Finite element solution for thermal shock phenomenon in multilayered cylindrical bodies

Jacob Nagler
School of Mechanical Engineering, Fluid Mechanics & Heat Transfer
Tel Aviv University
Tel-Aviv 69978
ISRAEL
syankitx@Gmail.com https://www.researchgate.net/profile/Jacob_Nagler

Abstract: - General finite element (F.E.) coupled transient-dynamic model of three layers cylinder in polar coordinates under stress plane assumption, will be introduced and discussed. We have found through an analytical analysis and literature comparison that four main factors might affect the solution qualitative and quantitative behavior: (1) Temperature-stress boundary conditions, (2) Model size and length, (3) Number of layers, and (4) Materials properties in each layer. Finally, good qualitative agreement was found between the F.E. analysis results and the relevant literature for the radial displacement, the tangential and radial stresses.

Key-Words: - thermal shock, multilayered cylinder, finite element model, coupled transient-dynamic model

1 Introduction

Thermal shock topic is the particular case of thermal stress analysis field (thermo elasticity). Thermal stress analysis is aided to model the coupling between the heat transfer energy equations and the stress-strain constitutive relations under specific thermal and mechanical conditions. However, thermal shock occurs when a thermal gradient causes differential expansion of the material stress and strain properties. At specific moment, the stress is likely to exceed the allowable material strength, forming a crack. This crack may lead to structure failure if nothing is done to avoid it. This phenomenon is likely to occur in very small time intervals duration ($\approx 10^{-3}, 10^{-4}[\mathrm{sec}]$) under high temperatures. In order to prevent it, one should use one or more of the following methods [1-3]: (1) Reducing the created thermal gradient by changing its temperature more slowly or increasing the material's thermal conductivity, (2) Reducing the material's coefficient of thermal expansion, (3) Increase the material's allowable ultimate tensile strength, (4) Increasing the compressive stress (tempered glass), (5) Decreasing its modulus of elasticity, and finally (6) Toughness increasing (crack deflection). Example of material that has the ability to absorb the thermal shock energy is the Borosilicate glass (reduced expansion coefficient and greater strength than the usual glass). Another example is the reinforced carbon-carbon that is characterized by high carbon fiber strength alongside graphite's extremely high thermal conductivity and low expansion coefficient. Hence, the robustness thermal shock parameter is defined as [1]:

$$R_T = \frac{k\sigma_T(1-\upsilon)}{\alpha E} \qquad (1)$$

where $k, \sigma_T, \upsilon, \alpha, E$ are thermal conductivity, the maximal tension stress, the Poisson ratio, thermal expansion coefficient and the modulus of elasticity, respectively.

Many applications of thermal shock stress exist in aerospace, biomedical and other engineering fields [2-5]. In the current essay, we will analyze analytically the particular case of multilayer cylindrical bodies. As mentioned above, thermal shock topic was derived from the thermo-elasticity branch [6], while fresh thermal stress analysis in the context of cylindrical vessels without the thermal shock has also been concerned and performed recently by Torabi et al. [7-8]. Moreover, it should be mentioned that also advanced numerical methods (like finite volume methods) have been employed to accomplish with the task of thermal shock in graded and layered cylindrical vessel material, as appear in [9-10]. Other geometries variations of thermal shock can be found on plate and sphere as discussed by Lue & Fleck [1] and Crandall & Ging [11].

Specifically, when dealing with thermal shock that is developed in cylindrical vessels, there are two kinds of materials (layered or composite materials, like Functionally Graded Materials (FGM)) and six frequently used thermo-elastic models. Balla [6] has reviewed these thermo-elastic models for one solid material according to the following partition. In the aspect of the equation of motion there are two types: coupled equation of motion and the non-coupled equation of motion – without inertia and coupled terms. In addition, there are also three main heat conduction models (general, without the coupled term and the third one is without the coupled term and second order time dependency). Accordingly, there are six different model permutations due to the coupling between the heat conduction and the equation of motion. Note that the particular case of one hollow cylinder (dynamic and quasi static models) subjected to convection boundary condition has been discussed by [12].

The distinction that is based on the material kind (layered or FGM) will be brought here, briefly. The analytic case of two layered cylinder under dynamic thermal shock with initial interface pressure using series expansion has been investigated by Xi [13]. Similar study that neglects the interface pressure was performed by Kudinov et al. [14]. The researchers [14] have developed an exact analytical solution of a multilayer shallow cylinder with constant physical properties in each layer. They have found that the circumferential compressive stresses appear on the external surface of the cylinder under thermal shock while the tensile stresses appear on the internal surface. Recent study in this context that will also be presented here was performed by Dai et al. [15]. They have investigated numerically (FDM, NM and iterative method) the thermo-viscoelastic dynamic behavior of a double-layered (viscoelastic and homogenous layers) hollow cylinder under thermal shocking.

Considering continuous orthotropic material model with varying properties along its radius, (or one may assume that the layer interface interactions may be neglected) lead us to the FGM material. Safari et al. [16] have performed analytical study (Laplace transform with series solution) of the dynamic behavior of thermos-elastic stresses in a finite-length functionally graded (FG) thick hollow cylinder under thermal shock loading. The thermo-mechanical properties have been assumed to vary continuously through the radial direction as a nonlinear power function, and the temperature shape was assumed to behave as the well-known sinus wave. Refs. [17-18] have also coped with the FGM cylinders using different analytical methods, while Ref. [18] defines the critical temperature criterion that may lead to the material failure due to tensile overloading.

Overlooking on the experimental studies, we will concentrate on Kingery [2], Schneider et al. [25], Zhang et al. [26], Wang et al. [27] and Li et al. [28] experimental studies. For instance, Kingery [2] has investigated the thermo-mechanical properties of slab geometry case. Recently, numbers of advanced studies that deals with ceramics material [25-28] have developed semi-empirical model that can implemented on fully or one layer cylinder. However, excluding Zhang et al. [26] the following studies [25, 27-28] have examined it, by using a probe on the outer layer, whereas Zhang et al. [26] have applied it on the inner layer and also compared it with the F.E. model. Moreover, most studies (experimental and numerical) are applied on a disk with geometry ratio (between the inner and the outer radii) of about 0.05 [25]. The considered thickness typical size that considered is about few millimeters (range: 0.2 - 5 mm, see [25-26]).

Until today, the author has investigated the case of homogenous cylindrical pressure vessels in a number of occasions [19-22]. In the current study, we will introduce the full dynamic stress equation of multi n-layered cylindrical bodies that are subjected to the general heat conduction model. Particularly, the transient-dynamic equations will be solved through finite element three layers cylinder case model, with data based on Lee et al. [29] for very small time instants.

2 General analytical model of multi layered cylinder model

Dynamic thermal-shock transient model equations will be presented along with the necessary boundary conditions using Refs. [6, 13-15]. Dealing with the analytical model, we will introduce multi- layer model of hollow semi-infinite long cylinder representative geometry with internal heat source generation as appear in Fig. 1. Whence the heat conduction general equation (that is coupled with the radial strain u over the time space t) in each layer (noted by i) for the one dimensional problem in polar coordinates (r,θ,z) is given by [6]:

$$k_i \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_i}{\partial r}\right)+\left(Q+\tau_i\dot{Q}\right)-\rho_i C_{p,i}\left(\frac{\partial T_i}{\partial t}+\tau_i\frac{\partial^2 T_i}{\partial t^2}\right)-\frac{E_i\alpha_i T_0}{(1-2\upsilon_i)k_i}\left(\frac{\partial^2 u_{r,i}}{\partial r\partial t}+\tau\frac{\partial^3 u_{r,i}}{\partial r\partial t^2}\right)=0 \quad (2)$$

where $k_i,\alpha_i,\rho_i,C_{p,i},E_i,Q,T,T_0,\upsilon_i,\tau_i$ are the material conductive heat transfer coefficient, linear thermal expansion, the mass density, the specific heat capacity at constant strain, the modulus of elasticity, the heat source, the absolute temperature, the natural temperature of the natural stress-free state, the Poisson's coefficient and the relaxation time in each i

layer, respectively. The appropriate general elasto-dynamic model with body forces and internal heat sources generation can be expressed by:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_{r,i}}{\partial r}\right)-\frac{(1+\upsilon_i)(1-2\upsilon_i)\rho_i}{(1-\upsilon_i)E_i}\frac{\partial^2 u_{r,i}}{\partial t^2}-...$$
$$-\frac{u_{r,i}}{r^2}-\alpha_i\left(\frac{1+\upsilon_i}{1-\upsilon_i}\right)\frac{\partial T_i}{\partial r}+\rho f_i=0 \quad (3)$$

where f_i represents the body force in each layer. Remark that the author has already solved the case of pressure cylinder under the influence of radial body forces in different study [22]. In addition, note that if one deals with the planar case and would like to achieve higher order solution, he might use the study of [23].

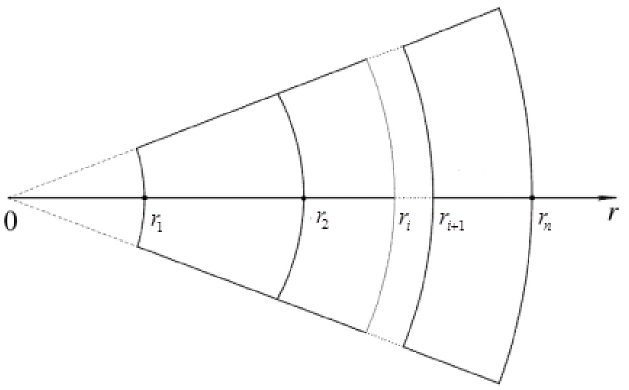

Figure 1. Multilayer hollow long cylinder illustration (Up view).

The cylinder geometrical properties are the radii that are noted by r_1 (inner), r_i (interface), r_n (outer). Now, boundary conditions will be defined in the inner region, interface and the outer region. The interface continuum boundary conditions (B.C.) may be described by the temperature and the heat flux equalities:

$$T_i\big|_{r=r_i}=T_{i+1}\big|_{r=r_i} \quad (4)$$

$$\alpha_i\frac{\partial T_i}{\partial r}\bigg|_{r=r_i}=\alpha_{i+1}\frac{\partial T_{i+1}}{\partial r}\bigg|_{r=r_i} \quad (5)$$

while $1<i<n$. The inner and outer B.C. will be prescribed by relatively high constant inner wall temperature and convective outer wall, as follows:

$$T_1(r_1,t)=f(t) \quad (6)$$

$$-k_n\frac{\partial T_n}{\partial r}\bigg|_{r=r_n}=-h\left(T_\infty-T_n(r_n,t)\right) \quad (7)$$

whereas $h\left[\frac{W}{K^0 m^2}\right], T_\infty[K^0]$ are the convective coefficient and the surrounding environment temperature. Additionally, the initial time condition will be prescribed by:

$$T_i(r,0)=T_\infty \quad (8)$$

Note that the parameters values will be numerically specified in the results section, continually. Also, $f(t)$ can be represented, following the models that appear in [6] and [14]:

$$f(t)=\begin{cases}\text{Sudden heating case}: T_0+(T_\infty-T_0)H(t)\text{ or } f(t)=T_0+(T_\infty-T_0)\delta(t)\\ \text{Ramp-type heating case: } T_0+(T_\infty-T_0)\frac{t}{t_0}\left[H(t)-H(t-t_0)\right]+(T_\infty-T_0)H(t-t_0)\\ \text{or } f(t)=T_0+(T_\infty-T_0)\frac{t}{t_0}\left[\delta(t)-\delta(t-t_0)\right]+(T_\infty-T_0)\delta(t-t_0)\end{cases} \quad (9)$$

where $H(t),\delta(t)$ are the unit step-function and the delta function, respectively. The constant time t_0 represents the time-lag while the surface temperature is reaching its stationary value T_0. The B.C. for the stress-strain relation are prescribed by the formulation of [6,14] as:

$$\sigma_r(r_1,t)=0,\ u_r(r_n,t)=0,\frac{\partial}{\partial r}u_r(r_n,t)=0$$
$$\text{or} \quad (10)$$
$$\sigma_r(r_1,t)=0,\sigma_r(r_n,t)=0$$

The initial stress-strain conditions (I.C.) are:

$$\sigma_r(r,0)=0 \text{ or } u_r(r,0)=0,\frac{\partial u_r(r,0)}{\partial t}=0 \quad (11)$$

with the stress-strain interface continuum B.C. as described by stress-strain equalities [14]:

$$\sigma_i\big|_{r=r_i}=\sigma_{i+1}\big|_{r=r_i} \quad (12)$$

$$u_i(r_i,t)=u_{i+1}(r_i,t) \quad (13)$$

We also know that [14, 24] in each layer (i), the stresses in the polar coordinates are as follows:

$$\sigma_{rr,i} = c_{1,i}\frac{\partial u_i}{\partial r} + c_{2,i}\frac{u_i}{r} - \lambda_i T(r)$$
$$\sigma_{\theta\theta,i} = c_{2,i}\frac{\partial u_i}{\partial r} + c_{1,i}\frac{u_i}{r} - \lambda_i T(r) \quad (14)$$
$$\sigma_{zz,i} = \upsilon_i\left(\sigma_{r,i} + \sigma_{\theta,i}\right) - E_i\alpha_i T(r)$$

where,

$$c_{1,i} = \frac{E_i(1-\upsilon_i)}{(1+\upsilon_i)(1-2\upsilon_i)}; c_{2,i} = \frac{E_i\upsilon_i}{(1+\upsilon_i)(1-2\upsilon_i)}; \lambda_i = \frac{E_i\alpha_i}{1-2\upsilon_i} \quad (15)$$

The full balance dynamic equilibrium equation with body forces is:

$$\frac{\partial\sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta} + \frac{\partial\sigma_{rz}}{\partial z} + \frac{\sigma_{rr}-\sigma_{\theta\theta}}{r} + f_r = \rho\frac{\partial^2 u}{\partial t^2} \quad (16)$$

Remark that by substituting (12) into (14) and by neglecting the longitudinal dimension and the shear stresses, we return back to Eq. (3).

3 F. E. model solution & discussion

Until now, we have brought the general transient-dynamic model of the multilayer hollow cylinder under sudden heat thermal shock (the reversed phenomenon is called, the 'cold' thermal shock). Moreover, the basic formulation of the analytical and numerical model can be found in Lee *et al*. [29].

In order to perform the F.E. analysis, we have used MARC 2016 Student Version/Edition program. The analysis type is non-linear analysis under the assumption of plane-stress, while heat sources and body forces were neglected. The coupled model is implemented through thermal/structural model using the category of Transient/Dynamic model. The number of steps was constant during the analysis (1000 steps) operation performance. The element type was Quad-8. Also, the number of elements and nodes that were used were 716 and 2248, as appear in Figs. 1(a)-(b), respectively. The axi-symmetrical cylinder was composed from three layers (Titanium, Al_2O_3, Steel) as appear in Fig. 1(c) and Table 1. The applied thermo-mechanical boundary conditions that appear in Fig.2 (a) are:

$$\sigma_r(r_1,t)=0, \sigma_r(r_3,t)=0,$$
$$T(r_1,t)=3000[K], -k_3\left.\frac{\partial T}{\partial r}\right|_{r=r_3} = -h\left(T_\infty - T(r_3,t)\right), h=200\left[\frac{W}{\mathrm{K}^0 m^2}\right] \quad (17)$$

Additionally, the thermo-mechanical initial conditions that appear in Fig. 3(b) are:

$$T(r,0)=T_\infty=298\lfloor \mathrm{K}^0 \rfloor, u_r(r,0)=u_\theta(r,0)=0 \quad (18)$$

Good qualitative agreement and consistency were obtained, as a result of the model solution comparison (Figs. 4-6) with Lee *et al*. [29]. As expected, due to the heat expansion in each layer, unique temperature distribution was obtained, while the continuous conditions were fulfilled in the middle interface (intermediate) layer, as shown in Fig. 4. It was found that due to the delayed material expansion properties response, the radial displacement profile values are increased as long as time instant values are increased, as appear in Fig. 5 (a). Note that the latter figure has slight inappropriate numerical boundary conditions adjustment in the radial displacement that might be improved using higher element order. Moreover, the analysis reveals gradual jump of the radial stress in the middle layer region (even earlier) in the radial direction, dependent on the time instant values, as illustrated in Fig. 5(b). One may observe that this jump is evolved over time, as shown in Fig. 5(b). Similar behavior was obtained also for the tangential stress, as shown in Fig. 6(a) while the total displacement (Fig. 6(b)) acts similarly to the radial displacement. In addition, Von Mises stress profile values have been increased with time instants values, whereas at some point a jump is likely to appear in the radial direction, as shown in Fig. 7.

Two more aspects that concerned with the cylinder size and the number of layers are presented in Figs. 8(a)-(c). The geometry dimension size ratio between the small and the large cylinders was 1:1000 while the number of layers that was examined for each cylinder was one and three layers cases. It was found that the sensitivity to the dynamic response is increased with the cylinder dimensional size increasing, which is consistent with Euler's theory. On the other hand, Von Mises stress is larger in the case of small cylinder (even with one layer) with respect to the large cylinder case. However, the radial displacement profile values of the large cylinder case with a single layer are smaller than the appropriate three layers case as appear in Fig. 8(b). Similar behavior was obtained for the small cylinder, as shown in Fig. 8(c).

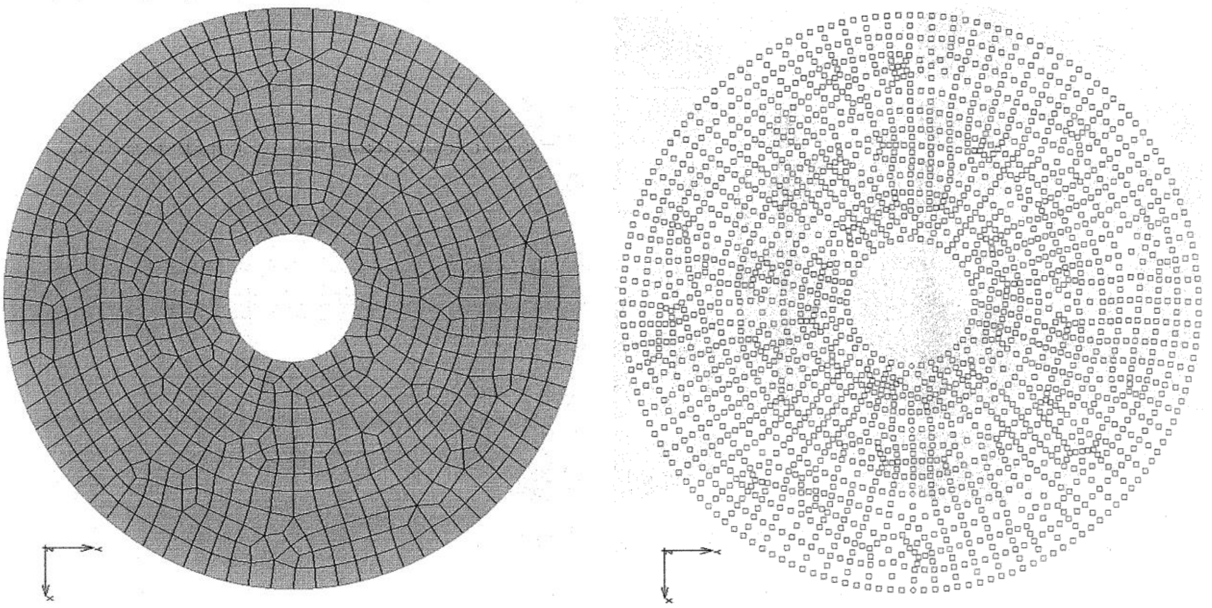

Fig. 1 (a) Quad 8 elements type model. (b) Quad 8 nodes type model.

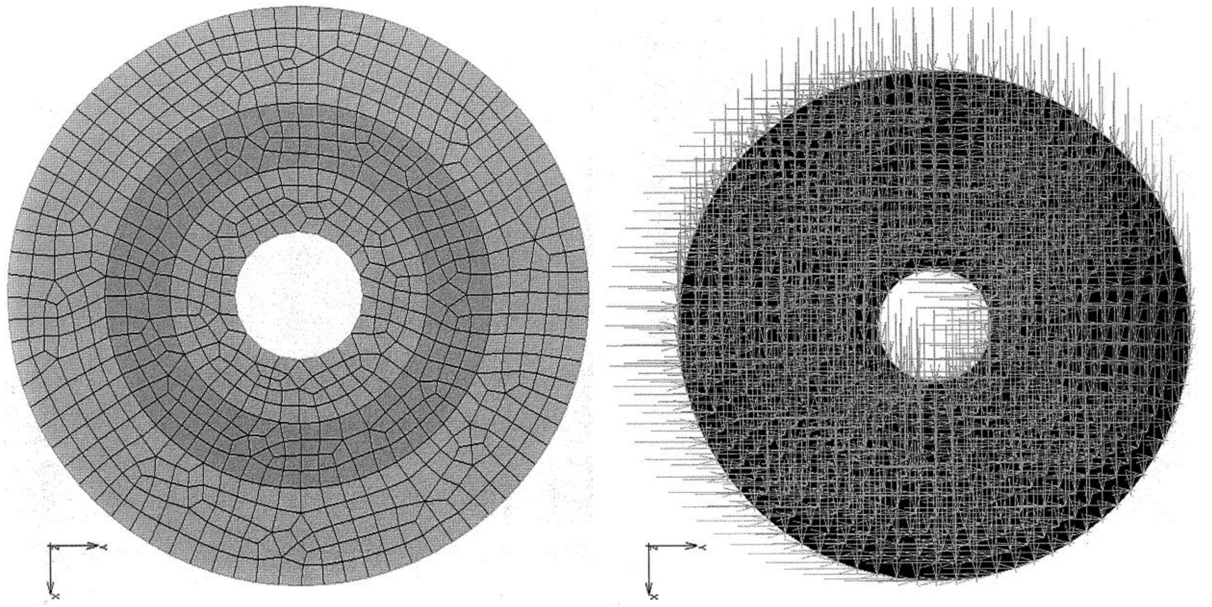

Fig. 2 (a) Three layer material model (Ti, Al203, Steel). (b) Quad 8 nodes type model.

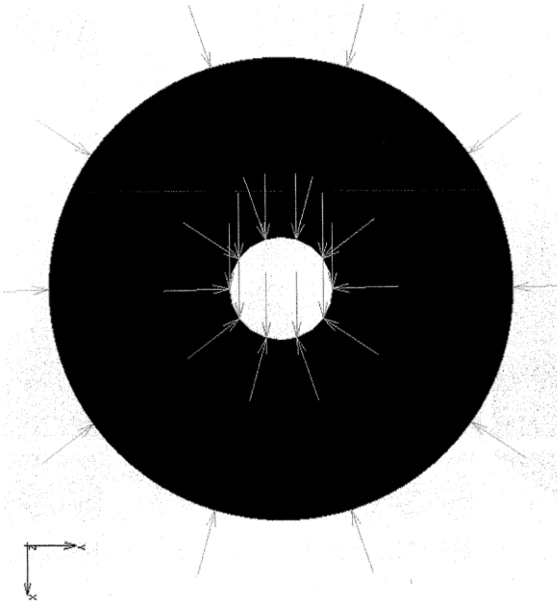

Fig. 3 boundary conditions model based on Eqs. (17)-(18).

Table 1: Material data as appear in Lee *et al*. [29]

Material properties	Layer 1 – Titanium	Layer 2 – Al_2O_3	Layer 3 - Steel
$E_r = E_\theta$ [Gpa]	108	390	207
$k_r = k_\theta$ [W/m·K^0]	20	6	17
$\upsilon_{r\theta} = \upsilon_{\theta r}$	0.3	0.23	0.3
$\alpha_r = \alpha_\theta \left[\mu m/\left(m\cdot K^0\right)\right]$	11	8	11
$\rho\,[kg/m^3]$	4000	3990	7800
$C_v\left[\frac{kJ}{kg\cdot K^0}\right]$	0.4	1.25	0.48

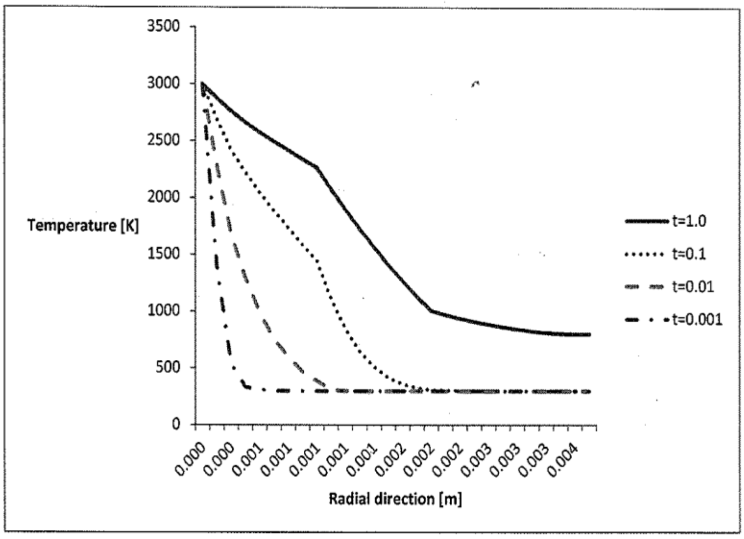

Fig. 4 Temperature distribution Vs. the radial distance for specific time instant values.

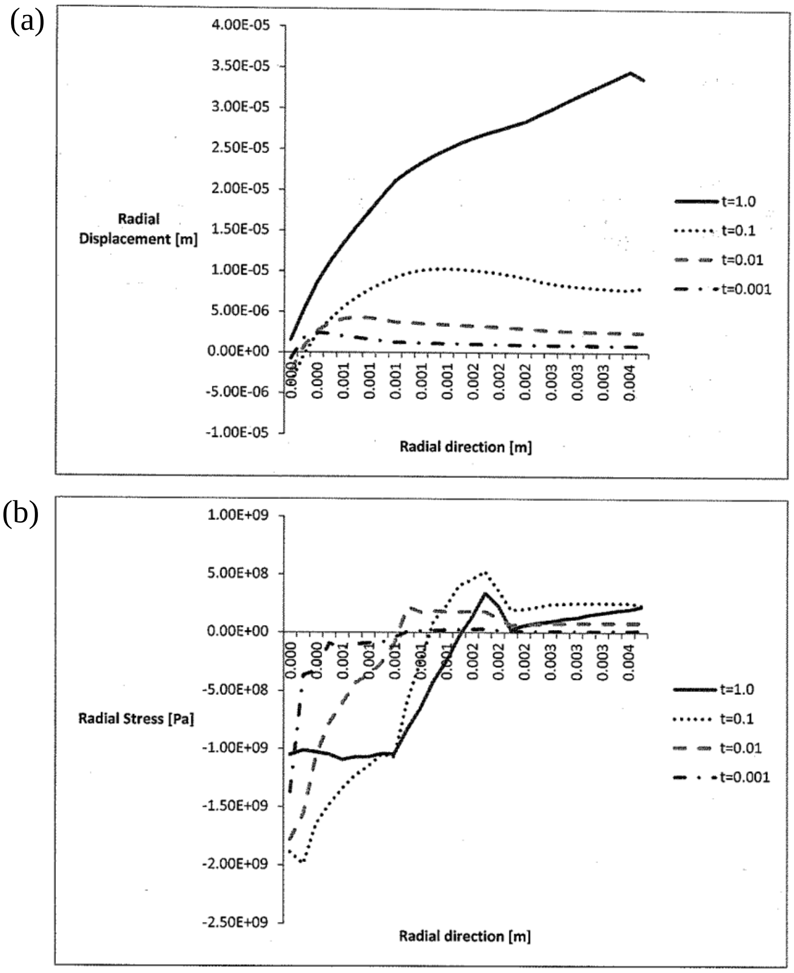

Fig. 5 (a) Radial displacement Vs. the radial direction for specific time instant values. (b) Radial stress Vs. the radial direction for specific time instant values.

(a)

(b)

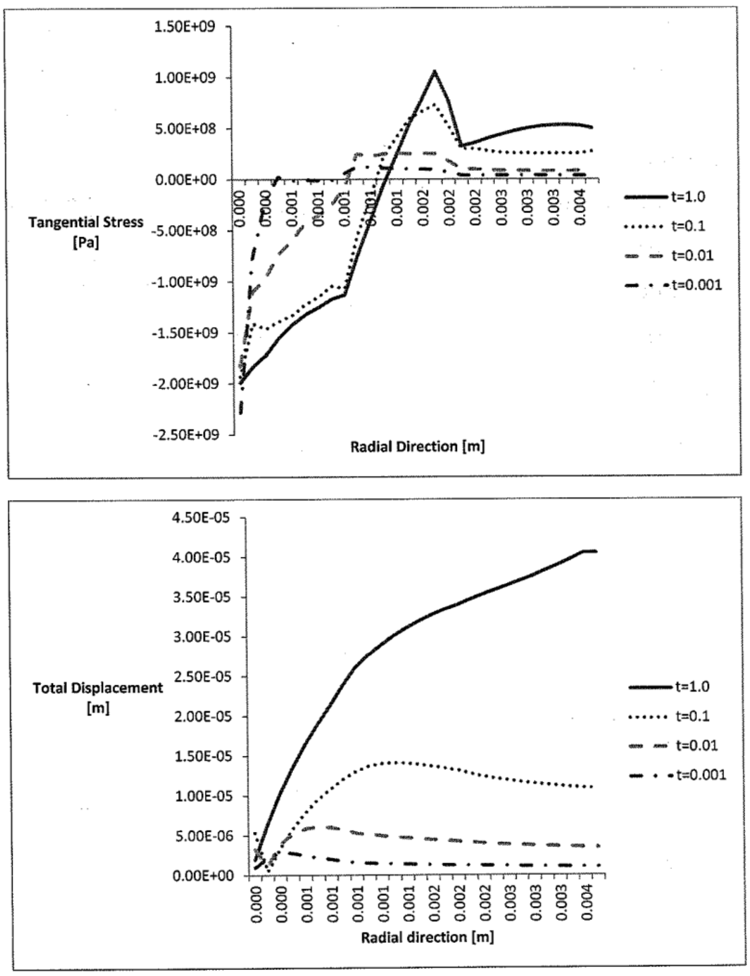

Fig. 6 (a) Tangential stress Vs. the radial direction for specific time instant values. (b) Total displacement Vs. the radial direction for specific time instant values.

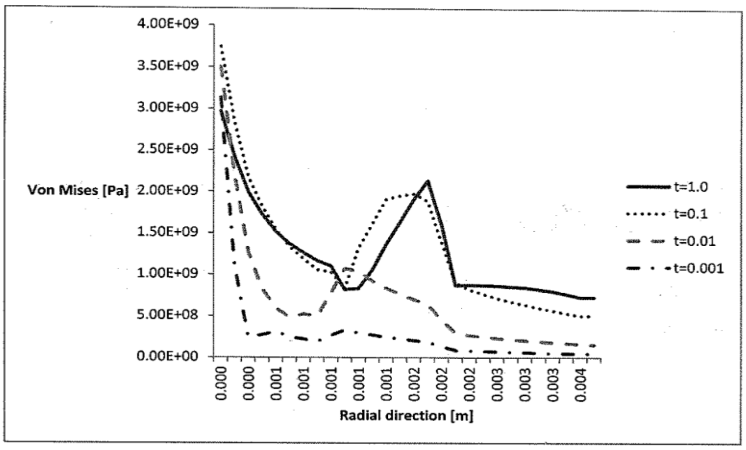

Fig. 7 Von Mises stress Vs. the radial direction for specific time instant values.

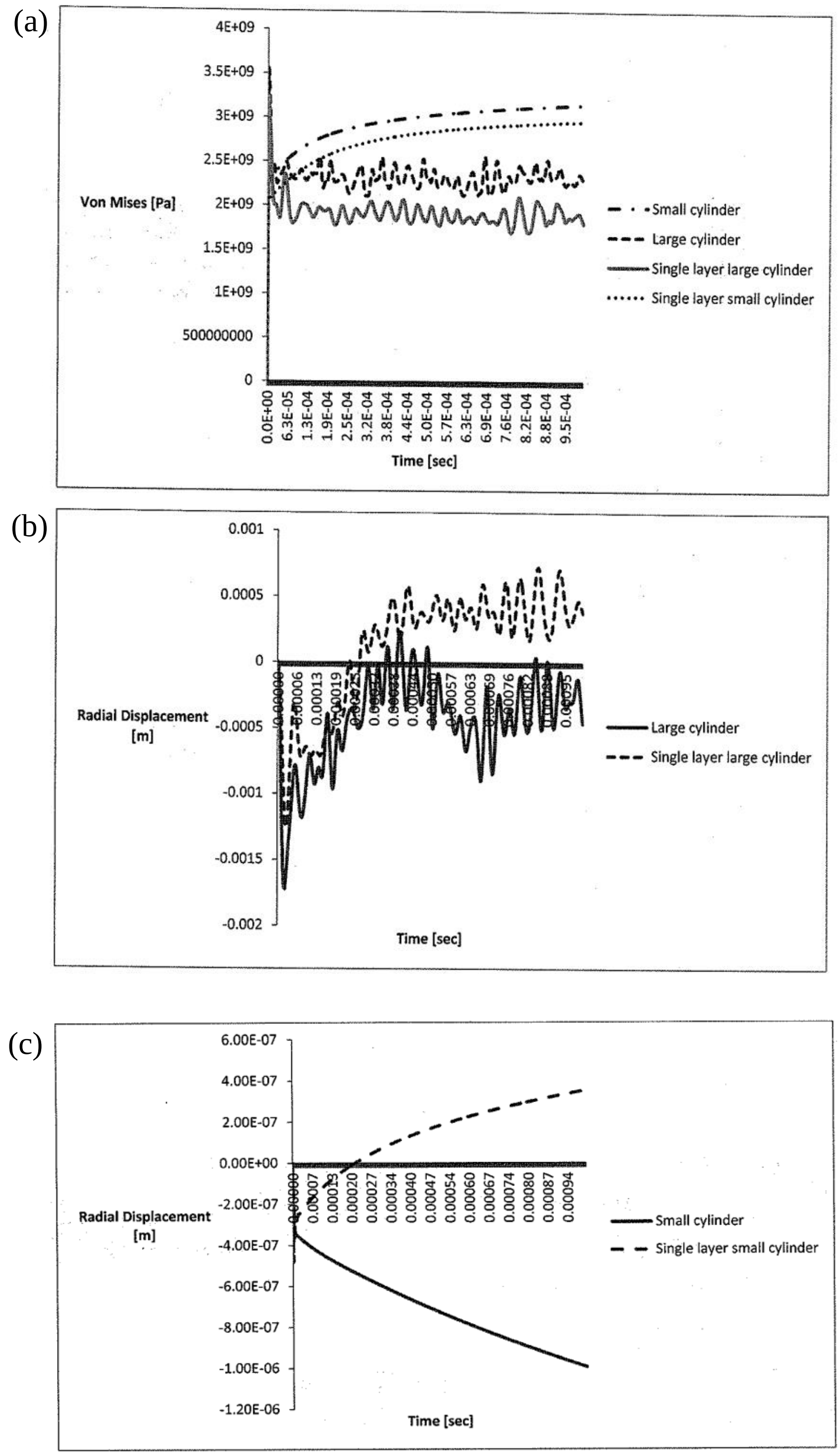

Fig. 8 (a) Von Mises Vs. time at the inner layer edge for different cylinders configurations. (b) Radial displacement Vs. time at the inner layer edge for different cylinders configurations. (c) Radial displacement Vs. time at the inner layer edge for different cylinders configurations.

4 Conclusion

General finite element (F.E.) coupled transient-dynamic model of three layers cylinder in polar coordinates under stress plane assumption, while thermal shock phenomenon is generated, without the effects of heat sources generation and body forces influence, was presented. The model material and geometrical properties data were obtained by using Lee et al. study [29]. We have found through the F. E. analysis and literature comparison that five main factors might influence mostly on the dynamic response: (1) Boundary conditions, (2) Model size and length, (3) Number of layers, (4) Material properties, and by literature review (5) Heat source distribution. Finally, good qualitative agreement was found between the F.E. analysis and Lee et al. [29] results for the radial displacement, tangential and radial stresses.

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