Modeling and simulation of heat transfer in conduction-convection systems

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Abstract: - The use of cooling systems in electronics is an important part of their design to provide a correct functioning of the device and improve its lifespan by controlling the temperature and avoiding overheating that could be of risk and cause any damage. One alternative to reduce the temperature is to use heat sinks that distribute the heat along their length by conduction and convection. The mathematical model of this process is important for design purposes as it can give information of the temperature distribution along the rod and the chip surface. Additionally, the use of dynamic models helps to know the influence of several parameters that affect the transfer of heat in the device, for example in the case of having specific dimensions of the rod. The present research contributes to explain the mathematical model of cooling fins in transistors. Equations for the temperature distribution along the length of the rod and the chip surface were obtained and used for simulation purposes. Accordingly, the evaluation of the ratio between the real heat transfer rate from the fin and he ideal heat transfer rate was performed.

Key-Words: - conduction, convection, cooling fins, heat transfer

1 Introduction

Integrated circuits are integral part of a diversity of electronics and electro-mechanical instruments. [1] The development of electronic devices is currently pushing the time rate of energy transfer per unit volume. Transistors are widely used as integral part of several electronic components consuming high amounts of electric power. In general, the failure rate of these devices is halved for each 10°C reduction in the junction operating temperature, consequently, the operating temperature at which each electronic devices work has to be kept below specific levels to minimize the risk of failure. As a result, new adjustments in element design are performed to facilitate the dissipation of energy, reduction of temperature and to provide alternatives for cooling [2]. Accordingly, a diversity of cooling techniques have been used, including air cooling and liquid cooling [3, 4]. Alternatively, schemes including single-phase high heat flux cooling along with hybrid microchannel/jet-impingement modules are also considered [5, 6]. Therefore it is important to take into account not only the electrical parameters but also thermal. Both of these parameters have equivalent laws that are used to perform calculations and develop dynamic models and numerical simulation. One of the most important parameters in thermal management of an electronic device and cooling is the temperature at the transistor chip surface. Precise information of this parameter is critical because functional properties of the device are diminished when care of temperature control is not considered [7]. Many integrated circuits are constituted from silicon. Technologies for embedding thinned integrated circuits in polyimide sheets are being performed to increase thermal resistance of chips; however it is inevitable that the operational life of the product is diminished when the devise is exposed to temperature increments. Integrated circuits are normally attached to materials including copper, ceramics or metal matrix composites. Due to differences in dilatation coefficients of the elements or compounds, differential thermal expansion occurs when the chip reaches higher temperatures, affecting the performance of the transistor. For these reasons, thermal management solutions are intensively sought [8], including engineering of fin geometry [9], i.e. optimization of heat transfer density is extensively investigated for the design of thin-film electronic circuits. Accordingly, natural and forced convection are commonly used to achieve a device's cooling requirements [10]. Air-cooled heat sinks are commonly used due to their low cost and reliability, except on cases of electronics with high heat flux density [11]. These fins are fabricated from materials with high thermal conductivity. They are attached to the transistor to decrease the operating temperature. Works on heat dissipation have been the interest of several researchers. Xu et al., for example, developed a thermal model for the power converter in switched reluctance motor drives under natural air cooling. They studied several cases of layout device and heat sink placement direction. They determined that symmetrical position of the devises on the heat sink is important to achieve the same power losses and that the cooling device requires to be in horizontal position with the fin opening up [12]. Other researchers performed studies on heat dissipation developing thermal resistance model of a heat sink with serpentine channels [13]. In their research, the authors simulated the thermal performance using a series of thermal resistance units connected by flow networks and three dimensional computational fluid dynamic simulations. Similarly a work by Choi and Anand used a two dimensional model to optimize the average Nusselt number and friction factor using the method of least squares by numerical simulation and experimental validation of turbulent heat transfer in serpentine channels [14].

In this work, we analyzed the cooling of a transistor surface studying the mathematical model of the heat transfer by conduction from the surface to the fin and further dissipation to the surrounding by convection.

2 Description of the mathematical model

The analysis of Newton's law of cooling gives important information to increase the rate of heat transfer. In general, this can be achieved by two different methods. The first one is to raise the coefficient of convection heat transfer by means, for example, of a fan. The second possibility is to increase the surface area, which can be achieved by connecting the surface with extended surfaces made of highly conductive materials, which are usually named fins. [15]

An alternative to achieve convective removal of heat from a surface is by using extensions on the surface of the transistor to increase the area [16, 17]. There exist several designs of fin, namely triangular, rectangular, cylindrical, circular, trapezoidal, concave parabolic, radial or even in the form of spiral tubes [18]. A fan is used to supply air to allow uniform dissipation of the heat that goes to the integrated circuits. Figure 1 shows a general scheme of the system under study. For simplification purposes, only one rod is represented, but in real practice, the surface is covered by several cylindrical rods. Cooling fins are generally used to improve heat transfer by increasing the surface area available for convection. Many geometric shapes can be used but the most common are rectangular, cylindrical and triangular. Initially, the surface of the chip is at temperature T_s, which will be normally greater than the surrounding temperature T_0. The length of the rod (L) is cooled by a fan [17]. The heat it conducted through the surface and dissipated to the surroundings by convection.

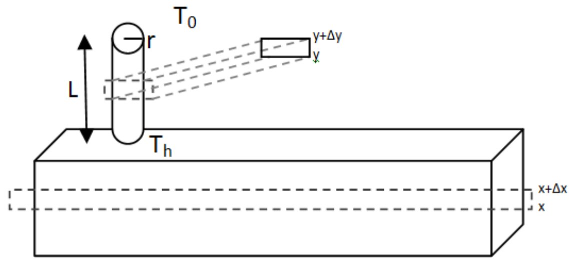

Fig. 1. Schematic diagram of the physical system under study

Fundamental assumptions are contemplated to propose the mathematical model. We consider an isotropic material with three constant parameters, namely the psychical properties of the material (conduction of heat, density, specific heat), the generation of heat in the dispositive, and finally the rate of heat energy transfer through the surface of contact per unit time and temperature difference [19].

We proceed to model the transfer of heat removal around the chip at position y in the rod by considering the rate of heat conduction into the element at y, the rate of heat conduction from the element at $y+\Delta y$ and the rate of heat convection from the element as represented in the steady state heat balance equations (1) and (2) describing the heat in the chip and the heat dissipated by the rod to the surroundings. [20]

$$Sq|_y - Sq|_{y+\Delta y} - q_L S' = 0 \quad (1)$$

$$\pi r^2 q|_y - \pi r^2 q|_{y+\Delta y} - (2\pi r \Delta y) h (T_r - T_0) = 0 \quad (2)$$

where S is the cross sectional area, S' is the lateral external area, r is the radius of the rod, q corresponds to the heat flux in y direction (W/m^2), q_L is the convection heat transfer from the rod to the surrounding h the convection heat transfer coefficient, T_r and T_0 are the temperatures in the rod and the surrounding respectively. Dividing equation (2) by $\pi r^2 \Delta y$ and taking the limit as $\Delta y \rightarrow 0$ leads to equations (3) and (4) respectively. Accordingly, considering the Fourier law, equation (4) is transformed into (5).

$$\frac{q|_y - q|_{y+\Delta y}}{\Delta y} - \frac{2h(T_r - T_0)}{r} = 0 \quad (3)$$

$$-\frac{dq}{dy} - \frac{2h(T_r - T_0)}{r} = 0 \quad (4)$$

$$k_r \frac{d^2 T_r}{dy^2} - \frac{2h(T_r - T_0)}{r} = 0 \quad (5)$$

Additionally, we can transform the previous equation into dimensionless variables of temperature, distance and heat transfer coefficient. Introducing the dimensionless variables represented in (6), equation (7) is obtained:

$$\xi = \frac{y}{L}, \text{for } \epsilon\ (0,1)\ ;\ \alpha = \sqrt{\frac{2hL^2}{rk_r}}\ ;\ \theta = \frac{T_r - T_0}{T_h - T_0} \quad (6)$$

$$\frac{d^2\theta}{d\xi^2} - \alpha\theta = 0 \quad (7)$$

The expression hL/K_r represents the Biot number, which comprises the interfacial convective transport with respect to conduction, while the expression $L\,/\,r$ concerns the geometrical aspect of the fin. Boundary conditions are required to evaluate the arbitrary constants A and B. They are expressed in equations (8) and (9).

$$\theta(0) = T_h \quad (8)$$

$$\frac{d\theta}{d\xi}(L) = 0 \quad (9)$$

Equation (8) express that the surface temperature at $\xi = 0$ corresponds to the temperature of the plate surface and equation (9) that the fin is insulated.

Solving the linear and homogenous second ordinary differential equation with constant coefficients by the method of characteristics, the roots for the heat balance equation are $m = \pm\alpha$.

Therefore, the solution of the equation leads to $\theta = Ae^{\alpha\xi} + Be^{-\alpha\xi}$. However, for the purpose of applying boundary conditions, it is often convenient to represent the solution of equation (7) using hyperbolic functions, i.e. $\theta = A\cosh\alpha\xi + B\sinh\alpha\xi$. As a result of applying boundary conditions, the value of the arbitrary constants A and B corresponds to A=1 and $B = -\frac{\sinh\alpha}{\cosh\alpha}$. Hence, substitution of the previous values into the solution of the differentia equation leads to (10).

$$\theta = \frac{(\cosh\alpha)(\cosh\alpha\xi) - (\sinh\alpha)(\sinh\alpha\xi)}{\cosh\alpha} \quad (10)$$

Using trigonometric identities, we obtain (11)

$$\theta = \frac{\cosh\alpha\,(1-\xi)}{\cosh\alpha}$$

And returning to original variables,

$$\frac{T_r - T_0}{T_h - T_0} = \frac{\cosh\sqrt{\frac{2hL^2}{rk_r}}\left(1 - \frac{y}{L}\right)}{\cosh\sqrt{\frac{2hL^2}{rk_r}}} \quad (11)$$

As a result, the temperature in the rod can be derived from previous equation (11). The next step, is to describe the transfer of heat around the computer chip at the position x with a thickness Δx. This is expressed in equation (12):

$$Sq|_x - Sq|_{x+\Delta x} + (S\Delta x)Q = 0 \quad (12)$$

where S is cross-sectional surface area of the chip, Q is the heat generated per unit volume and Δx the thickness. Dividing equation (12) by $S\Delta x$ we obtain:

$$\frac{q|_x - q|_{x+\Delta x}}{\Delta x} + Q = 0 \quad (13)$$

Taking the limit as $\Delta x \rightarrow 0$ gives the differential equation that represents the heat flux.

$$-\frac{dq}{dx} + Q = 0 \quad (14)$$

If the chip is homogenous, we can consider the vector form of Fourier's law of heat conduction which describes that the heat flux along the axis is proportional to the gradient in temperature, $q = -k\nabla T$. Therefore, equation (14) yields (15)

$$k_c \frac{d^2 T_c}{dx^2} + Q = 0 \quad (15)$$

where k_c and T_c corresponds to the thermal conductivity and temperature of the chip respectively.

The boundary conditions are set in equations (16) and (17)

$$\frac{dT_c}{dx}(0) = 0 \tag{16}$$

$$T_c(\delta) = T_h \tag{17}$$

Solution of equation (15) is represented in (18),

$$T_c = \frac{1}{k_c}\left(Cx + D - \frac{1}{2}Qx^2\right) \tag{18}$$

After application of the boundary condition (16) we obtain $C=0$ and hence $T_c = (1/k_c)(D - Qx^2/2)$. The value of D can be obtained using boundary condition (17) which corresponds to $= k_c T_h + Q\delta^2/2$. Substitution of the previous values of C and D leads to equation (19)

$$T_c = \frac{1}{k_c}\left(k_c T_h + \frac{Q\delta^2}{2} - \frac{1}{2}Qx^2\right) \tag{19}$$

And simplifying (19), the temperature in the chip surface can be expressed as :

$$T_c = T_h + \frac{Q}{2k_c}(\delta^2 - x^2) \tag{20}$$

From equation (20) it is evident that for the case of $x=\delta$, the temperature in the chip equals the exterior hot surface ($T_c=T_h$).

Once we know the temperature profile distribution, we can calculate the dissipated heat by convection from the fin, which is equal to the heat that enters at $x=0$. For an adiabatic fin, the rate of heat transfer is expressed in (22)

$$q = -k_r S \left.\frac{dT}{dy}\right|_{y=0} \tag{21}$$

$$q = k_r S(T_h - T_0)\sqrt{\frac{2h}{rk_r}}\tanh\sqrt{\frac{2h}{rk_r}}L \tag{22}$$

Accordingly, the fin efficiency can be determined by equation (23). This equation express the ratio between the real heat transfer rate from the fin and he ideal heat transfer rate from the fin.

$$\eta = \frac{q_{real}}{q_{ideal}} \tag{23}$$

$$\eta = \frac{k_r \pi r^2 (T_h - T_0)\sqrt{\frac{2h}{rk_r}}\tan h\sqrt{\frac{2h}{rk_r}}L}{(2\pi r L h)(T_h - T_0)} \tag{24}$$

$$\eta = \sqrt{\frac{rk_r}{2h}}\frac{\tan h\sqrt{\frac{2h}{rk_r}}L}{L} \tag{25}$$

The temperature of the fin decreases gradually from the surface toward the fin tip. This drop is caused by convection from the fin surface. In the case that there is no thermal resistance, the temperature of the fin would be uniform at the value of the surface temperature (T_h) with a maximum rate of heat transfer; however, there is always a decrease of temperature along the length (y) and drop in temperature difference $T_r - T_0$ towards the rod tip. The efficiency of the rod is used to determine the reduction in temperature on heat transfer.

3 Discussion

Thermal management in electronic devices is important to protect the correct functioning and operation of the dispositive. Accordingly, the knowledge of temperature distribution across the chip surface is a key factor also for operational safety reasons. In Figure 2 we present two cases of temperature distribution. The first case is when the temperature at the surface of the chip is 80°C and the second is 58°C. In both examples, the final temperature (near the surface that is in contact with the surrounding at y=0) approximates to 30°C. The information obtained from the plot is also important for the design and determination of rod geometry, i.e. to know the variation of temperature in the rod and the required length.

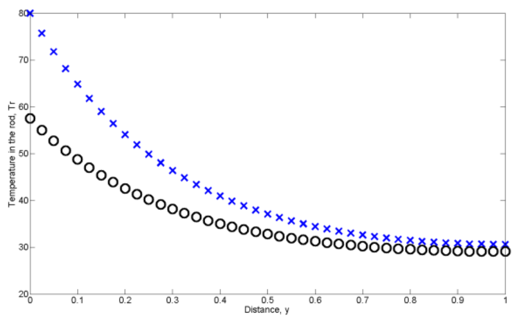

Fig. 2 Temperature profile in the rod according to distance from the base (contact with chip). T_h=80°C(×), T_h=58°C(O).

The temperature decreases along the fin exponentially, reaching the temperature of the surrounding after some distance. After this point, the rest of the fin does not contribute to heat transfer and is considered as a waste of material. Additionally, the increased size affects negatively the fluid motion and reduces the convection heat transfer coefficient [21]. Figure 3 presents the variation of heat transfer from a fin relative to that from an infinitely long fin. The plot shows that the variation of heat transfer is linear at the beginning but remains constant after reaching a plateau at $\sqrt{2h/rk_r}\,L = 5$. Therefore we can consider that a fin of $L = \frac{\sqrt{2h/rk_r}}{5}$ is considered an infinitely long fin.

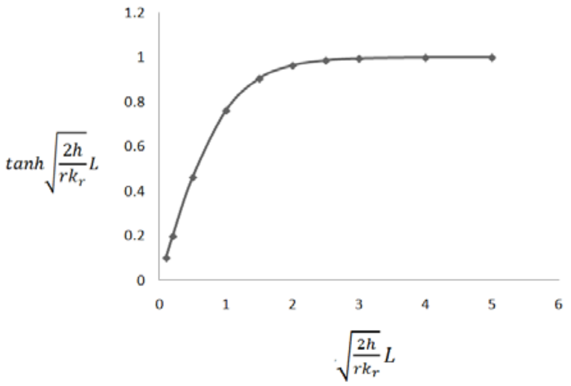

Fig. 3 Variation of heat transfer from a fin relative to an infinitely long fin.

It is also clear that at a value of $\sqrt{2h/rk_r}\,L = 2.5$, only 1% of heat transfer is decreased. Normally, a value of $\sqrt{2h/rk_r}\,L = 1$ would be satisfactory for heat transfer, economic and design purposes.

Figure 4 presents the variation of temperature in the fin at different values of transistor temperature using equation (11). The higher rate of heat transfer is presented in the region closer to the surface of the transistor. Similarly, Figure 5 shows the graphical representation of equation (20) where no considerable temperature difference is presented along the thickness of the chip.

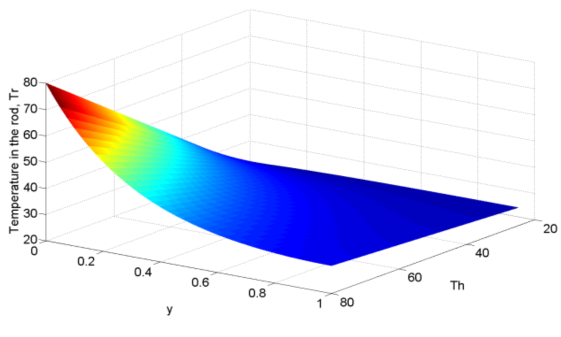

Fig. 4 Variation of temperature in the fin at different given values of chip temp. (T_h) and distance (y)

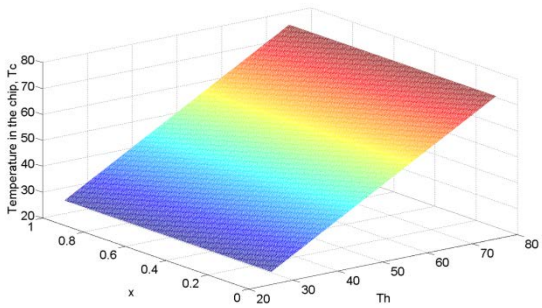

Fig. 5 Temperature variation in the chip along the distance (x)

We can also represent the variation of temperature in the rod in dimensionless units from equation (11) as shown in Figure 6 for any given initial values of temperature in the surface of the chip.

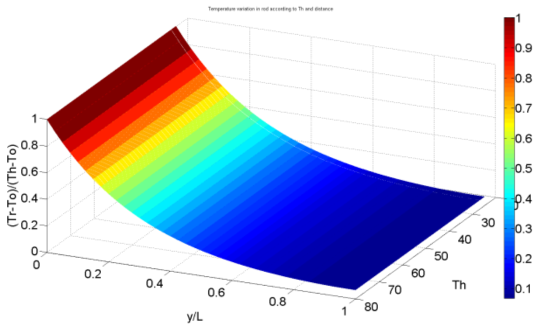

Fig. 6 Dimensionless representation of the variation of temperature in the rod as expressed in equation (11).

This represents a numerical and graphical example for which the formula obtained for the temperature profile in the rod was found to be useful. As a result, the use of cylindrical rods provided to be useful to aids in heat transfer by increasing the surface of the chip and allowing better heat distribution.

4 Conclusions

The development of microelectronic devices that are advanced in technology and smaller in size confront considerable challenges in thermal design aiming to dissipate the heat generated inside the device that could reduce reliability of an equipment, malfunctioning and problem of operation. An analysis of diffusion of energy due to random molecular motion (conduction) and energy diffusion due to the combined effect of random molecular motion and bulk motion (convection) system was performed in the present study to describe the cooling of a transistor surface by the use of a cooling rod. Mathematical model of these phenomena was performed to obtain the profile of temperature in the rod and the transistor surface.

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