Visco-Elastic MHD Boundary Layer Flow with Heat and Mass Transfer over a Continuously Moving Inclined Surface in Presence of Energy Dissipation

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Abstract: - The problem of two-dimensional free convective boundary layer flow of a visco-elastic fluid over a continuously moving inclined surface has been investigated in presence of transverse magnetic field and energy dissipation. Effects of heat and mass transfer are also analyzed in this paper. Heat is supplied from the plate to the fluid at a uniform rate. The suction at the plate is assumed to be constant and the plate is assumed to move continuously with a uniform velocity U in the upward direction. The visco-elastic fluid flow is characterized by Walters liquid (model B′). A uniform magnetic field of strength B_0 is applied in the direction perpendicular to the plate. Highly non-linear momentum boundary layer equation, thermal boundary layer equation and concentration equation are converted into non dimensional form and then solved analytically by using regular perturbation technique. The analytical expressions for velocity profile, temperature field, concentration field, shearing stress, rate of heat transfer and rate of mass transfer at the surface have been obtained. The effects of Grashof number (Gr), Hartmann number (M), Prandtl number (Pr) and Schmidt number (Sc) on velocity profiles and shearing stress at the surface have been illustrated graphically for various values of visco-elastic parameter in combination with other flow parameters.

Key-Words: - Visco-elastic, Walters liquid (model B$'$), MHD, heat and mass transfer, regular perturbation technique, Prandtl number, Schmidt number.

1 Introduction

One of the recent advances in the area of fluid dynamics is the mechanism of non-Newtonian fluid theory. Researchers have shown their interest in this emerging and dynamic field for its applications in geo-physics, chemical engineering (absorption, filtration), petroleum engineering, hydrology, soil-physics, bio-physics, paper and pulp technology. In non-Newtonian theory, the analysis of visco-elastic fluid lies on the fact that the viscosity of the fluid studies the energy dissipation and its elasticity analyses the energy stored during the flow.

The concept of simultaneous heat and mass transfer is used in various science and engineering problems. It is used in food processing, wet-bulb thermometer and polymer solution and also in various fluids flow related engineering problems. In our daily life, the combined heat and mass transfer phenomenon is observed in the formation of fog. The convective flow associated with the combined heat and mass transfer has many applications in various branches of science and engineering. Nature of vertical convection flows from the buoyancy effects of thermal and mass diffusion has been done by Gebhart and Pera [1]. Singh and Singh [2] have discussed the MHD free convection flow and mass transfer past a flat plate. Al-Qadat and Al-Azab [3] have studied the influence of chemical reaction on transient MHD free convective flow over a moving vertical plate. The study of combined heat and mass transfer in mixed convective MHD flow along a vertical plate in presence of heat source has been shown by Zueco and Ahmed [4]. Palani and Srikanth [5] have explained the mass transfer effects on MHD flow past a semi infinite vertical plate. Chaudhary and Jain [6] have analysed the combined heat and mass diffusion in a MHD free convective flow past a surface embedded in a porous medium. Effects of chemical reaction on transient MHD free convective flow over a vertical plate in slip-flow regime are explained by Sahin [7]. The investigation of boundary layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux in presence of transverse magnetic field has been done by Makinde [8].

The theory of MHD is extensively used in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, textile industry etc. The natural convection boundary layer flow of an electrically conducting fluid up a hot vertical wall in the presence of strong magnetic field has been studied by several authors such as Sparrow and Cess [9], Riley [10] and Kuiken [11] etc. Simultaneous occurrence of buoyancy and magnetic field forces in the flow of an electrically conducting fluid up a hot vertical flat plate in the presence of a strong cross magnetic field was studied by Singh and Cowling [12]. Crammer and Pai [13] have presented a similarity solution for the above problem with uniform heat flux by formulating it in terms of both a regular and inverse series expansions of characterizing co-ordinate that provided a link between the similarity state closed to and far from the leading edge. Hossain *et al.* [14] have studied the convection flow from an isothermal plate inclined at a small angle to the horizontal.

The problem of boundary layer flow past a stretching sheet are extensively used in various technological process like hot rolling, wire drawing, glass-fibre and paper production etc. Many researchers have done their work on the basis of the classical works done by Sakiadis [15, 16], *et.al.* [17] and Crane [18]. Ericksen *et al*. [19] have studied the combined heat and mass transfer on a moving continuous plate with suction or injection. Gupta and Gupta [20] have investigated the heat and mass transfer with suction and blowing. Vajravelu [21, 22] has analysed hydromagnetic flow with heat transfer and hydromagnetic convection respectively over a moving surface. A theoretical solution for hydromagnetic convection over a continuously moving vertical surface with uniform suction has been obtained by Kumar *et al.* [23]. The mechanisms of visco-elastic boundary layer flow are used in various manufacturing processes such as fabrication of adhesive tapes, extrusion of plastic sheets, coating layers into rigid surfaces etc. Various blood flow problems are also explained by using the visco-elastic boundary layer theory. The second-order visco-elastic fluid flow past a stretching sheet has been studied by Rajagopal *et al.* [24]. They [25] also have analyzed the non similar boundary layer flow on a stretching sheet of second-order visco-elastic fluid with uniform free stream. The investigation on visco-elastic fluid flow with heat transfer over a stretching sheet has been studied by Dhanpat and Gupta [26]. Walters liquid (Model $B^/$) is a type of visco-elastic fluid which resists shear flow and strains linearly with time under the application of an applied stress but when the stress is removed it quickly returns to its original position. Ahmed *et al.* [27] have studied the visco-elastic boundary layer flow characterized by Walters liquid past a stretching plate in presence of heat transfer. The study of visco-elastic (Walters liquid, Model $B^/$) flow past a stretching plate with suction has been done by Siddappa and Abel [28]. The combined analysis of heat and mass transfer of a visco-elastic, electrically conducting fluid past a continuous stretching sheet has been discussed by Kelly *et al*. [29]. Subhas *et al*. [30] have studied the MHD effects on visco-elastic fluid flow over a stretching surface in presence of heat transfer. A mathematical analysis of heat and mass transfer phenomena in a visco-elastic fluid over an accelerating stretching sheet in the presence of heat source/sink, viscous dissipation and suction/blowing has been analyzed by Sonth *et al*. [31]. Abel *et al.* [32] have investigated the visco-elastic boundary layer flow behaviour over a stretching sheet in presence of non-uniform heat source and energy dissipation. The nature of unsteady boundary layer flow over a stretching sheet has been studied by Ahmad and Mishra [33]. Choudhury and Das [34] have examined the flow behaviour of visco-elastic boundary layer flow on a continuously moving surface in presence of transverse magnetic field. Misra *et al*. [35] have investigated the unsteady boundary layer flow over a stretching sheet with variable thermal conductivity.

In this paper, we have studied the two dimensional steady free convective hydromagnetic boundary layer flow of visco-elastic over a continuously moving inclined surface in presence of heat and mass transfer. The dissipation of energy is also considered. The visco-elastic fluid flow is characterized by Walters liquid (Model $B^/$**).**

The constitutive equation for Walters liquid (Model $B^/$) is

$$\sigma_{ik} = -pg_{ik} + \sigma'_{ik} ,$$
$$\sigma'^{ik} = 2\eta_0 e^{ik} - 2k_0 e'^{ik} \qquad (1.1)$$

Where σ_{ik} is the stress tensor, p is isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x^i, v_i is the velocity vector, the contravarient form of e'^{ik} is given by

$$e'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}_{,m} - v^k_{,m} e^{im} - v^i_{,m} e^{mk}, \qquad (1.2)$$

It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v_{i,k} + v_{k,i} \qquad (1.3)$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau)d\tau \text{ and } k_0 = \int_0^\infty \tau N(\tau)d\tau \qquad (1.4)$$

$N(\tau)$ being the relaxation spectrum . This idealized model is a valid approximation of Walters liquid (Model B$'$) taking very short memories into account so that terms involving

$$\int_0^\infty t^n N(\tau)d\tau,\ n \geq 2 \qquad (1.5)$$

have been neglected.

2 Mathematical Formulations:

We consider a steady boundary layer free convective visco-elastic flow of an electrically conducting visco-elastic fluid over a continuously moving inclined surface, issuing from a slot and moving with a constant velocity U in a fluid. Heat is supplied to the plate at the constant rate. A magnetic field of uniform strength B_0 is applied in the transverse direction. The magnetic Reynolds number is assumed to be so small that the induced magnetic field is neglected. Let x' axis be taken along the plate in the upward direction and y' axis be taken normal to the surface. The inclination of the angle is assumed to so small that $\sin\alpha = 0$. The physical model of the problem is shown in Figure 1.

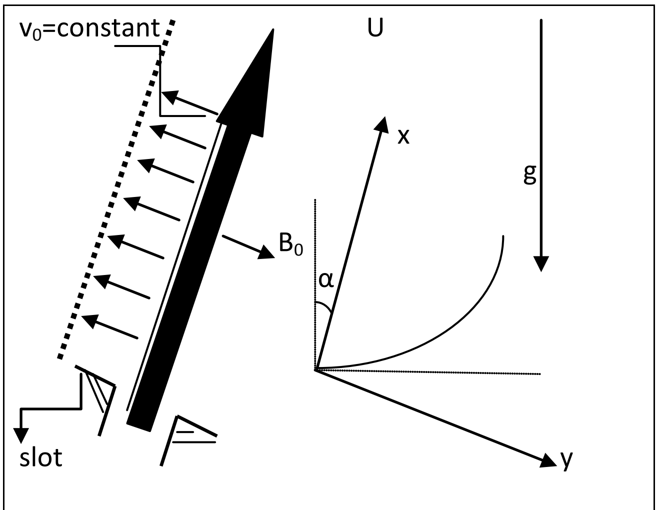

Figure 1: Physical Model of the problem

With these physical considerations and by using Boussinesq approximation, the equations governing the fluid flow are as follows:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \qquad (2.1)$$

$$\rho\left(u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'}\right) = \rho g\beta\cos\alpha\,(T' - T'_\infty) + \rho g\beta^*\cos\alpha\left(C' - C'_\infty\right) + \eta_0\frac{\partial^2 u'}{\partial y'^2} - k_0\left(v'\frac{\partial^3 u'}{\partial y'^3}\right) - \sigma B_0^2 u' \qquad (2.2)$$

$$\rho C_p\left(u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'}\right) = K_T\left(\frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'^2}\right) + \eta_0\left(\frac{\partial u'}{\partial y'}\right)^2 - k_0\frac{\partial u'}{\partial y'}v'\frac{\partial^2 u'}{\partial y'^2} \qquad (2.3)$$

$$u'\frac{\partial C'}{\partial x'} + v'\frac{\partial C'}{\partial y'} = D\left(\frac{\partial^2 C'}{\partial x'^2} + \frac{\partial^2 C'}{\partial y'^2}\right) \qquad (2.4)$$

The initial and boundary conditions are

$$y' = 0 : u' = U,\ v' = -v_0, \frac{\partial T'}{\partial y'} = -\frac{q}{k},\ C' = C'_w$$
$$y' \to \infty :\ u' \to 0,\ T' \to T'_\infty,\ C' \to C'_\infty \qquad (2.5)$$

3 Method of Solution:

Introducing the following non-dimensional quantities:

$$u = \frac{u'}{U}, y = \frac{y'v_0}{\nu}, \theta = \frac{T' - T'_\infty}{K_T\frac{q\nu}{v_0}}, \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty},$$

$$M = \frac{\sigma B_0^2\nu}{\rho v_0^2}, Gr = \frac{\nu g\beta\left(\frac{q\nu}{K_T v_0}\right)}{Uv_0^2}, Ec = \frac{U^2 K_T V_0}{q\nu C_p},$$

$$Gm = \frac{\nu g\beta^*(C'_w - C'_\infty)}{Uv_0^2}, Pr = \frac{\eta_0 C_p}{K_T},\ Sc = \frac{\nu}{D},\ k = \frac{k_0 v_0^2}{\rho\nu^2},$$

$$\frac{q}{K_T} = \frac{v_0}{\nu}\left(T' - T'_\infty\right) \qquad (3.1)$$

where Gr is the Grashof number for heat transfer, Gm is the Grashof number for mass transfer, Pr is the Prandtl number, M is the Hartmann number, k is the dimensionless visco-elastic parameter, Sc is the Schmidt number and Ec is the Eckert number.

We make use of the assumption that the velocity and temperature fields are independent of the distance parallel to the surface (as given in Schlichting 1968). Equations (2.2), (2.3) and (2.4) are reduced to the following ordinary differential equations.

$$k\frac{d^3u}{dy^3}+\frac{d^2u}{dy^2}+\frac{du}{dy}-Mu = -(Gr\theta+Gm\,\phi)\cos\alpha \qquad (3.2)$$

$$\frac{d^2\theta}{dy^2}+Pr\frac{d\theta}{dy}=-Ec\,Pr\left(\frac{\partial u}{\partial y}\right)^2-kEcPr\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} \qquad (3.3)$$

$$\frac{d^2\phi}{dy^2}+Sc\frac{d\phi}{dy}=0 \qquad (3.4)$$

The corresponding initial and boundary conditions in non-dimensional forms are

$$y=0:\quad u=1,\ \frac{\partial\theta}{\partial y}=-1,\ \phi=1$$

$$y\to\infty:\ u\to 0, \theta\to 0, \phi\to 0 \qquad (3.5)$$

Solving (3.4) with conditions (3.5) we get

$$\phi = e^{-Sc\,y} \qquad (3.6)$$

Again, the physical variables u, θ are expanded in the power of Eckert number (Ec) as Ec is very small as compared to unity for incompressible fluid. Thus the expressions for velocity, temperature are expressed as follows:

$$u(y)=u_0(y)+Ecu_1(y)+o(Ec^2)$$
$$\theta(y)=\theta_0(y)+Ec\theta_1(y)+o(Ec^2) \qquad (3.7)$$

Using (3.7) in equations (3.2) & (3.3) and equating the co-efficient of like powers of Ec, we obtain the following set of differential equations:

$$ku_0'''+u_0''+u_0'-Mu_0=-(Gr\theta_0+Gm\phi)\cos\alpha \qquad (3.8)$$

$$\theta_0''+Pr\theta_0'=0 \qquad (3.9)$$

$$ku_1'''+u_1''+u_1'-Mu_1=-Gr\theta_1 \qquad (3.10)$$

$$\theta_1''+Pr\theta_1'=-Pru_0'^2-kPru_0'u_0'' \qquad (3.11)$$

Subject to boundary conditions

$$y=0:\quad u_0=1,\ u_1=0, \frac{\partial\theta_0}{\partial y}=-1, \frac{\partial\theta_1}{\partial y}=0$$

$$y\to\infty:\ u_0\to 0,\ u_1\to 0,\ \theta_0\to 0,\ \theta_1\to 0 \qquad (3.12)$$

The solutions of the equations (3.9), subject to the boundary conditions given in (3.12) are as follows:

$$\theta_0=\frac{e^{-Pry}}{Pr} \qquad (3.13)$$

Again, to solve the equations (3.8), (3.10) and (3.11), we use the multi-parameter perturbation technique and the velocity and temperature components are expanded in the power of visco-elastic parameter k as k<<1 for small shear rate. Thus the expressions for velocity and temperature components are considered as

$$u_0=u_{00}+ku_{01}+o(k^2)$$
$$u_1=u_{10}+ku_{11}+o(k^2)$$
$$\theta_1=\theta_{10}+k\theta_{11}+o(k^2) \qquad (3.14)$$

Substituting the equations (3.14) into the equations (3.8), (3.10) and (3.11) and after equating the like powers of k, we get the following sets of ordinary differential equations:

$$u_{00}''+u_{00}'-Mu_{00} == -(Gr\theta_0+Gm\phi)\cos\alpha \qquad (3.15)$$

$$u_{01}''+u_{01}'-Mu_{01}=-u_{00}''' \qquad (3.16)$$

$$\theta_{10}''+Pr\theta_{10}'=-Pr(-\alpha_2A_2e^{-\alpha_2y}+PrA_3e^{-Pry}+ScA_4e^{-Scy})^2 \qquad (3.17)$$

$\theta_{11}''+Pr\theta_{11}'=-2Pr[(-\alpha_2A_2e^{-\alpha_2y}+PrA_3e^{-Pry}+$ ScA4e−Scy−α2A11e−α2y+PrA8e−Pry+ScA9e−Sc y−Pr−α2A2e−α2y+PrA3e−Pry+ScA4e−Scyα22A2 e−α2y−Pr2A3e−Pry−Sc2A4e−Scy
(3.18)

The modified boundary conditions are

$$y=0:\ u_{00}=1,\ u_{01}=u_{10}=u_{11}=0,$$
$$\frac{\partial\theta_{10}}{\partial y}=\frac{\partial\theta_{11}}{\partial y}=0,$$
$$y\to\infty:\quad u_{00}=u_{01}=u_{10}=u_{11}\to 0,$$
$$\theta_{10}\to 0,\quad \theta_{11}\to 0 \qquad (3.19)$$

The differential equations (3.15) to (3.18) are solved subject to the boundary condition (3.19). The solutions of the differential equations are not presented here for the sake of brevity.

4 Results and Discussion

The velocity profile is given by

$$u=u(y)=u_{00}+ku_{01}+Ec(u_{10}+ku_{11})$$

The non dimensional shearing stress at the plate is given by

$$\tau=\left(\frac{du}{dy}\right)_{y=0}+k\left(\frac{d^2u}{dy^2}\right)_{y=0}$$

The object of the present paper is to investigate the effects of visco-elasticity on free convective MHD boundary layer flow past an inclined moving surface in presence of heat and mass transfer. The visco-elastic effect is exhibited through the non-dimensional parameter k. The nonzero values of the parameter k characterize the visco-elastic fluid and k=0 represents the Newtonian fluid flow phenomenon. The velocity profile and the shearing stress at the plate are analyzed graphically for various values of flow parameters involved in the solution.

Figures 2 to 7 demonstrate the pattern of velocity profile against the displacement y for various values of visco-elastic parameter. These profiles conclude that speed diminishes as the fluid moves far away from the plate. Also it is visualized from these figures that the fluid decelerates rapidly in the neighbourhood of the plate. This flow behaviour is noticed in both Newtonian and visco-elastic fluid. Another interesting characteristic is noticed during the growth of visco-elasticity in the neighbourhood of the inclined surface. An accelerated flow is observed during the modification of visco-elasticity in comparison with the simple Newtonian fluid.

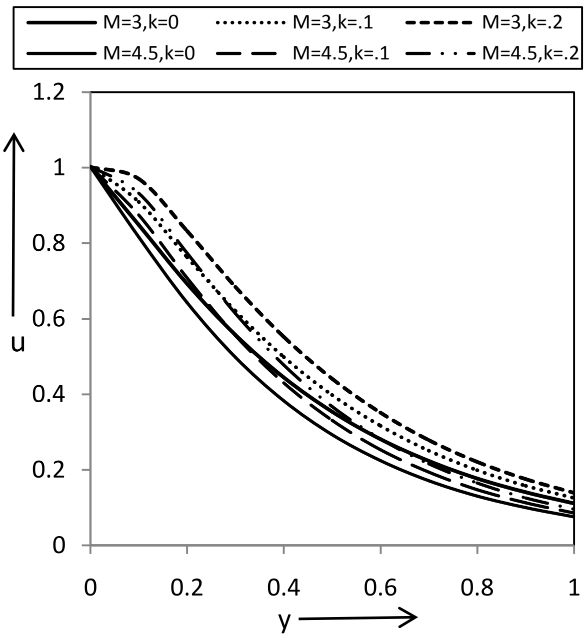

Figure 2: velocity profile u against y for Gr=7, Gm=10, Pr=10, Sc=10

The effects of Hartmann number in the boundary layer of both Newtonian and visco-elastic fluid are shown in figure 2. The application of transverse magnetic field generates a resistive force called Lorentz force which increases the viscosity of the fluid flow and hence a retarding trend is observed in the speed of the fluid flow. Both the visco-elastic fluid and Newtonian fluid experience a decreasing trend during the rise of magnetic parameter.

Grashof number studies the behaviour of free convection and it is defined as the ratio of buoyancy force to viscous force. It plays an important role in heat and mass transfer technology. Gr characterizes the free convection parameter for heat transfer and Gm characterizes the free convection parameter for mass transfer. In our study, the results are discussed for the flow past an externally cooled plate (Gr > 0) and flow past an externally heated plate (Gr < 0). Figure 3 studies the behaviour of various fluid flows during the positive values of free convection parameter. The enlargement of Grashof number for heat transfer reduces the speed for both Newtonian as well as visco-elastic fluid.

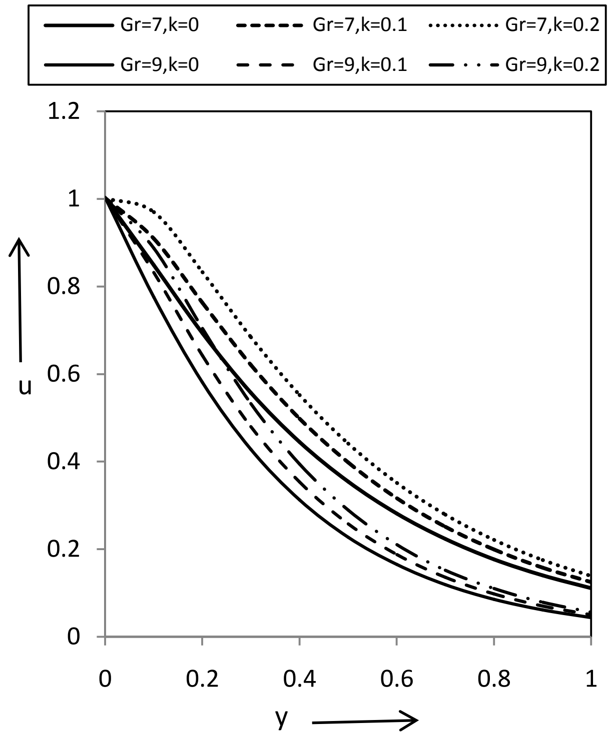

Figure 3: velocity profile u against y for M=3, Gm=10, Pr=10, Sc=10

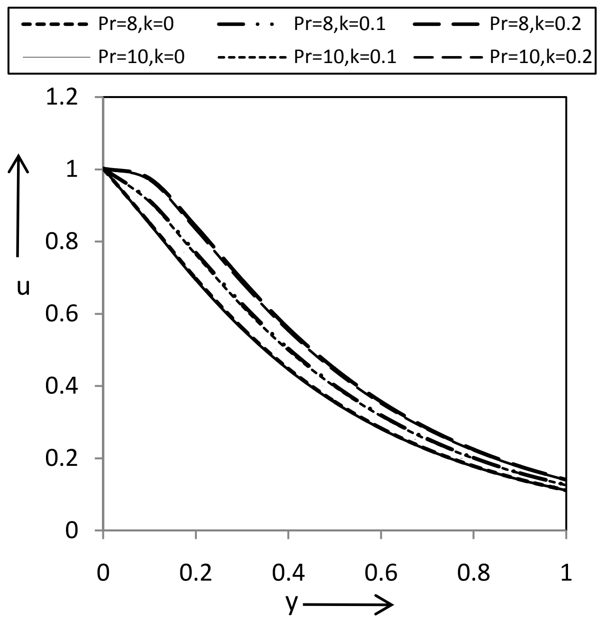

Figure 4: velocity profile u against y for M=3, Gm=10, Gr=7, Sc=10

Prandtl number plays a significant role in heat transfer flow problems as it helps to study the simultaneous effects of momentum and thermal diffusion in fluid flow. The effects of Prandtl number on both visco-elastic fluid and Newtonian fluid are analyzed in figure 4. It states that the ascending value of Prandtl number raises the viscosity of the fluid flow in case of cooling problems and the fluid becomes thick. Thus a reduction in speed is experienced for various fluid flow systems.

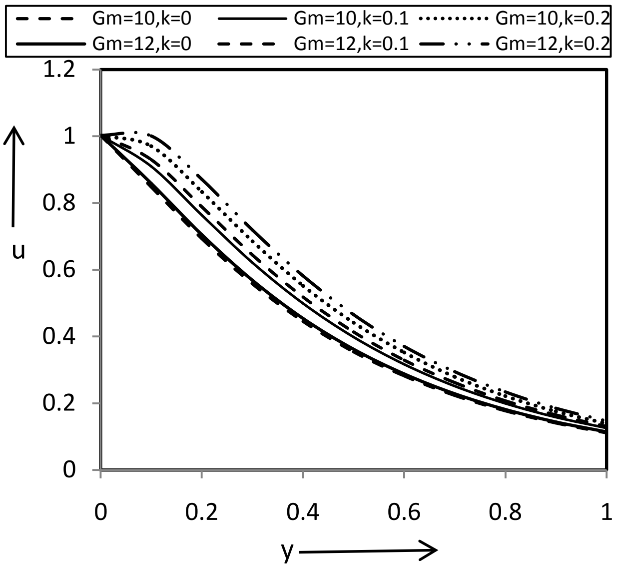

Figure 5: velocity profile u against y for M=3, Pr=10, Gr=7, Sc=10

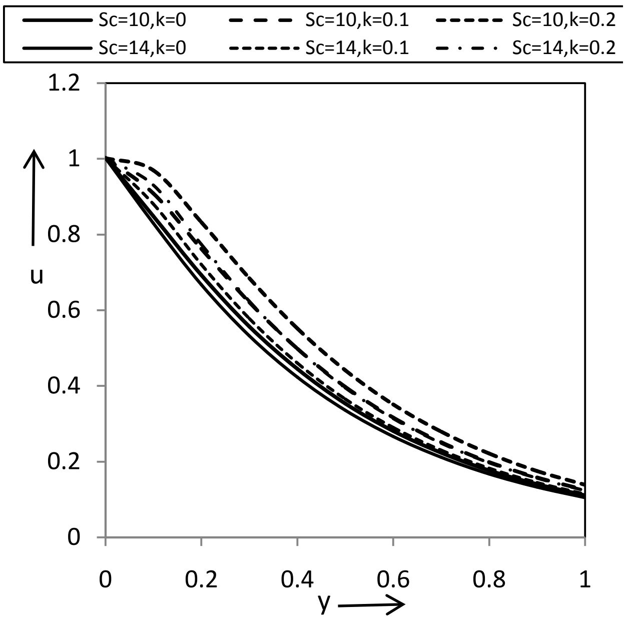

Figure 6: velocity profile u against y for M=3, Pr=10, Gr=7, Gm=10

Figure 5 represents the effect of Gm on both visco-elastic fluid and Newtonian fluid. Gm > 0 indicates that the free stream concentration is less than the concentration at the boundary surface. The increasing values of free convection parameter for mass transfer diminishes the viscosity of various fluid flow mechanisms and which in turn accelerates the flow of both simple Newtonian fluid and complex visco-elastic fluid flows.

In mass transfer problems, the importance of Schmidt number cannot be neglected as it studies the combined effect of momentum and mass diffusion. Figure 6 notifies the effect of Schmidt number in this paper. The rising nature of Schmidt number increases the viscosity of the fluid flow and hence it will slow down the speed of both Newtonian as well as visco-elastic fluid. The maximum effects of both Prandtl number and Schmidt number are detected in the neighbourhood of the inclined surface.

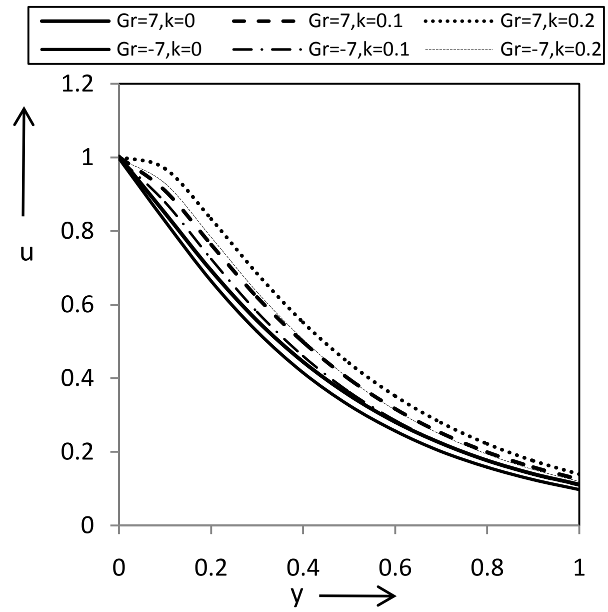

Figure 7: velocity profile u against y for M=3, Pr=10, Sc=10, Gm=10

The difference in the nature of various fluids for the flow past a heated plate and for the flow past a cooled plate is shown in figure 7. During visco-elastic fluid flows, maximum discrepancy is noticed in comparison with the Newtonian fluid flow. Also, a decelerating trend is observed in case of flow past an externally heated plate for both Newtonian and non-Newtonian fluid.

Knowing the velocity field, it is important from a practical point of view to know the effect of visco-elastic parameter on shearing stress or viscous drag. Figures 8 to 12 depict the shearing stress at the plate for the visco-elastic fluid in comparison with the

Newtonian fluid for various values of flow parameters involved in the solution. The graphical illustration has been given for both flow past an externally heated plate (Gr < 0) and externally cooled plate (Gr > 0). These graphs conclude that the growth of visco-elasticity raise the viscous drag at the surface. The shearing stress experienced by the visco-elastic fluids past a heated plate is lesser in magnitude in compared to the flow past a cooled plate.

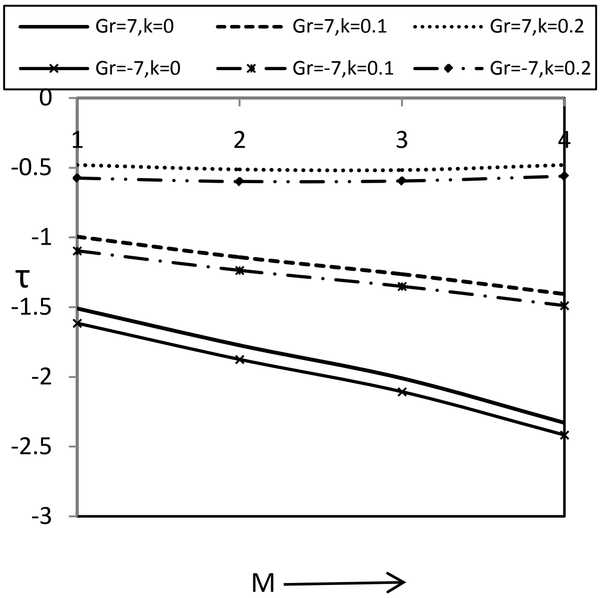

Figure 8: shearing stress against M for Pr=10, Gr=7,-7, Sc=10, Gm=10

The effects of Lorentz force on viscous drag are investigated in figure 8. The shearing stress experienced by the Newtonian fluid (k=0) experiences a declined trend. In non-Newtonian fluid flow mechanism, the visco-elastic fluid (k= 0.1) feels a reduction in shearing stress during the growth of Hartmann number (M), but when the elasticity factor k $\geq$ 0.2, an enhancement is noticed in shearing stress at the inclined surface.

Figure 9, illustrates the nature of shearing stress of both types of fluids against Prandtl number (Pr). During the flow past a cooled surface, the shearing stress produced at the surface will subdue its magnitude but a reverse behaviour is observed for the flow past a heated plate.

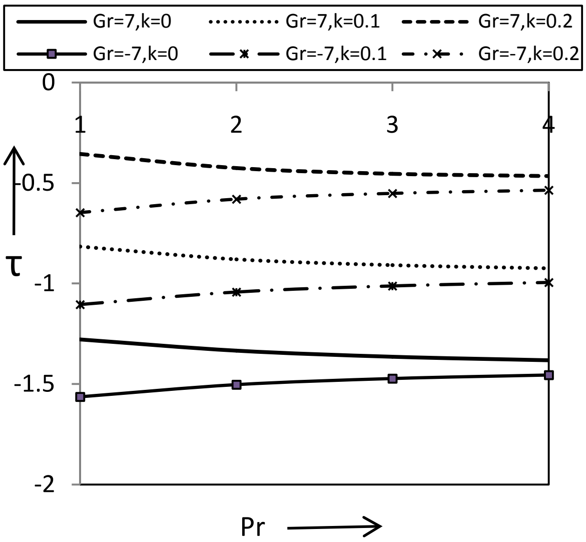

Figure 9: shearing stress against Pr for M=3, Gr=7,-7, Sc=10, Gm=10

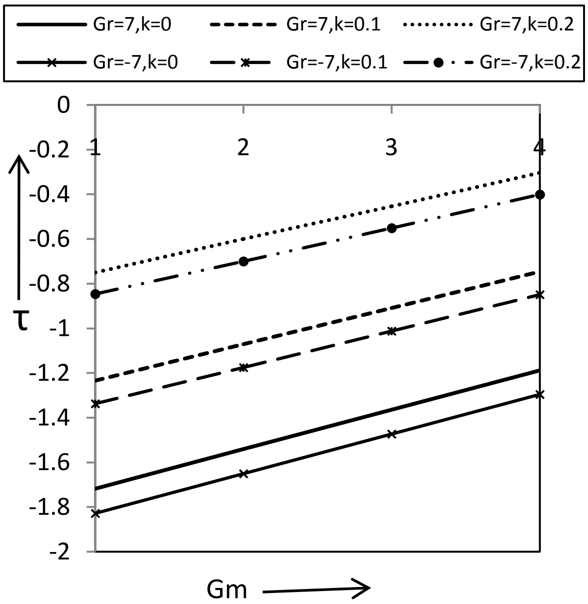

Figure 10: shearing stress against Gm for M=3, Gr=7, -7, Sc=10, Pr=10

On the other end, the shearing stress experiences a rising trend during the growth of Grashof number for mass transfer (figure 10).

Schmidt number characterizes the behaviour of simultaneous momentum and concentration diffusion. Figure 11 exhibits that the increasing value of Sc declines the shearing stress at the plate for both types of fluids along with the increasing values of visco-elastic parameter (k).

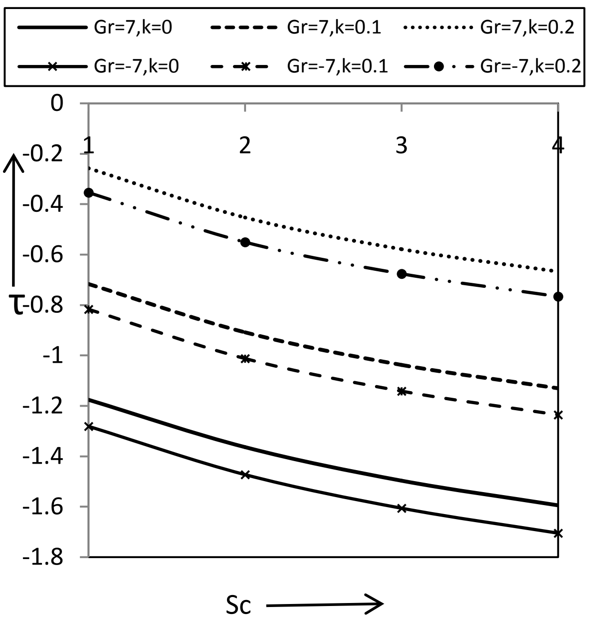

Fig 11: shearing stress against Sc for M=3, Gr=7,-7, Gm=10, Pr=10

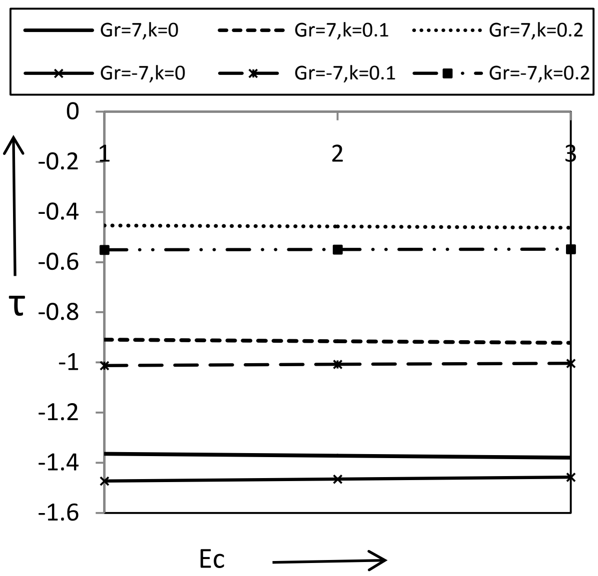

Fig 12: shearing stress against Ec for M=3, Gr=7,-7, Sc=10, Pr=10, Gm=10

Eckert number (Ec) characterizes the dissipation of mechanical energy into thermal energy due to the presence of viscosity. Figure 12, reveals the behaviour of shearing stress against Eckert number for various values of visco-elastic parameter. The viscous drag formed by both Newtonian and non-Newtonian fluid experience a diminishing trend during the growth of Eckert number.

The rate of heat transfer and the rate of mass transfer do not differ significantly affected by the visco-elastic parameter.

4 Conclusion

The behaviour of free convective MHD boundary flow of a visco-elastic fluid past an inclined moving surface in presence of heat and mass transfer has been investigated in this paper. Some of the important points are concluded as below:

1. The deceleration of the fluid flow is superior in the neighbourhood of the plate in comparison with fluid flowing at some distance from the plate.
2. The fluid flow is accelerated during the enhancement of visco-elastic parameter.
3. Both Newtonian and non-Newtonian fluids experience a decreasing trend under the amplified magnitude of Hartmann number.
4. A decelerating trend is observed in case of flow past an externally heated plate for both Newtonian and non-Newtonian fluid in comparison with the flow past an externally cooled plate.
5. The growth of visco-elasticity raises the viscous drag at the surface.
6. During the flow past a cooled surface, the shearing stress produced at the surface will subdue its magnitude.
7. The viscous drag formed by both Newtonian and non-Newtonian fluid experience a diminishing trend during the growth of Eckert number.

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