Numerical simulation of MHD natural convection flow in a wavy cavity filled by a hybrid Cu–Al_2O_3/water nanofluid with discrete heating

CORNELIA REVNIC
Faculty of Pharmacy,
University of Medicine and Pharmacy,
Pasteur Str., No 6, 400012, Cluj-Napoca
ROMANIA;
cornelia.revnic@umfcluj.ro

RADU TRÎMBIȚAȘ
Department of Applied Mathematics,
Babes Bolyai University
M. Kogalniceanu Str., No. 1, 400084, Cluj-Napoca
ROMANIA
tradu@math.ubbcluj.ro

Abstract: - The present paper investigates the magnetic field and heat transfer inside a square cavity with a wavy upper wall. The vertical and the upper walls are kept at lower constant temperature, the third part in the middle of the bottom wall is kept on high constant temperature and the remaining part of the bottom wall are considered to be adiabatic. The unsteady laminar convection model of hybrid Cu- Al_2O_3/water nanofluid was solved numerically using finite element method for different values of considered parameters. The effect of the magnetic field parameter Ha, of the Rayleigh number Ra, of the inclination magnetic field angle γ, of the nanoparticle volume fraction ϕ and of the undulation number of wave cavity n on the fluid flow, temperature and heat transfer are analysed. The important changes at the bottom wall is due to the high values in Rayleigh and Hartmann number while for the upper wall the changes come from taking different values for inclination magnetic field angle and of the undulation number of wave cavity.

Key-Words: - Magnetohydrodynamics, natural convection, hybrid nanofluid, heat transfer, bottom heater, wavy top enclosure.

1 Introduction

During the last several decades, numerous studies have focused on using nanoparticles in thermal systems in an attempt to enhance the inherent thermal conductivity of heat transfer fluids by suspending a certain concentration of nanoparticles in base fluids. It is already well-known that traditional working fluids such as water, oil or ethylene glycol have a low thermal conductivity. As a result, their heat transfer performance is inevitably limited. In an attempt to improve the heat transfer performance, Choi [1] dispersed metallic nanoparticles with a high thermal conductivity in traditional working fluids to form so-called nanofluids. Later on, many authors discussed the effects of nanoparticles for different fluid models, see for example, the books by Das et al. [2], Minkowycz et al. [3], Shenoy et al. [4] and Nield and Bejan [5], and in the review papers by Buongiorno et al. [6], Kakaç and Pramuanjaroenkij [7], Manca et al. [8], Mahian et al. [9-11], Sheikholeslami and Ganji[12], Groşan et al. [13], etc

After Huminic and Huminic [14], hybrid nanofluids are a new class of working fluids containing very small particles with sizes (under 100 nm) used in heat transfer applications. These fluids consis from two or three solid materials into conventional fluids (water, ethylene glycol or water ethylene glycol mixture, engine oil, kerosene, vegetable oil and paraffin oil). The solid nanoparticles used for heat transfer enhancement of working fluids are: Al_2O_3-Cu, Al_2O_3-Ag, $Cu-TiO_2$, $Cu-Cu_2O$, etc. In the last year's, these hybrid nanofluids were used in various heat transfer applications as heat pipes, micro-channel, mini channel heat sink, plate heat exchanger, air conditioning system, tubular heat exchanger, shell and tube heat exchanger, tube in tube heat exchanger and coiled heat exchanger, helical coil heat exchanger. Comprehensive reviews on hybrid nanofluids were presented by

Devi and Devi [15-17], Sarkarn et al. [18], Sidik et al. [19], Sundar et al. [20], Babu et al. [21], Tayebi and Chamkha [22], Yousef et al. [23], Aly and Pop [24], Waini et al. [25], Ghadikolaei et al. [26] Jana et al. [27], Hayat et al. [28], Ghadikolaei et al. [29], etc.

In the present work, nanoparticles of alumina Al_2O_3 with solid volume fraction ϕ_1 were added to the base fluid (water) as nanofluid (Al_2O_3/water). Then, copper (Cu) nanoparticles with solid volume fraction ϕ_2 are mixed to this nanofluid to get hybrid nanofluid (Cu-Al_2O_3/water).

2 Problem Formulation

Let us consider the unsteady laminar two-dimensional MHD flow and heat transfer of a wavy-walled enclosure with Cartesian coordinates (x,y) and velocity components (u,v) as shown in Fig. 1. The width and height of the enclosure is L and the fluid inside is hybrid Cu–Al_2O_3/water nanofluid. It is assumed that the top wavy wall and the vertical walls are maintained at low temperature (T_c). The temperature of the central part of the bottom wall is T_h, where $T_h > T_c$, whereas zero gradient of temperature is maintained at the rest part of the lower wall. Distance of the heat source from both the vertical walls is exactly the same. A uniform magnetic field with constant magnitude B_0 and the gravitational force g are applied vertically normal to the horizontal wall. It is considered that the upper wavy wall of the enclosure is defined by the relation:

$$f(x) = L + A\, sin\left(n\pi \frac{x}{L}\right) \tag{1}$$

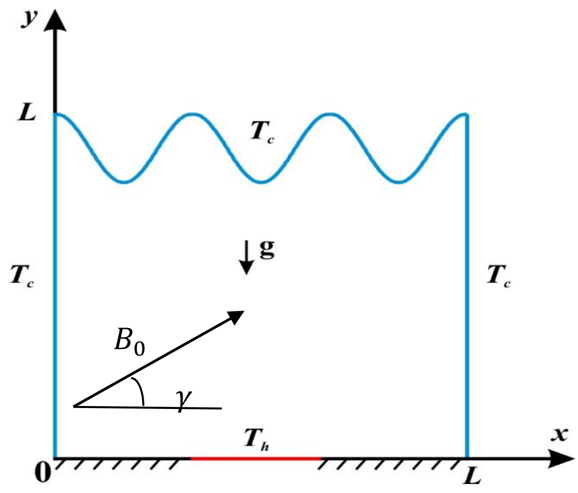

Fig. 1. The physical model and coordinate system.

The flow is considered to be laminar, unsteady and incompressible and the thermo-physical properties of base fluid and hybrid nanofluid are tabulated in Table 1. The governing non-linear partial differential equations of mass, momentum and energy can be written as follows (see Devi and Devi, 2006 and 2017)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho_{hnf}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu_{hnf}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \sigma_{hnf}B_0^2(u\sin^2(\gamma) - v\sin(\gamma)\cos(\gamma)) \tag{2}$$

$$\rho_{hnf}\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + +\mu_{hnf}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + (\rho\beta)_{hnf}g(T - T_0) -\sigma_{hnf}B_0^2(v\cos^2(\gamma) - u\sin(\gamma)\cos(\gamma)) \tag{3}$$

$$\rho_{hnf}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \alpha_{hnf}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

subject to the initial and boundary condition

$$\begin{aligned} &t = 0: u = 0, v = 0, T = T_c\ for\ \ 0 \le y \le L, \\ &\qquad x = 0, L \\ &t > 0: u = 0, v = 0, T = T_c\ for\ 0 \le x \le L, \\ &\qquad y = L \\ &u = 0, v = 0, T = T_h\ for\ \ 0.3L \le x \le 0.7L, \\ &\qquad y = 0 \end{aligned} \tag{5}$$

$$u = 0, v = 0, \partial T/\partial y = 0 \; for \;\; 0 \le x \le 0.3L, \quad 0.7L \le x \le L, \qquad y = 0$$

Here T is the temperature of the hybrid nanofluid, T_0 is the mean temperature of heated and cooled walls defined by $T_0 = \frac{T_h + T_c}{2}$, μ_{hnf} is the dynamic viscosity of the hybrid nanofluids, ρ_{hnf} is the density of the hybrid nanofluids, k_{hnf} is the thermal conductivity of the hybrid nanofluid, $(\rho C_p)_{hnf}$ is the heat capacity of the hybrid nanofluid, σ_{hnf} is the electrical conductivity of the hybrid nanofluids and C_p is the heat capacity at the constant pressure. Physical properties of nanofluids and hybrid nanofluids are given in Table 1.

Table 1. Thermal properties of nanofluids and hybrid nanofluids

Properties	Nanofluids	Hybrid nanofluids
Density	$\rho_{nf} = \phi\rho_s + (1-\phi)\rho_f$	$\rho_{hnf} = \phi_1\rho_{s1} + \phi_2\rho_{s2} + (1-\phi_{hnf})\rho_f$
Heat capacity	$(\rho C_p)_{nf} = \phi(\rho C_p)_s + (1-\phi)(\rho C_p)_f$	$(\rho C_p)_{hnf} = \phi_1(\rho C_p)_{s1} + \phi_2(\rho C_p)_{s2} + (1-\phi_{hnf})(\rho C_p)_f$
Dynamic viscosity	$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\phi)^{2.5}}$	$\mu_{hnf} = \mu_f\left(1-\phi_{Al_2O_2}-\phi_{Cu}\right)^{-2.5}$
Thermal conductivity	$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}$ $\frac{\sigma_{nf}}{\sigma_f} = \frac{\sigma_s + 2\sigma_f - 2\phi(\sigma_f - \sigma_s)}{\sigma_s + 2\sigma_f + \phi(\sigma_f - \sigma_s)}$	$\frac{k_{hnf}}{k_f} = \left\{\frac{\phi_{Al_2O_3}k_{Al_2O_3} + \phi_{Cu}k_{Cu}}{\phi_{Al_2O_3} + \phi_{Cu}} + 2k_f + 2\left(\phi_{Al_2O_3}k_{Al_2O_3} + \phi_{Cu}k_{Cu}\right) - 2(\phi_{Al_2O_3} + \phi_{Cu})k_f\right\} \times \left\{\frac{\phi_{Al_2O_3}k_{Al_2O_3} + \phi_{Cu}k_{Cu}}{\phi_{Al_2O_3} + \phi_{Cu}} + 2k_f - \left(\phi_{Al_2O_3}k_{Al_2O_3} + \phi_{Cu}k_{Cu}\right) + (\phi_{Al_2O_3} + \phi_{Cu})k_f\right\}^{-1}$
Electrical conductivity	$\frac{\sigma_{nf}}{\sigma_f} = \frac{\sigma_s + 2\sigma_f - 2\phi(\sigma_f - \sigma_s)}{\sigma_s + 2\sigma_f + \phi(\sigma_f - \sigma_s)}$	$\phi_{khf} = \phi_1 + \phi_2$ $\frac{\sigma_{hnf}}{\sigma_f} = \left\{\frac{\phi_{Al_2O_3}\sigma_{Al_2O_3} + \phi_{Cu}\sigma_{Cu}}{\phi_{Al_2O_3} + \phi_{Cu}} + 2\sigma_f + 2\left(\phi_{Al_2O_3}\sigma_{Al_2O_3} + \phi_{Cu}\sigma_{Cu}\right) - 2(\phi_{Al_2O_3} + \phi_{Cu})\sigma_f\right\} \times \left\{\frac{\phi_{Al_2O_3}\sigma_{Al_2O_3} + \phi_{Cu}\sigma_{Cu}}{\phi_{Al_2O_3} + \phi_{Cu}} + 2\sigma_f - \left(\phi_{Al_2O_3}\sigma_{Al_2O_3} + \phi_{Cu}\sigma_{Cu}\right) + (\phi_{Al_2O_3} + \phi_{Cu})\sigma_f\right\}^{-1}$

Here ϕ is the nanoparticle volume fraction ($\phi = 0$ correspond to a regular fluid, ϕ_1 correspond to Al_2O_3 and ϕ_2 correspond to Cu), ρ_f and ρ_s are the densities of the base fluid and the hybrid nanoparticle, respectively, k_f and k_s are the thermal conductivities of the base fluid and the hybrid nanoparticles, respectively, $(\rho C_p)_f$ and $(\rho C_p)_s$ are the heat capacitance of the base fluid and the hybrid nanoparticle, respectively, and σ_f and σ_s are the electrical conductivities of the base fluid and the nanofluid, respectively. The physical properties of the base fluid (water) and alumina-copper (Al_2O_3-Cu) nanoparticles are given in Table 2.

Table 2. The thermophysical properties of the base fluid and nanoparticles (Hayat et al., [28] or Devi and Devi, [17]).

Properties	Cu	Al_2O_3	Base fluid (water)
ρ (kg/m³)	8933	3970	997.1
Cp (J/kg.K)	385	765	4179.0
k (W/m.K)	400	40	0.613
$\alpha\times10^7$	1163.1	131.7	1.47
β (1/K)$\times10^6$	50.1	25.5	210
Nanoparticles size (nm)	20		-

According to Parveen and Mahapatra [30], we introduce the following dimensionless variables

$$X=\frac{x}{L},Y=\frac{y}{L},\tau=\frac{t\alpha_f}{L^2},U=\frac{uL}{\alpha_f},V=\frac{vL}{\alpha_f},\Omega=\frac{\omega L^2}{\alpha_f},\theta=\frac{T-T_0}{T_h-T_0},\Psi=\frac{\psi}{\alpha_f} \quad (6)$$

Using these parameters the Eqs. (1)-(4) become

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (7)$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{\mu_{hnf}}{\mu_f}\frac{\rho_f}{\rho_{hnf}}Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)-Ha^2Pr\frac{\mu_{hnf}}{\mu_f}\frac{\rho_f}{\rho_{hnf}}(U\sin^2(\gamma)-V\sin(\gamma)\cos(\gamma)) \quad (8)$$

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{\mu_{hnf}}{\mu_f}\frac{\rho_f}{\rho_{hnf}}Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+RaPr\frac{(\rho\beta)_{hnf}}{(\rho\beta)_f}\frac{\rho_f}{\rho_{hnf}}\frac{\partial\theta}{\partial X}-Ha^2Pr\frac{\mu_{hnf}}{\mu_f}\frac{\rho_f}{\rho_{hnf}}(V\cos^2(\gamma)-U\sin(\gamma)\cos(\gamma)) \quad (9)$$

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{k_{hnf}}{k}\frac{(\rho c_p)_f}{(\rho c_p)_{hnf}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \quad (10))$$

the initial and the boundary conditions (5) become

$$\tau=0: u=0, v=0, \theta=-1\ for$$
$$0\le Y\le 1, X=0,1$$
$$\tau>0: u=0, v=0, \theta=-1\ for \quad (11)$$
$$0\le X\le 1, Y=1$$
$$u=0, v=0, \theta=1\ for$$
$$0.3\le X\le 0.7, Y=0$$
$$u=0, v=0, \partial T/\partial Y=0\ for$$
$$0\le X\le 0.3,\quad 0.7\le X\le 1, Y=0$$

Here Pr is the Prandtl number, Ha is the Hartmann number and Ra is the Rayleigh number which are defined as

$$Pr=\frac{\nu_f}{\alpha_f}, Ha=\sqrt{\frac{\sigma_{hnf}}{\mu_{hnf}}}B_0L, Ra=\frac{\beta_f g\Delta T L^3}{\alpha_f\nu_f} \quad (12)$$

The parameters of physical interest are the local Nusselt number Nu and the average Nusselt number Nu_{avg}, which are defined as

$$Nu=-\frac{k_{hnf}}{k_f}\frac{\partial\theta}{\partial Y}, Nu_{avg}=\frac{1}{0.4}\int_{0.3}^{0.7}Nu\,dX \quad (13)$$

3. Numerical method and validation

The developed computational code has been verified using numerical data Yu et al. [31] for natural convection inside a square cavity with different temperature on the left and right boundary but adiabatic on the top and bottom boundary. The quantities presented here are the maximum absolute values of horizontal velocity $|u|_{max}$ and vertical velocity $|v|_{max}$ and the average Nusselt number on the left heated wall Nu_0. It is clear from Table 1 that the obtained results of the present work illustrates good agreement and therefore we get the confidence of the present numerical code.

Table 3. Comparison of present results with Yu et al. [31]

	$\|u\|_{max}$		$\|v\|_{max}$		Nu_0	
$Ra = 10^4$	Present	Yu.et.al.[31]	Present	Yu.et.al.[31]	Present	Yu.et.al.[31]
Ha=0	15.263	15.656	15.186	15.534	2.006	2.0212
Ha=10	11.683	11.762	11.832	11.883	1.7852	1.7862
Ha=50	2.4940	2.489	1.8973	1.905	1.0373	1.0374
Ha=100	0.9597	0.961	0.4806	0.483	1.0031	1.0032
$Ra = 10^5$	Present	Yu.et.al. [30]	Present	Yu.et.al.[30]	Present	Yu.et.al. [30]
Ha=0	43.132	44.595	43.441	44.725	3.4546	3.4704
Ha=30	24.281	24.383	28.069	28.702	2.7778	2.790
Ha=160	4.6495	4.657	2.1345	2.230	1.0502	1.0506
Ha=320	1.7193	1.707	0.5039	0.597	1.0037	1.0037

4. Results and discussions

In this study we have evaluated the MHD laminar natural convection of the hybrid Cu–Al_2O_3/water nanofluid in a wavy cavity heated on the third part in the middle of the bottom wall. The effects of different parameters, namely the Rayleigh number ($10^3 \leq Ra \leq 10^5$) and the Hartmann number ($0 \leq Ha \leq 60$)) as well as the effects of nanoparticles hybrid concentration ($\varphi_1 = 0.025, \varphi_2 = 0.025$) and magnetic field orientation ($0 \leq \gamma \leq \pi$), on the flow and heat transfer fields for the cases of hybrid nanofluid are investigated. The default values of Prandtl number and amplitude of wavy cavity are fixed to $Pr = 7$, A=0.2.

Figure 2 depicts the influence of Rayleigh number on streamlines and isotherms when $Ha = 30, \gamma = 0^0, A = 0.2, n = 3$. Because of heated the middle bottom wall, the warm fluid lifts from down-up and falls along the sides of the lower temperature vertical walls forming counter-rotating two vortices within the cavity. At lower values of $Ra (= 10^3)$, the strength of stream function is weak because the conduction is dominant while enhancement in $Ra(= 10^5)$, the magnitude of stream function increases because the buoyancy force ingresses.

Figure 3 shows the changes of isotherms and streamlines due to increasing of Hartmann number for fixed values of the following parameters $Ra = 10^5$, *Pr*=7, $A = 0.2, n = 3, \gamma = 0$. Comparing with the first case when Ra increases and the fluid spreads over the enclosure from down to up walls, now, increasing the Ha number, the fluid get down. This occurs because the maximum values of stream function decreases with the augmentation of Lorentz forces due to weak flow under the influence of magnetic field. The streamline boundary layers are found to be more compressed near the bottom wall and the center of the core become more flattened. It is observed from the isotherms that as Hartman number increases, effects of increasing Rayleigh number on temperature distribution decrease. This is in agreement with the results reported by Parveen R., et al. [30].

Figure 4 exposes the flow structure within the flow domain based on the different form of the upper wall. The isotherms have uniform pattern in the square cavity ($n = 0$), but the pattern of isotherms follows the form of upper wall for $n = 1$ forming one loop. Finally, these changes into two and three wavy for $n = 3$ and 5, respectively. It is seem that increasing in undulation number has a significant effect on the temperature gradient in the hybrid Cu-Al_2O_3/water nanofluid considered. Tacking a glance over these figures, we can conclude that the wave crests and troughs define the fluid flow intensity and the pattern of the isotherms.

Figure 5 shows the variation of fluid flow and isotherm with magnetic field angle. Therefore, the presence of the magnetic force tends to accelerate the fluid motion inside the cavity in the direction prescribed by the magnetic field angle. It is seems that the maximum velocity flux attained is when the direction of the magnetic field has the vertical direction ($\gamma = 90^0$). Also, these figures show that when the magnetic field direction is parallel with the first and the second bisecting lines ($\gamma = 45^0$ and 135^0, respectively) the core vortex is elongated along with these lines.

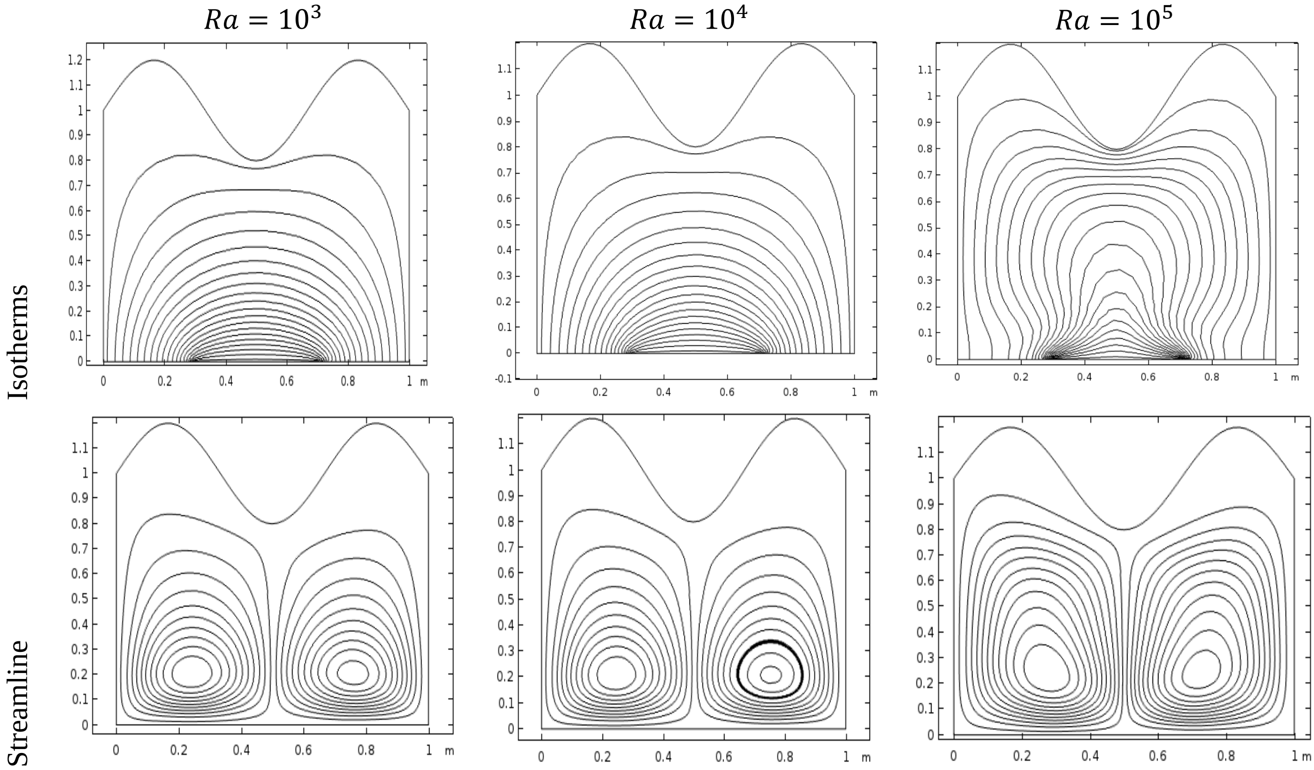

Fig. 2: Streamline and isotherms for different Ra at $Ha = 30$, Pr=7, $A = 0.2, n = 3, \gamma = 0$.

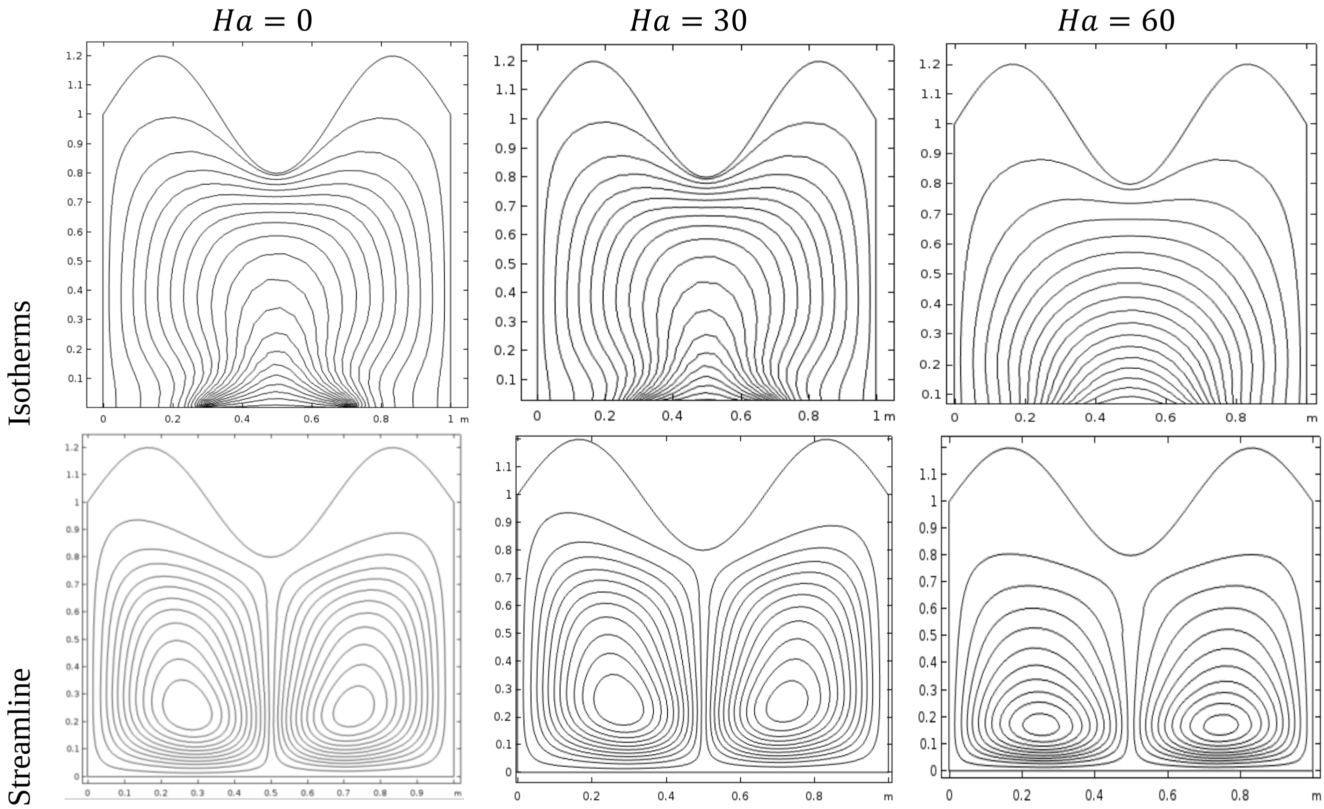

Fig. 3: Streamline and isotherms for different Ha at $Ra = 10^5$, Pr=7, $A = 0.2, n = 3, \gamma = 0$.

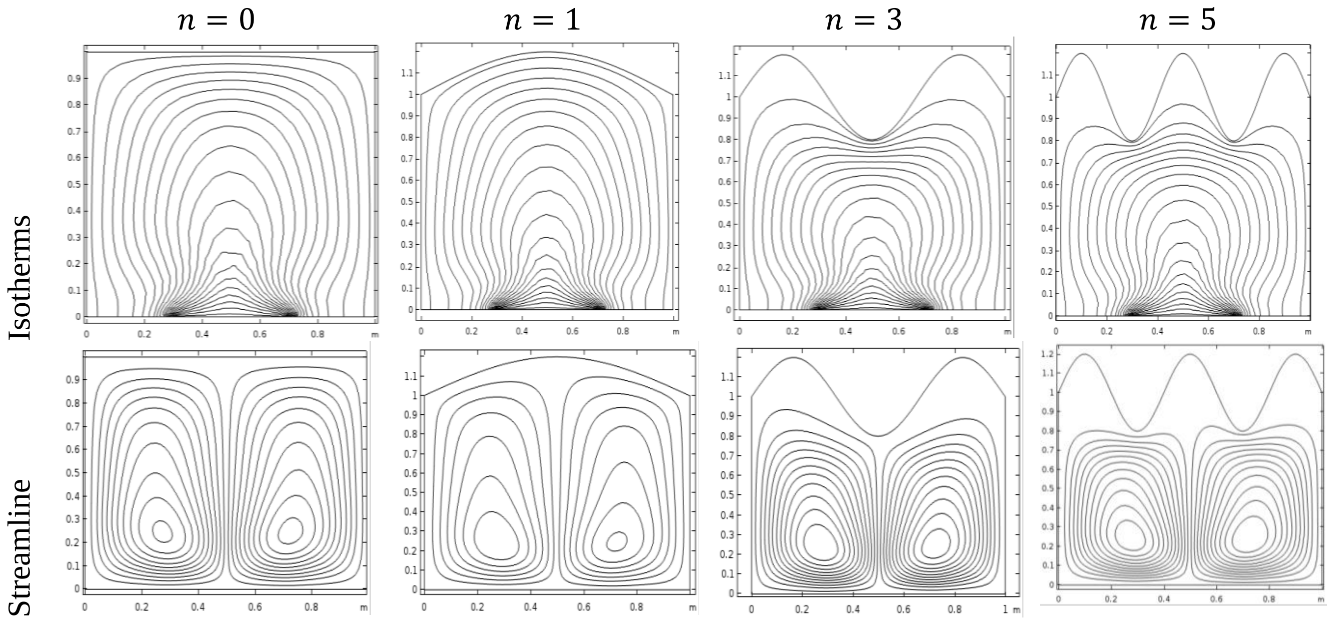

Fig. 4:Streamline and isotherms for different undulation numbers at $Ha = 30, Ra = 10^5$,Pr=7,$A = 0.2, \gamma = 0$.

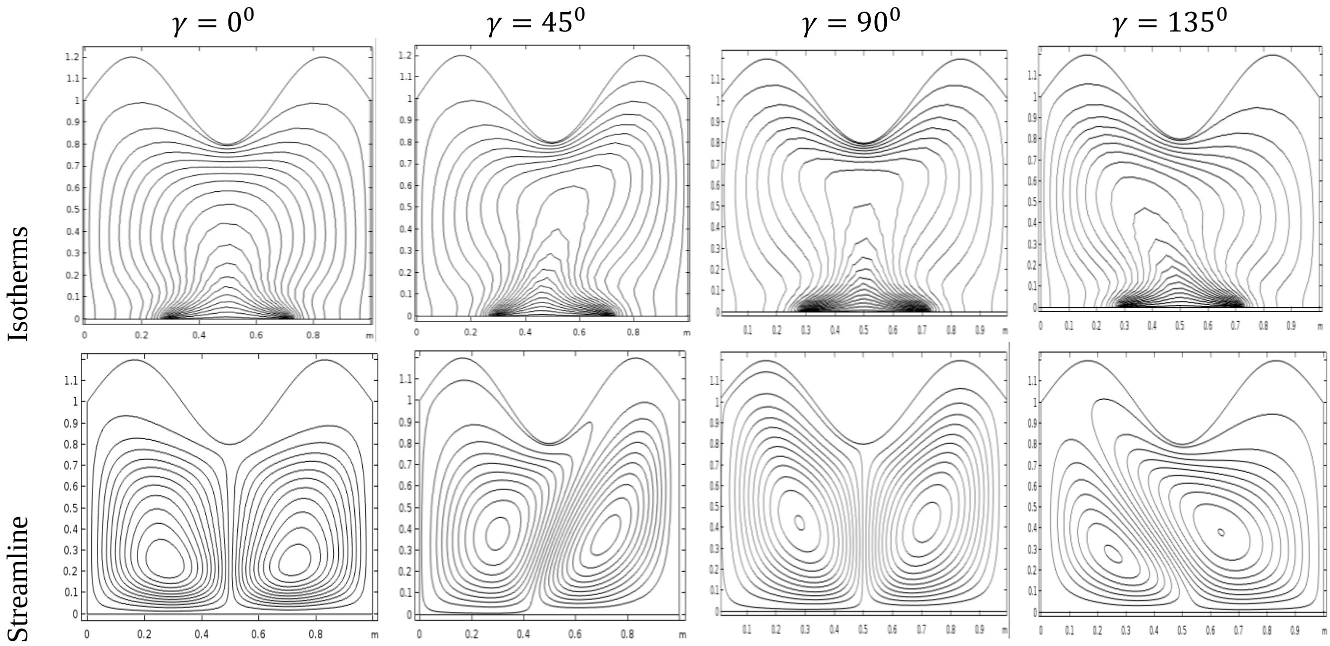

Fig. 5: Streamline and isotherms for different values of inclined angle at $Ha = 30$, $Ra = 10^5$, Pr=7, $A = 0.2$.

The heat rate is determined using the average Nusselt number in respect with Ha and for $\gamma = 0^0, Pr = 7, n = 3, A = 0.2$. Form Fig. 6, it can be seen that, the increase in Ra from 10^3 to 10^5 the heat transfer rate increase in respect with Ha. Also, Nusselt number attenuates with Ha because the presence of magnetic field intensifies the conduction mechanism and in consequently reduces the heat transfer rate. The influence of different proportions of nanoparticle volume fraction of the hybrid on heat transfer is illustrated in Table 4, by keeping the other parameters constant ($\gamma = 0^0, Pr = 7, n = 3, A = 0.2, Ha = 30$). The maximum heat transfer take place when the hybrid is composed from cooper ($\phi_2 = 0.05$) and the minimum heat transfer take place when the hybrid is half percent composed by Al_2O_3 and half percent composed by Cu ($\phi_1 = 0.05$), $\phi_2 = 0.05$).

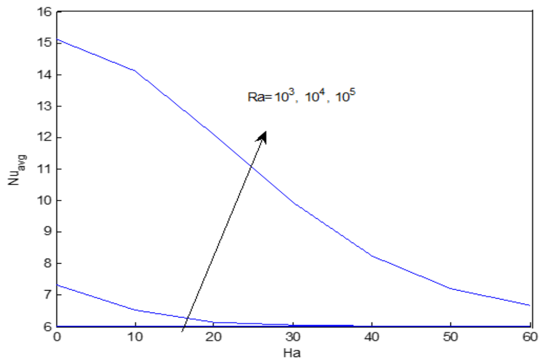

Fig. 6: Nusselt number in respect with Ha for different values of Ra and for $\gamma = 0^0, Pr = 7, n = 3, A = 0.2$

Table 4. Effect of nanoparticle volume fraction on heat transfer

ϕ_1/ϕ_2	0.025/0.025	0.05/0.00	0.00/0.05
Nu	9.91698	9.6591458	10.10171

5. Conclusions.

The present work investigates heat transfer in a wavy cavity heated from the bottom middle wall and filled with a hybrid Cu–Al_2O_3/water nanofluid. The vertical and the upper walls are maintained at a lower temperature while the remaining part of the bottom wall are considered to be adiabatic. The main results obtained can be summarized as follows:

- At lower values of Ra, the strength of stream function is weak because the conduction is dominant while enhancement in Ra, the magnitude of stream function increases because the buoyancy force ingresses.
- Also, it is observed that as Hartman number increases, the effect of increasing Rayleigh number on temperature distribution decrease.
- Convection mode increases with rise of Rayleigh number but it attenuates with rise of Lorentz forces
- The average Nusselt number increases with increase in Ra and attenuates with increase in Hartmann number.

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