Semi-Analytical Resolution of a Squeezing Unsteady Nanofluid Flow Between Two Parallel Plates Using Homotopy Perturbation Method (HPM)

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Abstract: In this current work, the heat transfer analysis for the unsteady squeezing flow of a viscous nanofluid between two parallel plates considering Fourier heat flux model have been explored. The partial differential equations representing flow model are reduced to nonlinear ordinary differential equations by introducing a similarity transformation. The dimensionless and nonlinear ordinary differential equations of the velocity and temperatures functions obtained are solved by employing The Homotopy Perturbation Method (HPM). The results found in this peper are verified by comparing it with the results obtained using the numerical method RK4, The results obtained are agree with this numerical solution. The effects of different parameters on the velocity and temperature profiles are examined graphically, and numerical calculations for the skin friction coefficient and local Nusselt number are tabulated. It is found an excellent agreement in the comparative study with literature results.

1. Introduction

Nanofluids are widely encountered in many industrial and technology applications, for example, melts of polymers, biological solutions, paints, asphalts and glues, etc. Nanofluids appear to have the potential to significantly increase heat transfer rates in a variety of areas, The word nanofluid represents the fluid in which particles of size with order of nanometer (diameter < 100 nm) are mixed in the base fluid. The nanoparticles used in nanofluids are generally made of metals (Al, Cu), oxides (Al2O3, CuO, TiO2, and SiO2), carbides (SiC), nitrides (AlN, SiN), and nonmetal (graphite, carbon nanotubes) and the base fluid is usually a conductive fluid, such as water or ethylene glycol. Other base fluids are toluene, oil, other lubricants, biofluids, and polymer solution. Nanoparticles are

present up to 5% volume fraction in nanofluids.The conventional heat transfer fluids are poor conductors of heat. Nanofluids make an edge over them because they have high heat transfer capability. Since these heating/cooling fluids play a vital role in the development of energy efficient heat transfer equipment for energy supply, to raise the thermal conductivity of these fluids, nanosized conducting metal particles are added to them. Therefore, their proper understanding is a must to use them efficiently in modern industry. Applications of nanofluids include microelectronics, fuel cells, and pharmaceutical processes, Choi and Eastman [1] were the first to propose the term nanofluid that represents the fluid in which nanoscale particles are suspended in the base fluid with low thermal conductivity such as water,

ethylene glycol, and oil. In recent years, many researchers have studied and reported nanofluid technology experimentally or numerically in the presence of heat transfer.

The study of heat and mass transfer in unsteady squeezing viscous nanofluid flow between two parallel plates is a stimulating topic of exploration because of its industrial use and intense biological situations, some of which include processing of polymer, compression, power transmitting, lubricant system, transient loading of mechanical components and the squeezed films in power transmission, food processing, and cooling water, modeling of synthetics transportation inside living bodies, hydromechanical machinery, chemical processing equipment, and crop destruction due to freezing. The first work on the squeezing flow under lubrication approximation was studied by Stefan [2].The flow analysis between two parallel plates of Cu-water squeezing nanofluid was investigated by Domairry and Hatami [3]. The squeezing flow of Cu-water (or kerosene) nanofluid between two parallel plates under the effects of viscous dissipation and velocity slipwas investigated by Khan et al. [4]. Heat transfer of a nanofluid flow which is squeezed between parallel plates was investigated analytically using Homotopy perturbation method (HPM) by Sheikholeslami and Ganji [5].They reported that Nusselt number has direct relationship with nanoparticle volume fraction, the squeeze number and Eckert number when two plates are separated but it has reverse relationship with the squeeze number when two plates are squeezed.

Most of engineering problems, especially some heat transfer equations are nonlinear, therefore

some of them are solved using numerical solution and some are solved using the different analytic method, such as perturbation method (PM), homotopy perturbation method (HPM), variational iteration method (VIM). Therefore, many different methods have recently introduced some ways to eliminate the small parameter. One of the semi-exact methods which does not need small parameters is the HPM. The homotopy perturbation method proposed and improved by He [6]. The method yields a very rapid convergence of the solution series in the most cases. The HPM proved its capability to solve a large class of nonlinear problems efficiently, accurately and easily with approximations convergency very rapidly to solution. Usually, few iterations lead to high accuracy solution. This method is employed for many researches in engineering sciences. HPM was used for solving meany problems such as : nonlinear MHD Jeffery Hamel problem by Moghimi et al. [7]. Mustafa et al. [8] analyzed the effect of squeezing nanofluid flow between parallel plates, Siddiqui et al. [9] used this method for solving non-linear problems involving Newtonian and non-Newtonian fluids, For a comprehensive account of the use of the HPM successfully to solve problems in fluid mechanics, please see [10, 11, 12].

The main objective of this present work is to investigate nanofluid flow and heat transfer between two parallel flat plates without presence of constant magnetic field. In this study, we have applied The Homotopy Perturbation method (HPM), to find the solution of nonlinear differential equations of velocity and temperature, The impacts of diverse active parameters such as :

the squeeze number (S), the volume fraction of nanofluid (φ), the Eckert number (Ec), the of Prandtl number (Pr) on flow and heat transfer characteristics are examined.

2. Mathematical Formulation

In this present study, analysis of flow and heat transfer in the unsteady squeezing two dimensional incompressible newtonian nanofluid flow allong two infinite parallel plates have been investigated. The flow configuration is modeled in such way that, the plates are separated by a distance $y = \pm h(t) = \pm l\sqrt{1 - \alpha t}$, where l is the initial location between the two plates (when time $t = 0$). However, the parallel plates are squeezed when $\alpha > 0$ till they reach $t = 1/\alpha$ and plates are separated when $\alpha < 0$. Cartesian coordinate system is considered to describe this problem, in which, x-axis is considered along the axial flow direction and y-axis is taken normal to the axial direction. **Figure 1** clearly describes the geometry and coordinate system of the considered problem. The continuity, momentum, vorticity and energy equations governing such type of flow are given as :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mu_{nf}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad (2)$$

$$\rho_{nf}\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial P}{\partial y} + \mu_{nf}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad (3)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}$$

$$= \frac{k_{nf}}{(\rho c_p)_{nf}}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad (4)$$

$$+ \frac{\mu_{nf}}{(\rho c_p)_{nf}}\left[4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right]$$

$$u\frac{\partial \square}{\partial x} + v\frac{\partial \square}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 \square}{\partial x^2} + \frac{\partial^2 \square}{\partial y^2}\right) \qquad (5)$$

equation **(5)** is called the vorticity equation is obtained by doing : $\frac{\partial (3)}{\partial x} - \frac{\partial (2)}{\partial y}$ with $\square = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$.

where u and v are the velocity components in the x and y directions, respectively, ρ_{nf} is the effective density of the nanofluid, μ_{nf} is the effective dynamic viscosity of the nanofluid, $(\rho c_p)_{nf}$ is the heat capacity of the nanofluid and k_{nf} is the thermal conductivity of the nanofluid, are given as :
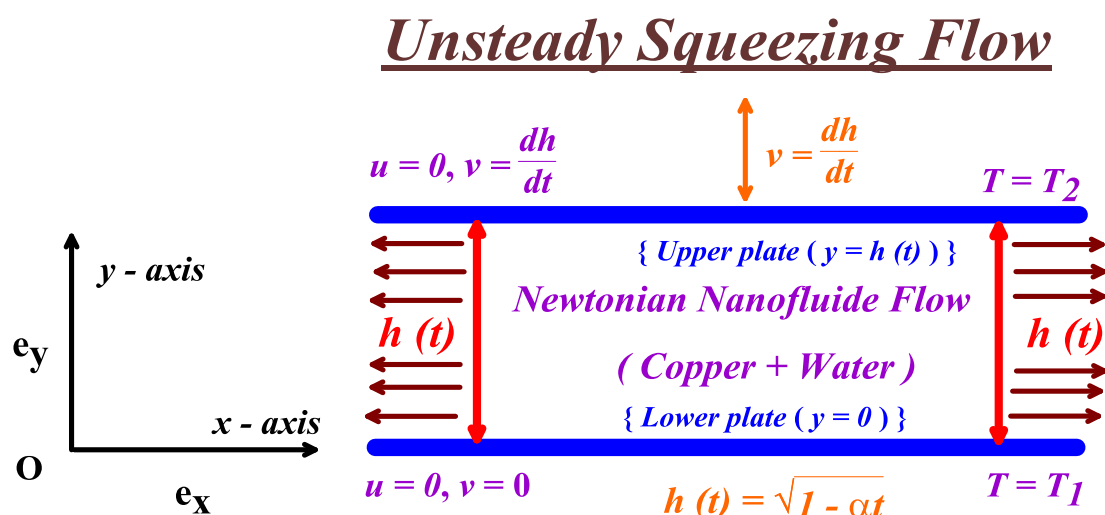

Unsteady Squeezing Flow

$u = 0, v = \dfrac{dh}{dt}$ $v = \dfrac{dh}{dt}$ $T = T_2$

{ Upper plate (y = h (t)) }

y - axis

$\mathbf{e_y}$

Newtonian Nanofluide Flow

$h (t)$ *(Copper + Water)* $h (t)$

{ Lower plate (y = 0) }

x - axis

O

$\mathbf{e_x}$ $u = 0, v = 0$ $h (t) = \sqrt{1 - \alpha t}$ $T = T_1$

Figure 1: Geometry of present work.

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_p \qquad (6)$$

$$(\rho Cp)_{nf} = (1 - \phi)(\rho Cp)_f + \phi(\rho Cp)_p \qquad (7)$$

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \qquad (8)$$

$$K_{nf} = \frac{K_s + 2K_f - 2\phi(K_f - K_s)}{K_s + 2K_f + 2\phi(K_f - K_s)} K_f \qquad (9)$$

The associated boundary conditions for the time dependent squeezing flow are as follows :

$$u = 0 , v = 0 , T = T_1 \quad en \quad y = 0 \qquad (10)$$

$$u = 0 , v = \frac{dh}{dt} , T = T_2 \quad en \quad y = h(t) \qquad (11)$$

In the Eq. (11), $v = \dfrac{dh}{dt}$ indicate the velocity with which upper plate is moving towards or away from the lower plate which is kept at a distance $y = 0$ from the upper plate at $y = h(t)$. Further, the value of the v is exam as $\left[\dfrac{-\alpha l}{2(1-\alpha t)^{\frac{1}{2}}}\right]$.

However, Eqs. (4)‑(5) along with conditions (10) and (11) governing the nanofluid squeezing flow are coupled highly nonlinear in nature and are not amenable to any analytical methods. Hence, in the system of partial differential equations, Eqs. (4)‑(5) are reduced to ordinary differential equations by using the following similarity transformations :

$$\eta = \frac{y}{l(1 - \alpha t)^{\frac{1}{2}}} = \frac{y}{h(t)} \qquad (12)$$

$$u = \frac{\alpha x}{2(1 - \alpha t)} f'(\eta) \qquad (13)$$

$$v = \frac{-\alpha l}{2(1 - \alpha t)^{\frac{1}{2}}} f(\eta) \qquad (14)$$

$$\theta = \frac{T - T_2}{T_1 - T_2} \qquad (15)$$

and substituting Eq. (12) in Eqs. (5) and (6), we obtain the reduced governing equations :

- The nonlinear fourth-order ordinary differential equation for the momentum equation :

$$f'''' - S\frac{A_1}{A_2}(3f'' + \eta f''' + f'f'' - ff''') = 0 \qquad (16)$$

- The second order differential equation for

$$\theta'' + SPr\frac{A_4}{A_3}(f\theta' - \eta\theta')$$
$$+ \left(P_r E_c \frac{A_2}{A_4}\right)(f''^2 + 4\delta^2 f'^2) = 0 \qquad (17)$$

the energy equation :

where A_1, A_2 and A_3 are dimensionless constants defined as follows :

$$A_1 = \frac{\rho_{nf}}{\rho_f} \qquad (18)$$

$$A_2 = \frac{\mu_{nf}}{\mu_f} \qquad (19)$$

$$A_3 = \frac{K_{nf}}{K_f} \qquad (20)$$

$$A_4 = \frac{(\rho c_p)_{nf}}{(\rho c_p)_f} \qquad (21)$$

The boundary conditions (10) in the terms of similarity variables (11) become :

$$f'(0) = 0 , f(0) = 0 , \theta = 1 \quad at \quad \eta = 0 \qquad (22)$$

$$f'(1) = 0 , f(1) = 1 , \theta = 0 \quad at \quad \eta = 1 \qquad (23)$$

where $S = \frac{\alpha l^2}{2\upsilon_f}$ is the squeeze number, $Pr = \frac{(\rho Cp)_f}{K_f}\upsilon_f$ is the Prandtl number and $E_c = \frac{\alpha^2 x^2}{(T_1 - T_2)c_{p_f}}$

The physical quantities of interest are the skin-friction coefficient C_f, the Nusselt number N_u, defined as :

$$c_f = \frac{\tau_w}{\rho_{nf}\upsilon_w} \tag{24}$$

$$N_u = \frac{lq_w}{k_f(T_1 - T_2)} \tag{25}$$

Where :

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{26}$$

$$q_w = -k_{nf}\frac{\partial T}{\partial y}\bigg)_{y=0} \tag{27}$$

Using **(7)** and **(11)** in **(10)**, we get :

$$C_f^* = \frac{x^2}{l^2}(1 - \alpha t)\, R_{e_x}\, C_f = \frac{f''(0)}{(1 - \phi)^{2.5}A_1} \tag{28}$$

$$N_{u_x}^* = \sqrt{1 - \alpha t}\, N_{u_x} = -A_3\theta'(0) \tag{29}$$

Where $R_{e_x} = \frac{\alpha l^5}{2x^3(1-\alpha t)^{\frac{1}{2}}\upsilon_f}$ the local Reynolds number.

3. Basic Idea of the Homotopy Perturbation Method

To illustrate the basic ideas of this method, we consider the following equation :

$$A(u) - f(r) = 0, \qquad r \in \Omega \tag{30}$$

with the boundary condition of :

$$B\left(u, \frac{\partial U}{\partial n}\right) = 0, \qquad r \in \Gamma \tag{31}$$

where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain Ω.

A can be divided into two parts which are L and N, where L is linear and N is nonlinear. Eq. (16) can therefore be rewritten as follows :

$$L(u) + N(u) - f(r) = 0 \tag{32}$$

Homotopy perturbation structure is shown as follows :

$$H(v,p) = (1 - p)[L(v) - L(u_0)] + p[L(v) + N(v) - f(r)] = 0 \tag{33}$$

Where :

$$v(r,p): \Omega \times [0,1] \to R \tag{34}$$

In Eq. (22), $p \in [0,1]$ is an embedding parameter and 0 u is the first approximation that satisfies the boundary condition. We can assume that the solution of Eq. (21) can be written as a power series in p, as following :

$$v = v_0 + pv_1 + p^2v_2 + p^3v_3 + \cdots \tag{35}$$

and the best approximation for solution is :

$$u = \lim_{p \to 1} v = v0 + v1 + v2 + \cdots \tag{36}$$

4. Implementation of the method

According to the so-called homotopy-perturbation method (HPM), we construct a homotopy suppose the solution of Eq. (17) has the form :

$$(1 - p)[f'''' - f_0''''(0)] + p[f'''' + S\,A_1(1 - \phi)^{2.5}[3f'' + \eta f''' + f'f'' - ff''']] = 0 \tag{37}$$

$$(1 - p)[\theta'' - \theta_0''(0)] + p\left\{\theta'' + S\,Pr\,\frac{A_2}{A_3}[f\theta' - \eta\theta']\right\} = 0 \tag{38}$$

We consider f and θ as follows :

$$f(\eta) = f_0(\eta) + f_1(\eta) + f_2(\eta) + f_3(\eta) + \cdots$$

$$= \sum_{i=0}^{N} f_i(\eta) \qquad (39)$$

$$\theta(\eta) = \theta_0(\eta) + \theta_1(\eta) + \theta_2(\eta) + \theta_3(\eta) + \cdots \qquad (40)$$

$$= \sum_{i=0}^{N} \theta_i(\eta)$$

with substituting f, θ from equations (20 - 21) into equations (18-19) and some simplification and rearranging based on powers of p − terms, we have :

$$p^0 :$$

$$f^{iv} = 0,$$

$$\theta_0'' = 0, \qquad (41)$$

And boundary conditions are :

$$f(0) = 0, f'(0) = 0, f(1) = 1, \ f'(1) = 0, \ \theta(0) = 1 \text{ et}$$

$$\theta(1) = 0 \qquad (42)$$

$$P^1 :$$

$$f_1^{iv} - 1.102087119 S(\eta f_0''' + 3f_0'' + f_0'f_0'' - f_0 f_0''')$$

$$= 0 \qquad (43)$$

$$\theta_1'' + 0.9201899729 Pr S(\theta_0' f_0 - \eta \theta_0') = 0$$

And boundary conditions are :

$$f(0) = 0, f'(0) = 0, f(1) = 0, \ f'(1) = 0, \ \theta(0) = 0$$
$$\text{et } \theta(1) = 0 \qquad (44)$$

Solving equations (22) and (24) with boundary conditions, we have :

$$f_0 = -2\eta^3 + 3\eta^2 \quad \theta_0 = -\eta + 1$$

$$f_1 = \frac{3306261357}{500000000} S\left(\frac{1}{105}\eta^7 - \frac{1}{30}\eta^6 - \frac{1}{60}\eta^5 + \frac{1}{8}\eta^4\right) \qquad (45)$$

$$- \frac{472323051}{625000000}\eta^3 S + \frac{157441017}{800000000}\eta^2 S$$

$$\theta_1 = -\frac{9201899729}{10000000000} S Pr\left(\frac{1}{10}\eta^5 - \frac{1}{4}\eta^4 + \frac{1}{6}\eta^3\right)$$

$$+ \frac{9201899729}{600000000000} S Pr\eta$$

The terms $f_i(\eta), \Theta_i(\eta)$ when $i > 2$ are too large that is mentioned graphically. The solution of this equation, when $p \to 1$, will be as follows :

$$f(\eta) = f_0(\eta) + f_1(\eta) + f_2(\eta) + f_3(\eta) + \cdots$$

$$= \sum_{i=0}^{N} f_i(\eta) \qquad (46)$$

$$\theta(\eta) = \theta_0(\eta) + \theta_1(\eta) + \theta_2(\eta) + \theta_3(\eta) + \cdots$$

$$= \sum_{i=0}^{N} \theta_i(\eta) \qquad (47)$$

5. Results and discussion

In this work, the unsteady squeezing nanofluid flow and heat transfer between two parallel plates in the attendance of viscous dissipations impact, and considering the Fourier heat flux model of heat conduction are studied Semianalytically by using method The Homotopy Perturbation Method (HPM). To verify the present analytical solution, we compared our results with results given by using

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Table 1 : Comparison the Present results with two different methods (HPM) and (RK).

| S | Present results ($\varphi = 0.02, Ec = 0.01, \text{Pr} = 6.2, \delta = 0.1$) | | | |
| | The Rung-Kutta Method (RK) | | The Homotopy Perturbation Method (HPM) | |
	$-f''(1)$	$-\theta'(1)$	$-f''(1)$	$-\theta'(1)$
-1.0	5.591711670	1.517392479	5.591715131	1.517339344
-0.5	5.799635516	1.453320934	5.799635615	1.453317763
-0.1	5.960500492	1.405664454	5.960500487	1.405664446
0.0	6.000000010	1.394213026	6.000000000	1.394213021
0.01	6.003934651	1.393077678	6.003934637	1.393077674
0.1	6.039222572	1.382938524	6.039222564	1.382938511
0.2	6.078172838	1.371837113	6.078172818	1.371837028
0.3	6.116855340	1.360905070	6.116855315	1.360904683
0.5	6.193434683	1.339534832	6.193434559	1.339531970
1.0	6.380494566	1.288834807	6.380491471	1.28879168
1.5	6.561671376	1.241727015	6.561648736	1.24151864
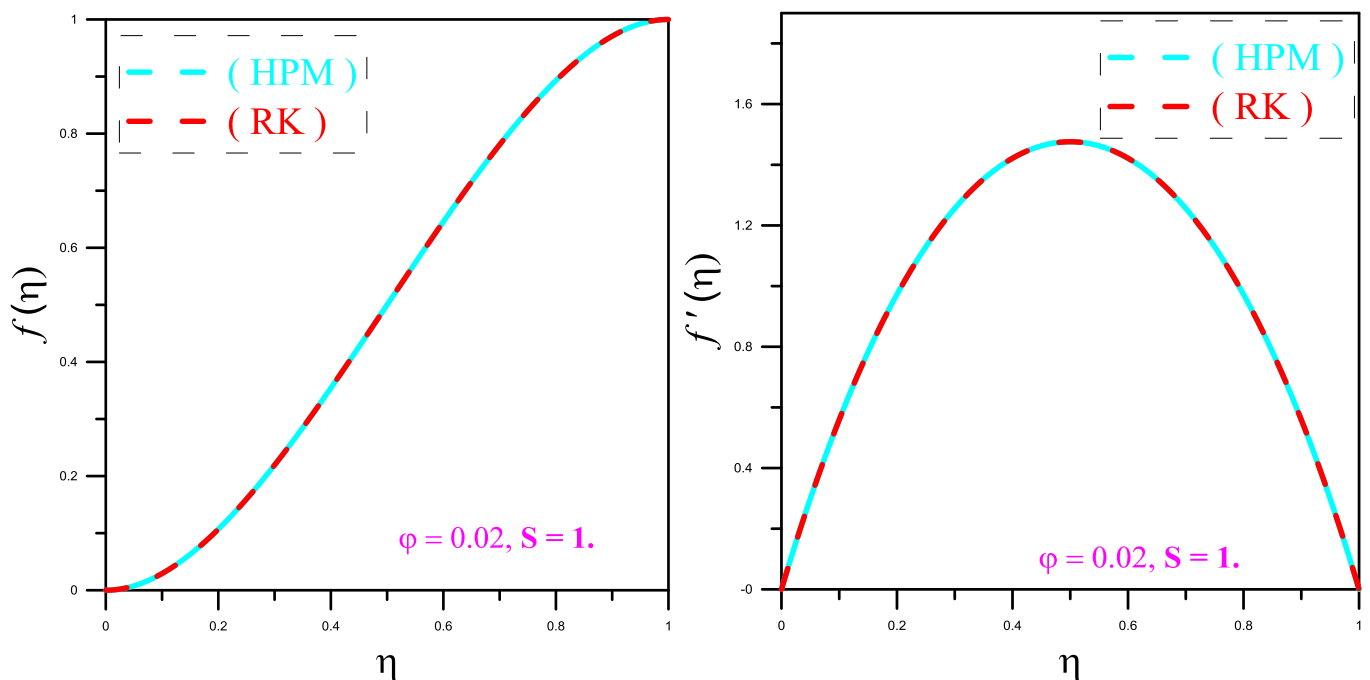
Figure 2 : comparison between the results given by (HPM) and (RK) for $f(\eta)$ and $f'(\eta)$.

Runge-Kutta. They are in an excellent agreement as they have been demonstrated in **Table 1** and **Figure 2 – Figure 3**.

To describe the physical insight of the present problem in depth, the thermodynamic flow behaviour of skin friction coefficient and Nusselt numbers along with velocity and temperature profiles in the flow region for different set physical parameters namely, squeezing parameter S, Eckert number Ec, Prandtl number Pr, are investigated. For better understanding of the numerical results, the computer generated numerical data are presented in the form of graphs and tables. The graphs are drawn for the copper

nanoparticles and water (Cu-Water). The thermophysical properties of nanofluid have been epitomized in **Table 2.**

Figure 4 – Figure 5 shows the effect of increasing the squeeze number S on the profile of the axial and radial velocity the latter increasing to the movable
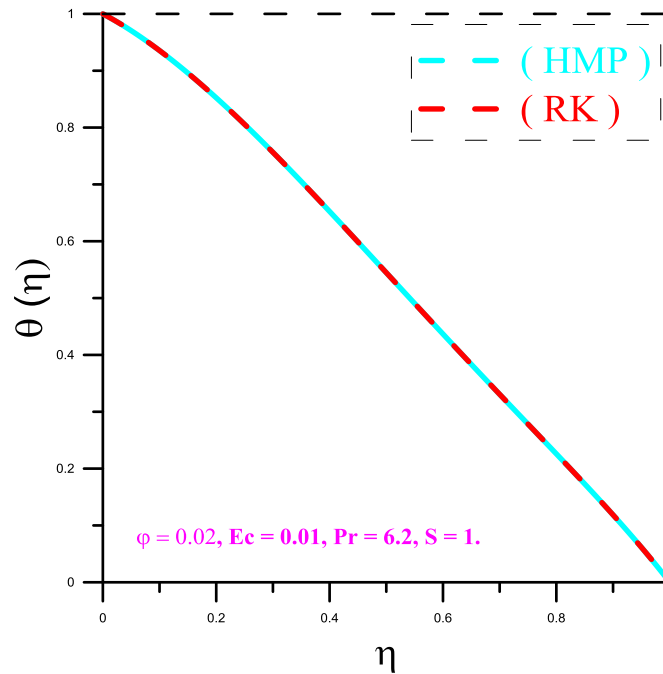

Figure 3 : comparison between the results given by (HPM) and (RK) for $\theta(\eta)$.

Table 2: Thermophysical properties of pure water and nanoparticles.

	ρ (Kg / m^3)	Cp (J / Kg)	K (W / mK)	σ (S m^{-1})
Copper (Cu)	8933	385	401	$5.96 * 10^7$
Pure water (H$_2$O)	997.1	4179	0.613	0.05
Alumina (Al$_2$ O$_3$)	3970	765.0	40.000	$1 * 10^{-10}$
Silver (Ag)	10.500	235	429	$6.3 * 10^7$
Gold (Au)	19.300	129	318	$4.52 * 10^7$

plate is in the middle of the initial distance between the plates, then it decreases slightly and tends towards zero when the two plates are glued together, and we also observe that the speed decreases with increasing values of S.

Figure 6 portrays the impact of the volume fraction φ on the velocity profile, Considering the curve of **Figure 6** we observe practically no change in the velocity profile when φ varies, but there is an effect well shown in the following **Table 3**, according to **Table 3** we observe that the

velocity rise when phi increases in the region $0 < \eta < 0.5$, and decreases in the region $0.5 < \eta < 1$.

The influence of Eckert number Ec on temperature profile is illustrated in the **Figure 7**. It is observed from **Figure 7** that, the temperature profile enhanced for the increasing values of Eckert number. However, this upsurge in temperature profile is expected because Ec has direct impact on the heat dissipation process and hence increases the temperature field in the flow region.

Similarly, the impact of Prandtl number Pr on temperature profiles is depicted in the **Figure 8**. However, it is remarked from the **Figure 8** that, the temperature profile enhanced for the increasing values of Prandtl number in the flow region. This variation in temperature profile is mainly due to the fact that, the larger Values of Pr considerably reduces the thermal diffusivity and decreases the thickness of the thermal boundary layer and which in turn responsible for the increment of thermal profile in the flow regime.

Figure 9 we observe practically change in the velocity profile when the squeese number S varies,
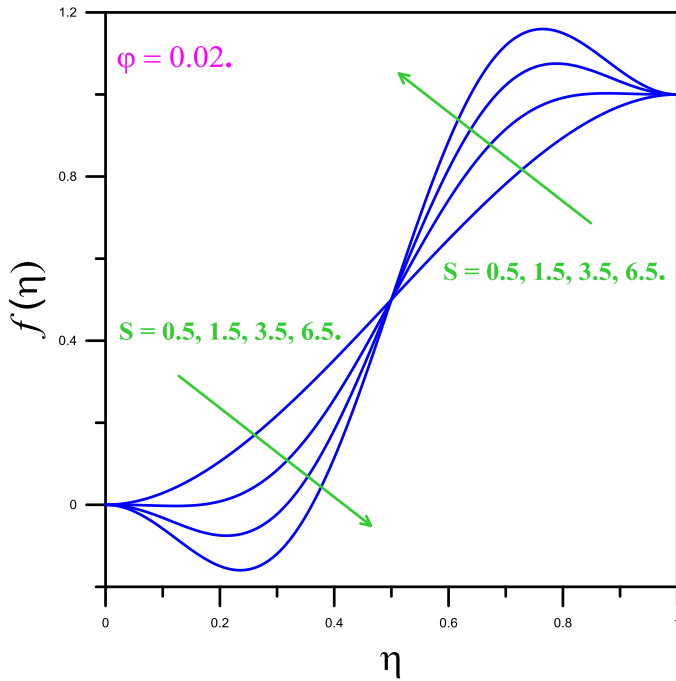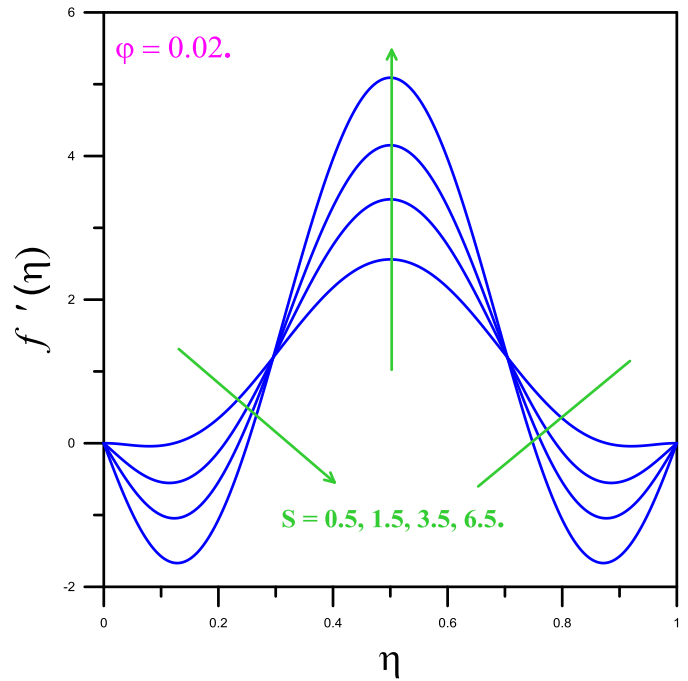

Figure 4 : Impact of S on $f(\eta)$.
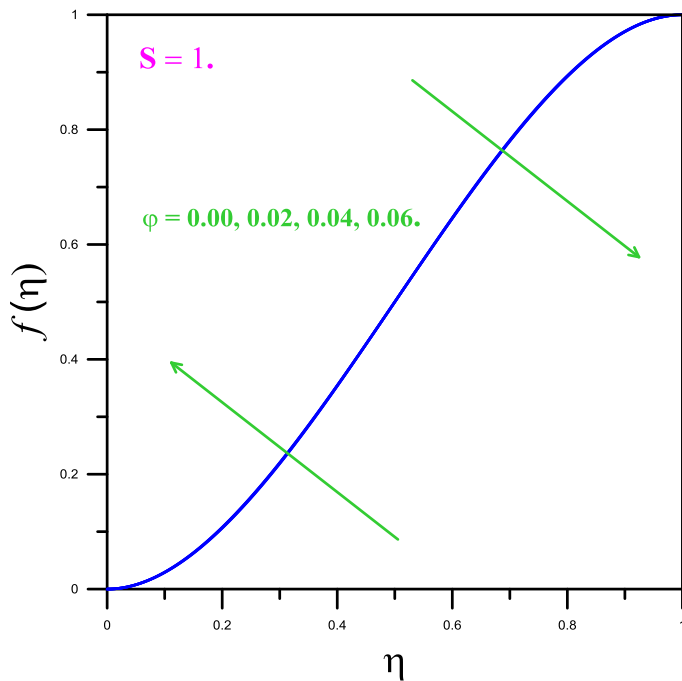

Figure 5 : Impact of S on $f(\eta)$.

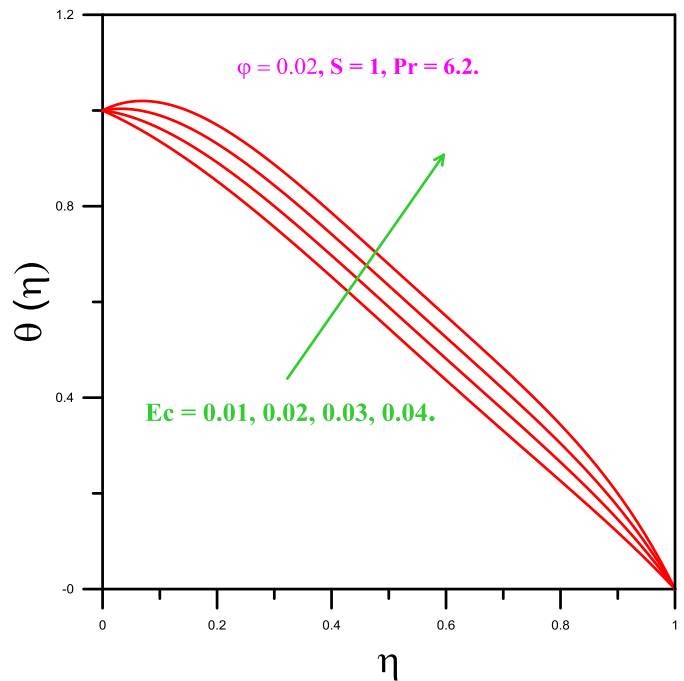**Figure 6 :** Impact of φ on $f(\eta)$.

Figure 7 : Impact of Ec on $\theta(\eta)$.

Table 3 : Velocity value for different values of φ.

η	$\varphi = 0.01$	$\varphi = 0.02$	$\varphi = 0.03$	$\varphi = 0.04$	$\varphi = 0.05$	$\varphi = 0.06$
			$S = 1.$			
	f	f	f	f	f	f
0	1	1	1	1	1	1
0.1	0.0291871774	0.0292404961	0.0292899321	0.0293356158	0.0293776727	0.0294162238
0.2	0.1068277623	0.1069539313	0.1070708472	0.1071788328	0.10727819797	0.1073692399
0.3	0.2192583687	0.2194030567	0.2195370789	0.2196608175	0.2197746383	0.2198788914
0.4	0.3541320709	0.3542264711	0.3543138908	0.3543945841	0.3544687942	0.3545367528
0.5	0.4999999998	0.4999999999	0.5000000000	0.4999999998	0.5000000000	0.4999999999
0.6	0.6458679288	0.6457735288	0.6456861091	0.6456054155	0.6455312058	0.6454632470
0.7	0.7807416309	0.7805969431	0.7804629210	0.7803391821	0.7802253617	0.7801211084
0.8	0.8931722373	0.8930460684	0.8929291527	0.8928211667	0.8927218021	0.8926307598
0.9	0.9708128220	0.9707595035	0.9707100677	0.9706643836	0.9706223274	0.9705837758
1	0	0	0	0	0	0

according to **Figure 9** we observe that the temperature decreases when phi increases in the region $0 < \eta < 0.5$, and rise in the region $0.5 < \eta < 1$.

The effect of the volume fraction φ on the temperature is well shown in **Figure 10**, from the **Figure 10** it is obvious that the temperature profile increased when the φ value increases because the rise in the volume fraction value increases the friction between the particles nanofluid so increase the temperature.

From **Table 4** it is evident that the coefficient of friction and the Nusselt number are proportional to S, and it is observed that the parameters C_f^* increase with increasing values of S and N_u^* decrease as S increases.

In **Table 5** it is evident that the coefficient of friction and the Nusselt number are inversely proportional to φ, C_f^* and N_u^* decreases as φ increases.

The comparison for metallic nanoparticles (Ag, Cu, and Au) and nonmetallic nanoparticles (Al$_2$ O$_3$) is done in **Table 6**.

Table 4: Variation of C_f^* and N_u^* for different values of S.

S	$Pr = 6.2, \varphi = 0.02, Ec = 0.01$	
	C_f^*	N_u^*
0.5	5.61973	0.88071
1	5.78950	0.84032
1.5	5.95406	0.80209
2	6.11394	0.76587

Table 5 : Variation of C_f^* and N_u^* for different values of φ.

Table 6 : Variation of C_f^* and N_u^* for different nanoparticles.

Nanoparticles	c_f	N_u
Gold (Au)	6.323437982	1.29475305
Copper (Cu)	6.380491471	1.28744224
Silver (Ag)	6.323381375	1.29476136
Alumina (Al$_2$ O$_3$)	6.348795072	1.28770311

6. Conclusions

The present paper deals with the numerical solution of heat transfer effect of an unsteady two dimensional flow of incompressible viscous nanofluids in middle of two parallel plates extended infinitely in the attendance of viscous dissipations impacts, The relevant nonlinear partial differential equations were transformed to a set of ordinary differential equations and then are solved analytically using the homotopy perturbation method (HPM), From the above discussion the eye-catching results are as follows.

φ	$Pr = 6.2, S = 1, Ec = 0.01$	
	C_f^*	N_u^*
0	6.34632	0.90222
0.01	6.04486	0.87082
0.02	5.78950	0.84032
0.03	5.57157	0.81069
0.04	5.38450	0.78190
0.05	5.22323	0.75391

- The temperature increases in Cu-water nanofluid with an increase in the volume fraction φ.

- The temperature increases in Cu-water nanofluid with an increase in the Eckert number Ec.

- The temperature increases in Cu-water nanofluid with an increase in the Prandtl number Pr.

- The coefficient of friction and the Nusselt number are proportional to squeese number S.

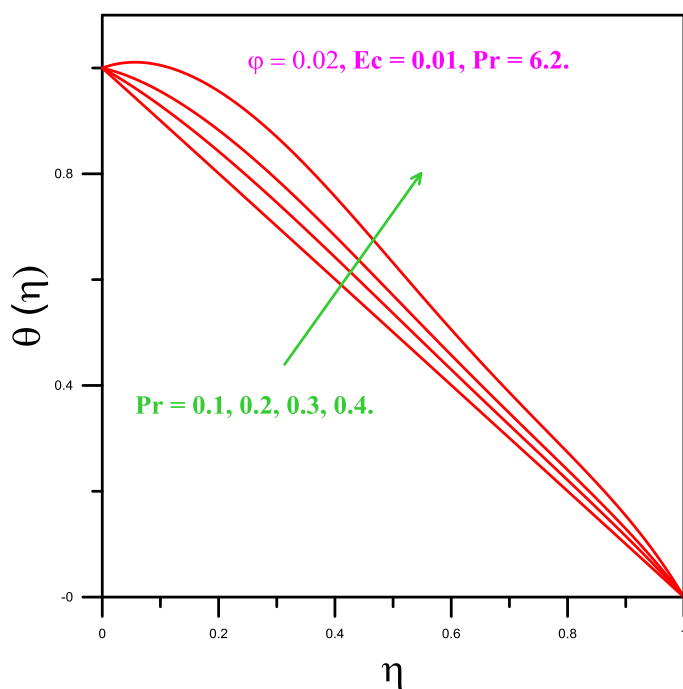- The coefficient of friction and the Nusselt number are inversely proportional to the volume fraction φ.
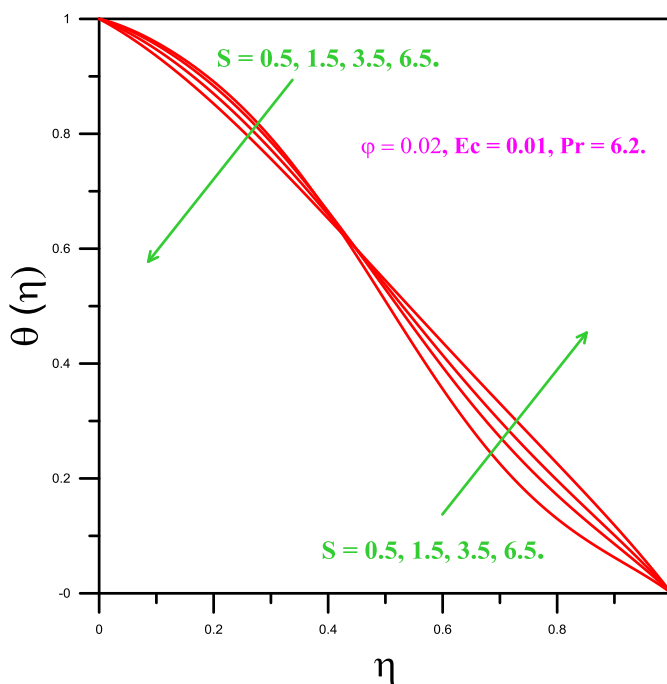

Figure 8 : Impact of Pr on $\theta(\eta)$.

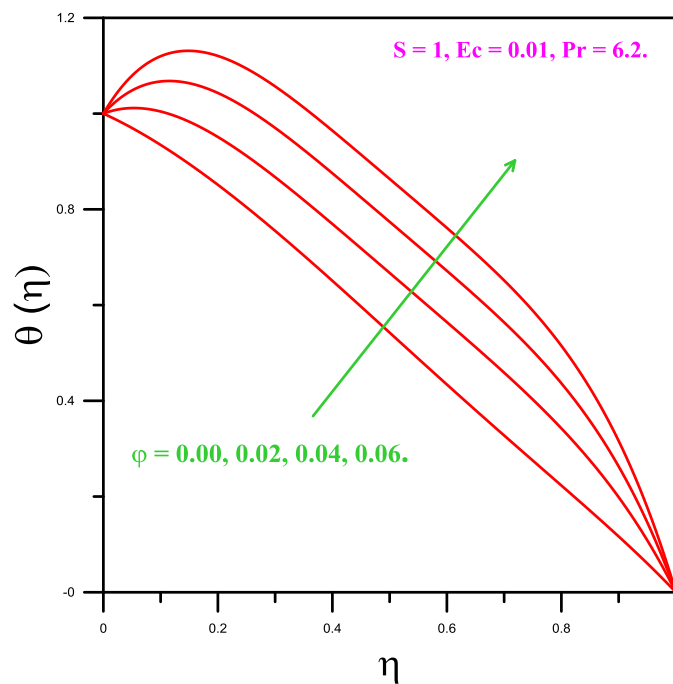**Figure 9 :** Impact of S on $\theta(\eta)$.

Figure 10 : Impact of φ on $\theta(\eta)$.

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