Viscous dissipation effects on unsteady natural convective flow past an infinite vertical plate with uniform heat and mass flux

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Abstract: - Viscous dissipation effects on the unsteady natural convective flow over an infinite vertical plate embedded in a porous medium subject to uniform heat and mass flux is investigated. A variable suction velocity is assumed to be normal to the plate. The governing unsteady, non-linear, coupled partial differential equations are reduced to ordinary differential equations by the method of perturbation. The effects of Prandtl number, Schmidt number, permeability parameter, thermal Grashof number, mass Grashof number, Eckert number on velocity, temperature and concentration are analyzed graphically. It is observed that the fluid velocity increases with an increase in porous medium parameter. An increase in viscous dissipation effects leads to an increase in fluid velocity and temperature. An increase in the combined effects of Prandtl number and Eckert number decelerates the fluid temperature. The fluids considered for the study are hydrogen, helium, oxygen, water vapor, carbon dioxide and ethyl benzene.

Key-Words: - viscous dissipation, natural convection, heat flux, mass flux, perturbation, porous medium.

1 Introduction

In real-world applications combined heat and mass transfer effects play a vital role. Some representative fields of interest are chemical processing equipments, cooling towers in power plant, dispersion of fog, food processing, moisture over agriculture fields, nuclear reactors, thermal insulators, heat exchangers, continuous casting, wire drawing, fiber drawing, and convective drying. In recent years significant increase in fundamental studies of heat and mass transfer in porous medium is of main attraction to researchers. Flows in porous medium have many applications in geothermal, geophysical and oil reservoirs etc. More simple application is, in cold climate buildings with their heat transfer effects forms a dissipative system results in a loss of energy which makes the interior climate cold.

An exact solution to the Navier-Stokes equation for the flow of a viscous incompressible fluid over an impulsively started infinite horizontal plate was first presented by Stokes [1]. Strewartson [2] studied the same kind of flow with semi infinite horizontal plate. The exact solution for viscous fluid flow past an impulsively started infinite vertical plate in its own plane was presented by Soundalgekar [3]. The simultaneous effects of heat and mass transfer concerned with a numerical study of transient natural convection flow past an impulsively started vertical plate with uniform heat and mass flux is investigated by Muthucumarasamy and Ganesan [4]. Pearson [5] presented similarity solution for plane channel flow of a high viscous fluid whose viscosity is exponentially dependent upon temperature by considering very high heat generation. Eckert and Faghri [6] observed the viscous heating of high Prandtl number fluids with temperature dependent viscosity for the sudden start of Coutte flow. Couette flow through a porous medium of a high Prandtl number fluid with temperature dependent viscosity is presented by Daskalakis [7]. Winter [8] presented viscous dissipation term in energy equation.

Viscous dissipation effects on unsteady free convective flow past an infinite vertical porous plate with constant suction was analysed by soundalgekar [9]. Later the same problem with variable suction was investigated by Soundalgekar [10]. Raptis and Vlohas [11] obtained the expressions for the velocity and temperature for unsteady hydrodynamic free convective flow through a porous medium. Raptis et al. [12] analysed steady two dimensional heat and mass transfer flow of an incompressible viscous fluid through a porous medium bounded by a vertical infinite surface with constant suction. Raptis and Tzivanidis [13] discussed the unsteady flow through a porous medium with the presence of mass transfer. Unsteady MHD convection flow of polar fluids past a vertical moving porous plate in a porous medium was investigated by Kim [14]. Same problem with MHD oscillatory flow of a micropolar fluid was studied analytically by Kim and Lee [15].

Recently radiation and mass transfer effects on an unsteady two dimensional laminar mixed convective boundary layer flow in the presence of porous medium and transverse magnetic field was presented by Ramachandra Prasad and Bhaskar Reddy [16] by taking into account the viscous dissipation effects. Heat and mass transfer effects on free convection infinite vertical porous plate with viscous dissipation and magnetohydro dynamics are investigated by Hemant Poonia and Chaudary [17]. Ramana Reddy et al. [18] studied the MHD free convective mass transfer flow past an infinite vertical porous plate with variable suction and soret effects.

In the present paper an attempt is made to study an unsteady, two dimensional, laminar flow with heat and mass transfer effects over an infinite vertical plate in the presence of porous medium along with viscous dissipation effects subject to uniform heat and mass flux. An approximate solution has been derived for velocity, temperature and concentration with perturbation technique. The behavior of velocity, temperature and concentration with respect to various parameters such as Prandtl number, Schimdt number, Eckert number, thermal Grashof number, mass Grashof number, permeability parameter for porous medium are presented graphically.

2 Formulation of the problem

Consider an unsteady, two-dimensional, laminar natural convective flow over an infinite vertical plate in the presence of porous medium under the influence of viscous dissipation, subject to uniform heat and mass flux. The plate is taken in the vertical direction which orients with the x' axis and the y' axis is taken perpendicular to it. Let u' and v' be the velocity components in their respective directions. The flow model is given in Fig.1. All the fluid properties are assumed to be constant except that the influence of the density variation. In the energy equation viscous dissipation term is taken into account. Under usual Boussinesq's approximation the governing continuity, momentum, energy and concentration equations are given by

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = g\beta(T' - T_\infty{}') + g\beta^*(\mathrm{c}' - \mathrm{c}_\infty{}') + \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\upsilon}{k'}u' \tag{2}$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \alpha\frac{\partial^2 T'}{\partial y'^2} + \frac{\upsilon}{c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{3}$$

$$\frac{\partial c'}{\partial t'} + v'\frac{\partial c'}{\partial y'} = D\frac{\partial^2 c'}{\partial y'^2} \tag{4}$$

Initial and boundary conditions are

$$t' > 0 \ : \ u' = 0, \frac{\partial T'}{\partial y'} = -\frac{q}{k^*}, \frac{\partial c'}{\partial y'} = -\frac{j''}{D} \text{ for } y' = 0.$$

$$u' \to 0, T' \to T_\infty{}', c' \to c_\infty{}' \text{ as } y' \to \infty. \tag{5}$$

In the above equations all the physical variables are functions of y' and t' alone as the plate is infinite. From equation (1), it is evident that v' is either a constant or a function of time. In this work, it is assumed that the suction velocity varies with time. It can be written as

$$v' = -v_0(1 + \varepsilon A e^{n't'}) \tag{6}$$

where v_0 is a scale of suction velocity, A is a real positive constant, ε and εA are very small.(<< 1).

On introducing the following non dimensional quantities:

$$u=\frac{u'}{v_0},t=\frac{t'v_0{}^2}{\upsilon},y=\frac{y'v_0}{\upsilon},T=\frac{T'-T_\infty{}'}{\left(\frac{q\upsilon}{k^*v_0}\right)},$$

$$Gr=\frac{\upsilon g\beta\left(\frac{q\upsilon}{k^*v_0}\right)}{v_0{}^3},\ Gc=\frac{\upsilon g\beta^*\left(\frac{j''\upsilon}{Dv_0}\right)}{v_0{}^3}$$

$$c=\frac{c'-c_\infty{}'}{\left(\frac{j''\upsilon}{Dv_0}\right)},\Pr=\frac{\upsilon}{\alpha},Sc=\frac{\upsilon}{D},\ Ec=\frac{k^*v_0{}^3}{c_p\upsilon q},\qquad(7)$$

$$k=\frac{k'v_0{}^2}{\upsilon^2},n=\frac{n'\upsilon}{v_0{}^2}$$

Equations (1) – (5) reduce to the following non dimensional form:

$$\frac{\partial u}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial u}{\partial y}=Gr\,T+Gc\,c+\frac{\partial^2u}{\partial y^2}-\frac{u}{k}\qquad(8)$$

$$\frac{\partial T}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial T}{\partial y}=\frac{1}{\Pr}\left(\frac{\partial^2T}{\partial y^2}\right)+Ec\left(\frac{\partial u}{\partial y}\right)^2\qquad(9)$$

$$\frac{\partial c}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial c}{\partial y}=\frac{1}{Sc}\left(\frac{\partial^2c}{\partial y^2}\right)\qquad(10)$$

Initial and boundary conditions are

$$t>0:\ u=0,\frac{\partial T}{\partial y}=-1,\frac{\partial c}{\partial y}=-1\ \text{for}\ \ y=0.$$

$$u\to 0,T\to 0,c\to 0\ \text{as}\ \ y\to\infty.\qquad(11)$$

3 Method of solution

The above partial differential equations (10) - (11) are reduced to a system of ordinary differential equations in dimensionless form. In order to reduce this, the velocity, temperature and concentration are taken as follows

$$u=u_0(y)+\varepsilon e^{nt}u_1(y)+o(\varepsilon^2)+...........\qquad(12)$$

$$T=T_0(y)+\varepsilon e^{nt}T_1(y)+o(\varepsilon^2)+...........\qquad(13)$$

$$c=c_0(y)+\varepsilon e^{nt}c_1(y)+o(\varepsilon^2)+...........\qquad(14)$$

Substituting equations (12) – (14) in equations (8) – (11) and equating the harmonic and non-harmonic terms, and neglecting the ε^2 and higher order terms, the following system of equations are obtained

$$u_0''+u_0'-N\,u_0=-Gr\,T_0-Gcc_0\qquad(15)$$

$$u_1''+u_1'-nu_1-Nu_1=-Gr\,T_1-Gcc_1-Au_0'\qquad(16)$$

$$T_0''+\Pr T_0'=-Ec\Pr u_0'^2\qquad(17)$$

$$T_1''+\Pr T_1'-n\Pr T_1=-2Ec\Pr u_0'u_1'-A\Pr T_0'\qquad(18)$$

$$c_0''+Sc\,c_0'=0\qquad(19)$$

$$c_1''+Sc\,c_1'-nSc\,c_1=-ASc\,c_0'\qquad(20)$$

Subject to the initial and boundary conditions

$$\left.\begin{array}{l} t>0:\begin{array}{l}u_0=0,\ u_1=0,\mathrm{T}_0'=-1,\\ \mathrm{T}_1'=0,c_0'=-1,c_1'=0\end{array}\ \text{for}\ y=0.\\ \\ \quad\begin{array}{l}u_0\to 0,\ u_1\to 0,\mathrm{T}_0\to 0,\\ \mathrm{T}_1\to 0,c_0\to 0,c_1\to 0\end{array}\ \text{as}\ \ y\to\infty.\end{array}\right\}(21)$$

Here primes denote differentiation with respect to y.

Equations (15) – (20) are coupled and non-linear. To solve these equations power series solution method is employed. Since the fluid is incompressible and the suction velocity is very small, the parameter Eckert number (Ec) is also small. Hence u_0,u_1,T_0,T_1 are expanded in powers of Ec and neglecting terms of $o(Ec^2)$. We have

$$u_0(y)=u_{01}(y)+Ecu_{02}(y)\qquad(22)$$

$$u_1(y)=u_{11}(y)+Ecu_{12}(y)\qquad(23)$$

$$T_0(y)=T_{01}(y)+EcT_{02}(y)\qquad(24)$$

$$T_1(y)=T_{11}(y)+EcT_{12}(y)\qquad(25)$$

Substituting eqns. (22) – (25) in (15) – (18) and equating the like terms and unlike terms and neglecting the terms containing Ec^2 and higher order terms, we get

$$u_{01}'' + u_{01}' - Nu_{01} = -GrT_{01} - Gc\,c_0 \quad (26)$$

$$u_{02}'' + u_{02}' - Nu_{02} = -GrT_{02} \quad (27)$$

$$u_{11}'' + u_{11}' - N_1 u_{11} = -GrT_{11} - Gc\,c_1 - Au_{01}' \quad (28)$$

$$u_{12}'' + u_{12}' - N_1 u_{12} = -GrT_{12} - Au_{02}' \quad (29)$$

$$T_{01}'' + \Pr T_{01}' = 0 \quad (30)$$

$$T_{02}'' + \Pr T_{02}' = -\Pr u_{02}'^2 \quad (31)$$

$$T_{11}'' + \Pr T_{11}' - n\Pr T_{11} = -A\Pr T_{01}' \quad (32)$$

$$T_{12}'' + \Pr T_{12}' - n\Pr T_{12} = -2\Pr u_{01}' u_{11}' - A\Pr T_{02}' \quad (33)$$

Subject to the initial and boundary conditions

$$t > 0: \begin{array}{l} u_{01} = 0,\ u_{02} = 0,\ u_{11} = 0,\ u_{12} = 0, \\ T_{01}' = -1, T_{02}' = 0,\ T_{11}' = 0, T_{12}' = 0 \end{array} \text{ for } y = 0.$$

$$\begin{array}{l} u_{01} \to 0,\ u_{02} \to 0,\ u_{11} \to 0,\ u_{12} \to 0, \\ T_{01} \to 0, T_{02} \to 0,\ T_{11} \to 0, T_{12} \to 0 \end{array} \text{ as } y \to \infty. \quad (34)$$

Solving the above eight equations with respect to the conditions in eq. (34), we get the following solutions

$$u_{01} = a_1 e^{-L_4 y} + b_1 e^{-\Pr y} + b_2 e^{-Scy}. \quad (35)$$

$$u_{02} = a_6 e^{-L_4 y} + b_{42} e^{-\Pr y} + b_{43} e^{-2L_4 y} + b_{44} e^{-2\Pr y} + b_{45} e^{-2Scy} + b_{46} e^{-m_1 y} + b_{47} e^{-m_2 y} + b_{48} e^{-m_3 y} \quad (36)$$

$$u_{11} = a_4 e^{-L_8 y} + b_{10} e^{-L_6 y} + b_{15} e^{-L_2 y} + b_{12} e^{-L_4 y} + h_2 e^{-\Pr y} + h_3 e^{-Scy}. \quad (37)$$

$$\begin{aligned} u_{12} = {} & a_7 e^{-L_8 y} + b_{49} e^{-L_6 y} + (b_{50} + b_{71}) e^{-m_1 y} \\ & + (b_{51} + b_{72}) e^{-m_2 y} + (b_{52} + b_{73}) e^{-m_3 y} \\ & + b_{53} e^{-m_4 y} + b_{54} e^{-m_5 y} + b_{55} e^{-m_6 y} \\ & + b_{56} e^{-m_7 y} + b_{57} e^{-m_8 y} + b_{58} e^{-m_9 y} \\ & + b_{59} e^{-m_{10} y} + b_{60} e^{-m_{11} y} + b_{61} e^{-m_{12} y} \\ & + (b_{62} + b_{68}) e^{-2L_4 y} + (b_{63} + b_{69}) e^{-2\Pr y} \\ & + (b_{64} + b_{70}) e^{-2Scy} + (b_{65} + b_{67}) e^{-\Pr y} \\ & + b_{66} e^{-L_4 y} \end{aligned}$$

. (38)

$$T_{01} = \frac{1}{\Pr} e^{-\Pr y}. \quad (39)$$

$$T_{02} = a_2 e^{-\Pr y} + b_3 e^{-2L_4 y} + b_4 e^{-2\Pr y} + b_5 e^{-2Scy} + b_6 e^{-m_1 y} + b_7 e^{-m_2 y} + b_8 e^{-m_3 y} \quad (40)$$

$$T_{11} = a_3 e^{-L_6 y} + b_9 e^{-\Pr y} \quad (41)$$

$$\begin{aligned} T_{12} = {} & a_5 e^{-L_6 y} + (b_{39} + h_4) e^{-m_1 y} + (b_{40} + h_5) e^{-m_2 y} \\ & + (b_{41} + h_6) e^{-m_3 y} + b_{17} e^{-m_4 y} + b_{18} e^{-m_5 y} \\ & + b_{19} e^{-m_6 y} + b_{23} e^{-m_7 y} + b_{24} e^{-m_8 y} \\ & + b_{25} e^{-m_9 y} + b_{29} e^{-m_{10} y} + b_{30} e^{-m_{11} y} \\ & + b_{31} e^{-m_{12} y} + (b_{20} + b_{36}) e^{-2L_4 y} \\ & + (b_{27} + b_{37}) e^{-2\Pr y} + (b_{34} + b_{38}) e^{-2Scy} \\ & + b_{35} e^{-\Pr y} \end{aligned} \quad (42)$$

$$c_0 = \frac{1}{Sc} e^{-Scy} \quad (43)$$

$$c_1 = h_1 e^{-L_2 y} + b_9 e^{-Scy} \quad (44)$$

Substituting the above solutions in equations (26) – (33) and then the resulting expressions in eqn. (12) – (14), we get the expressions for velocity and temperature. The constants are listed in the appendix.

4 Results and discussion

In order to get the physical insight of the problem the computed values of different parameters like Schmidt number, Prandtl number, permeability, thermal Grashof number, mass Grashof number and

Eckert number are presented graphically. For the calculation part changing the values of various parameters in the above expressions the numerical values of velocity, temperature and concentration are computed until they converge to free stream boundary conditions.

In fig. 2, velocity increases with increasing values of permeability parameter. Permeability is the measure of the materials ability to permit liquid or gas through its pores or voids. Filters made of soil and earth dams are very much based upon the permeability of a saturated soil under load. Permeability is a part of the proportionality constant in Darcy's law. Darcy's law relates the flow rate and fluid properties to the pressure gradient applied to the porous medium. This supports that as permeability increases velocity should also increase.

In fig. 3, velocity profile for various values of Prandtl number is shown. Prandtl number being the ratio of momentum diffusivity to thermal diffusivity will influence the fluid flow as long as the velocity field and the temperature field are coupled. Hear the velocity decreases with increasing values of Prandtl number. It is observed that the velocity increases rapidly near the wall and reaches the maximum at one point and then gradually reduces to the free stream velocity.

In fig. 4, velocity profile for various values of thermal Grashof number and mass Grashof number is presented. Grashof number is used in analyzing the velocity distribution in natural convection. Grashof number is the ratio of buoyancy force due to the spatial variation in the fluid density to the viscous force. When Gr and Gc (<<1) then viscous force is negligible compared to buoyancy force and inertial force. Grashof number in positive sence indicates the cooling effect near the plate. Due to the spatial variation in fluid density caused by the temperature difference, velocity increases with increasing values of thermal Grashof number and mass Grashof number.

In fig. 5, velocity profile for various values of Eckert number and permeability is plotted. Eckert number expresses the relation between kinetic energy and the enthalphy. It is used to characterize the dissipative process. Eckert number and permeability are the major effects of the problem together with uniform heat and mass flux. It is evident that viscous dissipative heat raises the velocity of the fluid. Hence, the velocity increases with increasing values of Eckert number and permeability.

In fig. 6, temperature profile for different values of permeability parameter is presented. Permeability is the proportionality constant in Darcy's law, which involves the pressure gradient caused by the temperature difference. Hence the velocity variation and temperature variation with respect to the permeability effect should show the same variation. It is concluded that the temperature increases with increasing values of permeability.

In fig. 7, temperature profile for different values of Prandtl number is presented. The size of the thermal boundary layer increases with decreasing Prandtl number because the Boussinesq's approximation in the momentum equation consists of assuming that density of the fluid varies with temperature linearly. Since the velocity and temperature are coupled. Similar type of behavior will be seen both in velocity and temperature. Here temperature profile gradually reduces to reach the free stream temperature.

In fig. 8, temperature profle for different values of Eckert number is presented. Dissipation process is the result of shear stresses in the fluid at the wall Eckert number which is the coefficient of viscous dissipation of energy is the basic parameter for determining the temperature. Eckert number plays an important role in representing the ratio of kinetic energy at the wall to the specific enthalpy difference between the wall and the fluid. It is obvious that thermal boundary layer increases with increasing values of Eckert number.

In fig. 9, combined effects of Eckert number and Prandtl number are visualized. Eckert number multiplied with Prandtl number is the key parameter in determining the viscous dissipation of energy. If the product of Ec and Pr is large then, the energy dissipation is an important parameter in the heat transfer process and the kinetic energy plays a significant role in determining the temperature distribution in the flow. In this fig. the size of thermal boundary layer increases with decreasing values of their product.

In fig. 10, Temperature for different values of thermal Grashof number is analysed. Grashof number is mainly used in the correlation of heat and mass transfer due to thermally induced natural convection. The significance of Grashof number represents the ratio between the buoyancy force due to density variation to the viscous force of the fluid. If the Grashof number increases the temperature should obviously increase. Hence it is visualized that the thermal boundary layer increases with increasing values of thermal Grashof number.

In fig. 11, Concentration for different values of Sc is plotted. Schmidt number represents the relationship between mass diffusivity and thermal diffusivity. It physically relates the hydrodynamic boundary layer and mass transfer boundary layer. As Schmidt number depends on mass diffusion rate concentration should decrease for increasing Schmidt number values. It shows that the mass transfer boundary layer decreases for increasing values of Schmidt number.

4 Conclusion

This paper presents the expressions for the velocity, temperature and concentration by perturbation technique. The various effects like permeability, thermal Grashof number, mass Grashof number, Prandtl number, Schmidt number and Eckert number are visualized and investigated graphically. The conclusions of the study are as follows

1. The velocity increases with increasing values of permeability, Grashof number and Eckert number.
2. For increasing values of Prandtl number, velocity boundary layer decreases.
3. Thermal boundary layer increases for increasing values of thermal Grashof number, Eckert number and permeability parameter.
4. Thermal boundary layer decreases for increasing values of Prandtl number.
5. The mass transfer boundary layer decreases for increasing values of Schmidt number.

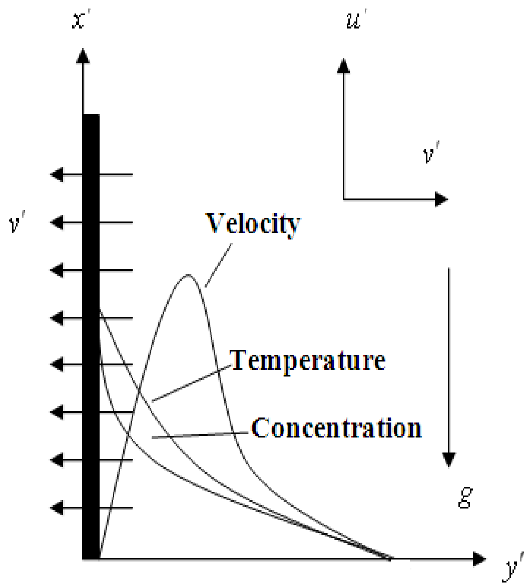

Fig.1 Physical model and co-ordinate system

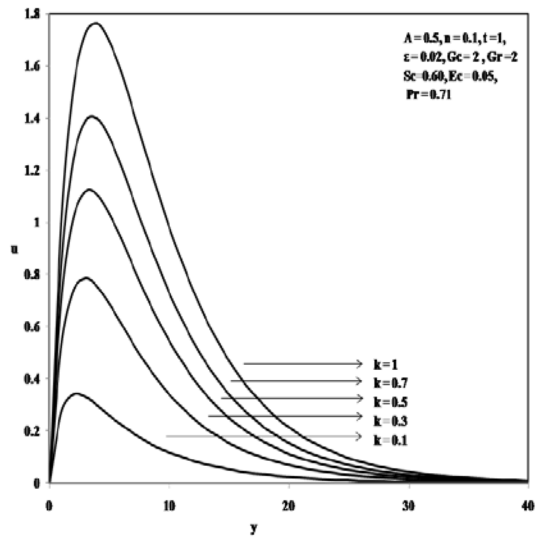

Fig.2 Velocity for different values of Permeability

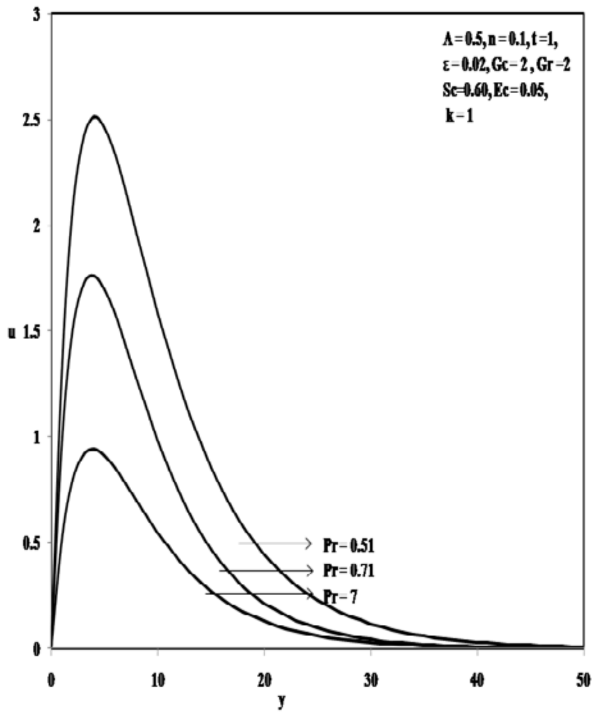

Fig.3. Velocity for various values of Prandtl number.

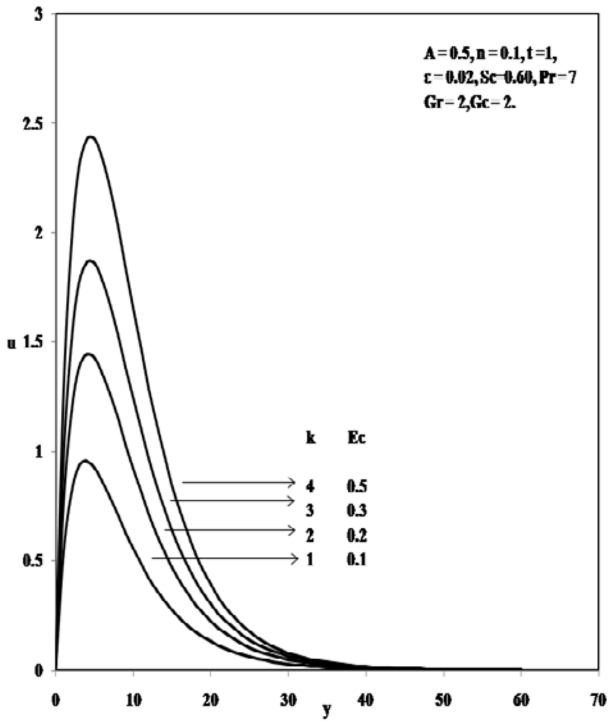

Fig. 5. Velocity profile for different values of permeability and Ec

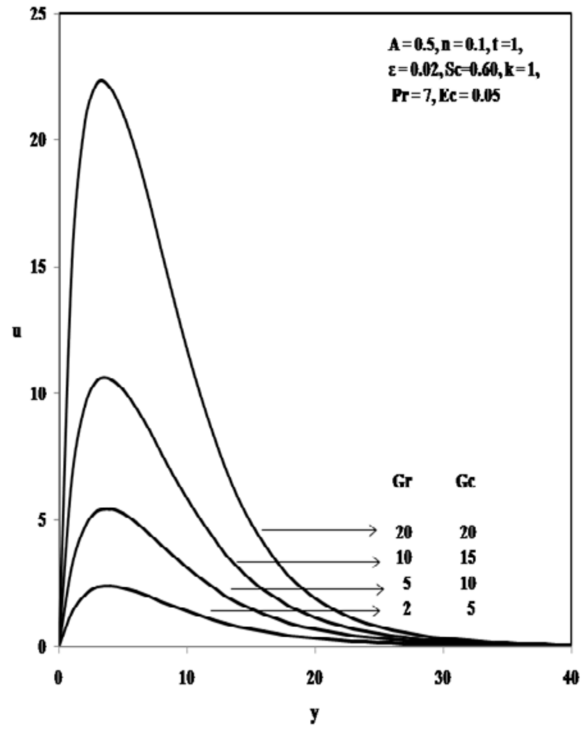

Fig. 4. Velocity for various values of Gr and Gc

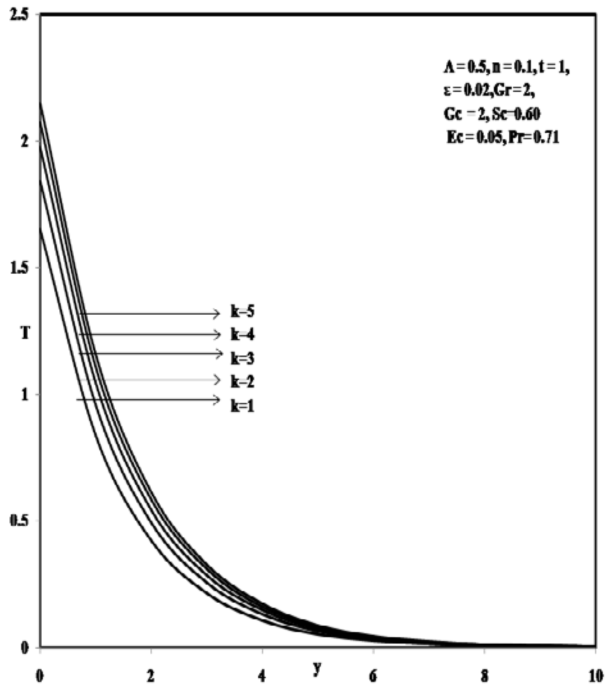

Fig. 6. Temperature for different Values of permeability

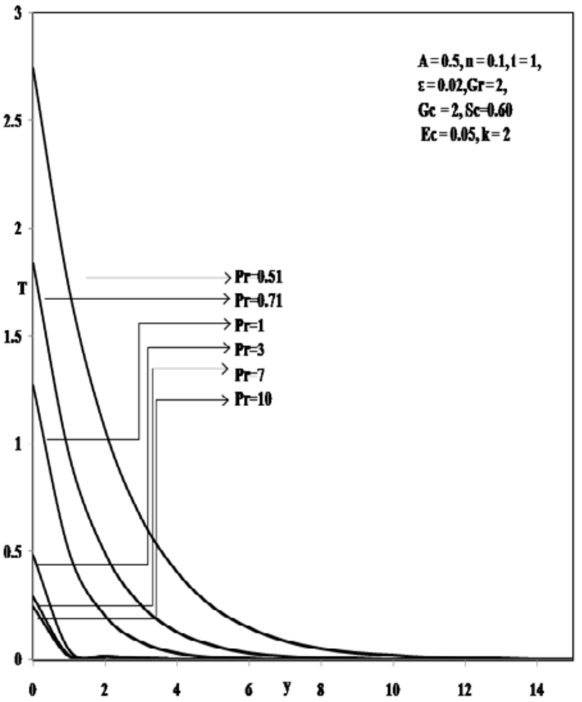

Fig. 7, Temperature for different Prandtl numbers

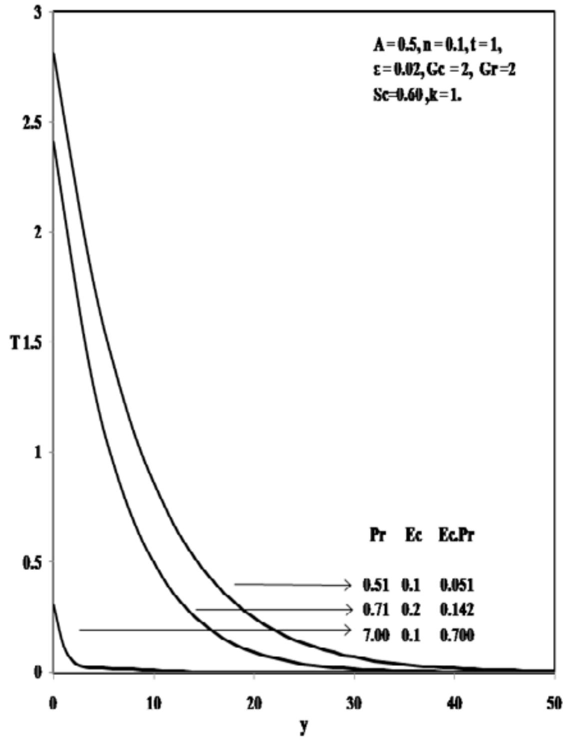

Fig. 9, Effects of Ec and Pr in temperature.

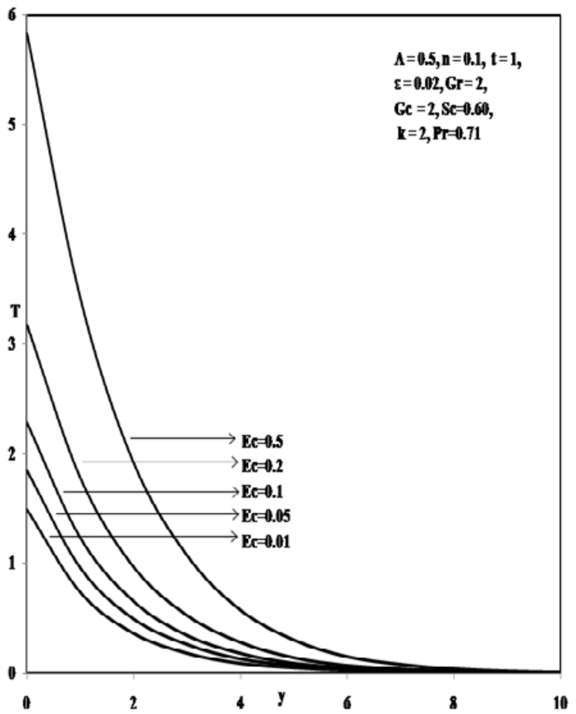

Fig. 8, Effect of Eckert numbers on temperature.

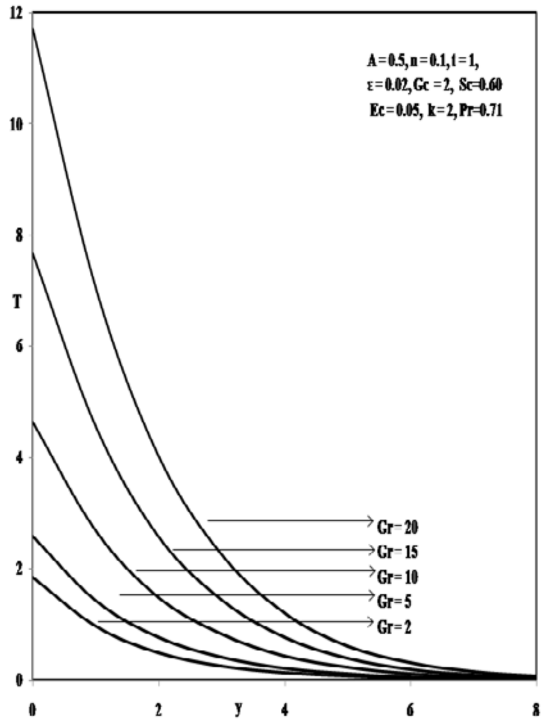

Fig. 10, Effects of Gr on temperature.

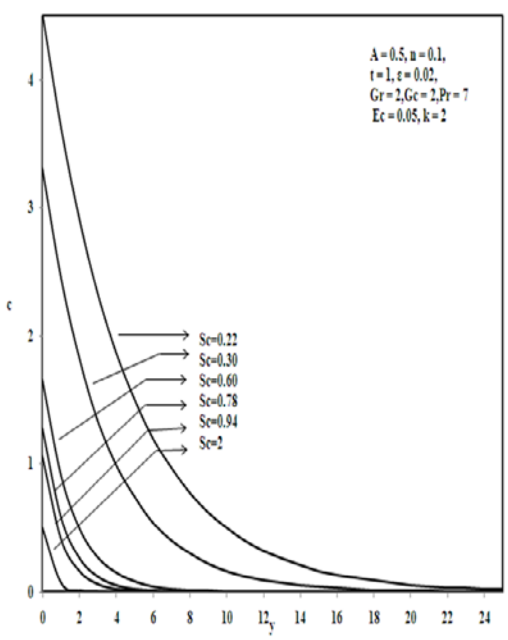

Fig. 11, Effects of Sc on concentration.

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APPENDIX

$$N=\frac{1}{k},\quad N_1=n+\frac{1}{k}$$

$$b_1=-\frac{Gr}{\Pr(\Pr^2-\Pr-\mathrm{N})},\quad b_2=-\frac{Gc}{\mathrm{Sc}(\mathrm{Sc}^2-Sc-\mathrm{N})}$$

$$L_2=\frac{Sc+\sqrt{Sc^2+4nSc}}{2},\quad L_4=\frac{1+\sqrt{1+4N}}{2},$$

$$L_6=\frac{\Pr+\sqrt{\Pr^2+4n\Pr}}{2},\quad L_8=\frac{1+\sqrt{1+4N_1}}{2}$$

$$m_1=L_4+\Pr,\quad m_2=Sc+\Pr,\quad m_3=L_4+Sc,$$

$$m_4=L_4+L_8,\quad m_5=L_4+L_6,\quad m_6=L_4+L_2$$

$$m_7=L_8+\Pr,\quad m_8=L_6+\Pr,\quad m_9=L_2+\Pr,$$

$$m_{10}=L_8+Sc,\quad m_{11}=L_6+Sc,\quad m_{12}=L_2+Sc$$

$$b_3=-\frac{\Pr a_1^2 L_4}{4L_4-2\Pr},\ b_4=-\frac{\Pr b_1^2}{2},\ b_5=-\frac{\Pr b_2^2 Sc}{4Sc-2\Pr},$$

$$b_6=-\frac{2\Pr^2 a_1 b_1 L_4}{m_1^2-\Pr m_1},\ b_7=-\frac{2\Pr^2 b_1 b_2 Sc}{m_2^2-\Pr m_2}$$

$$b_8=-\frac{2\Pr a_1 b_2 Sc L_4}{m_3^2-\Pr m_3},\quad a_1=-(\mathrm{b}_1+\mathrm{b}_2)$$

$$\mathrm{a}_2=-\frac{1}{\Pr}\left(2L_4b_3+2\Pr b_4+2Scb_5+m_1b_6+m_2b_7+m_3b_8\right),$$

$$\mathrm{a}_3=\frac{\mathrm{A}\Pr}{L_6 n},\quad b_9=-\frac{A}{n}$$

$$h_1=\frac{ASc}{L_2 n},\quad b_{10}=-\frac{Gra_3}{L_6^2-L_6-N_1},$$

$$b_{11}=-\frac{Grb_9}{\Pr^2-\Pr-N_1},\ b_{12}=\frac{a_1L_4A}{L_4^2-L_4-N_1}$$

$$b_{13}=\frac{b_1\Pr A}{\Pr^2-\Pr-N_1},\quad b_{14}=\frac{b_2ScA}{Sc^2-Sc-N_1},$$

$$b_{15}=-\frac{Gch_1}{L_2^2-L_2-N_1},\quad b_{16}=-\frac{Gcb_9}{Sc^2-Sc-N_1}$$

$$a_4=-(\mathrm{b}_{10}+\mathrm{b}_{11}+\mathrm{b}_{12}+\mathrm{b}_{13}+\mathrm{b}_{14}+\mathrm{b}_{15}+\mathrm{b}_{16}),$$

$$\mathrm{h}_2=b_{11}+b_{13},\quad h_3=b_{14}+b_{16}$$

$$s_1=-2\Pr a_1a_4L_4L_8,\quad s_2=-2\Pr a_1b_{10}L_4L_6,$$

$$s_3=-2\Pr a_1b_{15}L_4L_2,\quad s_4=-2\Pr a_1b_{12}L_4^2$$

$$s_5=-2\Pr^2 a_1h_2L_4,\ s_6=-2\Pr a_1h_3L_4\,\mathrm{Sc},$$

$$s_7=-2\Pr^2\mathrm{b}_1\,a_4L_8,\ s_8=-2\Pr^2\mathrm{b}_1\,b_{10}L_6$$

$$s_9=-2\Pr^2\mathrm{b}_1\,b_{15}L_2,\quad s_{10}=-2\Pr^2\mathrm{b}_1\,b_{12}L_4,$$

$$s_{11}=-2\Pr^3\mathrm{b}_1\,h_2,\quad s_{12}=-2\Pr\mathrm{b}_1\,h_3Sc\Pr$$

$$s_{13}=-2\Pr\mathrm{b}_2\,a_4ScL_8,\quad s_{14}=-2\Pr\mathrm{b}_2\,b_{10}L_6\,\mathrm{Sc},$$

$$s_{15}=-2\Pr\mathrm{b}_2\,b_{15}ScL_2,\quad s_{16}=-2\Pr^2\mathrm{b}_2\,h_2Sc$$

$$s_{17}=-2\Pr^2\mathrm{b}_2\,h_2Sc,\quad s_{18}=-2\Pr\mathrm{h}_3\,b_2Sc^2,$$

$$s_{19}=A\Pr^2\mathrm{a}_2,\quad s_{20}=2A\Pr\mathrm{L}_4\,b_3,\ s_{21}=2A\Pr^2 b_4,$$

$$s_{22}=2A\Pr\mathrm{Sc}\,b_5,\quad s_{23}=A\Pr\mathrm{m}_1\,b_6,\quad s_{24}=A\Pr\mathrm{m}_2\,b_7,$$

$$s_{25}=A\Pr\mathrm{m}_3\,b_8,\ s_{26}=-Gra_5,\ s_{27}=-Gr(h_4+\mathrm{b}_{39}),$$

$$s_{28}=-Gr(h_5+\mathrm{b}_{40}),\ s_{29}=-Gr(h_6+\mathrm{b}_{41}),$$

$$s_{30}=-Gr\,\mathrm{b}_{17},\quad s_{31}=-Gr\,\mathrm{b}_{18},\quad s_{32}=-Gr\,\mathrm{b}_{19},$$

$$s_{33}=-Gr\,\mathrm{b}_{23},\quad s_{34}=-Gr\,\mathrm{b}_{24},\ s_{35}=-Gr\,\mathrm{b}_{25},$$

$$s_{36}=-Gr\,\mathrm{b}_{29},\quad s_{37}=-Gr\,\mathrm{b}_{30},\ s_{38}=-Gr\,\mathrm{b}_{31},$$

$$s_{39}=-Gr(\mathrm{b}_{20}+\mathrm{b}_{36}),\ s_{40}=-Gr(\mathrm{b}_{27}+\mathrm{b}_{37}),$$

$$s_{41}=-Gr(\mathrm{b}_{34}+\mathrm{b}_{38}),\ s_{42}=-Gr\,\mathrm{b}_{35},\ s_{43}=L_4a_6\,\mathrm{A}$$

$$s_{44}=\Pr b_{42}\,\mathrm{A},\quad s_{45}=2L_4b_{43}\,\mathrm{A},\quad s_{46}=2\Pr b_{44}\,\mathrm{A},$$

$$s_{47}=2Scb_{45}\,\mathrm{A},\quad s_{48}=m_1b_{46}\,\mathrm{A},\ s_{49}=m_2b_{47}\,\mathrm{A},$$

$$s_{50}=m_3b_{48}\,\mathrm{A}.$$

$$b_{17}=\frac{s_1}{m_4^2-\Pr m_4-n\Pr},\ b_{18}=\frac{s_2}{m_5^2-\Pr m_5-n\Pr},$$

$$b_{19}=\frac{s_3}{m_6^2-\Pr m_6-n\Pr},\ b_{20}=\frac{s_4}{4L_4^2-2\Pr\mathrm{L}_4-n\Pr},$$

$$b_{21}=\frac{s_5}{m_1^2-\Pr m_1-n\Pr},\ b_{22}=\frac{s_6}{m_3^2-\Pr m_3-n\Pr}$$

$$b_{23}=\frac{s_7}{m_7^2-\Pr m_7-n\Pr},\ b_{24}=\frac{s_8}{m_8^2-\Pr m_8-n\Pr},$$

$$b_{25}=\frac{s_9}{m_9^2-\Pr m_9-n\Pr},\ b_{26}=\frac{s_{10}}{m_1^2-\Pr m_1-n\Pr},$$

$$b_{27}=\frac{s_{11}}{4\Pr^2-2\Pr^2-n\Pr},\ b_{28}=\frac{s_{12}}{m_2^2-\Pr m_2-n\Pr}$$

$$b_{29}=\frac{s_{13}}{m_{10}^2-\Pr m_{10}-n\Pr},\ b_{30}=\frac{s_{14}}{m_{11}^2-\Pr m_{11}-n\Pr},$$

$$b_{31}=\frac{s_{15}}{m_{12}^2-\Pr m_{12}-n\Pr},\ b_{32}=\frac{s_{16}}{m_3^2-\Pr m_3-n\Pr},$$

$$b_{33}=\frac{s_{17}}{m_2^2-\Pr m_2-n\Pr},\ b_{34}=\frac{s_{18}}{4Sc^2-2\Pr\mathrm{Sc}-n\Pr}$$

$$b_{35} = -\frac{s_{19}}{n\,\mathrm{Pr}},\ b_{36} = \frac{s_{20}}{4L_4^{\ 2} - 2\,\mathrm{Pr}\,L_4 - n\,\mathrm{Pr}},$$

$$b_{37} = \frac{s_{21}}{4\,\mathrm{Pr}^2 - 2\,\mathrm{Pr}^2 - n\,\mathrm{Pr}},\ b_{38} = \frac{s_{22}}{4Sc^2 - 2Sc\,\mathrm{Pr} - n\,\mathrm{Pr}},$$

$$b_{39} = \frac{s_{23}}{m_1^{\ 2} - \mathrm{Pr}\,m_1 - n\,\mathrm{Pr}},\ b_{40} = \frac{s_{24}}{m_2^{\ 2} - \mathrm{Pr}\,m_2 - n\,\mathrm{Pr}},$$

$$b_{41} = \frac{s_{25}}{m_3^{\ 2} - \mathrm{Pr}\,m_3 - n\,\mathrm{Pr}}.$$

$$h_4 = (\mathrm{b}_{21} + \mathrm{b}_{26}),\ h_5 = (\mathrm{b}_{28} + \mathrm{b}_{33}),\ h_6 = (\mathrm{b}_{22} + \mathrm{b}_{32})$$

$$a_5 = \frac{-1}{L_6}\begin{bmatrix} \{(h_4 + b_{39})m_1 + (h_5 + b_{40})m_2 + (h_6 + b_{41})m_3 \\ +b_{17}m_4 + b_{18}m_5 + b_{19}m_6 + b_{23}m_7 + b_{24}m_8 \\ +b_{25}m_9 + b_{29}m_{10} + b_{30}m_{11} + b_{31}m_{12} \\ +2(b_{20} + b_{36})L_4 + 2(b_{27} + b_{37})\,\mathrm{Pr} \\ +2(b_{34} + b_{38})\,\mathrm{Sc} + b_{35}\,\mathrm{Pr}\} \end{bmatrix}$$

$$b_{49} = \frac{s_{26}}{L_6^{\ 2} - L_6 - N_1},\quad b_{50} = \frac{s_{27}}{m_1^{\ 2} - m_1 - N_1},$$

$$b_{51} = \frac{s_{28}}{m_2^{\ 2} - m_2 - N_1},\quad b_{52} = \frac{s_{29}}{m_3^{\ 2} - m_3 - N_1},$$

$$b_{53} = \frac{s_{30}}{m_4^{\ 2} - m_4 - N_1},\quad b_{54} = \frac{s_{31}}{m_5^{\ 2} - m_5 - N_1},$$

$$b_{55} = \frac{s_{32}}{m_6^{\ 2} - m_6 - N_1},\quad b_{56} = \frac{s_{33}}{m_7^{\ 2} - m_7 - N_1},$$

$$b_{57} = \frac{s_{34}}{m_8^{\ 2} - m_8 - N_1},\quad b_{58} = \frac{s_{35}}{m_9^{\ 2} - m_9 - N_1},$$

$$b_{59} = \frac{s_{36}}{m_{10}^{\ 2} - m_{10} - N_1},\quad b_{60} = \frac{s_{37}}{m_{11}^{\ 2} - m_{11} - N_1},$$

$$b_{61} = \frac{s_{38}}{m_{12}^{\ 2} - m_{12} - N_1},\quad b_{62} = \frac{s_{39}}{4L_4^{\ 2} - 2L_4 - N_1},$$

$$b_{63} = \frac{s_{40}}{4\,\mathrm{Pr}^2 - 2\,\mathrm{Pr} - N_1},\quad b_{64} = \frac{s_{41}}{4Sc^2 - 2Sc - N_1},$$

$$b_{65} = \frac{s_{42}}{\mathrm{Pr}^2 - \mathrm{Pr} - N_1},\quad b_{66} = \frac{s_{43}}{L_4^{\ 2} - L_4 - N_1},$$

$$b_{67} = \frac{s_{44}}{\mathrm{Pr}^2 - \mathrm{Pr} - N_1},\quad b_{68} = \frac{s_{45}}{4L_4^{\ 2} - 2L_4 - N_1},$$

$$b_{69} = \frac{s_{46}}{4\,\mathrm{Pr}^2 - 2\,\mathrm{Pr} - N_1},\quad b_{70} = \frac{s_{47}}{4Sc^2 - 2Sc - N_1},$$

$$b_{71} = \frac{s_{48}}{m_1^{\ 2} - m_1 - N_1},\quad b_{72} = \frac{s_{49}}{m_2^{\ 2} - m_2 - N_1},$$

$$b_{73} = \frac{s_{50}}{m_3^{\ 2} - m_3 - N_1}$$

$$\begin{aligned} a_7 = -[\,& b_{49} + b_{50} + b_{51} + b_{52} + b_{53} + b_{54} + b_{55} + b_{56} \\ & + b_{57} + b_{58} + b_{59} + b_{60} + b_{61} + b_{62} + b_{63} + b_{64} \\ & + b_{65} + b_{66} + b_{67} + b_{68} + b_{69} + b_{70} + b_{71} + b_{72} \\ & + b_{73}] \end{aligned}$$