Transient Heat Transfer with Partial Boiling in System with Double Wall and Double Fins

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Abstract: - In this paper we present a transient model of transient heat conduction in a 2D system with double wall and double fins. Here we consider third type linear boundary conditions and a boiling condition. Conservative averaging and finite difference methods are applied to the given problem to construct numerical solution of the given problem.

Key-Words: - double wall with double fins, L-type domain, temperature fields, non-stationary heat conduction, initial-boundary value problem, conservative averaging method, difference scheme

1 Introduction

With this article we continue our study on systems with *double wall and double fins* (see [2] - [4]). By this definition we understand such structures that consist of a flat surface that is roughened by adding densely distributed vertical fin arrays, and then covered with some kind of coating (see Fig.1).

Fig.1: 2D system with fins

These artificial roughness elements are usually developed and used to enhance heat transfer performance (see, e.g., [10], [11]).

Here we propose a model that describes the heat conduction problem for 2D assembly with straight fins of rectangular profile when the process is transient, homogeneous and when partial boiling is present. Just like in the publications [2] - [8], we use conservative averaging method to reduce the given 2D problem to 1D one. The latter is solved numerically by finite difference method used in [7].

Our mathematical models differ quite a bit from those where relatively simple fin assemblies are considered (see, e.g., [1], [9], [12], [13], [15]).

2 Problem Formulation in 2D

Before formulating the problem, let’s divide the given figure into symmetrical parts. That allows us to describe the problem for only one of those (see Fig.2).

Fig.2: L-type domain

As shown in Fig.3, this L-shaped part can be represented by a union of five non-overlapping subdomains. Bear in mind, that $l > b, l_0$.

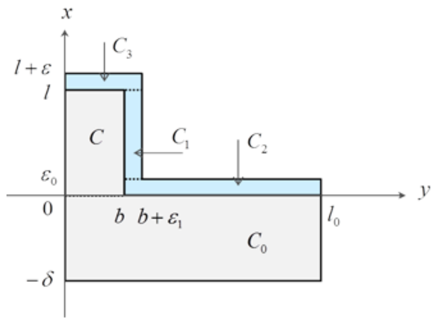

Fig.3: Definition of geometrical parameters for the sample

Let us denote the temperatures of the domains C_i by the symbols $V_i(x, y, t)$. The basic properties, such as thermal conductivity, heat transfer coefficient, specific heat and density, are constant and denoted by k_i, h_i, c_i, $\tilde{\rho}_i$, respectively. Here we take $k = k_0$ and $k_2 = k_3 = k_1$.

We are going to describe these non-stationary temperature fields by the following partial differential equations:

$$\frac{\partial^2 V_i}{\partial x^2} + \frac{\partial^2 V_i}{\partial y^2} = \frac{1}{\tilde{a}_i^2} \frac{\partial V_i}{\partial t}, \quad \tilde{a}_i^2 = \frac{k_i}{c_i \tilde{\rho}_i}, \quad x, y \in C_i,$$

with the initial conditions

$$V_i\big|_{t=0} = V_i^0(x, y),$$

and the boundary ones. As the geometry of interest has mirror symmetry along the lines $y = 0$ and $y = l_0$, we use the symmetry boundary conditions

$$\frac{\partial V_i}{\partial n} = 0$$

with n as the unit exterior normal to the boundary of the domains C_i.

Along the line $x = -\delta$ heat flux

$$\left.\frac{\partial V_0}{\partial x}\right|_{x=-\delta} = -Q_0(y, t)$$

is applied.

Assuming that at the $y = b + \varepsilon_1$ there is boiling occurring,

$$\left.\left(\frac{\partial V_1}{\partial y} + \beta_1^1 V_1^m\right)\right|_{y=b+\varepsilon_1} = 0,$$

where $\beta_1^1 = \dfrac{h_1}{k_1}$.

At the other sides there is heat exchange between the sample and its surroundings, that's why third type boundary conditions have to be specified here:

$$\frac{\partial V_i}{\partial n} + \beta_1^1 V_i = 0,$$

Along the lines connecting two neighbour domains the continuity of temperature and heat flux are ensured by

$$V_i\big|_{x=\ldots} = V_j\big|_{x=\ldots},$$

$$\left.\frac{\partial V_i}{\partial x}\right|_{x=\ldots} = \frac{k_j}{k_i} \left.\frac{\partial V_j}{\partial x}\right|_{x=\ldots},$$

$$V_i\big|_{y=\ldots} = V_j\big|_{y=\ldots},$$

$$\left.\frac{\partial V_i}{\partial y}\right|_{y=\ldots} = \frac{k_j}{k_i} \left.\frac{\partial V_j}{\partial y}\right|_{y=\ldots}.$$

3 Approximate Solution of Problem

As the outer layer is quite thin compared with the substrate, we may assume that the temperature variations across the layer thickness are so small as to be negligible. In this way the temperature can be taken constant here. Thus, owing to appropriate conjugation conditions, approximate expressions for calculating the temperatures in the upper layer are given by

$$V_2(x, y, t) = v_2(y, t) = V_0(0, y, t), \tag{1}$$

$$V_1(x, y, t) = v_1(x, t) = V(x, b, t), \tag{2}$$

$$V_3(x, y, t) = v_3(y, t) = V(l, y, t). \tag{3}$$

Now we are left with the heat conduction problem for the basic layer only:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{1}{\tilde{a}_0^2} \frac{\partial V}{\partial t}, \tag{4}$$

$$\frac{\partial^2 V_0}{\partial x^2} + \frac{\partial^2 V_0}{\partial y^2} = \frac{1}{\tilde{a}_0^2} \frac{\partial V_0}{\partial t}. \tag{5}$$

The boundary conditions imposed on all sides of the new domain are as follows:

$$\left.\frac{\partial V_0}{\partial x}\right|_{x=-\delta} = -Q_0(y, t), \tag{6}$$

$$\left.\frac{\partial V_0}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial V_0}{\partial y}\right|_{y=l_0}=0, \tag{7}$$

$$\left.\frac{\partial V}{\partial y}\right|_{y=0}=0. \tag{8}$$

To get boundary conditions specified at $x=0$, $x=l$, and $y=b$, let's use appropriate conjugation conditions and expressions (1) – (3). For example, at $x=0$ we have

$$\left.\left(\frac{\partial V_2}{\partial x}+\beta_1^1 V_2\right)\right|_{x=\varepsilon_0}=\frac{1}{k_1}\left.\left(k_0\frac{\partial V_0}{\partial x}+h_1 V_0\right)\right|_{x=0}=0$$

or

$$\left.\left(\frac{\partial V_0}{\partial x}+\beta_0^1 V_0\right)\right|_{x=0}=0,\ y\in\left(b,l_0\right). \tag{9}$$

But at $x=l,\ y=b$:

$$\left.\left(\frac{\partial V}{\partial x}+\beta_0^1 V\right)\right|_{x=l}=0, \tag{10}$$

$$\left.\left(\frac{\partial V}{\partial y}+\beta_0^1 V^m\right)\right|_{y=b}=0. \tag{11}$$

For the case of boiling the value of the index m should belong to the interval $\left[3;3\frac{1}{3}\right]$. It is usually taken to be equal to 3 or $3\frac{1}{3}$. The model is non-linear because of the boiling condition.

In addition we also have conjugation conditions which express the continuity of the temperature and heat flux at the interface $x=0$:

$$V_0\big|_{x=-0}=V\big|_{x=+0}, \tag{12}$$

$$\left.\frac{\partial V_0}{\partial x}\right|_{x=-0}=\left.\frac{\partial V}{\partial x}\right|_{x=+0}. \tag{13}$$

And finally, the initial conditions:

$$V(x,y,0)=V^0(x,y), \tag{14}$$

$$V_0(x,y,0)=V_0^0(x,y). \tag{15}$$

Using conservative averaging method (see [2] – [8], etc.), we are going to transform the given mathematical model into a more usable form.

3.1 Solution for the Fin

At first we are going to approximate the function $V(x,y,t)$ by its mean value over the interval $[0,b]$:

$$v(x,t)=\rho\int_0^b V(x,y,t)dy. \tag{16}$$

Namely,

$$V(x,y,t)\approx v(x,t). \tag{17}$$

When integrating the partial differential equation (4) from 0 to b, we get

$$\frac{\partial^2 v}{\partial x^2}+\rho\left.\frac{\partial V}{\partial y}\right|_{y=0}^{y=b}=\frac{1}{\tilde{a}_0^2}\frac{\partial v}{\partial t}.$$

The difference of the derivatives is found via the boundary conditions (8) and (11). Thereby the differential equation becomes

$$\frac{\partial^2 v}{\partial x^2}-\lambda^2 v^m(x,t)=\frac{1}{\tilde{a}_0^2}\frac{\partial v}{\partial t}, \tag{18}$$

where

$$\lambda^2=\rho\beta_0^1.$$

We apply the operator (16) on (10) and (14) to get boundary and initial conditions for (18):

$$\left.\left(\frac{\partial v}{\partial x}+\beta_0^1 v\right)\right|_{x=l}=0, \tag{19}$$

$$v(x,0)=\rho\int_0^b V^0(x,y)dy\equiv u^0(x). \tag{20}$$

3.1.1 Difference Scheme for the Fin

We are going to solve the stated 1D problem in numerical way. For approximating the solution we'll use finite-difference method.

Let's begin by defining grid points in space and time domains. The following notation is used:

$$x_i=ih_x,\ i=0..N,\ h_x=\frac{l}{N},$$

$$y_j=\begin{cases} jh_{y,1} & j=0..M_0\\ b+(j-M_0)h_{y,2} & j=M_0+1..M\end{cases},$$

$$h_{y,1}=\frac{b}{M_0},\ h_{y,2}=\frac{l_0-b}{M-M_0}.$$

Similarly we partition $[0,T]$ as

$$t_n=n\tau,\ n=0,1,..,\frac{T}{\tau}.$$

We denote the approximation of $v(x,t)$ at (x_i,t_n) by v_i^n, but $v_{0,j}^n$ is a difference approximation of $v_0(y_j,t_n)$. As the equation (18) is non-linear, we are going to solve it using iterations. When the process at the *n*- th time level has become stable, we don't use the iteration number.

For the differential equations we are going to apply the so called three-point scheme with nonnegative weights. Additionally we make a linearization for the non-linear term through iterative process. Hence for (18) we have the following finite difference scheme:

$$\frac{v_i^{n+1,k}-v_i^n}{\tilde{a}_0^2\tau}=\sigma\frac{v_{i+1}^{n+1,k}-2v_i^{n+1,k}+v_{i-1}^{n+1,k}}{h_x^2}$$
$$+(1-\sigma)\frac{v_{i+1}^n-2v_i^n+v_{i-1}^n}{h_x^2}$$
$$-\sigma\lambda^2\left(v_i^{n+1,k-1}\right)^{m-1}v_i^{n+1,k}-(1-\sigma)\lambda^2\left(v_i^n\right)^m, \qquad (21)$$

where $i=1,..,N-1$ and the weight $0\le\sigma\le1$.

We can rewrite the scheme (21) as

$$Av_{i-1}^{n+1,k}-C_i^{n+1,k-1}v_i^{n+1,k}+Bv_{i+1}^{n+1,k}=-F_i^n, \quad (22)$$

where

$$A=\frac{\sigma}{h_x^2}=B,$$
$$C_i^{n+1,k-1}=\frac{2\sigma}{h_x^2}+\lambda^2\sigma\left(v_i^{n+1,k-1}\right)^{m-1}+\frac{1}{\tilde{a}_0^2\tau},$$
$$F_i^n=(1-\sigma)\frac{v_{i+1}^n-2v_i^n+v_{i-1}^n}{h_x^2}$$
$$-\lambda^2(1-\sigma)\left(v_i^n\right)^m+\frac{v_i^n}{\tilde{a}_0^2\tau}.$$

To get second order approximation to the first derivative in the boundary condition (19), let's use the differential equation (18) and its approximation (21) (see [2], [3], [7], [8], and [14]):

$$\sigma\frac{v_N^{n+1,k}-v_{N-1}^{n+1,k}}{h_x}+(1-\sigma)\frac{v_N^n-v_{N-1}^n}{h_x}$$
$$+\frac{h_x}{2}\left[\begin{array}{l}\dfrac{v_N^{n+1,k}-v_N^n}{\tilde{a}_0^2\tau}+\sigma\lambda^2\left(v_N^{n+1,k-1}\right)^{m-1}v_N^{n+1,k}\\ +(1-\sigma)\lambda^2\left(v_N^n\right)^m\end{array}\right]$$
$$+\sigma\beta_0^1v_N^{n+1,k}+(1-\sigma)\beta_0^1v_N^n=0.$$

Let's rewrite this in the form

$$v_N^{n+1,k}=\varpi_1^{n+1,k-1}v_{N-1}^{n+1,k}+\varpi_2^{n+1,k-1} \qquad (23)$$

with coefficients

$$\varpi_1^{n+1,k-1}=\frac{\sigma}{h_x}\left(\begin{array}{l}\dfrac{\sigma}{h_x}+\sigma\beta_0^1\\ +\dfrac{h_x}{2}\left(\dfrac{1}{\tilde{a}_0^2\tau}+\sigma\lambda^2\left(v_N^{n+1,k-1}\right)^{m-1}\right)\end{array}\right)^{-1},$$

$$\varpi_2^{n+1,k-1}=\left(\begin{array}{l}\dfrac{\sigma}{h_x}+\sigma\beta_0^1\\ +\dfrac{h_x}{2}\left(\dfrac{1}{\tilde{a}_0^2\tau}+\sigma\lambda^2\left(v_N^{n+1,k-1}\right)^{m-1}\right)\end{array}\right)^{-1}\times$$
$$\left(\frac{h_x}{2}\frac{v_N^n}{\tilde{a}_0^2\tau}-(1-\sigma)\left(\begin{array}{l}\dfrac{v_N^n-v_{N-1}^n}{h_x}\\ +\left(\dfrac{h_x}{2}\lambda^2\left(v_N^n\right)^m+\beta_0^1v_N^n\right)\end{array}\right)\right).$$

Using new notations

$$\varsigma_N^{n+1,k-1}=\varpi_1^{n+1,k-1},\ \eta_N^{n+1,k-1}=\varpi_2^{n+1,k-1}, \quad (24)$$

(23) becomes

$$v_N^{n+1,k}=\varsigma_N^{n+1,k-1}v_{N-1}^{n+1,k}+\eta_N^{n+1,k-1}. \qquad (25)$$

Let's solve (22) for $v_i^{n+1,k}$ and take $i=N-1$:

$$v_i^{n+1,k}=\frac{A}{C_i^{n+1,k-1}}v_{i-1}^{n+1,k}+\frac{B}{C_i^{n+1,k-1}}v_{i+1}^{n+1,k}+\frac{F_i^n}{C_i^{n+1,k-1}}.$$

Substituting this in (25) and rearranging terms yields

$$v_N^{n+1,k}=\varsigma_N^{n+1,k-1}v_{N-1}^{n+1,k}+\eta_N^{n+1,k-1}$$

with

$$\varsigma_{N-1}^{n+1,k-1}=\frac{A}{C_{N-1}^{n+1,k-1}-B\varsigma_N^{n+1,k-1}},$$
$$\eta_{N-1}^{n+1,k-1}=\frac{B\eta_N^{n+1,k-1}+F_{N-1}^n}{C_{N-1}^{n+1,k-1}-B\varsigma_N^{n+1,k-1}}.$$

Doing the same procedure for the indices $i=N-2,..,1$, it follows that

$$v_{i+1}^{n+1,k}=\varsigma_{i+1}^{n+1,k-1}v_i^{n+1,k}+\eta_{i+1}^{n+1,k-1},\ i=0,..,N-1, \quad (26)$$

with (24) and

$$\varsigma_i^{n+1,k-1}=\frac{A}{C_i^{n+1,k-1}-B\varsigma_{i+1}^{n+1,k-1}},$$
$$\eta_i^{n+1,k-1}=\frac{B\eta_{i+1}^{n+1,k-1}+F_i^n}{C_i^{n+1,k-1}-B\varsigma_{i+1}^{n+1,k-1}},\ i=N-1,..,1.$$

Having used the recurrence relations to find the coefficients, the values of $v_i^{n+1,k}$ are easily obtained from (26). The only problem here is that we don't know $v_0^{n+1,k}$ yet. We'll find it in due course.

3.2 Solution for the Wall

We use the same method of conservative averaging to describe the 2D temperature field in the wall. Let's approximate $V_0(x,y,t)$ in the x-direction by

$$V_0(x,y,t)=g_0(y,t)$$

$$+\left(e^{-dx}-1\right)g_1(y,t)+\left(1-e^{dx}\right)g_2(y,t), \qquad (27)$$

with $d=\delta^{-1}$. Once again we use appropriate boundary conditions to solve for the unknown functions $g_i(y,t)$, $i=0,1,2$.

Before we proceed, let's introduce integral averaged value function defined by

$$v_0(y,t)=d\int_{-\delta}^{0}V_0(x,y,t)dx.$$

After integrating (27) over the segment $[-\delta,0]$, it gives

$$v_0(y,t)=$$
$$=g_0(y,t)+(e-2)g_1(y,t)+e^{-1}g_2(y,t). \qquad (28)$$

As the function $V_0(x,y,t)$ satisfies the boundary condition (6), we apply the condition to (27) to find $g_2(y,t)$:

$$g_2(y,t)=e\delta Q_0(y,t)-e^2g_1(y,t). \qquad (29)$$

Then by combining (28) and (29) together, we have

$$g_1(y,t)=$$
$$=\frac{1}{2}\left(g_0(y,t)-v_0(y,t)+\delta Q_0(y,t)\right). \qquad (30)$$

Putting the last two expressions, (29) and (30), in formula (27), it becomes

$$V_0(x,y,t)=$$
$$=g_0(y,t)\left(1+\frac{1}{2}\left(e^{-dx}-1\right)-e^2\frac{1}{2}\left(1-e^{dx}\right)\right)$$
$$+v_0(y,t)\left(-\frac{1}{2}\left(e^{-dx}-1\right)+e^2\frac{1}{2}\left(1-e^{dx}\right)\right)$$
$$+Q_0(y,t)\begin{pmatrix}\frac{1}{2}\delta\left(e^{-dx}-1\right)+\\ +\left(\delta e-e^2\frac{1}{2}\delta\right)\left(1-e^{dx}\right)\end{pmatrix}. \qquad (31)$$

Before continuing, we divide the wall into two parts, the right part of which occupies the domain $x\in(-\delta,0)$, $y\in(b,l_0)$, but the left one is for $x\in(-\delta,0)$, $y\in(0,b)$.

3.2.1 The Right Part of the Wall

Let us apply the boundary condition (9) to (31) to compute $g_0(y,t)$:

$$g_0(y,t)\left(-d+2\beta_0^1+de^2\right)+v_0(y,t)\left(d-de^2\right)$$
$$+Q_0(y,t)\left(-1-2e+e^2\right)=0.$$

And thus

$$g_0(y,t)=v_0(y,t)a_0+Q_0(y,t)b_0, \qquad (32)$$

where

$$a_0=\frac{d-de^2}{d-2\beta_0^1-de^2},\ b_0=-\frac{1+2e-e^2}{d-2\beta_0^1-de^2}.$$

Plugging (32) into the representation (31), we obtain

$$V_0(x,y,t)=$$
$$=v_0(y,t)\begin{pmatrix}a_0+\frac{1}{2}(a_0-1)\left(e^{-dx}-1\right)\\ +e^2\frac{1}{2}(1-a_0)\left(1-e^{dx}\right)\end{pmatrix}$$
$$+Q_0(y,t)\begin{pmatrix}b_0+\frac{1}{2}(b_0+\delta)\left(e^{-dx}-1\right)\\ +\left(\delta e-e^2\frac{1}{2}(\delta+b_0)\right)\left(1-e^{dx}\right)\end{pmatrix}. \qquad (33)$$

From (33) it follows that the function $V_0(x,y,t)$ now depends on one unknown - $v_0(y,t)$. Let's integrate the main equation (5) with respect to x:

$$d\left.\frac{\partial V_0}{\partial x}\right|_{x=-\delta}^{x=0}+\frac{\partial^2 v_0}{\partial y^2}=\frac{1}{\tilde{a}_0^2}\frac{\partial v_0}{\partial t}. \qquad (34)$$

Now, owing to the boundary conditions (6) and (9), and the formula (33), equation for the right part of the wall results in

$$\frac{\partial^2 v_0}{\partial y^2}-\kappa^2 v_0(y,t)=\frac{1}{\tilde{a}_0^2}\frac{\partial v_0}{\partial t}+\gamma Q_0(y,t), \qquad (35)$$

where

$$\kappa^2=d\beta_0^1a_0,\ \gamma=d\left(\beta_0^1b_0-1\right).$$

Integrating (7_2), the boundary condition for (35) becomes

$$\left.\frac{\partial v_0}{\partial y}\right|_{y=l_0}=0. \qquad (36)$$

For 1D initial condition we take (15) and integrate it with respect to x:

$$v_0(y,0)=d\int_{-\delta}^{0}V_0^0(x,y)dx=u_0^0(y). \qquad (37)$$

3.2.2 The Left Part of the Wall

We are still left with finding the unknown functions $g_0(y,t)$ and $v_0(y,t)$ for the temperature in the left part of the base. To get those we are going to use the conjugation conditions (12) and (13). From the first condition and the expression (17) for $V(x,y,t)$ we get that

$$g_0(y,t)=v(0,t).$$

But $v_0(y,t)$ is found solving 1D equation (34) that is modified for the left part of the base. As the functions $V_0(x,y,t)$, $V(x,y,t)$ satisfy (13) at $x=0$, from (13) and (21) it follows that

$$d\left.\frac{\partial V_0}{\partial x}\right|_{x=-0} = d\left.\frac{\partial V}{\partial x}\right|_{x=+0} = d\left.\frac{\partial v}{\partial x}\right|_{x=0}. \qquad (38)$$

So, now we can remove the term $d\left.\frac{\partial V_0}{\partial x}\right|_{x=-\delta}^{x=0}$ in (34) by using (38) and the boundary condition (6). And equation (34) becomes

$$\frac{\partial^2 v_0}{\partial y^2} + d\left.\frac{\partial v}{\partial x}\right|_{x=0} + dQ_0(y,t) = \frac{1}{\tilde{a}_0^2}\frac{\partial v_0}{\partial t}. \qquad (39)$$

In addition we have a boundary condition that we get from (7_1):

$$\left.\frac{\partial v_0}{\partial y}\right|_{y=0} = 0 \qquad (40)$$

and initial condition (37).

For simplicity we assume the function $Q_0(y,t)$ is constant, that is, $Q_0(y,t) = Q_0$.

3.2.3 Difference Scheme for the Right Part of the Wall

The differential equation (35) for the right part of the wall is approximated by the scheme

$$\frac{v_{0,j}^{n+1,k} - v_{0,j}^{n}}{\tilde{a}_0^2\tau} = \sigma_2 \frac{v_{0,j+1}^{n+1,k} - 2v_{0,j}^{n+1,k} + v_{0,j-1}^{n+1,k}}{h_{y,2}^2}$$
$$+\left(1-\sigma_2\right)\frac{v_{0,j+1}^{n} - 2v_{0,j}^{n} + v_{0,j-1}^{n}}{h_{y,2}^2}$$
$$-\sigma_2\kappa^2 v_{0,j}^{n+1,k} - (1-\sigma_2)\kappa^2 v_{0,j}^{n} - \gamma Q_0,$$

where $i = M_0+1,..,M-1$ and $0 \le \sigma_2 \le 1$. Which we rewrite as

$$A_2 v_{0,j-1}^{n+1,k} - C_2 v_{0,j}^{n+1,k} + B_2 v_{0,j+1}^{n+1,k} = -F_{2,j}^{n},$$

using following notations:

$$A_2 = \frac{\sigma_2}{h_{y,2}^2} = B_2,\ C_2 = \frac{2\sigma_2}{h_{y,2}^2} + \kappa^2\sigma_2 + \frac{1}{\tilde{a}_0^2\tau},$$
$$F_{2,j}^{n} = \left(1-\sigma_2\right)\frac{v_{0,j+1}^{n} - 2v_{0,j}^{n} + v_{0,j-1}^{n}}{h_{y,2}^2}$$
$$-\kappa^2\left(1-\sigma_2\right)v_{0,j}^{n} + \frac{v_{0,j}^{n}}{\tilde{a}_0^2\tau} - \gamma Q_0.$$

When approximating the boundary condition (36), we have

$$\sigma_2\frac{v_{0,M}^{n+1,k} - v_{0,M-1}^{n+1,k}}{h_{y,2}} + \left(1-\sigma_2\right)\frac{v_{0,M}^{n} - v_{0,M-1}^{n}}{h_{y,2}} +$$
$$+\frac{h_{y,2}}{2}\begin{pmatrix}\dfrac{v_{0,M}^{n+1,k} - v_{0,M}^{n}}{\tilde{a}_0^2\tau} + \sigma_2\kappa^2 v_{0,M}^{n+1,k} \\ +(1-\sigma_2)\kappa^2 v_{0,M}^{n} + \gamma Q_0\end{pmatrix} = 0$$

or

$$v_{0,M}^{n+1,k} = \varpi_{2,1}v_{0,M-1}^{n+1,k} + \varpi_{2,2}^{n}$$

with

$$\varpi_{2,1} = \frac{\sigma_2}{h_{y,2}}\left(\frac{\sigma_2}{h_{y,2}} + \frac{h_{y,2}}{2}\left(\sigma_2\kappa^2 + \frac{1}{\tilde{a}_0^2\tau}\right)\right)^{-1},$$
$$\varpi_{2,2}^{n} = -\left(\frac{\sigma_2}{h_{y,2}} + \frac{h_{y,2}}{2}\left(\sigma_2\kappa^2 + \frac{1}{\tilde{a}_0^2\tau}\right)\right)^{-1} \times$$
$$\left(\begin{array}{l}(1-\sigma_2)\dfrac{v_{0,M}^{n} - v_{0,M-1}^{n}}{h_{y,2}} \\ +\dfrac{h_{y,2}}{2}\left((1-\sigma_2)\kappa^2 v_{0,M}^{n} - \dfrac{v_{0,M}^{n}}{\tilde{a}_0^2\tau} + \gamma Q_0\right)\end{array}\right).$$

On carrying through the similar analysis as for the fin, the difference scheme for the right part of the wall can be expressed as

$$v_{0,j+1}^{n+1,k} = \varsigma_{2,j+1}v_{0,j}^{n+1,k} + \eta_{2,j+1}^{n},\ j = M_0,..,M-1, \qquad (41)$$

with

$$\varsigma_{2,M} = \varpi_{2,1},\ \eta_{2,M}^{n} = \varpi_{2,2}^{n},$$
$$\varsigma_{2,j} = \frac{A_2}{C_2 - B_2\varsigma_{2,j+1}},\ \eta_{2,j}^{n} = \frac{B_2\eta_{2,j+1}^{n} + F_{2,j}^{n}}{C_2 - B_2\varsigma_{2,j+1}}$$

for $j = M-1,..,M_0+1$.

3.2.4 Difference Scheme for the Left Part of the Wall

The scheme for (39) is extended in this way to give

$$\frac{v_{0,j}^{n+1,k} - v_{0,j}^{n}}{\tilde{a}_0^2\tau} = \sigma_1\frac{v_{0,j+1}^{n+1,k} - 2v_{0,j}^{n+1,k} + v_{0,j-1}^{n+1,k}}{h_{y,1}^2}$$
$$+\left(1-\sigma_1\right)\frac{v_{0,j+1}^{n} - 2v_{0,j}^{n} + v_{0,j-1}^{n}}{h_{y,1}^2} + dQ_0$$
$$+d\left(\sigma\frac{v_1^{n+1,k} - v_0^{n+1,k}}{h_x} + (1-\sigma)\frac{v_1^{n} - v_0^{n}}{h_x}\right)$$
$$-\frac{h_x}{2}d\begin{pmatrix}\dfrac{v_0^{n+1,k} - v_0^{n}}{\tilde{a}_0^2\tau} + \sigma\lambda^2\left(v_0^{n+1,k-1}\right)^{m-1}v_0^{n+1,k} \\ +(1-\sigma)\lambda^2\left(v_0^{n}\right)^{m}\end{pmatrix}. \qquad (42)$$

It depends on the values of $v_0^{n+1,k}$ and $v_1^{n+1,k}$. The latter can be computed from (26):

$$v_1^{n+1,k} = \varsigma_1^{n+1,k-1}v_0^{n+1,k} + \eta_1^{n+1,k-1}. \qquad (43)$$

Substituting (43) into (42), the difference scheme results in

$$A_1 v_{0,j-1}^{n+1,k} - C_1 v_{0,j}^{n+1,k} + B_1 v_{0,j+1}^{n+1,k} + D_1^{n+1,k-1} v_0^{n+1,k} = -F_{1,j}^{n,k-1}, \qquad (44)$$

where

$$A_1 = \frac{\sigma_1}{h_{y,1}^2} = B_1,\; C_1 = \frac{2\sigma_1}{h_{y,1}^2} + \frac{1}{\tilde{a}_0^2 \tau},$$

$$D_1^{n+1,k-1} = -d \left(\begin{array}{l} \dfrac{\sigma}{h_x} + \dfrac{h_x}{2}\left(\dfrac{1}{\tilde{a}_0^2\tau} + \sigma\lambda^2 \left(v_0^{n+1,k-1}\right)^{m-1} \right) \\ -\dfrac{\sigma}{h_x} \varsigma_1^{n+1,k-1} \end{array} \right),$$

$$F_{1,j}^{n,k-1} = \frac{v_{0,j}^n}{\tilde{a}_0^2\tau} + (1-\sigma_1)\frac{v_{0,j+1}^n - 2v_{0,j}^n + v_{0,j-1}^n}{h_{y,1}^2} + dQ_0$$

$$+ d \left(\begin{array}{l} \sigma \dfrac{\eta_1^{n+1,k-1}}{h_x} + (1-\sigma)\dfrac{v_1^n - v_0^n}{h_x} \\ -\dfrac{h_x}{2}\left(-\dfrac{v_0^n}{\tilde{a}_0^2\tau} + (1-\sigma)\lambda^2 \left(v_0^n\right)^m \right) \end{array} \right).$$

The boundary condition (40) has the following finite difference representation:

$$\sigma_1 \frac{v_{0,1}^{n+1,k} - v_{0,0}^{n+1,k}}{h_{y,1}} + (1-\sigma_1)\frac{v_{0,1}^n - v_{0,0}^n}{h_{y,1}}$$

$$+ d\,\frac{h_{y,1}}{2} \left(\begin{array}{l} \sigma \dfrac{v_1^{n+1,k} - v_0^{n+1,k}}{h_x} + (1-\sigma)\dfrac{v_1^n - v_0^n}{h_x} \\ -\dfrac{h_x}{2} \left(\begin{array}{l} \dfrac{v_0^{n+1,k} - v_0^n}{\tilde{a}_0^2\tau} \\ +\sigma\lambda^2 \left(v_0^{n+1,k-1}\right)^{m-1} v_0^{n+1,k} \\ +(1-\sigma)\lambda^2 \left(v_0^n\right)^m \end{array} \right) \end{array} \right)$$

$$-\frac{h_{y,1}}{2}\frac{v_{0,0}^{n+1,k} - v_{0,0}^n}{\tilde{a}_0^2\tau} + \frac{h_{y,1}}{2} dQ_0 = 0.$$

Which allows to be rewritten as

$$v_{0,0}^{n+1,k} = \varpi_{1,1} v_{0,1}^{n+1,k} - \varpi_{1,3}^{n+1,k-1} v_0^{n+1,k} + \mu_1^{n,k-1}.$$

Using $\varpi_{1,1} = \dfrac{\sigma_1}{h_{y,1}}\left(\dfrac{\sigma_1}{h_{y,1}} + \dfrac{h_{y,1}}{2\tilde{a}_0^2\tau} \right)^{-1}$,

$$\varpi_{1,3}^{n+1,k-1} = \left(\frac{\sigma_1}{h_{y,1}} + \frac{h_{y,1}}{2\tilde{a}_0^2\tau} \right)^{-1} \times$$

$$d\,\frac{h_{y,1}}{2} \left(\begin{array}{l} \dfrac{\sigma}{h_x}\left(1 - \varsigma_1^{n+1,k-1}\right) \\ +\dfrac{h_x}{2}\left(\dfrac{1}{\tilde{a}_0^2\tau} + \sigma\lambda^2 \left(v_0^{n+1,k-1}\right)^{m-1} \right) \end{array} \right),$$

$$\mu_1^{n,k-1} = \left(\frac{\sigma_1}{h_{y,1}} + \frac{h_{y,1}}{2\tilde{a}_0^2\tau} \right)^{-1} \times$$

$$\left(\begin{array}{l} (1-\sigma_1)\dfrac{v_{0,1}^n - v_{0,0}^n}{h_{y,1}} + \dfrac{h_{y,1}}{2} dQ_0 + \dfrac{h_{y,1}}{2}\dfrac{v_{0,0}^n}{\tilde{a}_0^2\tau} \\ + d\,\dfrac{h_{y,1}}{2} \left(\begin{array}{l} \dfrac{\sigma}{h_x}\eta_1^{n+1,k-1} + (1-\sigma)\dfrac{v_1^n - v_0^n}{h_x} \\ +\dfrac{h_x}{2}\left(\dfrac{1}{\tilde{a}_0^2\tau} v_0^n - (1-\sigma)\lambda^2 \left(v_0^n\right)^m \right) \end{array} \right) \end{array} \right).$$

After some substitutions and algebraic manipulations the difference scheme (44) for equation (39) becomes

$$v_{0,j}^{n+1,k} = \xi_{j+1} v_{0,j+1}^{n+1,k} - \psi_{j+1}^{n+1,k-1} v_0^{n+1,k} + \chi_{j+1}^{n,k-1} \qquad (45)$$

for $j = M_0 - 1,..,0$, with coefficients

$$\xi_1 = \varpi_{1,1},\; \psi_1^{n+1,k-1} = \varpi_{1,3}^{n+1,k-1},\; \chi_1^{n,k-1} = \mu_1^{n,k-1},$$

and

$$\xi_{j+1} = \frac{B_1}{C_1 - A_1\xi_j},$$

$$\psi_{j+1}^{n+1,k-1} = \frac{A_1\psi_j^{n+1,k-1} - D_1^{n+1,k-1}}{C_1 - A_1\xi_j},$$

$$\chi_{j+1}^{n,k-1} = \frac{A_1\chi_j^{n,k-1} + F_{1,j}^{n,k-1}}{C_1 - A_1\xi_j}$$

for $j = 1..M_0 - 1$.

3.3 Conjugation of Solutions

We see that the schemes (26), (41) and (45) can only be used once the values of $v_0^{n+1,k}$ and $v_{0,M_0}^{n+1,k}$ are found. They could be obtained using these requirements:

$$V(0,b,t) = V_0^r(0,b,t), \qquad (46)$$

$$\left.\frac{\partial v_0^l}{\partial y}\right|_{y=b} = \left.\frac{\partial v_0^r}{\partial y}\right|_{y=b}, \qquad (47)$$

that state that the temperatures $V_0(x,y,t)$, $V(x,y,t)$ must coincide at the contact point $x=0, y=b$ between the fin and the right part of the wall.

So, from (17) and (33) it follows that

$$V(0,b,t) = v(0,t),$$
$$V_0^r(0,b,t) = a_0 v_0^r(b,t) + b_0 Q_0,$$

and (46) becomes

$$v_0^{n+1,k} = a_0 v_{0,M_0}^{n+1,k} + b_0 Q_0. \tag{48}$$

But for the condition (47) we have

$$\sigma_1 \frac{v_{0,M_0}^{n+1,k} - v_{0,M_0-1}^{n+1,k}}{h_{y,1}} + (1-\sigma_1)\frac{v_{0,M_0}^{n} - v_{0,M_0-1}^{n}}{h_{y,1}}$$
$$+\frac{h_{y,1}}{2}\left(\frac{v_{0,M_0}^{n+1,k} - v_{0,M_0}^{n}}{\tilde{a}_0^2\tau} - dQ_0\right)$$
$$-d\,\frac{h_{y,1}}{2}\left(\begin{array}{l} \sigma\dfrac{v_1^{n+1,k} - v_0^{n+1,k}}{h_x} + (1-\sigma)\dfrac{v_1^n - v_0^n}{h_x} \\ -\dfrac{h_x}{2}\left(\begin{array}{l} \dfrac{v_0^{n+1,k} - v_0^n}{\tilde{a}_0^2\tau} \\ +\sigma\lambda^2\left(v_0^{n+1,k-1}\right)^{m-1} v_0^{n+1,k} \\ +(1-\sigma)\lambda^2\left(v_0^n\right)^m \end{array}\right) \end{array}\right) =$$
$$= \sigma_2 \frac{v_{0,M_0+1}^{n+1,k} - v_{0,M_0}^{n+1,k}}{h_{y,2}} + (1-\sigma_2)\frac{v_{0,M_0+1}^{n} - v_{0,M_0}^{n}}{h_{y,2}}$$
$$-\frac{h_{y,2}}{2}\left(\begin{array}{l} \dfrac{v_{0,M_0}^{n+1,k} - v_{0,M_0}^n}{\tilde{a}_0^2\tau} + \sigma_2\kappa^2 v_{0,M_0}^{n+1,k} \\ +(1-\sigma_2)\kappa^2 v_{0,M_0}^n + \gamma Q_0 \end{array}\right). \tag{49}$$

According to formulas (26), (41) and (45) when $i=0$, $j=M_0$ and $j=M_0-1$, respectively,

$$v_1^{n+1,k} = \varsigma_1^{n+1,k-1} v_0^{n+1,k} + \eta_1^{n+1,k-1},$$
$$v_{0,M_0+1}^{n+1,k} = \varsigma_{2,M_0+1} v_{0,M_0}^{n+1,k} + \eta_{2,M_0+1}^{n},$$
$$v_{0,M_0-1}^{n+1,k} = \xi_{M_0} v_{0,M_0}^{n+1,k} - \psi_{M_0}^{n+1,k-1} v_0^{n+1,k} + \chi_{M_0}^{n,k-1}.$$

Let us insert these into (49) and thereafter compute

$$H_1 v_{0,M_0}^{n+1,k} + J_1^{n+1,k-1} v_0^{n+1,k} = -G_1^{n,k-1}, \tag{50}$$

where

$$H_1 = \sigma_1 \frac{(1-\xi_{M_0})}{h_{y,1}} + \frac{h_{y,1}}{2\tilde{a}_0^2\tau}$$
$$-\sigma_2 \frac{(\varsigma_{2,M_0+1} - 1)}{h_{y,2}} + \frac{h_{y,2}}{2}\left(\frac{1}{\tilde{a}_0^2\tau} + \sigma_2\kappa^2\right),$$

$$J_1^{n+1,k-1} = d\,\frac{h_{y,1}}{2}\left(\begin{array}{l} \dfrac{h_x}{2}\left(\dfrac{1}{\tilde{a}_0^2\tau} + \sigma\lambda^2\left(v_0^{n+1,k-1}\right)^{m-1}\right) \\ +\dfrac{\sigma}{h_x}\left(1-\varsigma_1^{n+1,k-1}\right) \end{array}\right) +$$
$$+\frac{\sigma_1}{h_{y,1}}\psi_{M_0}^{n+1,k-1},\ G_1^{n,k-1} = -\frac{\sigma_1}{h_{y,1}}\chi_{M_0}^{n,k-1} +$$
$$+(1-\sigma_1)\frac{v_{0,M_0}^n - v_{0,M_0-1}^n}{h_{y,1}} - \frac{h_{y,1}}{2}\left(\frac{v_{0,M_0}^n}{\tilde{a}_0^2\tau} + dQ_0\right)$$
$$-d\,\frac{h_{y,1}}{2}\left(\begin{array}{l} \dfrac{\sigma}{h_x}\eta_1^{n+1,k-1} + (1-\sigma)\dfrac{v_1^n - v_0^n}{h_x} \\ +\dfrac{h_x}{2}\left(\dfrac{v_0^n}{\tilde{a}_0^2\tau} - (1-\sigma)\lambda^2\left(v_0^n\right)^m\right) \end{array}\right)$$
$$-\frac{\sigma_2}{h_{y,2}}\eta_{2,M_0+1}^n - (1-\sigma_2)\frac{v_{0,M_0+1}^n - v_{0,M_0}^n}{h_{y,2}}$$
$$+\frac{h_{y,2}}{2}\left(-\frac{v_{0,M_0}^n}{\tilde{a}_0^2\tau} + (1-\sigma_2)\kappa^2 v_{0,M_0}^n + \gamma Q_0\right).$$

It is easy to eliminate $v_0^{n+1,k}$ from (50) and (48):

$$H_1 v_{0,M_0}^{n+1,k} + J_1^{n+1,k-1} a_0 v_{0,M_0}^{n+1,k} = -J_1^{n+1,k-1} b_0 Q_0 - G_1^{n,k-1}.$$

Solving this equation for $v_{0,M_0}^{n+1,k}$, we get

$$v_{0,M_0}^{n+1,k} = -\frac{G_1^{n,k-1} + b_0 Q_0 J_1^{n+1,k-1}}{H_1 + a_0 J_1^{n+1,k-1}}. \tag{51}$$

Substitution into the expression (48) for $v_0^{n+1,k}$ then gives

$$v_0^{n+1,k} = -a_0 \frac{G_1^{n,k-1} + b_0 Q_0 J_1^{n+1,k-1}}{H_1 + a_0 J_1^{n+1,k-1}} + b_0 Q_0. \tag{52}$$

Then given the resulting values (51) and (52) for $v_0^{n+1,k}$ and $v_{0,M_0}^{n+1,k}$, equations (26), (41), (45) together with approximations of the initial conditions (20), (37) that give the values for all nodes at the first time level,

$$v_i^0 = u_i^0,\ i = 0,..,N,$$
$$v_{0,j}^0 = u_{0,j}^0,\ j = 0,..,M,$$

constitute the approximate solution of the given system in 1D.

4 Numerical Results

To get some kind of notion if this model could describe the actual situation in computer cooling systems for L-shaped micro elements, we used the following geometrical parameters:

$$\delta = 5\mu m,$$
$$l = 1\mu m,$$
$$b = 5\cdot 10^{-2}\,\mu m,$$
$$l_0 = 1\cdot 10^{-1}\,\mu m.$$

But for the termophysical properties we chose:

$$h_1 = 4.48 \cdot 10^{-7} W \mu m^{-2} K^{-1}$$
$$k_0 = 1.412 \cdot 10^{-4} W \mu m^{-1} K^{-1} \text{ (for silicon)}$$
$$Q_0 = 10 K \mu m^{-1}$$
$$c_0 = 700 Jkg^{-1} K^{-1} \text{ (for silicon)}$$
$$\tilde{\rho}_0 = 2.33 \cdot 10^{-15} kg \mu m^{-3} \text{ (for silicon)}$$
$$v(x,0) = 25^{\circ} C,\ v_0(y,0) = 35^{\circ} C,\ T = 8s.$$

The temperature distributions are calculated for both transient and stationary problem with linear 3rd type boundary conditions.

x\y	0	0.01	0.02	0.03	0.04	0.05
1	9.80	9.80	9.80	9.79	9.79	9.78
0.9	9.98	9.98	9.98	9.97	9.97	9.96
0.8	10.34	10.34	10.34	10.33	10.33	10.32
0.7	10.88	10.88	10.88	10.87	10.87	10.86
0.6	11.60	11.60	11.60	11.59	11.59	11.58
0.5	12.50	12.50	12.50	12.49	12.49	12.48
0.4	13.58	13.58	13.58	13.57	13.57	13.56
0.3	14.84	14.84	14.84	14.84	14.83	14.82
0.2	16.29	16.29	16.29	16.28	16.28	16.27
0.1	17.92	17.92	17.92	17.91	17.91	17.90
0	19.73	19.73	19.73	19.73	19.72	19.71

Table 1. Temperature distribution in the fin, in C°; transient case

x\y	0	0.01	0.02	0.03	0.04	0.05
0	19.73	19.73	19.73	19.73	19.72	19.71
-0.5	20.37	20.37	20.37	20.37	20.37	20.36
-1	21.39	21.39	21.39	21.38	21.38	21.38
-1.5	22.78	22.78	22.78	22.78	22.78	22.78
-2	24.57	24.57	24.57	24.57	24.57	24.57
-2.5	26.78	26.78	26.78	26.78	26.78	26.78
-3	29.42	29.42	29.42	29.42	29.43	29.43
-3.5	32.53	32.53	32.53	32.53	32.53	32.54
-4	36.12	36.12	36.13	36.13	36.13	36.14
-4.5	40.25	40.25	40.25	40.26	40.26	40.27
-5	44.95	44.95	44.95	44.96	44.96	44.96

Table 2. Temperature distribution in the left part of the base, in C°; transient case

x\y	0.05	0.06	0.07	0.08	0.09	0.1
0	19.71	19.71	19.71	19.71	19.71	19.71
-0.5	20.36	20.36	20.36	20.36	20.36	20.36
-1	21.38	21.38	21.38	21.38	21.38	21.38
-1.5	22.78	22.78	22.78	22.78	22.78	22.78
-2	24.57	24.58	24.58	24.58	24.58	24.58
-2.5	26.78	26.79	26.79	26.79	26.79	26.79
-3	29.43	29.43	29.43	29.43	29.43	29.43
-3.5	32.54	32.54	32.54	32.54	32.54	32.54
-4	36.14	36.14	36.14	36.14	36.14	36.14
-4.5	40.27	40.27	40.27	40.27	40.27	40.27
-5	44.96	44.97	44.97	44.97	44.97	44.97

Table 3. Temperature distribution in the right part of the base, in C°; transient case

x\y	0	0.01	0.02	0.03	0.04	0.05
1	9.81	9.80	9.80	9.80	9.79	9.78
0.9	9.99	9.98	9.98	9.98	9.97	9.96
0.8	10.34	10.34	10.34	10.34	10.33	10.32
0.7	10.88	10.88	10.88	10.88	10.87	10.86
0.6	11.60	11.60	11.60	11.60	11.59	11.58
0.5	12.50	12.50	12.50	12.50	12.49	12.48
0.4	13.58	13.58	13.58	13.58	13.57	13.56
0.3	14.85	14.85	14.84	14.84	14.83	14.83
0.2	16.29	16.29	16.29	16.29	16.28	16.27
0.1	17.92	17.92	17.92	17.92	17.91	17.90
0	19.74	19.74	19.73	19.73	19.72	19.72

Table 4. Temperature distribution in the fin, in C°; stationary case

x\y	0	0.01	0.02	0.03	0.04	0.05
0	19.74	19.74	19.73	19.73	19.72	19.72
-0.5	20.38	20.38	20.38	20.37	20.37	20.36
-1	21.39	21.39	21.39	21.39	21.39	21.38
-1.5	22.79	22.79	22.79	22.79	22.78	22.78
-2	24.58	24.58	24.58	24.58	24.58	24.58
-2.5	26.78	26.78	26.78	26.79	26.79	26.79
-3	29.43	29.43	29.43	29.43	29.43	29.43
-3.5	32.53	32.53	32.53	32.53	32.54	32.54
-4	36.13	36.13	36.13	36.13	36.14	36.14
-4.5	40.26	40.26	40.26	40.26	40.26	40.27
-5	44.96	44.96	44.96	44.96	44.96	44.97

Table 5. Temperature distribution in the left part of the base, in C°; stationary case

x\y	0.05	0.06	0.07	0.08	0.09	0.1
0	19.72	19.72	19.72	19.72	19.72	19.72
-0.5	20.36	20.36	20.37	20.37	20.37	20.37
-1	21.38	21.38	21.38	21.38	21.39	21.39
-1.5	22.78	22.78	22.78	22.79	22.79	22.79
-2	24.58	24.58	24.58	24.58	24.58	24.58
-2.5	26.79	26.79	26.79	26.79	26.79	26.79
-3	29.43	29.43	29.43	29.44	29.44	29.44
-3.5	32.54	32.54	32.54	32.54	32.54	32.54
-4	36.14	36.14	36.14	36.14	36.14	36.14
-4.5	40.27	40.27	40.27	40.27	40.27	40.27
-5	44.97	44.97	44.97	44.97	44.97	44.97

Table 6. Temperature distribution in the right part of the base, in C°; stationary case

From the tables we can conclude that at the end of the process there isn't much difference between the two cases. So, the transient temperature tends to the stationary one.

5 Conclusion

Here in the paper we have formulated a transient heat conduction problem for double wall with double fins in 2D when partial boiling is present.

Conservative averaging method and finite difference method is used to construct numerical solution.

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References:

[1] A.H.M. Al-Essa, One Dimensional Finite Element Solution of the Rectangular Fin with Rectangular Perforations, *WSEAS Transactions on Heat and Mass Transfer*, Vol.1, No.10, 2006, pp. 762-768.

[2] T. Bobinska, M. Buike, A. Buikis, Heat Transfer with Full Boiling in System with Double Wall and Double Fins, *Proc. of the 11th WSEAS Int. Conf. on Heat Transfer, Thermal Engineering and Environment, Vouliagmeni, Greece*, WSEAS Press, 2013, pp. 148-153.

[3] T. Bobinska, M. Buike, A. Buikis, Heat Transfer with Partial Boiling in System with Double Wall and Double Fins, *Proc. of the 11th WSEAS Int. Conf. on Heat Transfer, Thermal Engineering and Environment, Vouliagmeni, Greece*, WSEAS Press, 2013, pp. 132-137.

[4] T. Bobinska, M. Buike, A. Buikis, H.H. Cho, Stationary Heat Transfer in System with Double Wall and Double Fins, *Proc. of the 11th WSEAS Int. Conf. on Data Networks, Communications, Computers and the 5th WSEAS Int. Conf. on Materials Science, Sliema, Malta*, WSEAS Press, 2012, pp. 260-265.

[5] M. Buike, Simulation of steady-state heat process for the rectangular fin-containing system, *Mathematical Modelling and Analysis*, Vol.4, 1999, pp. 33-43.

[6] M. Buike, A. Buikis, Approximate Analytical Two-Dimensional Solution for a Longitudinal Fin of Rectangular Profile, *Acta Universitatis Latviensis*, Vol.612, 1998, pp. 33-44.

[7] A. Buikis, M. Buike, N. Ulanova, Transient solution for heat transfer in periodical system with fins (А.Буйкис, М.Буйке, Н.Уланова. Нестационарные решения для теплопереноса в периодических системах с ребрами), Proceedings of the Ukrainian Congress of Mathematics, Section 8, 2002, pp.15-23 (In Russian).

[8] A. Buikis, M. Buike, N. Ulanova, Analytically-Numerical Solution for Transient Process in the System with Rectangular Fins, *Proc. of the 5th WSEAS Int. Conf. on HEAT and MASS TRANSFER, Acapulco, Mexico*, WSEAS Press, 2008, pp. 31-36.

[9] D.Q. Kern, A.D. Kraus, *Extended Surface Heat Transfer*, McGraw Hill Book Company, 1972.

[10] B.S. Kim, S. Shin, S.J. Shin, K.M. Kim, H.H. Cho, Control of Superhydrophilicity/ Superhydrophobicity using Silicon Nanowires via Electroless Etching Method and Fluorine Carbon Coatings, *Langmuir*, Vol.27, No.16, 2011, pp. 10148-10156.

[11] B.S. Kim, S. Shin, S.J. Shin, K.M. Kim, H.H. Cho, Micro-nano hybrid structures with manipulated wettability using a two-step silicon etching on a large area, *Nanoscale Research Letters, a SpringerOpen Journal*, Vol.6 (ID:333), 2011.

[12] A.D. Kraus, *Analysis and evaluation of extended surface thermal systems*, Hemisphere Publishing Corporation, 1982.

[13] M. Manzoor. *Lecture Notes in Engineering. Heat Flow through Extended Surfaces Heat Exchangers*. Edited by C.A. Brebbia and S.A. Orszag. Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1984.

[14] A.A. Samarskii, *The Theory of Difference Schemes (Pure and Applied Mathematics Vol.240)*, New York, Marcel Dekker, 2001.

[15] A.S. Wood, G.E. Tupholme., M.I.H. Bhatti, P.J. Heggs, Performance indicators for steady-state heat transfer through fin assemblies, *Trans. ASME Journal of Heat Transfer*, Vol.118, 1996, pp. 310-316.