Entropy Generation Analysis of Transient Heat Conduction in a Solid Slab with Fixed Temperature Boundary Conditions

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Abstract: **-** Analysis of entropy generation rate for transient heat conduction, taking place in homogeneous solid slab with fixed temperature boundary conditions, is presented. The initial condition is defined as a constant temperature. The exact solution is solved and used to analyze the local and total entropy generation rates. Two cases, heating and cooling processes, are considered in this study. It is found that the local entropy generation rate dramatically changes at small time and slowly approaches the local entropy generation rate of steady state case. The location of minimum local entropy generation rate can be found in the slab for both heating and cooling cases. For the total entropy generation rate, it shows very high value at small time. This is due to high temperature gradient. It, then, sharply reduces and converges to the total entropy generation rate of steady state case. However, the minimum total entropy generation rate is found for heating case, while it cannot be detected for cooling case. The minimum total entropy generation rate, found in heating case, is observed that it is lower than that of steady state case. The boundary conditions clearly express the effect on the local and total entropy generation rates for transient heat conduction. However, the effect of initial condition on the entropy generation rates can be neglected, especially at large time.

Key-Words: - Entropy Generation Rate, Second Law Analysis, Transient Heat Conduction, Steady State Heat Conduction, Boundary Condition of the First Type, Exact Solution

1. Introduction

Systems in thermal processes usually involve fluid flow and heat transfer phenomena. To analyze the energy transfer in thermal systems, the first law of thermodynamics is often used, for example, to find the efficiency of the system or to investigate the energy loss from the system. For the second law analysis, it evaluates the irreversibility, which destroys the available work of the system, and it can be quantified by exergy destruction or entropy generation (so called entropy production). In a thermal system, heat transfer process is one of the most important irreversibility sources. Therefore, it has been widely interested by researchers.

For steady state heat conduction, Kolenda et al. [1] investigated the entropy generation taking place in this system. They concluded that minimization of entropy generation in conduction process is always possible by introducing additional heat sources, which can be arbitrarily chosen as positive or negative. The entropy generation of steady state heat conduction was also studied by Ibanez et al. [2]. The system analyzed is a solid slab having uniform internal heating. The boundary condition of the third kind was applied to the system in both surfaces. They found that Biot numbers (Bi) has significant effect on the global (or total) entropy generation rate and the minimum point of this function can be found.

For transient heat conduction, Bautista et al. [3] have been shown that the only source of irreversibility in the system is the presence of thermal gradient. Moreover, the spatial average of the nondimensional entropy generation rate decreases with time. Strub et al. [4] analyzed the entropy generation of heat conduction through a wall which was submitted to periodic temperature fluctuation on one face and constant temperature on the other face. Moreover, they also analyzed the problem using ideal Carnot cycle. They found that the two approaches lead to different sometime opposite results.

Recently, the minimum entropy generation for steady state conduction with temperature dependent thermal conductivity and asymmetric thermal boundary conditions was studied by Aziz and Khan [5]. Three model geometries, a plane wall, a hollow cylinder, and a hollow sphere, were focused. They summarized that the dimensionless entropy rates for both classical as well minimum entropy generation (MEG) models were found to be higher in a hollow sphere compared with those in the hollow cylinder.

As observed from Refs. [6 and 7], the entropy generation rate has been widely analyzed for combined heat and fluid flow system. However, for the entropy generation analysis of heat conduction, a few researches have been found.

In this study, the entropy generation of transient heat conduction with fixed temperature boundary conditions is investigated. Heating case (the boundary temperature is higher than the initial temperature) and cooling case (the boundary temperature is lower than the initial temperature) are assumed and the local and total entropy generation rates are investigated. The minimum points of local and total entropy generation rates are focused. The entropy generation rate of steady state heat conduction problem is analyzed and compared with that of transient heat conduction problem with the same boundary condition.

2. Problem Formulation

In this study, the one-dimensional transient conduction through a slab is investigated. The solid slab is a homogeneous material with a constant thermal conductivity, k. The temperatures of both surfaces of the slab are fixed at constants (boundary condition of the first type). The schematic diagram of the slab with boundary conditions is shown in Fig. 1. The heat equation for this problem can be written as:

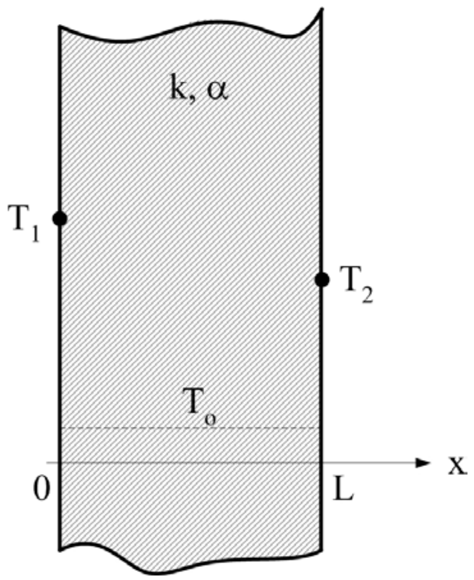

Fig. 1 Schematic representation of the slab and boundary conditions

$$\frac{\partial T^2}{\partial x^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

with the initial and boundary conditions as:

$$T(x,0)=T_o \tag{2}$$

$$T(0,t)=T_1 \tag{3}$$

$$T(L,t)=T_2 \tag{4}$$

where α is thermal diffusivity and t is time. To find the solution or the temperature distribution, the above problem can be separated into the steady state component and transient component with homogeneous boundary conditions [8]. The steady state component is defined as:

$$\frac{\partial T_S^{\ 2}}{\partial x^2}=0 \tag{5}$$

$$T_S(0)=T_1 \tag{6}$$

$$T_S(L)=T_2 \tag{7}$$

The steady state solution can be obtained by integrating and substituting the boundary conditions. It results:

$$T_S(x)=\left(T_2-T_1\right)\frac{x}{L}+T_1 \tag{8}$$

The transient solution, $T_T(x,t)$, satisfies:

$$\frac{\partial T_T^{\ 2}}{\partial x^2}=\frac{1}{\alpha}\frac{\partial T_T}{\partial t} \tag{9}$$

$$T_T(x,0)=T_o-T_S(x) \tag{10}$$

$$T_T(0,t)=0 \tag{11}$$

$$T_T(L,t)=0 \tag{12}$$

This problem can be solved using separation of variables method and it gives:

$$T_T(x,t)=\sum_{n=1}^{\infty}A_n\sin\left(\frac{n\pi x}{L}\right)\exp\left[-\frac{\alpha t}{L^2}\left(n\pi\right)^2\right] \tag{13}$$

A_n can be obtained using orthogonality relation and the result is:

$$A_n = \frac{2}{L}\int_0^L \left\{ T_o - \left[\left(T_2 - T_1\right)\frac{x'}{L} + T_1 \right] \right\} \times \sin\left(\frac{n\pi x'}{L}\right) dx' \tag{14}$$

Therefore, the complete solution of investigated problem is:

$$\begin{aligned} T(x,t) &= T_S(x) + T_T(x,t) \\ &= T_1 + \frac{\left(T_2 - T_1\right)}{L}x + \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{T_2\cos(n\pi) - T_1}{n} \times \sin\left(\frac{n\pi}{L}x\right)\exp\left[-\frac{\alpha t}{L^2}\left(n\pi\right)^2\right]\right) + \\ &\quad \frac{2}{L}\sum_{n=1}^{\infty}\left(\sin\left(\frac{n\pi}{L}x\right)\exp\left[-\frac{\alpha t}{L^2}\left(n\pi\right)^2\right] \times \frac{T_o L}{n\pi}\left[1 - \cos(n\pi)\right]\right) \end{aligned} \tag{15}$$

3. Entropy Generation Analysis

The local entropy generation rate per unit volume (in W/m^{3}K) of one-dimensional heat conduction without internal heat generation can be defined as [9, 10]:

$$\dot{s}'''_{gen} = k\frac{\left(\nabla T\right)^2}{T^2} \tag{16}$$

It should be mentioned here that the local entropy generation rate per unit volume is often referred as the local entropy generation rate through this paper. For the temperature gradient, ∇T, it is obtained by finding derivative of Eq. (15) respect to x and the result is:

$$\begin{aligned} \nabla T &= \frac{\left(T_2 - T_1\right)}{L} + \frac{2}{L}\sum_{n=1}^{\infty}\left(\frac{T_2\cos(n\pi) - T_1}{n} \times \cos\left(\frac{n\pi}{L}x\right)\exp\left[-\frac{\alpha t}{L^2}\left(n\pi\right)^2\right]\right) + \\ &\quad \frac{2T_o}{L}\sum_{n=1}^{\infty}\left(\cos\left(\frac{n\pi}{L}x\right)\exp\left[-\frac{\alpha t}{L^2}\left(n\pi\right)^2\right]\left(1 - \cos(n\pi)\right)\right) \end{aligned} \tag{17}$$

If A is defined as the area normal to the direction of heat flow and it is assumed to be constant, the total entropy generation rate (in W/K) can be calculated by integrating Eq. (16) over the slab volume or:

$$\dot{s}_{gen} = \int_0^L \dot{s}'''_{gen} A dx \tag{18}$$

The nondimensional local entropy generation rate and the nondimensional total entropy generation rate can be formed, respectively, as:

$$N_{loc} = \frac{\dot{s}'''_{gen}}{k/L^2} \tag{19}$$

$$N_{tot} = \frac{\dot{s}_{gen} L}{kA} \tag{20}$$

As well known that when time increases and approaches infinity, the transient heat conduction system can be approximately characterized by steady state conduction which has the same boundary conditions. Therefore, the entropy generation rate of steady state heat conduction is also interested in this study. The local entropy generation rate of steady state conduction problem can be obtained using Eqs. (16) and (8).

$$\dot{s}'''_{gen,steady} = \frac{k}{L^2}\frac{\left(T_2 - T_1\right)^2}{\left[\left(T_2 - T_1\right)\frac{x}{L} + T_1\right]^2} \tag{21}$$

The nondimensional local entropy generation of steady state heat conduction matching with the above problem is:

$$N_{loc} = \frac{\left(T_2 - T_1\right)^2}{\left(\left(T_2 - T_1\right)x/L + T_1\right)^2} = \frac{\left(1 - \theta\right)^2}{\left(\left(1 - \theta\right)x/L + \theta\right)^2} \tag{22}$$

where $\theta = T_1/T_2$. For A=1m^2, the nondimensionless total entropy generation rate can be expressed as:

$$N_{tot} = \left(\sqrt{\frac{T_1}{T_2}} - \sqrt{\frac{T_2}{T_1}}\right)^2 = \left(\sqrt{\theta} - \sqrt{\frac{1}{\theta}}\right)^2 \tag{23}$$

It is worth to mentioned here that the local and total entropy generation rates shown in Eqs. (22) and (23) are away positive. Moreover, from Eq. (23), it can be presented in terms of heat transfer rate, $\dot{q}$, through the slab as:

$$N_{tot} = \frac{\dot{s}_{gen,steady} L}{kA} = \frac{\dot{q} L}{kA}\left(-\frac{1}{T_1} + \frac{1}{T_2}\right) \tag{24}$$

where

$$\dot{q} = kA\frac{\left(T_1 - T_2\right)}{L} \tag{25}$$

From Eq. (24), the total entropy generation rate, $\dot{s}_{gen,steady}$, can be expressed as

$\dot{s}_{gen,steady} = \dot{q}\left(-1/T_1 + 1/T_2\right)$

and this is the same as the entropy generation due to heat transfer, which can be found in thermodynamics textbooks [11, 12].

4. Results and Discussions

The temperature distribution, observed from Eq. (15), was expressed as a series of summation. Even through the number of summation terms, n, should be infinity, it is not common practice to generate a computer code for infinite summation because it consumes a lot of calculation time without any significant accuracy improvement. However, the temperature shows fluctuation, especially at small time, if only a few terms are used. Fig. 2 illustrates that, at t=5s, the fluctuation of temperature value cannot be noticed when n is equal to 30. If the temperature distribution at t=1s is considered, using n=30 cannot provide the stable result. The fluctuation of temperature value is still observed when t is small. Therefore, to compromise between the accuracy of calculation and the calculation time consumed, the number of summation terms employed in this study is 200. Comparing to a similar problem discussed in Ref. [13], Hahn and Ozisik illustrated that for a transient heat conduction problem with symmetric boundary condition, at t=0s, the temperature at center of slab after summing 100 terms causes the error of 0.29%. Moreover, they also mentioned that 200 summation terms is sufficient for convergence in their presented problem.

4.1 Heating Case

To study the local entropy generation rate in the mentioned problem, the initial temperature was set as 50^{o}C (323K) and the surface temperatures at x=0 and x=L are 200^{o}C (473K) and 20^{o}C (293K), respectively. The thermophysical properties, used in this analysis, are assumed to be constants. The thermal conductivity and thermal diffusivity of copper at 300K are adopted (k =401 W/m·K and $\alpha = 117 \times 10^{-6}$ m^2/s) [14]. The temperature distribution and the local entropy generation rate per unit volume were computed and the results are presented in Figs. 2 and 3.

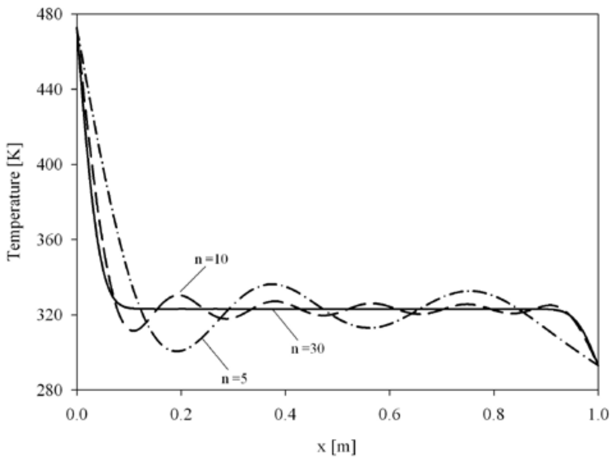

Fig. 2 The effect of summation term on the temperature at t=5s

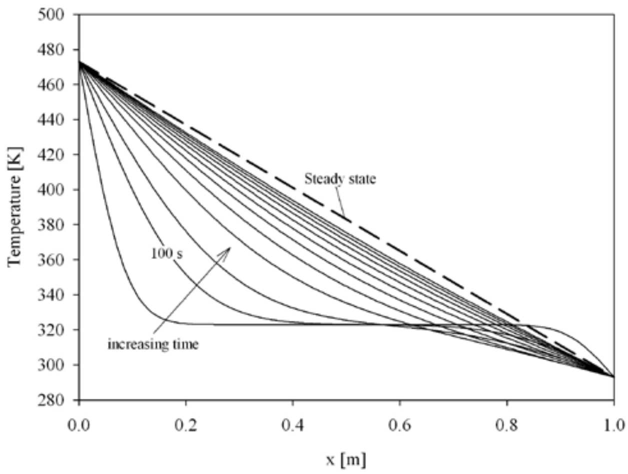

Fig. 3 Temperature distribution for different times

From Fig. 3, it can be observed that, at small time, the temperature near the left surface (x=0) dramatically changes with x. It means that the absolute temperature gradient is high near the surface. At a specific location, where heat penetration still does not reach, the temperature remains at the initial temperature, which is a constant (323K). Therefore, the temperature gradient is zero. The absolute temperature gradient through the slab is observed to reduce with time and the temperature distribution slowly approaches a uniform distribution, the steady state case, indicated by dash line.

In Fig. 4, the local entropy generation rate, presented in dimensionless form, is expressed as a function of distance, x, for different times. This figure reveals that the maximum nondimensional entropy generation rate takes place near the left surface and its value reduces with increase of time. This is because of high temperature gradient near the left surface. At a certain time, the dimensionless entropy generation rate reduces with x. For example, at t=100s, the dimensionless entropy generation rate is noticed that it is zero for x between 0.55 and 0.6. It can be explained that the temperature gradient is zero in this range due to the absence of heat penetration, as discussed before. Moreover, the temperature gradient near the right surface can be also observed. Consequently, the local entropy generation rate grows up again. As the time increases, the nondimensional local entropy generation rate finally approaches that of the steady state case.

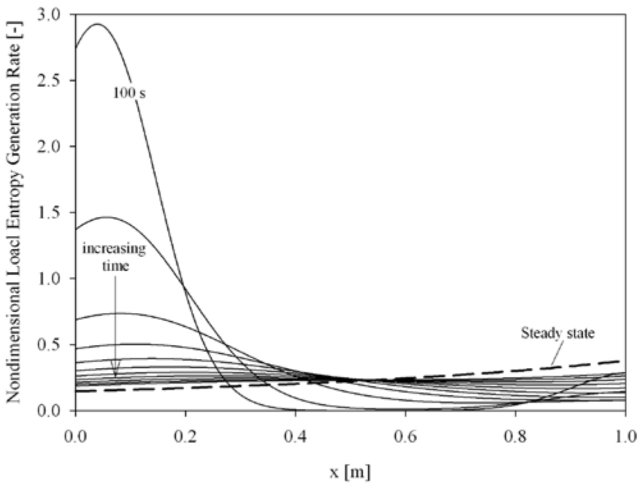

Fig. 4 Variation of nondimensional local entropy generation rates for heating case

There is an interesting issue obtained from Fig. 4, that is the minimum point of the local entropy generation rate. Fig. 5 illustrates the nondimensional local entropy generation rate at 400s, 600s, 800s and 1400s. In the figure, the circles mark the location of the minimum values of the local entropy generation rate at defined times. It can be found that the minimum entropy generation moves forward to the right surface. However, after a specific time, the location of minimum entropy generation, then, moves far from the right surface until it takes place at x=0 when time approaches infinity, as observed from steady state case. Moreover, the local entropy generation rate for small time is higher than that for large time. This is different when x increases. For example, the local entropy generation rate of 800s is being higher than that of 600s when x is higher that 0.41 m. It is worth to notice that the area under the curve, called the total entropy generation rate, at the large time is possible to be higher than that of the small time.

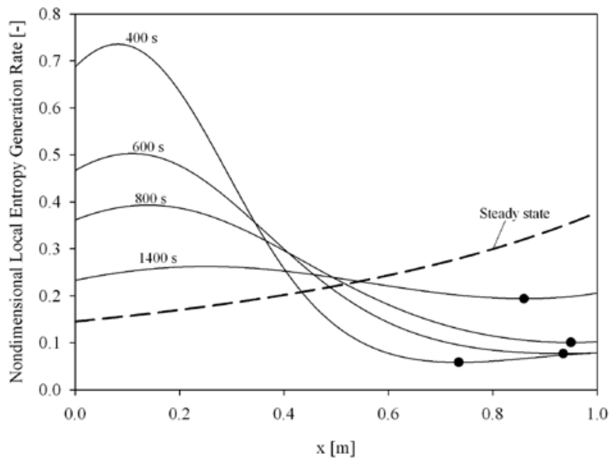

Fig. 5 Nondimensional local entropy generation rates and their minimum locations

To find the total entropy generation rate, the numerical integration method, called the trapezoidal rule [15], was used. If the local entropy generation rate at a specific time is defined as f(x), the total entropy generation rate can be figured by trapezoidal rule as:

$$\int_a^b f(x)dx \cong \frac{b-a}{2M}\left[f(a)+2\sum_{i=1}^{M-1} f\left(a+i\frac{b-a}{M}\right)+f(b)\right] \quad (26)$$

where a and b are lower limit and upper limit of integration. For this study, a = 0 and b = 1. The local entropy generation rate was divided into M subintervals of equal length. The number of M was varied and the results were computed and compared in order to make sure that the number of M, used in this study, is large enough (or the subintervals are small enough) to do not significantly affect on the result.

Fig. 6 shows the nondimensional total entropy generation rate of the above case. It sharply decreases from 45.1 at t=0 (It is not presented in the range of Fig. 6) to 1.2 in 20s. From the calculation, the nondimensional total entropy generation rate can be found the minimum value, 0.2294 at t=1680s. This value is lower than the nondimensional total entropy generation rate of steady state case (see magnified figure), whose value is 0.2338, indicated by dash line in Fig. 6. After t=1680s, the total entropy generation rate increases again and finally approaches the steady state value. As discussed before, even through the local entropy generation rate near the left surface significantly reduces with time, but it increases at the location far from the left surface. For large time, this possibly causes increase of the area under the curve or the total entropy generation rate if the reducing area at the front part is smaller that the increasing area at the rear part. Fig. 7 confirms the above discussion. In order to easily compare the minimum nondimensional total entropy generation rate with others, the trapezoidal rule was used again to find the nondimentional total entropy generation rates at t=1480s and t=1880s. At t=1480s, the area under the curve or the nondimensional total entropy generation rate is 0.2297 while it is 0.2296 for t=1880s. Both nondimensional total entropy generation rates found higher than that at t=1680s. Consequently, the minimum total entropy generation rate is exist.

4.2 Cooling Case

In the previous section, the slab is heated. Even through it has the right boundary temperature lower than the initial temperature, the same trend of result can be obtained for the case that the right boundary temperature higher than the initial temperature. In this section, the cooling case, the boundary temperatures are lower than the initial temperature, is investigated. The initial temperature was set as 500^{o}C (973K) while the boundary temperatures are defined as the same as the heating case.

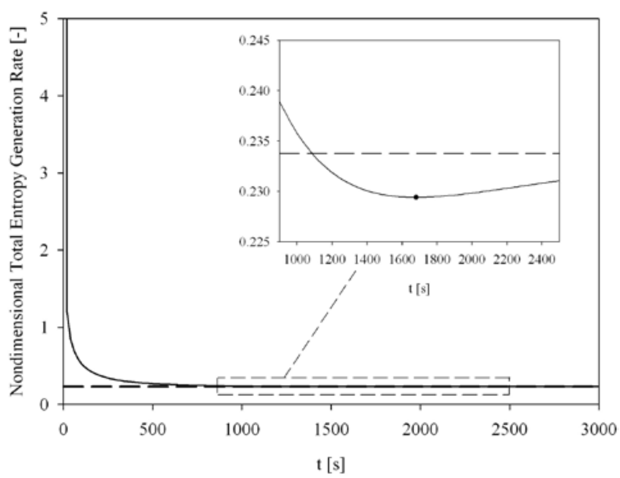

Fig. 6 Nondimensional total entropy generation rates for heating case

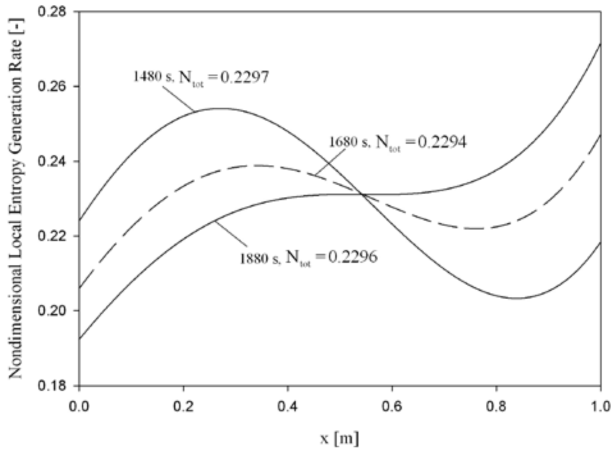

Fig. 7 Comparison of the minimum nondimensional total entropy generation rate with others

Fig. 8 shows that the nondimensional local entropy generation rate through the slab reduces with time and finally approaches to the steady state case. The minimum points of the local entropy generation rate at different times are clearly observed in the figure. It is also found that the minimum point of the local entropy generation rate shifts to the left surface, at which the initial-boundary temperature different is smaller than that of right surface. When time tends to infinity, the lowest entropy generation rate can be, then, found at the left surface, as expressed in steady state case. Moreover, from Fig. 8, it can be implied that if the system is symmetric temperature boundaries, the minimum local entropy generation rate should take place at the middle of slab.

For the total entropy generation rate of cooling case, Fig. 9 presents this value in dimensionless form. In the figure, the minimum point of total entropy generation rate cannot be found during considered period. Although, the period of time is expanded, it also shows the same result. The total entropy generation rate monotonically decreases with increasing time. It tends to a fixed value that is the total entropy generation rate of steady state case, as indicated by a circle in Fig. 9.

4.3 Effect of Boundary Temperatures

To study the effect of boundary temperatures on the total entropy generation rate, the parameter θ, defined as the ratio of T_1 and T_2, is focused. For this section, the initial temperature was fixed at $2T_2$. The calculation was done and the result is demonstrated in Fig. 10.

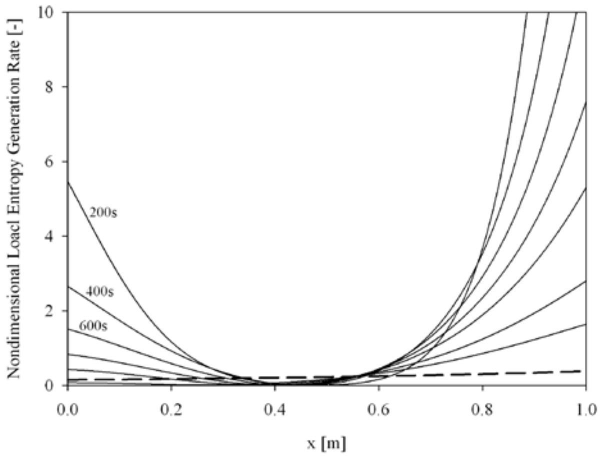

Fig. 8 Variation of nondimensional local entropy generation rates for cooling case

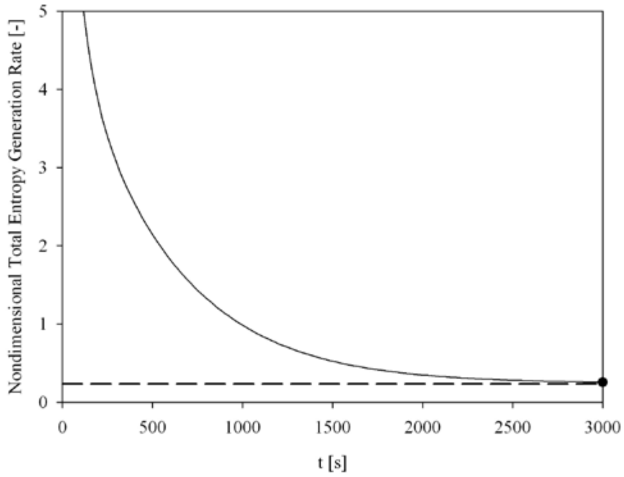

Fig. 9 Nondimensional total entropy generation rates for cooling case

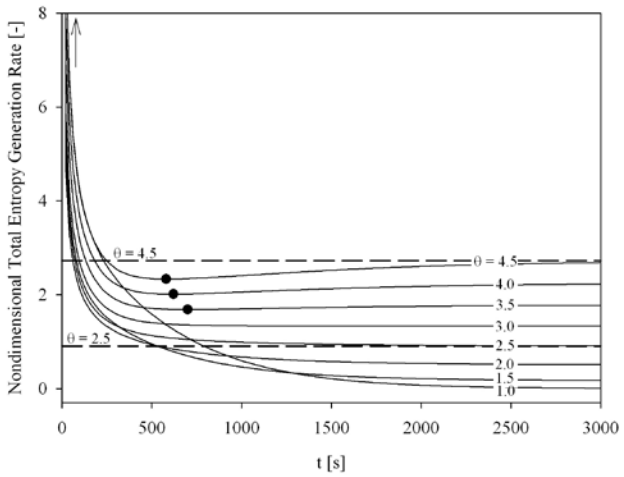

Fig. 10 Variation of nondimensional total entropy generation rates for different θ

From the calculation, it is found that the nondimensional total entropy generation rate sharply increases when time approaches zero. However, the higher value of nondimensional total entropy generation rate was omitted to show. From the figure, it is observed that the nondimensional total entropy generation rate increases with θ. Although the minimum point of the nondimensional total entropy generation rate is difficult to find using analytically method because of the infinite summation. Fig. 10 shows that the minimum of the total entropy generation rate can be noticed during t =0 to 3000s when θ is equal to 3.5 and higher. Moreover, the minimum point takes place at smaller time when θ increases. In order to discuss more on the effect of boundary temperatures on the total entropy generation rate, the case of θ =4.5 is considered. For this case, the minimum point of nondimensional total entropy generation rate is 2.333 and it is 14.3% lower compared to the nondimensional total entropy generation rate of steady state case. Comparing with θ =2.5, the nondimensional total entropy generation rate dramatically decreases and approaches that of steady state case without minimum value presented in the figure.

4.4 Effect of Initial Temperature

In this subsection, the effect of initial temperature on the total entropy generation rate is investigated. The initial temperature was varied from 323K to 443K with increment of 30K. The nondimensional total entropy generation rates, associated with initial temperatures, were calculated and the results are shown in Fig. 11.

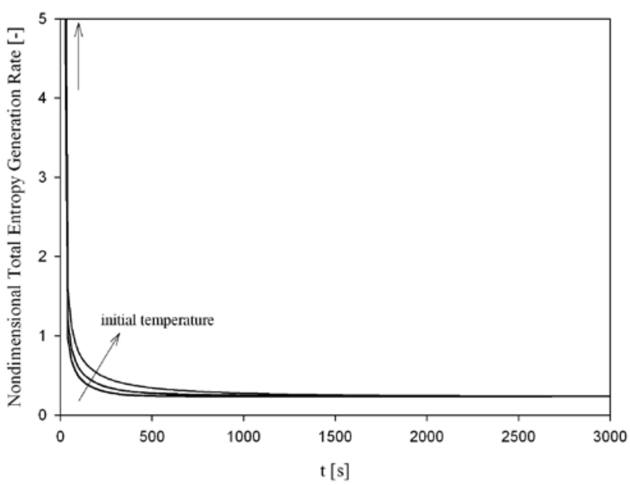

Fig. 11 Effect of initial temperature on the nondimensional total entropy generation rate

In the figure, there are 5 nondimensional total entropy generation rates for 5 cases of initial temperature. However, it is difficult to notice the distinction between these nondimensional total entropy generation rates. Thus, it can be mentioned that the initial temperature has weak effect on the total entropy generation rate, especially, at large time. However, the effect of initial temperature can be detected at t=0. High value of initial temperature causes higher total entropy generation rate at staring time. Finally, the total entropy generation rates converge when time increases.

4.5 Steady State Case

According to the results discussed above, it can be mentioned that when time increases the effect of initial condition slowly disappears. Therefore, it can be proved that the total entropy generation rates for different initial temperature cases are not much difference, when time increases. Moreover, if time approaches infinity, the temperature distribution, the local entropy generation rate, and the total entropy generation rate of transient problem are approach those of steady state problem. Thus, this section contributes to discussion the steady state case that is asymptotic value of the transient problem.

Fig. 12 shows the relative percentage difference of temperature distributions between transient problem and steady state case (% Diff-T). The boundary and initial conditions, used in the calculated, are obtained from heating case. From the figure, the maximum % Diff-T can be found around the middle of slab. At t=2000s, the maximum % Diff-T is about 2% while the average value of % Diff-T at t=3000s is 0.4%. From the calculation, the average value of % Diff-T is less that 1% when t is higher that 2200s which is equal to $F_o = \alpha t/L^2 = 0.2574$.

As presented in Eqs. (15) and (16), the local entropy generation rate can be rewritten as:

$$\begin{aligned} \dot{s}'''_{gen} &= k\frac{\left[\nabla\left(T_S + T_T\right)\right]^2}{T^2} \\ &= k\frac{\nabla T_S^2}{T^2} + 2k\frac{\nabla T_S \nabla T_T}{T^2} + k\frac{\nabla T_T^2}{T^2} \end{aligned} \tag{27}$$

From the right hand side of Eq. (27), it can be observed that the first and the third terms are positive. For the second term, it can be positive or negative value depending on the temperature gradient of steady and transient sub-problems. However, when time increases, the second and the third terms decrease and approximately become zero due to exponential terms (see Eqs. (15) and (17)). Therefore, only the first term provides significant value and it is approximately equal to the value obtained from steady stated case.

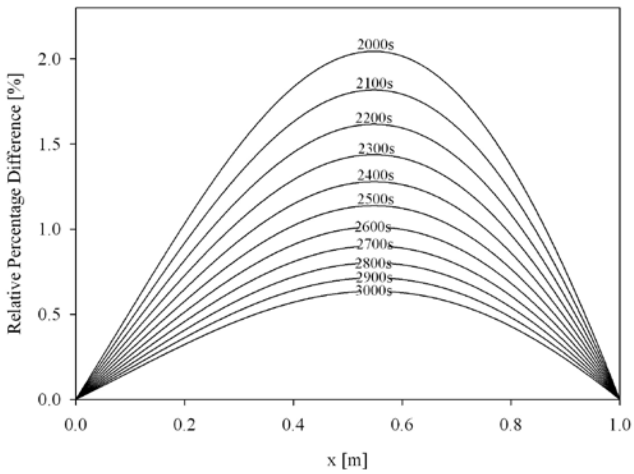

Fig. 12 Relative percentage difference of temperature distributions between transient and steady state problems

Fig. 13 shows the relative percentage difference of the local entropy generation rate between transient and steady state problems (% Diff-sgen). It can be observed that the percentage difference is high even through it is at 3000s. The maximum % Diff-sgen at 3000s is about 8.9% and the average % Diff-sgen is 5.3%. However, it can be noticed that % Diff-sgen is about zero near the middle of slab because of that the local entropy generation rates cross each other at point (see Fig. 5).

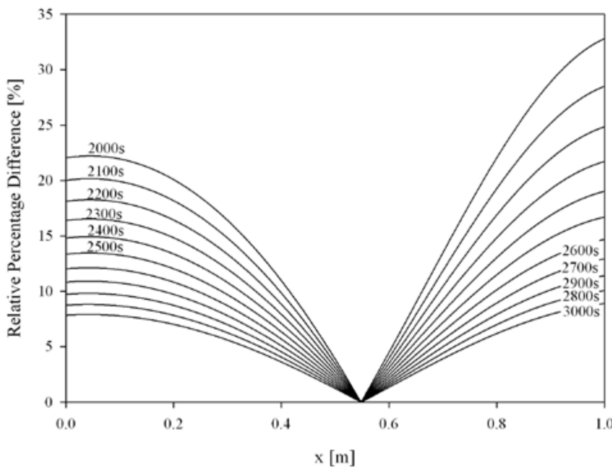

Fig. 13 Relative percentage difference of the local entropy generation rates between transient and steady state problems

From Figs. 12 and 13, it can be summarized that, at a specific time, even through the temperature distribution of steady stat case can be employed to estimate that of transient case with acceptable error, the entropy generation rate of steady state case at that time may not be used to estimate that of transient case with low error.

As observed from Eq. (22), if $\theta \to \infty$, $N_{loc}(x=0) \to 1$ while $N_{loc}(x=L) \to \infty$. On the other hand, if $\theta \to 0$, $N_{loc}(x=0) \to \infty$ and $N_{loc}(x=L) \to 1$. Fig. 14 shows the results discussed above. From the figure, the nondimensional local entropy generation sharply decreases from x =0 and approaches to 1 at x = 1 for θ=0.005 ($\theta \to 0$). This variation is found, but it takes place in opposite boundary for θ=300 ($\theta \to \infty$). The minimum point of local entropy generation rate cannot be investigated. This can be also confirmed by using basic calculus. The differentiation of Eq. (22) is done and set the result to be equal to zero. It gives:

$$x_m = \frac{-\theta}{(1-\theta)} \tag{28}$$

Considering Eq. (28), If $\theta > 1$, it can be implied that the location of x_m is always higher than 1. Moreover, if the value of θ is high, the result of x_m is about 1. However, if $\theta < 1$, the result of x_m is away negative. Therefore, it can be summarized that the minimum value of local entropy generation cannot be found.

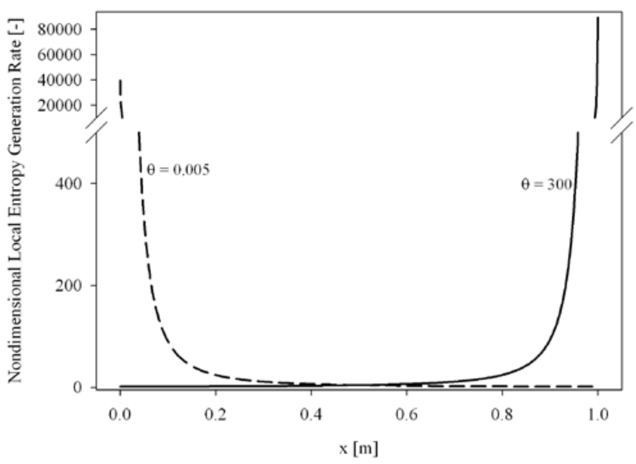

Fig. 14 The nondimensional total entropy generation rates for steady state case

For the nondimensionless total entropy generation rate, Eq. (23) expressed that if $\theta \to \infty$, $N_{tot} \to \theta$. However, if $\theta \to 0$, $N_{loc} \to 1/\theta$.

5. Conclusions

In this study, the transient heat conduction in solid slab is analyzed based on the second law of thermodynamic. The local entropy generation rate is found the minimum value for heating and cooling cases. For the total entropy generation rate analyzed in heating case, it is also observed the minimum point during the considered period. Moreover, the minimum value is less than the total entropy generation rate of steady state case with the same boundary conditions. For cooling case, the minimum total entropy generation rate cannot be detected. The boundary conditions have significant effect on the entropy generation rate, while the effect of initial condition on the entropy generation rate can be neglected, especially at large time.

References:

[1] Z. Kolenda, J. Donizak, and J. Hubert, On the minimum entropy production in steady state heat conduction processes. *Energy*, Vol.29, No.12-15, 2004, pp. 2441–2460.

[2] G. Ibanez, S. Cuevas, M. Lopez de Haro, Minimization of entropy generation by asymmetric convective cooling. *International Journal of Heat and Mass Transfer*, Vol.46, No.8, 2003, pp. 1321–1328.

[3] O. Bautista, F. Mendez, and I. Campos, Transient heat conduction in a solid slab using multiple-scale analysis. *Heat and Mass Transfer*, Vol.42, No.2, 2005, pp. 150–157.

[4] F. Strub, J. Castaing-Lasvignottes , M. Strub, M. Pons and F. Monchoux, Second law analysis of periodic heat conduction through a wall. *International Journal of Thermal Sciences*, Vol.44, No.2, 2005, pp.1154–1160.

[5] A Aziz and W.A. Khan, Classical and minimum entropy generation analyses for steady state conduction with temperature dependent thermal conductivity and asymmetric thermal boundary conditions: Regular and functionally graded materials, *Energy*, Vol. 36, No.10, 2011.

[6] R. Ben-Mansour and A.Z. Sahin. Entropy generation in developing laminar fluid flow through a circular pipe with variable properties, *Heat and Mass Transfer*, Vol. 42, No. 1, 2005, pp.1-11.

[7] S. Jarungthammachote, Entropy generation analysis for fully developed laminar convection in

hexagonal duct subjected to constant heat flux, *Energy*, Vol.35 , No.12 , 2010, pp. 5374-5379.
[8] H.S. Carslaw and J.C. Jaeger, *Conduction of Heat in Solid*, Clarendon Press, Oxford, 1959.
[9] V. Arpaci, *Conduction Heat Transfer*, Addison-Wesley, Reading, MA, 1996.
[10] A. Bejan, *Entropy generation through heat and fluid flow*. Wiley, New York, 1982.
[11] M. J. Moran and H. N. Shapiro, *Fundamentals of Engineering Thermodynamics*, John Wiley and Sons, New York, 2007.
[12] Y. A. Cengel, M. A. Boles, *Thermodynamics: An Engineering Approach*, McGraw-Hill, New York, 2007.
[13] D.W. Hahn and M.N. Ozisik, *Heat Conduction*, John Wiley and Sons, New York, 2012.
[14] F.P. Incropera, D.P. Dewitt, T.L. Bergman and A.S. Lavine. *Fundamentals of heat and mass transfer*, John Wiley and Sons, New York, 2007.
[15] S. Chapra and R. Canale, *Numerical Methods for Engineers with Programming and Software Applications*, McGraw-Hill, New York, 2002.