# Soret and Dufour effects of Convective Boundary layer flow over a moving permeable cylinder

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*Abstract:* - The present analysis deals with the numerical investigation of a time dependent laminar free convective flow of a viscous isochoric fluid past a moving semi-infinite vertical cylinder entrenched in a porous medium under the influence of Cross-diffusion. The problem consists of a system of coupled nonlinear boundary layer equations together with appropriate boundary conditions are solved using an implicit Crank-Nicholson finite difference scheme. The flow characteristics of velocity, temperature, concentration, wall shear stress, Nusselt number, Sherwood number based on the pertinent parameters are anatomized and are illustrated graphically. The comportment of streamlines and isolines for the fluid is in motion is also depicted graphically.

*Key-Words:* - moving semi-infinite vertical cylinder, Soret and Dufour effect, porous medium, finite difference method

## **1** Introduction

Heat and mass transfer associated with convective flows have various applications in chemical industries, solar energy porous water collector systems, and nuclear waste disposal and also in the devices of heat exchanger. Many researchers have given a remarkable contribution in the area related to flow through porous media. In manufacturing plants like refining process of petroleum and water purification, porous materials are used for filtration. Natural convective flow through a vertical cylinder ensconced in a porous medium was anatomized [1, 2]. The results procured from this study will be applied to estimate the rate of cooling for intrusive bodies and also to calculate the duration of life time for geothermal reservoir.

If transfer of heat and mass occurs concurrently and complementing each other then the effect of cross diffusion takes place. It leads to the case where the heat flux caused by both temperature and mass diffusion gradients. The intensity of heat flow rate generated by mass diffusion gradients is called the Dufour or diffusion-thermo effect. On the other side, mass flux generated by temperature gradients is called the Soret or thermal-diffusion effect. The diffusion-thermo effect and thermal-diffusion effect are also being known as double diffusion or crossdiffusion effects. In nature, this situation happens in many transport process.

In general, the cross diffusion effects are of nanoscopic order of magnitude compare to the effects described by Fourier's law of heat conduction and Fick's law for mass diffusion and are often neglected during the process of heat and mass transfer. However there are some exceptions where these effects cannot be neglected. If the fluids with light molecular weight or medium molecular weight then the Soret and Dufour effects are predominant and such effects cannot be neglected. To cite as evidence, the thermal-diffusion effect is useful in isotope separation, and in mixture between gases with very light molecular weight (H2, He) and of medium molecular weight (N2, air) [3]. Soret and Dufour effects have been evolved from the kinetic theory of gases and the necessary formulae has been derived to evaluate the thermal-diffusion coefficient and thermal-diffusion factor for monatomic gases or for polyatomic gas mixtures [4, 5].

Heat and mass transfer takes place due to natural convection in the presence of permeable medium along with cross diffusion effects have applications in chemical process of engineering and aerospace technology. Postelnicu [6, 7] anatomized the impact of Soret and Dufour effect on heat and mass transfer by natural convection from a vertical surface entrenched in a permeable medium along with the effect of a magnetic field and under the influence of chemical changes exist in the fluid.

Alam et al. [8] exemplified the double diffusion effects on a time independent free convective flow

past a semi-infinite vertical porous plate. Later on, the similar class of problem [9-11] was analyzed for a time dependent electrically conducting fluid and also under the impact of chemical changes exist in the thermally radiative fluid using finite element method and explicit finite difference method.

Soret and Dufour effects on a flat plate for Non-Darcy and Darcy permeable medium was inspected [12, 13] and it was concluded that the processes involving vigorous free convection will be dominated by double diffusion effects. Hayat et al. [14] used HAM to get the solution for double diffusion effects on mixed convective flow of a viscoelastic fluid over a stretchable vertical cylinder in the presence of permeable medium. It was inferred that when the direct coupling takes place between temperature and concentration, crossdiffusion is not negligible. Chamkha et al. [15] applied explicit finite difference method to get the solution for a time dependent doubly diffusive free convective flow of a fluid which is electrically conducting about a vertical cylinder along with the conditions of cross-diffusion and chemical changes exist in the thermally radiative fluid.

An analysis was made to find the heat and mass transfer of natural convection under the influence of double diffusion effects about a vertical surface embedded in a porous medium subject to variable viscosity by Moorthy et al. [16]. The extended work of this problem for stretching sheet along with the effects of MHD and time-dependent viscosity was considered by Pal et al. [17]. Unsteady free convective flow over a moving plate ensconced in a permeable medium under the influence of double diffusion, radiation and chemical reaction was proposed by Ibrahim [18] and the results were obtained using shooting technique.

Further, an analysis on convective flow over a cylinder in a permeable medium along with crossdiffusion effects has not received much attention among researchers. Only few authors have made analysis on such type of flows, but it has several applications in chemical engineering and industrial processes and also in the field of aerospace technology. Rani and Kim [19] computed the numerical solutions for transient natural convective flow about a vertical cylinder along with double diffusion effects in which the surface of the cylinder is maintained at a uniform temperature and concentration. Cheng [20] analyzed the same form of problem where the cylinder is ensconced in a saturated permeable medium. El-Kabeir [21] inspected about free convective flow through a stretchable cylinder in a porous medium along with the effects of Soret Dufour and chemical changes exist in the fluid.

Alao et al. [22] studied the effects of Soret and Dufour on a time dependent MHD free convective flow past a semi-infinite moving vertical plate with viscous dissipation and thermal radiation. In this analysis, Spectral relaxation method was adopted to get the solution for the problem. Combined effects of transfer of heat and mass on a time independent natural convection of a chemically reacting and electrically conducting fluid past a moving vertical plate with suction or injection in the presence of thermal diffusion and diffusion-thermo effects has been explored by Titiloye et al. [23].

Khan et al. [24] inspected the effects of thermodiffusion on convective heat transfer of a steady incompressible flow of a nanofluid over permeable sheet embedded in a porous medium which has applications in aircraft. Sreedevi et al. [25] scrutinized the effects of transfer of heat and mass due to natural convection past a permeable stretching sheet with the existence of thermodiffusion and diffusion-thermo effects, viscous dissipation, chemical reaction, MHD and thermal radiation.

Recently Ahmed et al. [26] anatomized numerically the effects of radiation and chemical reaction along with cross-diffusion gradients of time dependent buoyancy driven convective flow of a stretchable surface. Pandya et al. [27] proposed an implicit finite difference scheme and discussed the influence of cross-diffusion effects on an inclined plate with varying temperature and mass diffusion along with the effects of radiation and chemical reaction. Sharma and Aich [28] analyzed the influence of Soret Dufour effects about the boundary layer flow of a chemically reacting fluid over a stretchable cylinder entrenched in a permeable medium.

No investigation has been made to study the influence of Soret and Dufour effects on a time dependent free convective flow over an impulsively started vertical cylinder entrenched in a porous medium. Hence, this article discusses about the cross diffusion effects on time dependent buoyancy driven flow about a moving vertical cylinder entrenched in a permeable medium.

## 2 **Problem Formulation**

Consider the time dependent buoyancy driven convective flow of a Newtonian viscous isochoric fluid past a semi-infinite moving vertical cylinder entrenched in a porous medium. The radius of the cylinder is taken as  $r_0$ . The fluid motion is measured vertically upward along the axial direction of the cylinder is taken as *x*-axis and the axis measured perpendicular to the axis of the cylinder is taken as *r*-axis (radial co-ordinate). The action of shear forces between the layers of the fluid is insignificant because of viscous dissipation effect is negligible in the energy equation. In this problem  $T_{\infty}^{'}, C_{\infty}^{'}, T_{w}^{'}$  and  $C_{w}^{'}$  are taken as free stream temperature, free stream concentration, wall temperature and wall concentration, respectively. The schematic model of the problem is depicted in Figure 1.



#### Fig. 1 Schematic representation of the problem

Originally, both cylinder and the fluid are retained to be at rest. Therefore, it is apparent that the temperature and concentration of the fluid remains free stream temperature  $T_{\infty}^{'}$  and free stream mass diffusion level  $C_{\infty}$ . The cylinder begins to move in the vertical direction with uniform velocity  $u_0$  at t' > 0. At this time, the level of temperature and concentration on the surface of the  $T_w > T_\infty$ cylinder are raised to be and  $C'_{w} > C'_{\infty}$  respectively. Ultimately, the fluid experiences the heat and mass transfer near the wall. If the fluid flow is distant from the cylinder then the temperature and concentration drops to free stream cases and the velocity gradually declines to zero. The diffusion-thermo effect (Dufour effect) and thermo-diffusion effect (Soret effect) are taken into account in order to study the amalgamation of heat and mass transfer occurs concurrently and influencing each other. The acceleration takes place due to gravity g is acting downward. Beneath all these assumptions jointly with the Boussinesq approximation, the equations of continuity, momentum, thermal energy and mass diffusion for the boundary layer formed by a laminar flow using Schlichting and Gersten [29] can be written as

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t'} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = g\beta(T' - T'_{\infty}) + g\beta^{*}(C' - C'_{\infty}) + \frac{\upsilon}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r}\right) - \frac{\upsilon}{\lambda^{*}} u$$
<sup>(2)</sup>

$$\frac{\partial T'}{\partial t'} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial r} = \frac{\alpha}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T'}{\partial r} \right) + \frac{D_m K_T}{C_s C_p} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C'}{\partial r} \right)$$
(3)

$$\frac{\partial C}{\partial t'} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial r} = \frac{D_m}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C}{\partial r} \right) + \frac{D_m K_T}{T'_m} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T'}{\partial r} \right)$$
(4)

The initial and boundary conditions are

$$t' \leq 0 \quad u = 0, v = 0, T' = T'_{\infty}, C' = C'_{\infty}$$
  
for all  $x, r \geq 0$   
$$t' > 0 \quad u = u_0, v = 0, T' = T'_{w}, C' = C'_{w}$$
  
at  $r = r_0$   
$$u = 0, v = 0, T' = T'_{\infty}, C' = C'_{\infty} \text{ at } x = 0$$
  
and  $r \geq r_0$   
$$u \to 0, v \to 0, T' \to T'_{\infty}, C' \to C'_{\infty} \text{ as } r \to \infty$$
 (5)

where *u* and *v* are velocity components in *x* and *r* directions respectively, *t*' is the time, *T*' is dimensional temperature, *C*' is dimensional mass diffusion,  $\beta$  is the volumetric coefficient of thermal expansion,  $\beta^*$  is the volumetric coefficient of thermal expansion with mass diffusion, *v* is the kinematic viscosity,  $\lambda^*$  is the dimensional permeability,  $\alpha$  is the thermal diffusion coefficient,  $D_m$  is the mass diffusivity,  $K_T$  is the thermal diffusion ratio,  $C_s$  is the concentration susceptibility,  $C_p$  is the specific heat at constant pressure and  $T_m'$  is the mean fluid temperature.

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Introducing the following non-dimensional quantities

$$X = \frac{x\upsilon}{r_{0}^{2}u_{0}}, R = \frac{r}{r_{0}}, U = \frac{u}{u_{0}},$$

$$V = \frac{vr_{0}}{\upsilon}, t = \frac{t'\upsilon}{r_{0}^{2}}, \lambda = \frac{\lambda^{*}}{r_{0}^{2}},$$

$$T = \frac{T' - T_{\infty}'}{T_{w}' - T_{\infty}'}, Gr = \frac{g\beta r_{0}^{2}(T_{w}' - T_{\infty}')}{\upsilon u_{0}},$$

$$C = \frac{C' - C_{\infty}'}{C_{w}' - C_{\infty}'}, Gc = \frac{g\beta^{*}r_{0}^{2}(C_{w}' - C_{\infty}')}{\upsilon u_{0}},$$

$$Pr = \frac{\upsilon}{\alpha}, Du = \frac{D_{m}K_{T}}{\upsilon C_{s}C_{p}} \frac{(C_{w}' - C_{\infty}')}{(T_{w}' - T_{\infty}')},$$

$$Sc = \frac{\upsilon}{D_{m}}, Sr = \frac{D_{m}K_{T}}{\upsilon T_{m}'} \frac{(T_{w}' - T_{\infty}')}{(C_{w}' - C_{\infty}')}$$
(6)

Equations (1) - (4) are abridged to the following form.

$$\frac{\partial(RU)}{\partial X} + \frac{\partial(RV)}{\partial R} = 0 \tag{7}$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial R} = GrT + GcC + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial U}{\partial R} \right) - \frac{U}{\lambda}$$
(8)

$$\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial R} = \frac{1}{\Pr R} \frac{\partial}{\partial R} \left( R \frac{\partial T}{\partial R} \right) + Du \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial C}{\partial R} \right)$$
(9)

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial R} = \frac{1}{Sc R} \frac{\partial}{\partial R} \left( R \frac{\partial C}{\partial R} \right) + Sr \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial T}{\partial R} \right)$$
(10)

The corresponding initial and boundary conditions in non-dimensional quantities are given by

$$t \le 0: \quad U = 0, \ V = 0, \ T = 0, \ C = 0$$
  
for all X,  $R \ge 0$   
 $t > 0: \quad U = 1, \ V = 0, \ T = 1, \ C = 1 \text{ at } R = 1$   
 $U = 0, \ V = 0, \ T = 0, \ C = 0 \text{ at } X = 0$   
and  $R \ge 1$   
 $U \to 0, \ V \to 0, \ T \to 0, \ C \to 0$   
as  $R \to \infty$  (11)

where U and V are velocity components in dimensionless axial coordinate X and radial coordinate R respectively, t is the dimensionless time, T is the dimensionless temperature, C is the dimensionless mass diffusion,  $Gr, Gc, \lambda, Pr$ , Du, Sc, Sr are the Grashof number for heat transfer, Grashof number for mass transfer, permeability parameter of the porous medium, Prandtl number, Dufour effect, Schmidt number and Soret number respectively.

Using the values of velocity, temperature and mass diffusion, the local and mean values of skinfriction coefficient, Nusselt number and Sherwood number is calculated and these values are defined as follows.

$$\tau_X = -\left(\frac{\partial U}{\partial R}\right)_{R=1} \tag{12}$$

$$\overline{\tau} = -\int_{0}^{1} \left(\frac{\partial U}{\partial R}\right)_{R=1} dX$$
(13)

$$Nu_X = -X \left(\frac{\partial T}{\partial R}\right)_{R=1} \tag{14}$$

$$\overline{Nu} = -\int_{0}^{1} \left(\frac{\partial T}{\partial R}\right)_{R=1} dX$$
(15)

$$Sh_X = -X \left(\frac{\partial C}{\partial R}\right)_{R=1}$$
 (16)

$$\overline{Sh} = -\int_{0}^{1} \left(\frac{\partial C}{\partial R}\right)_{R=1} dX$$
(17)

where  $\tau_X$  is the local skin-friction coefficient,  $\overline{\tau}$  is the mean skin-friction coefficient,  $Nu_X$  is the local Nusselt number,  $\overline{Nu}$  is the mean Nusselt number,  $Sh_X$  is the local Sherwood number and  $\overline{Sh}$  is the mean Sherwood number.

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The non-dimensional Stream function  $\psi$ , provides the path of motion of fluid particles which is a function of *X* and *R*. For a two dimensional flow of a fluid over a moving vertical cylinder, the relation between *U*, *V* and  $\psi$  which satisfies Eq. (6) is given as follows.

$$U = \frac{1}{R} \frac{\partial \psi}{\partial R}, \quad V = -\frac{1}{R} \frac{\partial \psi}{\partial X}$$
(18)

#### **3** Numerical Technique

Eqs. (7) - (10) comprises of a system of non-linear, coupled partial differential equations. These equations accompanied by the initial and boundary conditions are given in Eq. (11) have been solved using Crank Nicholson method. The finite difference solution procedure employed for the above equations are discussed in detail by Carnahan et al. [30] and Ganesan and Loganathan [31].

Consider the integrating region as a rectangle with  $X_{\text{max}}$  (=1.0) along the axial direction and  $R_{\text{max}}$  (=15.0) along the radial direction, where  $R_{\text{max}}$  have a close similarity to  $R = \infty$  which lies distant from the boundary layers of momentum, temperature and mass diffusion. The mesh independence test has been performed to fix the lattice sizes for the problem. The lattice sizes are fixed at the level  $\Delta X = 0.02$ ,  $\Delta R = 0.20$ , with time step  $\Delta t = 0.01$ . Here  $\Delta X$ ,  $\Delta R$  and  $\Delta t$  denotes the mesh sizes in axial coordinate, radial coordinate and time respectively. In this case, spatial lattice sizes are declined by 50% in one direction and later in both directions and the results are compared then the lattice sizes are fixed. If the comparative differences between the current and previous iterative values are found to differ only in the fifth decimal place then the iteration process gets terminated. Therefore, the above lattice sizes have been considered as appropriate.

The truncation error involved in the finitedifference approximation is  $O(\Delta t^2 + \Delta R^2 + \Delta X)$  and this tends to zero as  $\Delta t$ ,  $\Delta X$  and  $\Delta R \rightarrow 0$ . Thus the scheme under consideration is convergent.

Five-point approximation formula is used to evaluate the derivatives occur in equations (12) to (17) and Newton Cotes formula is applied to evaluate the integrals involved in equations (12) to (17). Further, the stream function  $\psi$  involved in Eq. (18) is simulated using an explicit finite difference scheme.

#### **4** Results and Discussion

The values of velocity, temperature and mass diffusion are evaluated numerically for the different values of the parameters under consideration and those values are interpreted to the physical situation of the problem. Solutions are obtained for a nonchemically reacting fluid hydrogen-air mixture and also obtained for water and water vapor. In these cases, the values of Prandtl number and Schmidt number are Pr = 0.7 (air), Pr = 7.0 (water), Sc = 0.2 (hydrogen) and Sc = 0.6 (water vapor). The value of Schmidt number is specified to represent hydrogen at approx.  $25^{\circ}C$  and 1 atmospheric pressure by Gebhart [32]. The dimensionless numbers such as Du (Dufour number) and Sr (Soret number) are taken in a manner that their product should be retained as constant, provided that the mean temperature  $T_{m}$  and the reference temperature  $T_{\infty}^{'}$  are also retained as

constant which was explained by Kafoussias and Williams [33]. The values of Grashof number for heat transfer and Grashof number for mass transfer are chosen to be positive and large due to natural convection.

In order to examine the accuracy, the solutions of temperature and mass diffusion are compared with Chen and Yuh [34] where Pr = 0.7, Sc = 0.2, N = 1.0, Gr = Gc = 1.0, Du = Sr = 0 and  $\lambda \rightarrow \infty$  and it is found to be an excellent match as shown in Figure 2.



Fig. 2 Comparison of temperature/mass diffusion distribution for Sc = 0.2, Pr = 0.7, Gr = 1.0 and Gc = 1.0

Steady state velocity profiles for Pr = 0.7 and Pr = 7.0 along with different values of Gr, Gc, Du, Sr and  $\lambda$  at X = 1.0 are interpreted in Figure 3. It is perceived that lessening the values of Pr causes a rise in velocity. For declining values of Pr, the time consumed to attain steady state increases. The velocity escalates at a slow pace with time, acquires a temporal maximum at t = 1.570 and becomes steady at t = 6.380. It is also observed that rising the value of permeability parameter  $\lambda$ , increases the velocity. This implies that, if the permeability of the surface is enlarged then the fluid flows quickly. Also, the time consumed to achieve steady state increases with increasing values of  $\lambda$ . From this, it is perceived that the influence of the permeability parameter is highly significant in the velocity profile. It is examined that increasing the values of Gr alone or Gc alone or both Gr and  $G_c$  shows a notable increment in the velocity U. Time taken to reach steady state decreases for large values of Gr and Gc. The effect of buoyancy force is predominant during natural convection. Also, the effect of Sr is high compare to Du in the velocity profile. This is because, the amalgamated effects of thermal and mass buoyancy force enhances the convection velocity.



for various values of Gr, Gc, Du and Sr

Steady state temperature distribution for different values of Gr, Gc, Du, Sr and  $\lambda$  are shown in Figure 4. The influence of diffusion-thermo effect is highly dominating in the temperature profile. This is due to the occurrence of energy flux driven by the concentration gradient from high temperature region to low temperature region. It is inspected that enhancing the value of Du improves the

temperature. Also, the time taken to attain steady state becomes slow for large values of Du. It is also observed that increasing the values of  $\lambda$ , Sr, Gr and Gc decreases the temperature. Time consumed to reach steady state decreases with increasing values of  $\lambda$ , Sr, Gr and Gc. From this, it is inferred that the temperature profile is independent of the effects of  $\lambda$ , Sr, Gr and Gc.



Fig. 4 Effect of  $\lambda$  and Du on temperature distribution for Pr = 0.7 and Sc = 0.2



Fig. 5 Effect of  $\lambda$  and *Sr* on mass diffusion distribution for Pr = 0.7 and Sc = 0.2

Steady state mass diffusion distribution for different values of Gr, Gc, Du, Sr and  $\lambda$  are shown in Figure 5. The influence of thermal-diffusion effect is high compare to diffusion-thermo effect in concentration profile. This is due to the production of mass flux driven by the temperature gradient

from high concentration region to the low concentration region. It is inspected that increasing the value of Sr increases the mass diffusion. For large values of Sr, the time to hit steady state gets decreased. It is also noticed that decreasing the values of  $\lambda$ , Du, Gr and Gc increases the mass diffusion. Also, the time taken to reach steady state decreases with the increasing values of  $\lambda$ , Du, Gr and Gc.



Fig. 6 Effect of various values of Gr, Gc, Du, Sr,  $\lambda$  and Pr on local wall shear stress for Sc = 0.2



Fig. 7 Effect of various values of Gr, Gc, Du, Sr,  $\lambda$  and Pr on local Nusselt number for Sc = 0.2

Local wall shear stress at steady state is shown in Figure 6. The local skin-friction become smaller as

X gets larger. It is discerned that viscous drag decreases with increasing values of Gr and Gc. It is also examined that rising the value of Du leads to a fall in local skin-friction. Wall shear stress grows, as Pr decays. Increasing the value of permeability parameter  $\lambda$  and Soret number Sr declines the magnitude of local skin-friction coefficient.



Fig. 8 Effect of various values of Gr, Gc, Sc, Du, Sr,  $\lambda$  and Pr on local Sherwood number for Pr = 0.7



Fig. 9 Effect of various values of Gr, Gc, Sc, Du, Sr,  $\lambda$  and Pr on mean wall shear stress

Local Nusselt number at steady state for various values of Pr is shown in Figure 7. Local Nusselt number for Pr = 0.7 and Pr = 7.0 are analyzed. The local rate of heat transfer increases with incrementing values of X. It is observed that the

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local rate of heat transfer gets enhanced for large values of Pr. This shows that for large values of Pr, the thermal boundary layer does not widen much. It is also spotted that raising the value of Soret number leads to a magnification in the local heat transfer. The local Nusselt number elevates as  $\lambda$  expands. Increasing the value of Gr, Gc and Sr increases the heat transfer locally.

Local Sherwood number at steady state for a set of specified values of  $S_c$  is shown in Figure 8. Local Sherwood number escalates as X uplifts. The impact of  $S_c$  is prepotent in local Sherwood number. It is discerned that the local Sherwood number becomes larger as  $S_c$  gets smaller. Local mass transfer rate also gets elevated as  $\lambda$  increases. It is also inspected that increasing the values of Gr, Gc and Du increases the local mass transfer rate. Increasing the value of Dufour number Du, elevates the local Sherwood number.

Mean wall shear stress for two specific values of Pr is plotted in Figure 9 as a function of time. Mean Skin-friction increases if the permeability of the surface  $\lambda$  is small. An increase in Pr, generates an increase in Mean skin-friction. It is perceived that increasing the values of Gr, Gc and Du decreases the average shear stress rate. Lessening the Soret number improves the mean skin-friction coefficient.



Fig. 10 Effect of various values of Gr, Gc, Sc, DuSr,  $\lambda$  and Pr on mean Nusselt number

Mean Nusselt number for two specific values of Pr with respect to time are shown in Figure 10. It is perceived that increasing the Prandtl number Pr, increments the rates of heat transfer. The value of Gr is more significant compare to Gc for heat

transfer. When the value of Gr raises then it gives rise to an increase in the rate of heat transfer because of high influence of buoyancy force in free convection. Increasing the value of Soret number Sr leads to an improvement in the rate of heat transfer.



Fig. 11 Effect of various values of Gr, Gc, Sc, Du, Sr,  $\lambda$  and Pr on mean Sherwood number



Fig. 12 Soret and Dufour effects on streamlines for the values of Pr = 0.71, Sc = 0.22, Gr = 5.0, Gc = 5.0,  $\lambda = 1.0$ 

Mean Sherwood number for two specific values of  $S_c$  with respect to time t are shown in Figure 11. It is examined that the mass transfer rate increases as  $S_c$  increases. The rate of mass transfer gets elevated for raising values of  $G_c$ . In this case, the value of  $G_c$  is predominant compare to  $G_r$ . It is also perceived that the value of mean Sherwood number increases for increasing value of Du.

Figure 12 depicts the Soret and Dufour effects on the stream function for Pr = 0.71, Sc = 0.22, Gr = 5.0, Gc = 5.0 and  $\lambda = 1.0$ . Streamlines traces out the flow path of the fluid particles which moves from one point to another. Further, the flow pattern of the streamlines changes with time, if the fluid is unsteady. The Soret effect is momentos compare to the Dufour effect in stream functions. It is discerned that the flow path of the stream function having high values of *Sr* allows the fluid particles to move faster from one point to another compare to the higher values of *Du*.



Fig. 13 Impact of permeability parameter on Isolines of velocity for Pr = 0.71, Sc = 0.22, Gr = 5.0, Gc = 5.0, Du = 0.15, Sr = 0.15

Figure 13 illustrates the level plot representation of velocity for different values of permeabiliy parameter  $\lambda$ . The permeability parameter has the ability of permitting the fluid to flow rapidly. Hence, the isolines of velocity for  $\lambda = 7.0$  develops faster than for  $\lambda = 1.0$ .

Figure 14 represents the level plots of temperature for different values of Dufour number. For elevating values of Du, the temperature gets escalated. When the chemical system experiences a concentration gradient, the heat flow rate of intensity gets amplified for binary gas mixutures. Due to this fact that the temperature on the surface of the cylinder improves for high values of Du.

Figure 15 explains the isolines of temperature together with the impact of different values of Prandtl number. Enlarging the values of Prandtl number reduces the depth between the level plots of temperature. This causes a fall in the temperature for rising values of Pr. The same trend is observed for growing values of Sc in the case of contour plots of concentration which is explored in Figure 16. Strengthening the values of Sc, weakens the mass diffusion exists in the system.



Fig. 14 Impact of Dufour number on Isolines of temperature for Pr = 0.71, Sc = 0.22, Gr = 5.0, Gc = 5.0, Du = 0.15, Sr = 0.15



Fig. 15 Impact of Prandtl number on isolines of temperature for Sc = 0.22, Gr = 5.0, Gc = 5.0, Du = 0.15, Sr = 0.15,  $\lambda = 1.0$ 

Figure 17 decribes the level plots of mass diffusion for different values of Soret number. If the temperature gradient acts as a driving force for mass diffusion then the thermophoresis effect is preponderant. Therefore, the growth in the values of Sr encounters an upliftment in the value of concentration.



Fig. 16 Impact of Schmidt number on isoline plots of concentration for Pr = 7.0, Gr = 5.0, Gc = 5.0, Du = 0.15, Sr = 0.15,  $\lambda = 1.0$ 



Fig. 17 Impact of Soret number on contour plots of concentration for Pr = 0.71, Gr = 5.0, Gc = 5.0,  $\lambda = 1.0$ 

### 4 Conclusions

In this paper, a numerical analysation is executed to study about the time dependent buoyancy driven convective flow over a moving semi-infinite vertical cylinder entrenched in a porous medium under the influence of double diffusion effects. The nondimensional form of momentum, energy and mass diffusion equations are solved numerically using finite difference method of Crank Nicholson type. The following results are obtained.

- 1. The time consumed to attain steady state in velocity, temperature and mass diffusion distribution is less when Gr and Gc values are taken as large.
- 2. The velocity increases, when the permeability the material gets increased. of The temperature and mass diffusion is less when the value of the permeability parameter  $\lambda$  is taken as large. It is also perceived that increasing the value of permeability parameter  $\lambda$  increases the heat and mass transfer rate due to the influence of crossdiffusion effects respectively.
- 3. The influence of Soret number is more in the velocity and mass diffusion distribution. If the Soret effect gets developed then the growth takes in the distribution of velocity as well as in mass diffusion. But in the case of temperature profile the influence of Dufour number is highly significant.
- 4. Skin friction is more for higher values of *Pr*, since viscous effects are highly significant.
- 5. Strengthening the generation of mass flux caused by temperature gradients leads to an elevation in the values of Soret number which in turn enhances the rate of heat transfer.
- 6. The mass transfer rate gets enhanced for escalating values of Dufour number.
- 7. Streamlines clarifies the flow visualization of the fluid particles and the isolines taken into account explains the distribution of velocity, temperature and concentration profiles.

#### Conflict of Interest: None

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