Camera self-Calibration with Varying Parameters from Two views

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Abstract: - This work presents a practical and new approach of self-calibration of cameras with varying parameters, by an unknown planar scene. We show that the estimation of the different parameters of the cameras used can be made from only two matches between two images of the planar scene. The strong point of our method resides at minimizing constraints on the self-calibration system (firstly we estimate the different parameters from only two images, secondly we use only two matches between these images, and thirdly we use the cameras having the varying parameters). The principal idea of our approach is based on demonstrating the relationship between two matches which have a best correlation score ZNCC and the relationship between images of absolute conic for the couple of images. These relations permit to formulate a non-linear cost function, its resolution provides the intrinsic parameters of the cameras used. The robustness of our method in terms of simplicity, stability, accuracy and convergence is shown by the experiment results and the simulations realized.

Key-Words: Interest points, Matching, Homography, Self-Calibration, Varying parameters, Non-linear optimization.

1 Introduction

The self-calibration of cameras is generally used in image processing and in several areas of computer vision, especially the 3D reconstruction, robotics, cinema, medical imaging. The self-calibration methods ([1], [3], [5], [7], [8], [9], [11], [12], [13], [14], [15], [16], [17], [18], [19], [21], [22], [24]) can automatically calibrate the cameras used, but without any prior knowledge on the scene. These are different to calibration methods ([25], [26], [27]), where the camera parameters are determined from the known patterns. The general idea of most of the self-calibration methods is to look for equations according to the invariants in the image and the intrinsic parameters of cameras, these equations are generally non-linear, and need two phases to solve them: initialization and optimization of a cost function.

The major problem of self-calibration methods is the proposal of several constraints on the selfcalibration system (scenes, images, cameras), these constraints limit several studies in literature, for example the use of multiple images provides a large number of equations which require on one hand powerful algorithms to solve them, and on the other hand require a long computation time to converge to the optimal solution. In addition, the studies based on self-calibration of cameras with constant intrinsic parameters remain restricted and limited in the domain of self-calibration of cameras characterized by varying intrinsic parameters.

We present in this paper a practical and new approach of camera self-calibration with varying parameters, by an unknown planar scene. The main idea of this approach is based on using two matches that have a best correlation score ZNCC. The projection matrices of two points of the planar scene in different images are estimated by solving a system of linear equations. This system is formulated from the two matches used and the homography matrix (which is determined from four matches by RANSAC algorithm [10]) between the pair of images. The projection matrices are used with the homography matrix to demonstrate the relationship between two matches and the relationship between images of absolute conic. These relations allow formulating a non-linear cost function. And the intrinsic parameters of the cameras are obtained from the resolution of this function by the Levenberg-Marquardt algorithm [6]. The strong point of this method resides at minimizing constraints on the selfcalibration system (firstly we estimate the different parameters from only two images, secondly we use only two matches between these images, and thirdly we use the cameras having the varying parameters).

This work is organized as follows: The second part presents the related work. The camera model is explained in the third part. The fourth part presents the camera self-calibration. The experiment results are presented in the fifth part and the conclusion is presented in the sixth part.

2 Related Work

The cameras self-calibration consists on determining the parameters of the transformation between the 3D scene coordinates and the image coordinates, and vice versa by an unknown scene. Several studies have been made in this axis: A particular movement of camera is used in [1] to estimate the plan at infinity, and finally to calibrate the camera having the constant intrinsic parameters. The paper [3], is based on self-calibration of cameras with constant intrinsic parameters by a planar scene, the idea of this method is the estimation of these parameters by the projection of two circular points in each image, and the determination of the homographies between the different images (five images at least). A detailed study up to 2003 on methods of camera self-calibration is presented in [5]. this study includes methods based on constant intrinsic parameters and those based on varying parameters. A practical method of self-calibration of cameras with varying intrinsic parameters is proposed in [7], this method can retrieve the metric reconstruction from a sequence of images, and the authors showed that the absence of skew alone is sufficient to calibrate the cameras used. An approach in [8] is based on a circle to estimate the varying intrinsic parameters of cameras. This estimation can be made by resolving a cost non-linear function which is formulated between at least four images. The problem addressed in [9] is the self-calibration of camera with varying focal length from the views of a planar scene whose Euclidean structure is unknown, the main idea of this method is to calculate both the camera intrinsic parameters and those related to the Euclidean structure by resolving a nonlinear cost function, this function causes the problems with the initialization

of the focal length, to solve these problems the authors proposed a new formulation that is independent of the focal length. The use of geometric constraints is the idea discussed in [11]. this method calculates the initial solution of the intrinsic and extrinsic parameters of the camera using geometric constraints on the first image, and the use of the second image permits to optimize the initial solution. In [12], the camera parameters are obtained by using the Kruppa equations. A new method in [13] based on the movement of the camera which is characterized by constant intrinsic parameters except the focal length that varies freely between the different views. A new simple method [14] based on the fundamental matrix to estimate the focal lengths of two cameras, the authors assumed that the pixels are squared and the principal point is known (the centre of the image). In [15] a selfcalibration of cameras with varying intrinsic parameters from image sequences of an object is based on a constant movement between images of the rotating object around a single axis, the relationship between the projection matrices and those of the fundamental matrices provides camera parameters by solving a system of non-linear equations. In [16], a low-complexity multistage method of self-calibration is the main idea to calibrate the cameras having the varying intrinsic parameters, the optimization of an objective function provides the cameras parameters, and a multistage procedure is used to refine the estimation. A new method [17] of camera self-calibration is based on the relative distance of the scene and on the homography matrix that converts the projective reconstruction to the metric one, and whose elements depend on the camera intrinsic parameters. These parameters and 3D structure are obtained by minimizing an error function that is related to the relative distance. A camera self-calibration method [18] with a positive tri-prism is based on circular points which are obtained from properties of triprism, the camera intrinsic parameters can be determined linearly after computing the vanishing points of each edges of tri-prism and the coordinates of circular points. In [19], an approach of selfcalibration of cameras with constant intrinsic parameters using vanishing line, the main idea of this method is to compute the vanishing line by solving three linear equations based on circles and their respective centres, and the theory of these lines and the circular points are used to calculate the intrinsic parameters of the camera. The problem addressed in [21] is self-calibration of cameras with varying intrinsic parameters, this method is based on the transformation of the image of the absolute dual quadric; this transformation is performed on all elements of the image of the absolute dual quadric to obtain the same magnitude for all these elements, which can make the solutions more stable. In [22], a method of self-calibration of cameras with varying intrinsic parameters, based on the quasi-affine reconstruction, the homography of the plane at infinity can be determined, and used with constraints on the image of the absolute conic to estimate the intrinsic parameters of cameras used. A new practical approach of camera self-calibration is processed in [24]. The camera parameters are determined from the analysis of matches between three (at least) images. These images are taken with different camera orientations from the same point. This approach requires no prior knowledge of the camera orientation.

3 Camera Model

We use in this paper the pinhole model of camera (Figure 1) which represents geometrically the perspective projection; this model is used to project the scene in the image plane. It is characterized by a 3×4 matrix, for the camera *i*, this matrix is defined by $K_i(R_i t_i)$, with: $(R_i t_i)$ represents the matrix of extrinsic parameters, such as: R_i is the rotation matrix and t_i is the translation vector of the camera *i* in space. And K_i represents the matrix of intrinsic parameters such as:

$$K_{i} = \begin{pmatrix} f_{i} & s_{i} & u_{0i} \\ 0 & \varepsilon_{i} f_{i} & v_{0i} \\ 0 & 0 & 1 \end{pmatrix}$$
(1)

With: f_i is the focal length, ε_i is the scale factor, s_i is the image skew and $(u_{0i} v_{0i})$ are the image coordinates of the principal point.



Figure 1. Pinhole model of camera.

4 Camera Self-calibration

4.1 Interest points and matching

The interest points are detected in this work by the Harris approach [4] and matched by the correlation measure *ZNCC* [2].

4.2 Estimation of projection matrices

We note by G_1 and G_2 two points of the planar scene (Figure 2), such as its projections in the images, respectively, are two matches (g_{i1}, g_{j1}) and (g_{i2}, g_{j2}) which verify a very high measure ZNCC between the couple of images. The projection matrices $(P_i \text{ and } P_j)$ of the two points $(G_1 \text{ and } G_2)$ are determined from the homography matrices and the two matches. Therefore, to calculate them: we consider a segment $[G_1G_2]$ with middle O and d = $G_1G_2/2$. We note by π the plane of this scene, and we consider a Euclidean reference $\mathcal{R}(O \ X \ Y \ Z)$ fixed on the planar scene and associated to the segment $[G_1G_2]$ such as: The centre O of the reference coincides with the midpoint of segment $[G_1G_2]$ and $Z \perp \pi$. The homogeneous coordinates of two points G_1 and G_2 in the reference $(O \ X \ Y)$ are given as follows: $G_1 = (dcos\theta, dsin\theta, 1)^T$ and

 $G_2 = (-d\cos\theta, -d\sin\theta, 1)^T$ With: θ is the angle between the line (G_1G_2) and the x-axis (X).



Figure 2. Projection of two points G_1 and G_2 in the plans of the images i and j.

The two points G_1 and G_2 are projected in the images *i* and *j* by the matrices H_i and H_j , such as:

$$g_{im} \sim H_i G_m \tag{2}$$

$$g_{jm} \sim H_j G_m \tag{3}$$

Where ~ indicates equality up to multiplication by a non-zero scale factor, m = 1, 2 and g_{im}, g_{jm} represent respectively the points in the images *i* and *j* which are the projections of two vertices G_1 and G_2 of the segment, and H_n represents the homography matrix defined by:

$$H_n \sim K_n R_n \begin{pmatrix} 1 & 0 \\ 0 & 1 & R_n^T t_n \\ 0 & 0 \end{pmatrix}, \quad n = i, j$$
 (4)

The relations (2) and (3) can be written as:

$$g_{im} \sim H_i DG'_m \tag{5}$$

$$g_{jm} \sim H_j DG'_m \tag{6}$$

With:

$$D = \begin{pmatrix} dcos\theta & 0 & 0\\ 0 & dsin\theta & 0\\ 0 & 0 & 1 \end{pmatrix}, \quad G'_1 = \begin{pmatrix} 1\\ 1\\ 1 \end{pmatrix}, \text{ and} \\ G'_2 = \begin{pmatrix} -1\\ -1\\ 1 \end{pmatrix}$$

We put:

$$P_n \sim H_n D, \quad n = i, j \tag{7}$$

With P_i and P_j represent the projections matrices of two points G'_1 and G'_2 in images *i* and *j*.

From expression (7) we can write:

$$P_i \sim H_{ij} P_i \tag{8}$$

With:

$$H_{ij} \sim H_j H_i^{-1} \tag{9}$$

 H_{ij} is the homography between the images *i* and *j*. The expressions (5), (6) and (7) give:

$$g_{im} \sim P_i G'_m \tag{10}$$

$$g_{jm} \sim P_j G'_m \tag{11}$$

Furthermore, from equations (8) and (11) we can write:

$$g_{jm} \sim H_{ij} P_i G'_m \tag{12}$$

The expressions: (10) and (12) are given according to eight unknowns of P_i , each of these expressions gives four equations with eight unknowns.

So, we can estimate the P_i parameters from these eight equations with eight unknowns.

The P_i matrix is estimated from the expression (8).

4.3 Self-calibration equations

We present in this part the important step of our method, the general idea is to show the relationship between two matches (g_{im}, g_{jm}) of each pair of

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images and the relationship between images of the absolute conic (ω_i and ω_j). A non-linear cost function is obtained by these relations; its resolution provides the intrinsic parameters of the cameras used.

Expressions (4) and (7) give:

$$P_n \sim K_n R_n \begin{pmatrix} 1 & 0 \\ 0 & 1 & R_n^T t_n \\ 0 & 0 \end{pmatrix} D, \quad n = i, j$$
(13)

Therefore, from the previous expression, we can write:

$$K_{i}^{-1}P_{i} \sim R_{i} \begin{pmatrix} 1 & 0 \\ 0 & 1 & R_{i}^{T}t_{i} \\ 0 & 0 \end{pmatrix} D$$
(14)

If we develop this expression, we obtain:

$$P_i^T \omega_i P_i \sim \begin{pmatrix} D'^T D' & D'^T R_i^T t_i \\ t_i^T R_i D' & t_i^T t_i \end{pmatrix}$$
(15)

With:

$$D' = \begin{pmatrix} dcos\theta & 0\\ 0 & dsin\theta\\ 0 & 0 \end{pmatrix} \text{ and } \omega_i = (K_i K_i^T)^{-1}$$

is the image of the absolute conic.

The same for P_i :

$$P_j^T \omega_j P_j \sim \begin{pmatrix} D'^T D' & D'^T R_j^T t_j \\ t_j^T R_j D' & t_j^T t_j \end{pmatrix}$$
(16)

From the expressions (10) and (12), we have:

$$g_{jm} \sim H_{ij} g_{im} \tag{17}$$

Therefore:

$$\lambda_{ijm}g_{jm} = H_{ij}g_{im} \tag{18}$$

With:

$$H_{ij} = \begin{pmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{pmatrix}, \ g_{im} = \begin{pmatrix} u_{im} \\ v_{im} \\ 1 \end{pmatrix},$$

$$g_{jm} = \begin{pmatrix} u_{jm} \\ v_{jm} \\ 1 \end{pmatrix}$$

And:

$$\lambda_{ijm} = H_{31}u_{im} + H_{32}v_{im} + H_{33}$$

Therefore, from the relation (18) we can write:

$$\lambda_{ijm}g'_{jm} = H_{ij}g'_{im} \tag{19}$$

With:

$$g'_{jm} = \begin{pmatrix} u_{jm} & \frac{H_{12}}{\lambda_{ijm}} & \frac{H_{13}}{\lambda_{ijm}} \\ v_{jm} & \frac{H_{22}}{\lambda_{ijm}} & \frac{H_{23}}{\lambda_{ijm}} \\ 1 & \frac{H_{32}}{\lambda_{ijm}} & \frac{H_{33}}{\lambda_{ijm}} \end{pmatrix}, \quad \text{and}$$
$$g'_{im} = \begin{pmatrix} u_{im} & 0 & 0 \\ v_{im} & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

The relation (19) gives:

$$H_{ij} \sim g'_{jm} g'^{-1}_{im}$$
 (20)

The relations (8) and (16) give:

$$(H_{ij}P_i)^T \omega_j (H_{ij}P_i) \sim \begin{pmatrix} D'^T D' & D'^T R_j^T t_j \\ t_j^T R_j D' & t_j^T t_j \end{pmatrix}$$
(21)

Using the last two expressions, we can write:

$$(g'_{jm}g'_{im}^{-1}P_{i})^{T}\omega_{j}(g'_{jm}g'_{im}^{-1}P_{i}) \sim \begin{pmatrix} D'^{T}D' & D'^{T}R_{j}^{T}t_{j} \\ t_{j}^{T}R_{j}D' & t_{j}^{T}t_{j} \end{pmatrix} (22)$$

According to the last expression, we can deduce that the matrices $(g'_{j1}g'_{i1}^{-1}P_i)^T \omega_j (g'_{j1}g'_{i1}^{-1}P_i)$ and $(g'_{j2}g'_{i2}^{-1}P_i)^T \omega_j (g'_{j2}g'_{i2}^{-1}P_i)$ are identical.

We note by:
$$A = \begin{pmatrix} a_{11j} & a_{12j} & a_{13j} \\ a_{12j} & a_{22j} & a_{23j} \\ a_{13j} & a_{23j} & a_{33j} \end{pmatrix}$$
 the

corresponding

matrix

$$(g'_{j1}g'_{i1}^{-1}P_i)^T \omega_j (g'_{j1}g'_{i1}^{-1}P_i).$$

And: $B = \begin{pmatrix} b_{11j} & b_{12j} & b_{13j} \\ b_{12j} & b_{22j} & b_{23j} \\ b_{13j} & b_{23j} & b_{33j} \end{pmatrix}$ the matrix

corresponding to $(g'_{j2}g'_{i2}^{-1}P_i)^T \omega_j (g'_{j2}g'_{i2}^{-1}P_i)$. So, we can write:

$$\begin{cases} a_{12j} = 0, b_{12j} = 0, \frac{b_{11j}}{a_{11j}} = \frac{b_{13j}}{a_{13j}}, \\ \frac{b_{13j}}{a_{13j}} = \frac{b_{22j}}{a_{22j}}, \frac{b_{22j}}{a_{22j}} = \frac{b_{23j}}{a_{23j}}, \frac{b_{23j}}{a_{23j}} = \frac{b_{33j}}{a_{33j}} \end{cases}$$
(23)

The previous formulas give a system of six equations which is expressed as follows:

$$\begin{cases}
 a_{12j} = 0, b_{12j} = 0 \\
 b_{11j}a_{13j} - a_{11j}b_{13j} = 0 \\
 b_{13j}a_{22j} - a_{13j}b_{22j} = 0 \\
 b_{22j}a_{23j} - a_{22j}b_{23j} = 0 \\
 b_{23j}a_{33j} - a_{23j}b_{33j} = 0
\end{cases}$$
(24)

According to expressions (15) and (22) (if : m = 1), we conclude that the first two rows and columns of matrices: $P_i^T \omega_i P_i$ and $(g'_{j1}g'^{-1}_{i1}P_i)^T \omega_j (g'_{j1}g'^{-1}_{i1}P_i)$ are identical.

We note by: $E = \begin{pmatrix} e_{11i} & e_{12i} \\ e_{12i} & e_{22i} \end{pmatrix}$ the matrix corresponding to the first two rows and columns of $P_i^T \omega_i P_i$.

Therefore:

$$\begin{pmatrix} e_{11i} & e_{12i} \\ e_{12i} & e_{22i} \end{pmatrix} \sim \begin{pmatrix} a_{11j} & a_{12j} \\ a_{12j} & a_{22j} \end{pmatrix}$$
(25)

The previous formula gives:

$$e_{12i} = 0, \frac{e_{11i}}{a_{11j}} = \frac{e_{22i}}{a_{22j}}$$
(26)

The formula (26) gives a system of two equations which is expressed as follows:

$$e_{12i} = 0, e_{11i}a_{22j} - a_{11j}e_{22i} = 0$$
⁽²⁷⁾

Similarly, from expressions (15) and (22) (if: m = 2), the first two rows and columns of matrices: $P_i^T \omega_i P_i$ and

$$(g'_{j2}g'_{i2}^{-1}P_i)^T \omega_j (g'_{j2}g'_{i2}^{-1}P_i)$$
 are identical.

Therefore:

to

$$\begin{pmatrix} e_{11i} & e_{12i} \\ e_{12i} & e_{22i} \end{pmatrix} \sim \begin{pmatrix} b_{11j} & b_{12j} \\ b_{12j} & b_{22j} \end{pmatrix}$$
(28)

The previous formula gives:

$$\frac{e_{11i}}{b_{11j}} = \frac{e_{22i}}{b_{22j}} \tag{29}$$

This gives:

$$e_{11i}b_{22j} - b_{11j}e_{22i} = 0 \tag{30}$$

According to expressions (15) and (16), we conclude that the first two rows and columns of matrices:

 $P_i^T \omega_i P_i$ and $P_j^T \omega_j P_j$ are identical.

We note by: $C = \begin{pmatrix} C_{11j} & C_{12j} \\ C_{12j} & C_{22j} \end{pmatrix}$ the matrix corresponding to the first two rows and columns of $P_j^T \omega_j P_j$.

Therefore:

$$\begin{pmatrix} e_{11i} & e_{12i} \\ e_{12i} & e_{22i} \end{pmatrix} \sim \begin{pmatrix} c_{11j} & c_{12j} \\ c_{12j} & c_{22j} \end{pmatrix}$$
(31)

This gives:

$$c_{12j} = 0, \frac{e_{11i}}{c_{11j}} = \frac{e_{22i}}{c_{22j}}$$
(32)

So:

$$c_{12j} = 0, e_{11i}c_{22j} - c_{11j}e_{22i} = 0$$
(33)

The expressions (24), (27), (30) and (33) give:

$$\begin{cases}
 a_{12j} = 0, b_{12j} = 0 \\
 b_{11j}a_{13j} - a_{11j}b_{13j} = 0 \\
 b_{13j}a_{22j} - a_{13j}b_{22j} = 0 \\
 b_{22j}a_{23j} - a_{22j}b_{23j} = 0 \\
 b_{23j}a_{33j} - a_{23j}b_{33j} = 0 \\
 e_{12i} = 0, e_{11i}a_{22j} - a_{11j}e_{22i} = 0 \\
 e_{11i}b_{22j} - b_{11j}e_{22i} = 0 \\
 c_{12j} = 0, e_{11i}c_{22j} - c_{11j}e_{22i} = 0
\end{cases}$$
(34)

This system consists of eleven equations with ten unknowns which are the ω_i and ω_j elements (five for ω_i and five for ω_j). It is non-linear; therefore, to solve it we seek to minimize the following non-linear cost function by the Levenberg-Marquardt algorithm [6]:

$$\min_{\omega_{i,j}} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \begin{pmatrix} \alpha_j^2 + \beta_j^2 + \gamma_j^2 + \lambda_j^2 + \\ \eta_j^2 + \phi_j^2 + \tau_i^2 + \\ \psi_{ij}^2 + \chi_{ij}^2 + \delta_j^2 + \varphi_{ij}^2 \end{pmatrix}$$
(35)

With:

$$\begin{aligned} \alpha_{j} &= a_{12j}, \beta_{j} = b_{12j}, \gamma_{j} = b_{11j}a_{13j} - a_{11j}b_{13j} \\ \lambda_{j} &= b_{13j}a_{22j} - a_{13j}b_{22j} \\ \eta_{j} &= b_{22j}a_{23j} - a_{22j}b_{23j} \\ \phi_{j} &= b_{23j}a_{33j} - a_{23j}b_{33j} \\ \tau_{i} &= e_{12i}, \psi_{ij} = e_{11i}a_{22j} - a_{11j}e_{22i} \\ \chi_{ij} &= e_{11i}b_{22j} - b_{11j}e_{22i} \\ \delta_{j} &= c_{12j}, \phi_{ij} = e_{11i}c_{22j} - c_{11j}e_{22i} \end{aligned}$$

And: *n* is the number of images.

This optimization algorithm is non-linear; therefore, it requires an initialization phase. For this, we propose the following constraints on the selfcalibration system to obtain the initial solution:

• The pixels are squared, therefore $\varepsilon_i = \varepsilon_j = 1$, $s_i = s_j = 0$.

• The principal point is in the centre of the image, therefore: $u_{0i} = v_{0i} = u_{0j} = v_{0j} = 256$ (because the size of images used is 512×512). And the focal lengths (f_i, f_j) are determined by replacing the parameters ($u_{0i}, v_{0i}, \varepsilon_{i}, s_i, u_{0j}, v_{0j}, \varepsilon_j, s_j$) in the expression (34) above.

5 Experiment Results

5.1 Simulations

In this part, we took a sequence of 22 images (512 \times 512) of an unknown planar scene by a camera CCD from different views. This sequence is simulated to test the performance of the approach presented in this paper. Gaussian noise of standard deviation σ is added to the image coordinates when projecting the planar scene in the different planes of images. The interest points are detected by Harris's approach [4] and matched in each pair of images by the correlation measure ZNCC [2]. The projection matrices of two points of the planar scene in different images are estimated by solving a system of linear equations. This system is formulated from the two matches used and the homography matrix (which is determined from four matches by RANSAC algorithm [10]) between the pair of images. The projection matrices are used with the homography matrix to demonstrate the relationship between two matches and the relationship between images of absolute conic. These relations allow formulating a non-linear cost function. And the intrinsic parameters of the cameras are obtained from the resolution of this function by the Levenberg-Marquardt algorithm [6]. The figure 3 and 4 show the relatives errors on the focal lengths, and the coordinates of the principal point according to number of images and noises respectively. And the figure 5 presents the computation time according to the number of images by the three methods.



Figure 3. Relative error on f, u_0 and v_0 according to number of images.



Figure 4. Relative error on f, u_0 and v_0 according to noise.



Figure 5. The Computation time according to the number of images by the three methods

According to figure 3, we can conclude that:

The relatives errors in the focal length and the coordinates of the principal point decrease almost linearly with the increment of the number of images (2 to 6 images), but we note that if the number of images used is between 6 and 9, the relative error decreases slowly and becomes almost stable if the number of images exceeds 9. We arrived in this work to estimate the different parameters of cameras used from two images only, but in practice we have shown that the number of images has relatively a simple effect on the outcome, therefore, to obtain more accurately and with a minimum relative error, it is preferable to increase the number of images.

To test the robustness of our approach to noise, we added to the projected image points a Gaussian noise with zero mean and a standard deviation (1 pixel), such as $\sigma \leq 5$ pixels. From Figure 4, we note that the relatives errors of the focal length and the coordinates of the principal point increase slowly if $\sigma \leq 3$, by cons, the relative error increases almost linearly if $\sigma > 3$.

And to evaluate the performance of our approach compared to other, we compared our results with those obtained by two methods (Jiang [22] and Zhang [23]) well-established, and were given good results. According to the simulations realized, we can conclude that our method firstly gives satisfactory results compared to Jiang for the estimation of the focal length and the principal point, while adding noise, and secondly it gives similar results to those obtained by Zhang.

We presented in Figures 3 and 4 the relative errors corresponding to focal length, and principal point, relative to the number of images, and the Gaussian noise, in the same way we can represent the curves corresponding to the errors related to the skew factor and the scale factor.

Figure 5 shows that the computation time of the three methods increases if the number of images increases. In addition, from this figure we can conclude that the computation times obtained by our method are lower to those obtained by Jiang [22] and they are closer to those obtained by Zhang [23].

5.2 Real data

In this section, we present the experiment results of the different algorithms used in our approach. For this, we took two images (512×512) of unknown planar scene (shown in Figure 5) from two different views by a CCD camera characterized by varying parameters. These experiment results are presented in figure 7, figure 8 and Table 1.

The figure 6 bellow shows the two images used in this article.









Figure 7. The interest points detected in two images (red color)



The figure 8 bellow shows the matches between the couple of images.

Figure 8. The matching between the couple of two images.

The table 1 below represents the results of experiments, of initial and optimal solutions of intrinsic parameters of the cameras obtained by our approach.

Table 1.	Initialization and optimization of				
cameras parameters in the case of real scenes.					

		f	З	S	u_0	v_0
image 1	Initial solution	1124	1	0	256	256
	Optimal solution	1158	0,91	0,02	260	258
image 2	Initial solution	1126	1	0	256	256
	Optimal solution	1145	0,93	0,03	265	253

We compare our experiments results obtained in the above table with those presented in Jiang [22] and Zhang [23], we can conclude that our estimation of the various parameters is closer to that obtained by Zhang, and it is a little different than that obtained by Jiang. So, the result shows that our method provides a very close robustness to other methods which were given good results. In addition, our approach has several strengths such as simplicity, stability due to the optimization of our cost function, and the minimization of the constraints on the selfcalibration system (on one hand, the cameras are characterized by varying intrinsic parameters, and on the other hand the use of only two images to estimate these parameters). And the use of only two images, gives a strong point to our approach in terms of computation time, and the speed of convergence to the optimal solution.

The different algorithms (Harris, *ZNCC*, Ransac, Levenberg-Marquardt,...) used in this article are implemented by the object-oriented programming language that is Java.

6 Conclusion

In this work, we presented a practical and new approach of self-calibration of cameras with varying parameters, by an unknown planar scene. The strong point of our method resides at minimizing constraints on the self-calibration system (firstly we estimate the different parameters from only two images, secondly we use only two matches between these images, and thirdly we use the cameras having the varying parameters). The relationship between the two matches which have a best correlation score ZNCC and the relationship between images of absolute conic for each pair of images allow formulating a non-linear cost function. And the intrinsic parameters of the cameras are obtained from the resolution of this function by the Levenberg-Marquardt algorithm. The results of experiments and the simulations realized show the robustness of our method in terms of simplicity, stability, accuracy and convergence.

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