, the SHINDEX, day trading volume, the volume of transactions. Data come from the software named Access to the Letter.

2/3 data is trained on different kernel functions of SVM, then the rest 1/3 data (the last 140 trading days' data) is predicted to test the prediction effect of the model by using the trained model.

## 5.1 Data Preprocessing

Use Eq. (12) to do the normalization process of the study data, make sure the data normalized into the range [0, 1]：

$$x_k = \frac{x_k - X_{\min}}{X_{\max} - X_{\min}} \left( Y_{\max} - Y_{\min} \right) + Y_{\min} \qquad (12)$$

Where $x_k$ denotes the $k$ value of the vector $X$ which need to be normalized, $X_{\max}$ and $X_{\min}$ denote the minimum and maximum values of the vector $X$, set $Y_{\min}$ as 1 and $Y_{\max}$ as 2.

## 5.2 Test and Results Analysis

Use the SVD method on time series after preprocessing to do the denoising. In this paper, we use the mean value as a threshold to isolate the noise, through this the obtained principal component accounts for 90% of the original sequence.

A new sequence $\{\tilde{x}(t), t = 1, 2, \cdots 420\}$ is obtained by noise reduction. SVD denoising results are shown in Fig. 2.

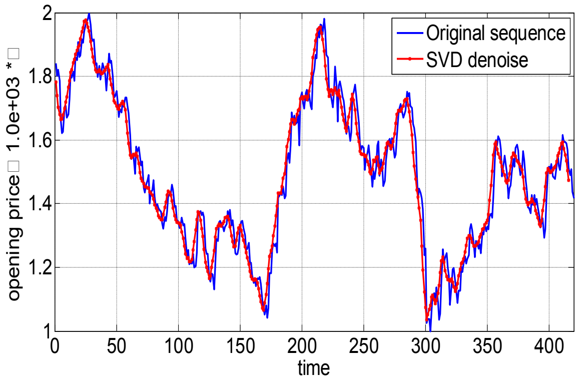


Fig. 2 The original sequence and the denoised sequence

As can be seen from Fig. 2 , after data reduction, the main trend component and oscillation period in the original time series has been extracted and noise decreased significantly.

Further process the new time series by C-C algorithm, the results are shown in Fig. 3.

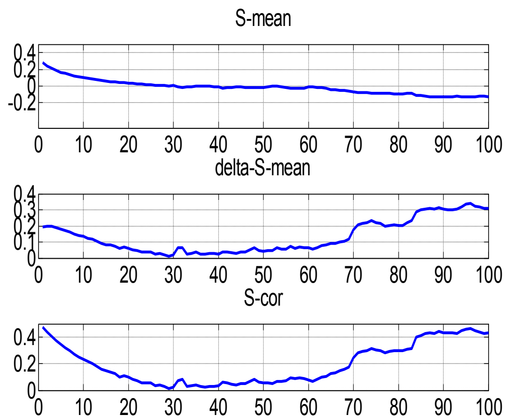


Fig.3 The C-C method of calculating time delay

From Fig.3 we can get the first minimum point in $delta\text{-}S\text{-}mean(\Delta S(t))$ corresponding to the value of t is 29, so the time delay τ is 29. From the minimum point corresponding to t=29 in S cor, we get the delay time window is 29. According to $\tau_w = (m-1)\tau$ , we get the embedding dimension m=2.

Then, according to m and τ, we can see the chaos characteristic of the stock market system using the GP algorithm. From Fig.4 we can see, the curve $\ln C(r) - \ln r$ in the figure for embedding dimension increased gradually converge to a saturated value instead of always increasing. This indicates that fractal structure exists in the stock system, which belongs to the chaos system. So the process above is effective and acceptable. The correlation dimension of the system is 1.6900 by GP algorithm.

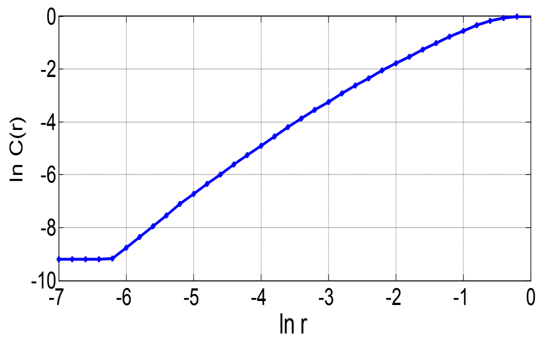


Fig. 4 The GP algorithm of calculating the correlation dimension

When m=2, we use m as the input characteristics node of SVM to establish the training samples of SSE Composite Index:

$$X = \begin{bmatrix} x_1 & x_2 & \cdots & x_m \\ x_2 & x_3 & \cdots & x_{m+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n-m} & x_{n-m+1} & \cdots & x_{n-1} \end{bmatrix}, Y = \begin{bmatrix} x_{m+1} \\ x_{m+2} \\ \cdots \\ x_n \end{bmatrix}$$

In this paper, we use the LIBSVM toolbox for SVM training. We select four common used kernel functions. Using the regression and prediction function SVMcgForRegress to analyze and to select the optimal parameter g and the penalty parameter c. By using the optimal parameters, we train SVM with different kernel functions of SVM with different kernel functions by the first 2/3 trading days' data. The training function is model=svmtrain (TS1, TSX1, cmd), which model is the obtained model after training, TS1 is the desired output, TSX1 is training input, cmd is parameter setting.

Then, we use the trained model to predict the rest 140 data to prove the prediction effect of the model. The prediction results of the four kernel functions are shown in Fig. 5, in which the blue dotted line represents the original data and the red solid line represents the predicted data.

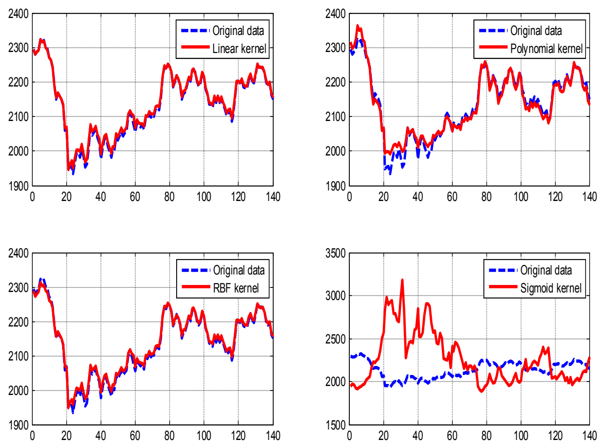


Fig.5 The prediction results of four kernel functions of SVM

### 5.3 Model Evaluation

Prediction results are evaluated by the two index of mean square error (MSE) and squared correlation coefficient (R).

MSE can evaluate the change degree of data. The smaller the MSE value, the better accuracy prediction model has.

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(observed_t - predicted_t)^2 \qquad (13)$$

R is a statistical index to reflect the relationship of the close degree between variables. The model precision is high when R is close to 1.

$$R = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2 \cdot \sum_{i=1}^{n}(y_i-\bar{y})}} = \frac{n\sum_{i=1}^{n}x_iy_i - \sum_{i=1}^{n}x_i \cdot \sum_{i=1}^{n}y_i}{\sqrt{n\sum_{i=1}^{n}x_i^2-(\sum_{i=1}^{n}x_i)^2} \bullet \sqrt{n\sum_{i=1}^{n}y_i^2-(\sum_{i=1}^{n}y_i)^2}} \qquad (14)$$

According to the schemes of whether to do the phase space reconstruction and four different kernel function, Table 1 shows the results of two evaluation indicators -- MSE and R under different schemes.

Table 1 Schemes comparison

| Kernel Function \ Schemes | | Without phase space reconstruction | Phase space reconstruction |
|---|---|---|---|
| Linear kernel function | MSE | 0.000963528 | 0.00835903 |
| | R | 96.5038% | 91.0325% |
| Polynomial kernel function | MSE | 0.000377189 | 0.00174009 |
| | R | 99.893% | 97.2052% |
| RBF kernel function | MSE | 0.000901894 | 0.000507609 |
| | R | 95.9743% | 99.7985% |
| Sigmoid kernel function | MSE | 0.900824 | 1.0845 |
| | R | 13.8517% | 12.7942% |

Through the comparison, the phase space reconstruction can significantly improve the accuracy of SVM prediction. The SVM with linear kernel function and RBF kernel function are better than the other two kinds of kernel function in error and correlation. Sigmoid kernel function is especially not suitable for regression prediction on financial time series. The experimental results show that, this method of combining phase space reconstruction and SVM as the stock price prediction model can achieve good results in the regression prediction.

## 6 Conclusion

Based on denoising the original stock time series, this paper do phase space reconstruction on the new sequence, and determine the time delay and the optimal embedding dimension m using C-C algorithm, and combine this two method by use m as the input characteristics node of SVM. These two methods form the regression prediction model are used to train the four kernel functions of SVM and then predict the Chinese stock volatility. The experimental results show that, the model can significantly improve the prediction ability of the system and the prediction precision is high. Although Chinese stock market is influenced by many internal and external factors and it's more difficult to predict than other mature markets, the model still has good prediction effect. This provides an effective method for the prediction of financial time series.

Although this method has achieved good results, we still have to face some problems. In the application of phase space reconstruction to solve economic problems, we are often faced with the following difficulties:

(1)The accuracy of the original data of many economic indicators is hard to ensure;

(2)The original data is not enough or the data is not comprehensive;

(3)Data time interval is too large.

Above is the problem from economic data itself. In the preprocessing of original data, we will encounter various external random factors. So the model usually will get good prediction results that coincide with the actual economic indicator of future better only in the state of relative stability economy period.

The method has great potential on economy prediction. There are not so much studies on the application of phase space reconstruction method in economic indicators prediction currently, so it is a topic worthy of study.

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